

Quantitative Robustness for Vulnerability Assessment

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Introduction

Quantitative Robustness and QRSE

Reduction to *f-e-majsat*

Experimental Results

Conclusion

Bonus

Bug-finding techniques are really good at finding bugs!

Fuzzing



Syzkaller / Syzbot



Other successful techniques

- ▶ symbolic execution (Klee, Angr, Binsec)
 - ▶ abstract interpretation (Frama-C, Infer)
 - ▶ ...

Not all bugs are created equal

Bug impacts

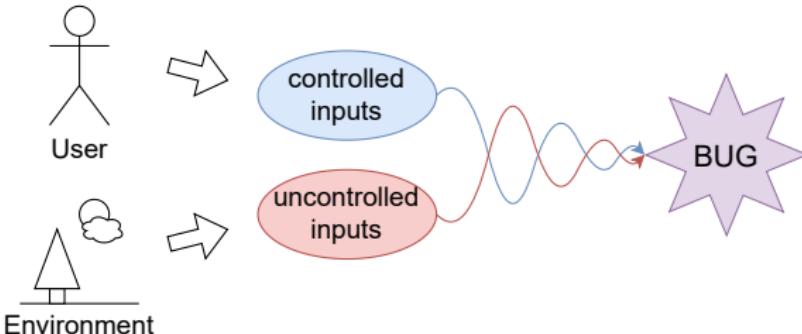
infinite loop, memory corruption...



Bug reproducibility

dependency on uncontrolled inputs

⇒ randomness, stack canaries, scheduling, undefined behaviour, uninitialized memory...

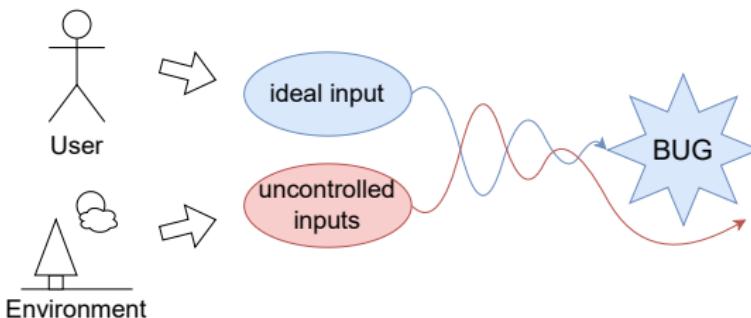


Evaluating bug reproducibility with robust reachability

Girol et al., CAV 2021, FMSD 2022

Robust reachability

\exists a *controlled input* triggering the bug \forall *uncontrolled input*



in the real-world: CVE-2019-20839, CVE-2019-15900,
CVE-2019-19307...

What about mostly-robust bugs?

Two different bugs

```
int controlled = INPUT;  
int uncontrolled = NONDET;  
...  
if(uncontrolled - controlled == 1)  
    //bug 1  
...  
if(uncontrolled & controlled == 1)  
    //bug 2
```

- ▶ **bug 1:** extremely unlikely ($\frac{1}{2^{31}}$)
- ▶ **bug 2:** very likely with
 $controlled = 1$ ($\frac{1}{2}$)

