

Maths Problems for 2017 — CHEN10072
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Week 6

1. Solve the following linear ODEs by finding an integrating factor:

(a) $\frac{dy}{dx} + y = \exp(-x)$; $y = 2$ when $x = 0$.

(b) $\frac{dy}{dx} + y \cos x = \cos x$; $y = 1$ when $x = 0$.

(c)^{*} $\frac{dy}{dx} + \frac{y}{x} = \sin x$; $y = 0$ when $x = 0$.

2. Let C be concentration of dissolved Oxygen in bioreactor and C_s concentration of dissolved Oxygen at saturation, and $D = C_s - C$ the ‘deficit’. Let L be the constant Biological Oxygen Demand of organisms in the reactor. The following differential equation is given as a model

$$\frac{dD}{dt} = k_d L - k_r D$$

where k_d and k_r are constants and t is time.

- (a) What does this equation mean?
- (b) Solve the differential equation, assuming $D = D_0$ at $t = 0$
- (c) What shape is this curve?