A The Type System of NeColus

▶ **Definition 32** (Type translation).

$$\begin{aligned}
|\mathsf{Nat}| &= \mathsf{Nat} \\
|\top| &= \langle\rangle \\
|A \to B| &= |A| \to |B| \\
|A \,\&\, B| &= |A| \times |B| \\
|\{l : A\}| &= \{l : |A|\}
\end{aligned}$$

▶ **Definition 33** (Meta-functions $[\![\cdot]\!]_\top$ and $[\![\cdot]\!]_\&$).

$$\begin{aligned}
[\![[]]\!]_\top &= \mathsf{top} \\
[\![\{l\}, \mathcal{L}]\!]_\top &= \{l : [\![\mathcal{L}]\!]_\top\} \circ \mathsf{top}_{\{l\}} \\
[\![A, \mathcal{L}]\!]_\top &= (\mathsf{id} \to [\![\mathcal{L}]\!]_\top) \circ ((\mathsf{top} \to \mathsf{top}) \circ (\mathsf{top}_\to \circ \mathsf{top}))
\end{aligned}$$

$$\begin{aligned}
[\![[]]\!]_\& &= \mathsf{id} \\
[\![\{l\}, \mathcal{L}]\!]_\& &= \{l : [\![\mathcal{L}]\!]_\&\} \circ \mathsf{dist}_{\{l\}} \\
[\![A, \mathcal{L}]\!]_\& &= (\mathsf{id} \to [\![\mathcal{L}]\!]_\&) \circ \mathsf{dist}_\to
\end{aligned}$$

$\boxed{A <: B \rightsquigarrow c}$ *(Declarative subtyping)*

$$\textsc{S-refl}\ \frac{}{A <: A \rightsquigarrow \mathsf{id}} \qquad \textsc{S-trans}\ \frac{A_2 <: A_3 \rightsquigarrow c_1 \qquad A_1 <: A_2 \rightsquigarrow c_2}{A_1 <: A_3 \rightsquigarrow c_1 \circ c_2} \qquad \textsc{S-top}\ \frac{}{A <: \top \rightsquigarrow \mathsf{top}}$$

$$\textsc{S-rcd}\ \frac{A <: B \rightsquigarrow c}{\{l : A\} <: \{l : B\} \rightsquigarrow \{l : c\}} \qquad \textsc{S-arr}\ \frac{B_1 <: A_1 \rightsquigarrow c_1 \qquad A_2 <: B_2 \rightsquigarrow c_2}{A_1 \to A_2 <: B_1 \to B_2 \rightsquigarrow c_1 \to c_2}$$

$$\textsc{S-andl}\ \frac{}{A_1 \,\&\, A_2 <: A_1 \rightsquigarrow \pi_1} \qquad \textsc{S-andr}\ \frac{}{A_1 \,\&\, A_2 <: A_2 \rightsquigarrow \pi_2} \qquad \textsc{S-and}\ \frac{A_1 <: A_2 \rightsquigarrow c_1 \qquad A_1 <: A_3 \rightsquigarrow c_2}{A_1 <: A_2 \,\&\, A_3 \rightsquigarrow \langle c_1, c_2\rangle}$$

$$\textsc{S-distArr}\ \frac{}{(A_1 \to A_2) \,\&\, (A_1 \to A_3) <: A_1 \to A_2 \,\&\, A_3 \rightsquigarrow \mathsf{dist}_\to}$$

$$\textsc{S-distRcd}\ \frac{}{\{l : A\} \,\&\, \{l : B\} <: \{l : A \,\&\, B\} \rightsquigarrow \mathsf{dist}_{\{l\}}} \qquad \textsc{S-topArr}\ \frac{}{\top <: \top \to \top \rightsquigarrow \mathsf{top}_\to}$$

$$\textsc{S-topRcd}\ \frac{}{\top <: \{l : \top\} \rightsquigarrow \mathsf{top}_{\{l\}}}$$