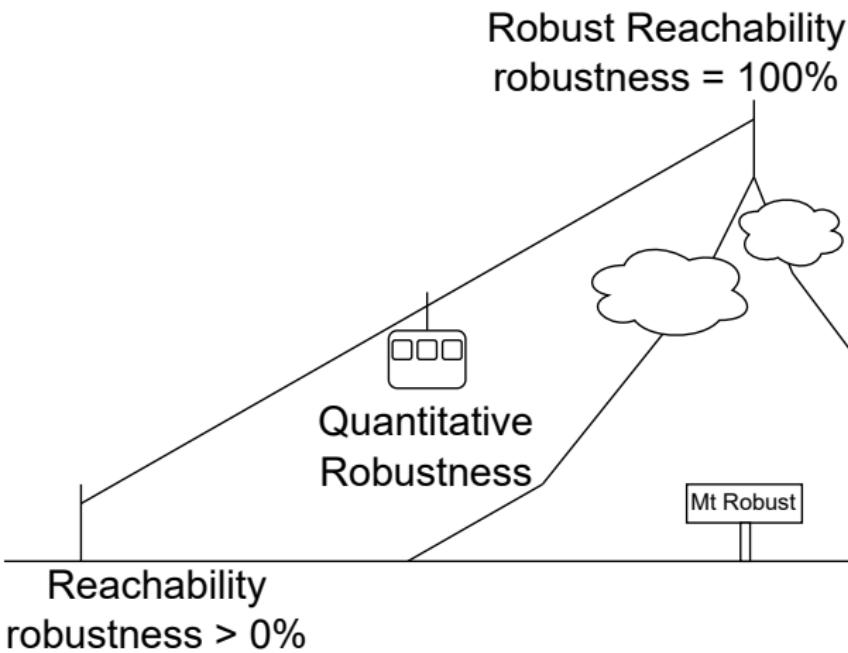
- ▶ both are reachable



- ▶ none are robustly
reachable



We need a quantitative measure of robustness



Goals and challenges

Goals

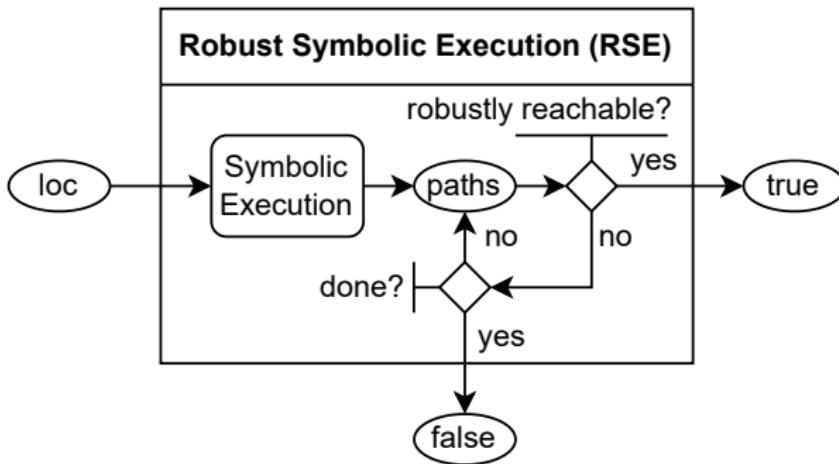
- ▶ formally define quantitative robustness
- ▶ design algorithms to measure it
- ▶ automation + scalability

Challenges

- ▶ scalability of quantitative analysis (ex: model counting)
- ▶ improve performance over robust symbolic execution (RSE)

