$$= pI + (1-p)\sum_{i=1}^{n-1} \frac{1}{n-1} \sigma_i^x \otimes \sigma_{i+1}^x$$