$\boxed{\Gamma \vdash E \Rightarrow A \rightsquigarrow e}$ *(Inference)*

$$\textsc{T-top}\ \frac{}{\Gamma \vdash \top \Rightarrow \top \rightsquigarrow \langle\rangle} \qquad \textsc{T-lit}\ \frac{}{\Gamma \vdash i \Rightarrow \mathsf{Nat} \rightsquigarrow i} \qquad \textsc{T-var}\ \frac{x : A \in \Gamma}{\Gamma \vdash x \Rightarrow A \rightsquigarrow x}$$

$$\textsc{T-app}\ \frac{\Gamma \vdash E_1 \Rightarrow A_1 \to A_2 \rightsquigarrow e_1 \qquad \Gamma \vdash E_2 \Leftarrow A_1 \rightsquigarrow e_2}{\Gamma \vdash E_1\, E_2 \Rightarrow A_2 \rightsquigarrow e_1\, e_2} \qquad \textsc{T-anno}\ \frac{\Gamma \vdash E \Leftarrow A \rightsquigarrow e}{\Gamma \vdash E : A \Rightarrow A \rightsquigarrow e}$$

$$\textsc{T-merge}\ \frac{\Gamma \vdash E_1 \Rightarrow A_1 \rightsquigarrow e_1 \qquad \Gamma \vdash E_2 \Rightarrow A_2 \rightsquigarrow e_2 \qquad A_1 * A_2}{\Gamma \vdash E_1 \,,,\, E_2 \Rightarrow A_1 \,\&\, A_2 \rightsquigarrow \langle e_1, e_2 \rangle}$$

$$\textsc{T-rcd}\ \frac{\Gamma \vdash E \Rightarrow A \rightsquigarrow e}{\Gamma \vdash \{l = E\} \Rightarrow \{l : A\} \rightsquigarrow \{l = e\}} \qquad \textsc{T-proj}\ \frac{\Gamma \vdash E \Rightarrow \{l : A\} \rightsquigarrow e}{\Gamma \vdash E.l \Rightarrow A \rightsquigarrow e.l}$$

$\boxed{\Gamma \vdash E \Leftarrow A \rightsquigarrow e}$ *(Checking)*

$$\textsc{T-abs}\ \frac{\Gamma, x : A \vdash E \Leftarrow B \rightsquigarrow e}{\Gamma \vdash \lambda x.\, E \Leftarrow A \to B \rightsquigarrow \lambda x.\, e} \qquad \textsc{T-sub}\ \frac{\Gamma \vdash E \Rightarrow B \rightsquigarrow e \qquad B <: A \rightsquigarrow c}{\Gamma \vdash E \Leftarrow A \rightsquigarrow c\, e}$$

$\boxed{A * B}$ *(Disjointness)*

$$\textsc{D-topL}\ \frac{}{\top * A} \qquad \textsc{D-topR}\ \frac{}{A * \top} \qquad \textsc{D-arr}\ \frac{A_2 * B_2}{A_1 \to A_2 * B_1 \to B_2} \qquad \textsc{D-andL}\ \frac{A_1 * B \qquad A_2 * B}{A_1 \,\&\, A_2 * B}$$

$$\textsc{D-andR}\ \frac{A * B_1 \qquad A * B_2}{A * B_1 \,\&\, B_2} \qquad \textsc{D-rcdEq}\ \frac{A * B}{\{l : A\} * \{l : B\}} \qquad \textsc{D-rcdNeq}\ \frac{l_1 \neq l_2}{\{l_1 : A\} * \{l_2 : B\}} \qquad \textsc{D-axNatArr}\ \frac{}{\mathsf{Nat} * A_1 \to A_2}$$

$$\textsc{D-axArrNat}\ \frac{}{A_1 \to A_2 * \mathsf{Nat}} \qquad \textsc{D-axNatRcd}\ \frac{}{\mathsf{Nat} * \{l : A\}} \qquad \textsc{D-axRcdNat}\ \frac{}{\{l : A\} * \mathsf{Nat}} \qquad \textsc{D-axArrRcd}\ \frac{}{A_1 \to A_2 * \{l : A\}}$$

$$\textsc{D-axRcdArr}\ \frac{}{\{l : A\} * A_1 \to A_2}$$

$\boxed{\mathcal{C} : (\Gamma \;\Rightarrow\; A) \mapsto (\Gamma' \;\Rightarrow\; B) \rightsquigarrow \mathcal{D}}$ *(Context typing I)*