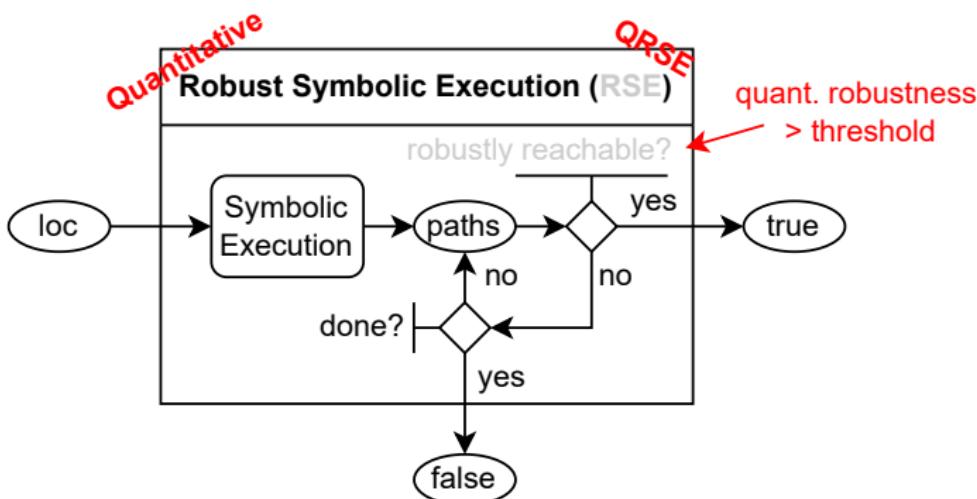
Our approach

robust reachability



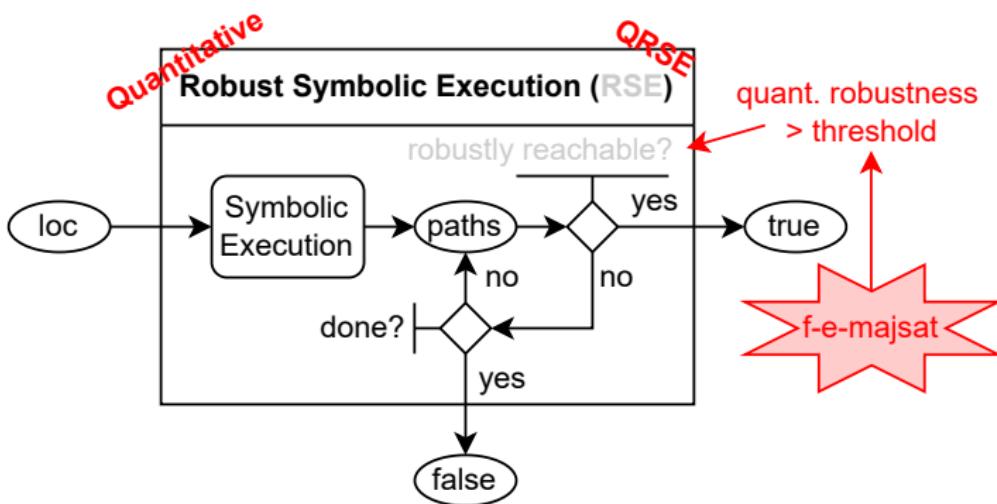
Our approach

robust reachability \Rightarrow quantitative robustness



Our approach

robust reachability \Rightarrow quantitative robustness



$f\text{-}e\text{-majsat}$: counting + optimization problem related to AI
 \sim unknown in security

Contributions

Quantitative robustness

formal definition + theorems

QRSE

- ▶ quantitative version of RSE
- ▶ path-wise quantitative robustness reduced to *f-e-mjsat*
⇒ security application of *f-e-mjsat*
- ▶ *Relax*, a new approximate *f-e-mjsat* solving algorithm

Implementation

- ▶ **BINSEC/QRSE**: binary-level QRSE
- ▶ **Popcon**: front-end for *f-e-mjsat* solvers (bitvectors)
- ▶ experiments with realistic security-related case studies

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Defining Quantitative robustness

Threat model (Girol et al., CAV 2021)

program \mathcal{P} , targeted location loc

- ▶ **controlled inputs** $\in \mathcal{A}$ countable
- ▶ **uncontrolled inputs** $\in \mathcal{X}$ countable, uniformly distributed

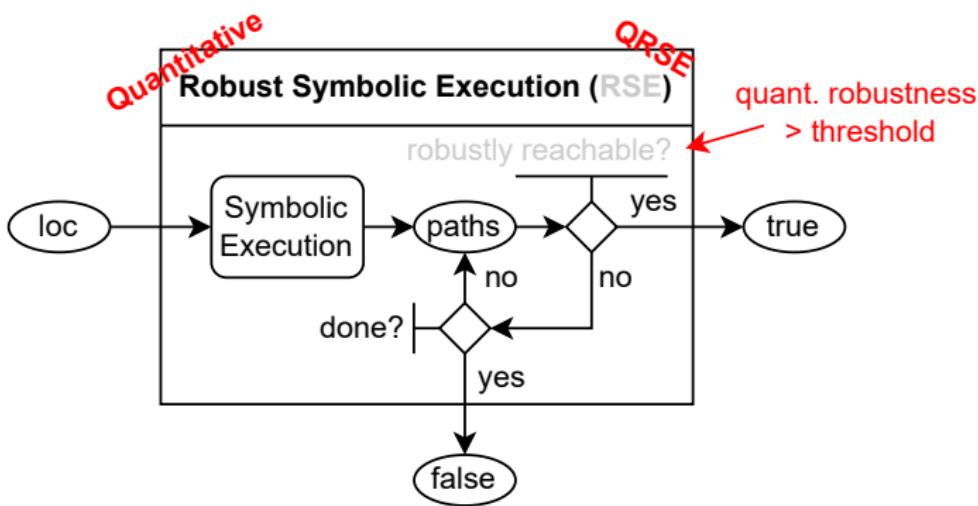
Quantitative robustness (finite input space)

