

**Maths Problems for 2017 — CHEN10072**  
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## Week 6

1. Solve the following linear ODEs by finding an integrating factor:

(a)  $\frac{dy}{dx} + y = \exp(-x)$  ;  $y = 2$  when  $x = 0$ .

(b)  $\frac{dy}{dx} + y \cos x = \cos x$  ;  $y = 1$  when  $x = 0$ .

(c)<sup>\*</sup>  $\frac{dy}{dx} + \frac{y}{x} = \sin x$  ;  $y = 0$  when  $x = 0$ .

2. Let  $C$  be concentration of dissolved Oxygen in bioreactor and  $C_s$  concentration of dissolved Oxygen at saturation, and  $D = C_s - C$  the ‘deficit’. Let  $L$  be the constant Biological Oxygen Demand of organisms in the reactor. The following differential equation is given as a model

$$\frac{dD}{dt} = k_d L - k_r D$$

where  $k_d$  and  $k_r$  are constants and  $t$  is time.

- (a) What does this equation mean?
- (b) Solve the differential equation, assuming  $D = D_0$  at  $t = 0$
- (c) What shape is this curve?