$$\textsc{CTyp-empty1}\ \frac{}{[\cdot] : (\Gamma \;\Rightarrow\; A) \mapsto (\Gamma \;\Rightarrow\; A) \rightsquigarrow [\cdot]}$$

$$\textsc{CTyp-appL1}\ \frac{\mathcal{C} : (\Gamma \;\Rightarrow\; A) \mapsto (\Gamma' \;\Rightarrow\; A_1 \to A_2) \rightsquigarrow \mathcal{D} \qquad \Gamma' \vdash E_2 \Leftarrow A_1 \rightsquigarrow e}{\mathcal{C}\, E_2 : (\Gamma \;\Rightarrow\; A) \mapsto (\Gamma' \;\Rightarrow\; A_2) \rightsquigarrow \mathcal{D}\, e}$$

$$\textsc{CTyp-appR1}\ \frac{\Gamma' \vdash E_1 \Rightarrow A_1 \to A_2 \rightsquigarrow e \qquad \mathcal{C} : (\Gamma \;\Rightarrow\; A) \mapsto (\Gamma' \;\Leftarrow\; A_1) \rightsquigarrow \mathcal{D}}{E_1\, \mathcal{C} : (\Gamma \;\Rightarrow\; A) \mapsto (\Gamma' \;\Rightarrow\; A_2) \rightsquigarrow e\, \mathcal{D}}$$

$$\textsc{CTyp-mergeL1}\ \frac{\mathcal{C} : (\Gamma \;\Rightarrow\; A) \mapsto (\Gamma' \;\Rightarrow\; A_1) \rightsquigarrow \mathcal{D} \qquad \Gamma' \vdash E_2 \Rightarrow A_2 \rightsquigarrow e \qquad A_1 * A_2}{\mathcal{C} \,,,\, E_2 : (\Gamma \;\Rightarrow\; A) \mapsto (\Gamma' \;\Rightarrow\; A_1 \,\&\, A_2) \rightsquigarrow \langle \mathcal{D}, e \rangle}$$

$$\textsc{CTyp-mergeR1}\ \frac{\Gamma' \vdash E_1 \Rightarrow A_1 \rightsquigarrow e \qquad \mathcal{C} : (\Gamma \;\Rightarrow\; A) \mapsto (\Gamma' \;\Rightarrow\; A_2) \rightsquigarrow \mathcal{D} \qquad A_1 * A_2}{E_1 \,,,\, \mathcal{C} : (\Gamma \;\Rightarrow\; A) \mapsto (\Gamma' \;\Rightarrow\; A_1 \,\&\, A_2) \rightsquigarrow \langle e, \mathcal{D} \rangle}$$

$$\textsc{CTyp-rcd1}\quad \frac{\mathcal{C} : (\Gamma \Rightarrow A) \mapsto (\Gamma' \Rightarrow B) \rightsquigarrow \mathcal{D}}{\{l = \mathcal{C}\} : (\Gamma \Rightarrow A) \mapsto (\Gamma' \Rightarrow \{l : B\}) \rightsquigarrow \{l = \mathcal{D}\}}$$

$$\textsc{CTyp-proj1}\quad \frac{\mathcal{C} : (\Gamma \Rightarrow A) \mapsto (\Gamma' \Rightarrow \{l : B\}) \rightsquigarrow \mathcal{D}}{\mathcal{C}.l : (\Gamma \Rightarrow A) \mapsto (\Gamma' \Rightarrow B) \rightsquigarrow \mathcal{D}.l}$$

$$\textsc{CTyp-anno1}\quad \frac{\mathcal{C} : (\Gamma \Rightarrow B) \mapsto (\Gamma' \Leftarrow A) \rightsquigarrow \mathcal{D}}{\mathcal{C} : A : (\Gamma \Rightarrow B) \mapsto (\Gamma' \Rightarrow A) \rightsquigarrow \mathcal{D}}$$