max proportion of x reaching loc for a fixed a

$$q_{loc} \triangleq \frac{\max_{a \in \mathcal{A}} |\{x \in \mathcal{X} \text{ s.t. } \mathcal{P}(a, x) \text{ reaches } loc\}|}{|\mathcal{X}|}$$

(read the paper for the infinite input space definition)

From RSE to QRSE

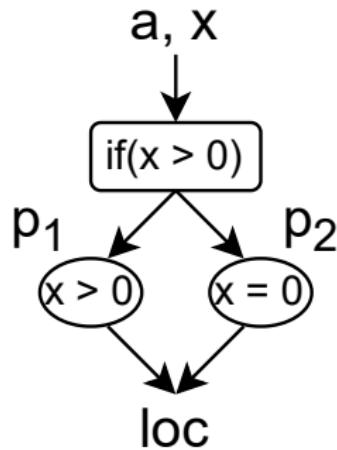


- ▶ QRSE+: QRSE with path merging
- ▶ correctness: ✓
- ▶ k-completeness (path lengths $\leq k$): QRSE+ ✓

Path merging and deduction power

Path merging

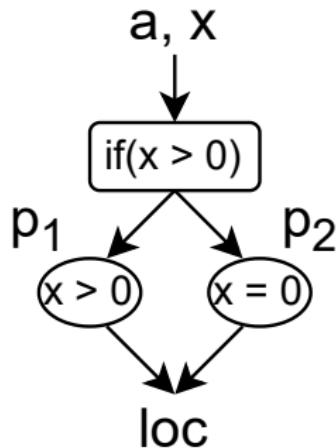
- ▶ multi-path constraint
- ▶ $p_1 \vee p_2 \sim \text{true}$
- ▶ added complexity \Rightarrow **scalability issues**



Path merging and deduction power

Path merging

- ▶ multi-path constraint
- ▶ $p_1 \vee p_2 \sim \text{true}$
- ▶ added complexity \Rightarrow **scalability issues**
- ▶ **SE:** deduction power =
 $\text{reach}(p_1), \text{reach}(p_2), \text{reach}(p_1 \vee p_2)$
- ▶ **RSE:** deduction power ↗↗↗
 $\neg RSE(p_1), \neg RSE(p_2), RSE(p_1 \vee p_2)$
- ▶ **QRSE:** deduction power ↗
 $QRSE(p_1) > 99\% \Rightarrow q_{loc} > 99\%$
+ guarantees on robustness lower bound
after branches



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What quantitative robustness is *not*

context: binary analysis \Rightarrow bitvectors \Rightarrow finite input space

Model counting (#sat)

- ▶ **application:** probabilistic reachability
- ▶ **here:** # reaching inputs
- ▶ **issue:** controlled inputs are not random

Projected model counting

- ▶ **application:** quantitative information flow, channel capacity...
- ▶ **here:** # reaching uncontrolled inputs (any controlled inputs)
- ▶ **issue:** choice of best controlled input

(also not weighted maxSMT)

What quantitative robustness *is*

f-e-majsat (Littman et al., JAIR 1998)

f: propositional formula

$$\text{femajsat}_{\mathcal{A}}(f) \triangleq \max_{a \in \mathcal{A}} \#(f|_a)$$



known applications: probabilistic planning, Bayesian networks...

