Multi-variable Markov Process: a matrix product states treatment

Collaboration:

Bin Xu – Physics
Peiqi Wang – ORFE
Liangsheng Zhang – Physics
Peter Gjeltema – MAE
Jun Xiong – Physics

CONTENTS

- Transitional matrix method of Markov process
- Matrix product states
- Algorithm
- Program structure
- Some results

MARKOV PROCESS

- An emotional boy: happy or angry
- IF he is happy:
  - 50% -> Angry
  - 50% -> Remain happy
- IF he is angry:
  - 50% -> Remain angry
  - 50% -> Happy

Transitional Matrix

$$\begin{pmatrix} 0.5P_{Angry} + 0.5P_{Happy} \\ 0.5P_{Angry} + 0.5P_{Happy} \end{pmatrix} = \begin{pmatrix} 0.5 & 0.5 \\ 0.5 & 0.5 \end{pmatrix} \begin{pmatrix} P_{Angry} \\ P_{Happy} \end{pmatrix}$$

MARKOV PROCESS

- 2 emotional boys
- With probability p, they will change their states TOGETHER
- Otherwise, they remain their states

Transitional Matrix

$$\begin{pmatrix} 1-p & 0 & 0 & p \\ 0 & 1-p & p & 0 \\ 0 & p & 1-p & 0 \\ p & 0 & 0 & 1-p \end{pmatrix} \begin{pmatrix} P_{AA} \\ P_{AH} \\ P_{HA} \\ P_{HH} \end{pmatrix}$$

$$T = (1-p)I + p\sigma^X \otimes \sigma^X$$

$$\sigma^X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

MARKOV PROCESS

- N emotional boys
- With probability p, they will change their states TOGETHER
- Otherwise, they remain their states

Transitional Matrix

$$T = pI + (1-p)\sum_{i=1}^{n-1} \frac{1}{n-1} \sigma_i^x \otimes \sigma_{i+1}^x$$

$$\sigma_i^x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

The T matrix is 2^n dimensional!

MARKOV PROCESS

- How do we describe a state of N boys?

$$\begin{pmatrix} P(AAA\cdots AA) \\ P(AAA\cdots AH) \\ P(AAA\cdots HA) \\ P(AAA\cdots HH) \\ \cdots \\ P(HHH\cdots HH) \end{pmatrix}$$

- Read as a function

$$P : \{A, H\}^{\otimes N} \to \mathbb{R}$$

- How do we represent a function?
  - Table of value
  - Analytical relations
    - (Taylor expansion, etc.)
  - Algebraic relations
    - (product of matrices)

MATRIX PRODUCT STATES (MPS)

- Algebraic representation of the "state function"

$$P : \{A, H\}^{\otimes N} \to \mathbb{R}$$
$$\sigma_1\sigma_2\cdots\sigma_N \mapsto M^{[1]}_{\sigma_1}M^{[2]}_{\sigma_2}M^{[3]}_{\sigma_3}\cdots M^{[N]}_{\sigma_N}$$

- We need to store 2N functions instead of 2^N numbers

- for example:

$$P(AHA\cdots HA) = M^{[1]}_A M^{[2]}_H M^{[3]}_A \cdots M^{[N-1]}_H M^{[N]}_A$$

- In the memory

$M^{[1]}_A$	$M^{[2]}_A$	$\cdots$	$M^{[N]}_A$
$M^{[1]}_H$	$M^{[2]}_H$	$\cdots$	$M^{[N]}_H$

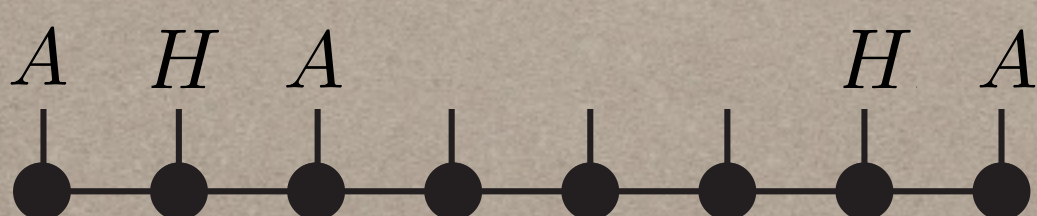
MATRIX PRODUCT STATES (MPS)

- Algebraic representation of the "state function"

$$P : \{A, H\}^{\otimes N} \to \mathbb{R}$$
$$\sigma_1 \sigma_2 \cdots \sigma_N \mapsto M^{[1]}_{\sigma_1} M^{[2]}_{\sigma_2} M^{[3]}_{\sigma_3} \cdots M^{[N]}_{\sigma_N}$$

- Pictorial representation

$$P(AHA \cdots HA) = M^{[1]}_A M^{[2]}_H M^{[3]}_A \cdots M^{[N-1]}_H M^{[N]}_A$$



- In the memory

$M^{[1]}_A$	$M^{[2]}_A$	$\cdots$	$M^{[N]}_A$
$M^{[1]}_H$	$M^{[2]}_H$	$\cdots$	$M^{[N]}_H$

MATRIX PRODUCT OPERATORS (MPO)

- The transitional matrix can also be represented as a product of matrices, whose matrices operate on single sites

$$T = pI + (1-p)\sum_{i=1}^{n-1} \frac{1}{n-1}\sigma_i^x \otimes \sigma_{i+1}^x$$

$$T = T^{[1]}T^{[2]}\dots T^{[N]}$$

$$T^{[1]} = \begin{bmatrix} pI & (1-p)\sigma_1^X & I \end{bmatrix}$$

$$T^{[i]} \begin{bmatrix} I & 0 & 0 \\ \sigma^X & 0 & 0 \\ 0 & (1-p)\sigma_i^X & I \end{bmatrix}$$

$$T^{[N]} = \begin{bmatrix} I \\ \sigma_N^X \\ 0 \end{bmatrix}$$

- Pictorial representation