$\boxed{\mathcal{C} : (\Gamma \Leftarrow A) \mapsto (\Gamma' \Leftarrow B) \rightsquigarrow \mathcal{D}}$ *(Context typing II)*

$$\textsc{CTyp-empty2}\quad \frac{}{[\cdot] : (\Gamma \Leftarrow A) \mapsto (\Gamma \Leftarrow A) \rightsquigarrow [\cdot]}$$

$$\textsc{CTyp-abs2}\quad \frac{\mathcal{C} : (\Gamma \Leftarrow A) \mapsto (\Gamma', x : A_1 \Leftarrow A_2) \rightsquigarrow \mathcal{D} \qquad x \notin \Gamma'}{\lambda x.\, \mathcal{C} : (\Gamma \Leftarrow A) \mapsto (\Gamma' \Leftarrow A_1 \to A_2) \rightsquigarrow \lambda x.\, \mathcal{D}}$$

$\boxed{\mathcal{C} : (\Gamma \Leftarrow A) \mapsto (\Gamma' \Rightarrow B) \rightsquigarrow \mathcal{D}}$ *(Context typing III)*

$$\textsc{CTyp-appL2}\quad \frac{\mathcal{C} : (\Gamma \Leftarrow A) \mapsto (\Gamma' \Rightarrow A_1 \to A_2) \rightsquigarrow \mathcal{D} \qquad \Gamma' \vdash E_2 \Leftarrow A_1 \rightsquigarrow e}{\mathcal{C}\, E_2 : (\Gamma \Leftarrow A) \mapsto (\Gamma' \Rightarrow A_2) \rightsquigarrow \mathcal{D}\, e}$$

$$\textsc{CTyp-appR2}\quad \frac{\Gamma' \vdash E_1 \Rightarrow A_1 \to A_2 \rightsquigarrow e \qquad \mathcal{C} : (\Gamma \Leftarrow A) \mapsto (\Gamma' \Leftarrow A_1) \rightsquigarrow \mathcal{D}}{E_1\, \mathcal{C} : (\Gamma \Leftarrow A) \mapsto (\Gamma' \Rightarrow A_2) \rightsquigarrow e\, \mathcal{D}}$$

$$\textsc{CTyp-mergeL2}\quad \frac{\mathcal{C} : (\Gamma \Leftarrow A) \mapsto (\Gamma' \Rightarrow A_1) \rightsquigarrow \mathcal{D} \qquad \Gamma' \vdash E_2 \Rightarrow A_2 \rightsquigarrow e \qquad A_1 * A_2}{\mathcal{C} \,,,\, E_2 : (\Gamma \Leftarrow A) \mapsto (\Gamma' \Rightarrow A_1 \,\&\, A_2) \rightsquigarrow \langle \mathcal{D}, e \rangle}$$

$$\textsc{CTyp-mergeR2}\quad \frac{\Gamma' \vdash E_1 \Rightarrow A_1 \rightsquigarrow e \qquad \mathcal{C} : (\Gamma \Leftarrow A) \mapsto (\Gamma' \Rightarrow A_2) \rightsquigarrow \mathcal{D} \qquad A_1 * A_2}{E_1 \,,,\, \mathcal{C} : (\Gamma \Leftarrow A) \mapsto (\Gamma' \Rightarrow A_1 \,\&\, A_2) \rightsquigarrow \langle e, \mathcal{D} \rangle}$$

$$\textsc{CTyp-rcd2}\quad \frac{\mathcal{C} : (\Gamma \Leftarrow A) \mapsto (\Gamma' \Rightarrow B) \rightsquigarrow \mathcal{D}}{\{l = \mathcal{C}\} : (\Gamma \Leftarrow A) \mapsto (\Gamma' \Rightarrow \{l : B\}) \rightsquigarrow \{l = \mathcal{D}\}}$$

$$\textsc{CTyp-proj2}\quad \frac{\mathcal{C} : (\Gamma \Leftarrow A) \mapsto (\Gamma' \Rightarrow \{l : B\}) \rightsquigarrow \mathcal{D}}{\mathcal{C}.l : (\Gamma \Leftarrow A) \mapsto (\Gamma' \Rightarrow B) \rightsquigarrow \mathcal{D}.l}$$

$$\textsc{CTyp-anno2}\quad \frac{\mathcal{C} : (\Gamma \Leftarrow B) \mapsto (\Gamma' \Leftarrow A) \rightsquigarrow \mathcal{D}}{\mathcal{C} : A : (\Gamma \Leftarrow B) \mapsto (\Gamma' \Rightarrow A) \rightsquigarrow \mathcal{D}}$$