What quantitative robustness *is*

f-e-majsat (Littman et al., JAIR 1998)

f: propositional formula

$$\text{femajsat}_{\mathcal{A}}(f) \triangleq \max_{a \in \mathcal{A}} \#(f|_a)$$



known applications: probabilistic planning, Bayesian networks...

Existing algorithms

Algorithm	Author(s)	Conference
DC-SSAT	Majercik et al.	AAAI 2005
Constrained Complan	Huang	ICAPS 2006
Complan	Huang	ICAPS 2006
Complan+	Pipatsrisawat et al.	IJCAI 2009
MaxCount	Fremont et al.	AAAI 2017
SsatABC	Lee et al.	IJCAI 2018

⇒ untested on quantitative robustness (-like) instances

Efficient approximation of *f-e-majsat*

Basic exact approach

- ▶ compile constraints to decision-DNNF form
- ▶ **additional constraint:** $(\mathcal{A}, \mathcal{X})$ -layering
- ▶ model counting in linear time (Darwiche, 2001)

issue: compilation is *hard* ($|\mathcal{X}| \nearrow \Rightarrow$ speed \searrow)

Our *Relax* algorithm

- ▶ **relaxation:** $(\mathcal{A} \cup \mathcal{R}, \mathcal{X} \setminus \mathcal{R})$ -layering
- ▶ interval
- ▶ $\text{Relax}_-(f) \leq \text{femajsat}_{\mathcal{A}}(f) \leq \text{Relax}_+(f) \leq 2^{|\mathcal{R}|} \text{ Relax}_-(f)$

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Research questions

- ▶ Is QRSE more precise than RSE in practice?
- ▶ Can we avoid path merging?
- ▶ What are the best f-e-mjsat solvers for quantitative robustness?

Is QRSE more precise than RSE in practice?

RSE benchmark ($> 20\%$)

Method	OK	FN	T
RSE	37	9	2
RSE+	40	6	2
QRSE	47	0	1
QRSE+	46	0	2

- ▶ no more false negatives!
- ▶ distinguishes *nearly robust* from *nearly unrepeatable*

Fault analysis benchmark

q_{loc}	SE	RSE	QRSE
100%	-	0/0	0/0
$> 20\%$	-	-	2/2
$[10^{-6}; 20\%]$	-	-	10/10
$< 10^{-6}$	-	-	27/27
$> 0\%$	39/39	-	39/39

Can we avoid path merging?

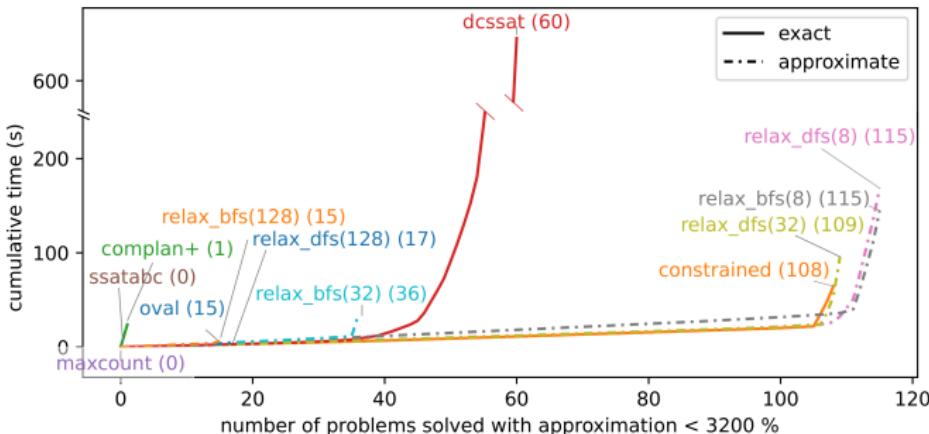
Case study: stack buffer overflow in libvncserver (CVE-2019-20839)

canary	SE	RSE	RSE+	QRSE	QRSE+
no	> 0%			1 path > 20%	> 20%
yes	> 0%			all paths < 20%	

⇒ useful results without path merging!

What are the best f-e-mjsat solvers for quantitative robustness?

