Maths Problems for 2017 — CHEN10072
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Week 6

1. Solve the following linear ODEs by finding an integrating factor:

   (a) $\dfrac{\mathrm{d}y}{\mathrm{d}x} + y = \exp(-x)\ ; \quad y = 2 \text{ when } x = 0.$

   (b) $\dfrac{\mathrm{d}y}{\mathrm{d}x} + y\cos x = \cos x\ ; \quad y = 1 \text{ when } x = 0.$

   (c)* $\dfrac{\mathrm{d}y}{\mathrm{d}x} + \dfrac{y}{x} = \sin x\ ; \quad y = 0 \text{ when } x = 0.$

2. Let C be concentration of dissolved Oxygen in bioreactor and C_s concentration of dissolved Oxygen at saturation, and $D = C_s - C$ the 'deficit'. Let L be the constant Biological Oxygen Demand of organisms in the reactor. The following differential equation is given as a model

$$\frac{\mathrm{d}D}{\mathrm{d}t} = k_d L - k_r D$$

   where k_d and k_r are constants and t is time.

   (a) What does this equation mean?

   (b) Solve the differential equation, assuming $D = D_0$ at $t = 0$

   (c) What shape is this curve?