Benchmark with 117 formulas from previous experiments



- ▶ **surprise 1:** almost all existing algorithms perform poorly 😞
- ▶ **surprise 2:** naive algorithm *Constrained* 😊 (108/117)
- ▶ *Relax* with $|\mathcal{R}| = 8$ 😊 (115/117)

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Conclusion

- ▶ Bug replicability is important!
- ▶ Quantitative robustness \Rightarrow precise indicator of replicability
- ▶ Measured with QRSE, reduction to *f-e-majsat*
- ▶ best algorithms: *Constrained* and *Relax*
- ▶ BINSEC/QRSE



Conclusion

- ▶ Bug replicability is important!
- ▶ Quantitative robustness \Rightarrow precise indicator of replicability
- ▶ Measured with QRSE, reduction to *f-e-majsat*
- ▶ best algorithms: *Constrained* and *Relax*
- ▶ BINSEC/QRSE



Possible improvements

concept to handle	Definitions	SE	Solver
non-uniform input distrib.	easy	hard	hard
hyper-safety properties	easy	easy	easy
hyperproperties, liveness	hard	hard	hard
string & LIA theories	ok	medium	hard
dense input spaces	easy	hard	hard

The end

Thank you for your attention.
Any questions?

k-completeness of QRSE+

k-completeness

$P| \leq k$ = restriction of P to traces of length $\leq k$

k-complete for $P \iff$ complete for $P| \leq k$

- ▶ 0 or 1 path of length $\leq k$ per input
 \Rightarrow finite number of finite paths
- ▶ QRSE+ can explore and merge them all
 \Rightarrow constraint for reaching *loc* in $P| \leq k \subset$ final constraint
- ▶ $q_{P| \leq k, loc} \geq Q \Rightarrow \text{QRSE+}(P, loc) \geq Q$
(assuming no timeouts or errors)

Branching theorem

Quantitative robustness pseudo-conservation

p_1, \dots, p_n paths in P , $P|^{p_1, \dots, p_n}$ = restriction of P to p_1, \dots, p_n

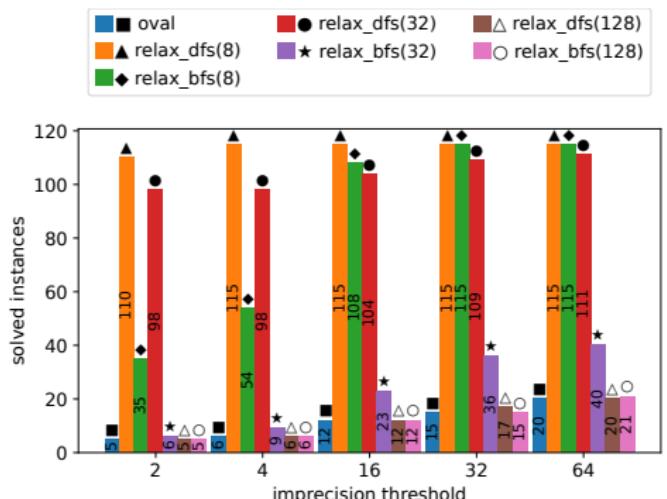
$$\exists i \text{ s.t. } q_{P|^{p_i}, \text{loc}} \geq \frac{1}{n} q_{P|^{p_1, \dots, p_n}, \text{loc}}$$

idea of the proof:

- ▶ show $q_{P|^{p, p'}, \text{loc}} \leq q_{P|^{p}, \text{loc}} + q_{P|^{p'}, \text{loc}}$
- ▶ contradiction: $q_{P|^{p_i}, \text{loc}} < \frac{1}{n} q_{P|^{p_1, \dots, p_n}, \text{loc}} \quad \forall i$
 $\Rightarrow q_{P|^{p_1, \dots, p_n}, \text{loc}} < n \times \frac{1}{n} q_{P|^{p_1, \dots, p_n}, \text{loc}}$

Practical precision of approximate solvers

Solved problems function of imprecision threshold



- ▶ Relax: $|\mathcal{R}| = 8$ with 4x imprecision ⇒ 😊
- ▶ better than theoretical bounds!