$\boxed{\mathcal{C} : (\Gamma \Rightarrow A) \mapsto (\Gamma' \Leftarrow B) \rightsquigarrow \mathcal{D}}$ *(Context typing IV)*

$$\textsc{CTyp-abs1}\quad \frac{\mathcal{C} : (\Gamma \Rightarrow A) \mapsto (\Gamma', x : A_1 \Leftarrow A_2) \rightsquigarrow \mathcal{D} \qquad x \notin \Gamma'}{\lambda x.\, \mathcal{C} : (\Gamma \Rightarrow A) \mapsto (\Gamma' \Leftarrow A_1 \to A_2) \rightsquigarrow \lambda x.\, \mathcal{D}}$$

$\boxed{\mathcal{L} \vdash A \prec: B \rightsquigarrow c}$ *(Algorithmic subtyping)*

$$\textsc{A-And}\quad \frac{\mathcal{L} \vdash A \prec: B_1 \rightsquigarrow c_1 \qquad \mathcal{L} \vdash A \prec: B_2 \rightsquigarrow c_2}{\mathcal{L} \vdash A \prec: B_1 \,\&\, B_2 \rightsquigarrow [\![\mathcal{L}]\!]_{\&} \circ \langle c_1, c_2 \rangle}$$

$$\textsc{A-Arr}\quad \frac{\mathcal{L}, B_1 \vdash A \prec: B_2 \rightsquigarrow c}{\mathcal{L} \vdash A \prec: B_1 \to B_2 \rightsquigarrow c}$$

$$\textsc{A-Rcd}\quad \frac{\mathcal{L}, \{l\} \vdash A \prec: B \rightsquigarrow c}{\mathcal{L} \vdash A \prec: \{l : B\} \rightsquigarrow c}$$

$$\textsc{A-Top}\quad \frac{}{\mathcal{L} \vdash A \prec: \top \rightsquigarrow [\![\mathcal{L}]\!]_{\top} \circ \mathsf{top}}$$

$$\textsc{A-ArrNat}\quad \frac{[] \vdash A \prec: A_1 \rightsquigarrow c_1 \qquad \mathcal{L} \vdash A_2 \prec: \mathsf{Nat} \rightsquigarrow c_2}{A, \mathcal{L} \vdash A_1 \to A_2 \prec: \mathsf{Nat} \rightsquigarrow c_1 \to c_2}$$

$$\textsc{A-RcdNat}\quad \frac{\mathcal{L} \vdash A \prec: \mathsf{Nat} \rightsquigarrow c}{\{l\}, \mathcal{L} \vdash \{l : A\} \prec: \mathsf{Nat} \rightsquigarrow \{l : c\}}$$

$$\textsc{A-Nat}\quad \frac{}{[] \vdash \mathsf{Nat} \prec: \mathsf{Nat} \rightsquigarrow \mathsf{id}}$$

$$\textsc{A-AndN1}\quad \frac{\mathcal{L} \vdash A_1 \prec: \mathsf{Nat} \rightsquigarrow c}{\mathcal{L} \vdash A_1 \,\&\, A_2 \prec: \mathsf{Nat} \rightsquigarrow c \circ \pi_1}$$

$$\textsc{A-AndN2}\quad \frac{\mathcal{L} \vdash A_2 \prec: \mathsf{Nat} \rightsquigarrow c}{\mathcal{L} \vdash A_1 \,\&\, A_2 \prec: \mathsf{Nat} \rightsquigarrow c \circ \pi_2}$$

B The Type System of λ_c

$\boxed{\Delta \vdash e : \tau}$ *(Target typing)*

$$\textsc{typ-unit}\quad \frac{}{\Delta \vdash \langle\rangle : \langle\rangle}$$

$$\textsc{typ-lit}\quad \frac{}{\Delta \vdash i : \mathsf{Nat}}$$

$$\textsc{typ-var}\quad \frac{x : \tau \in \Delta}{\Delta \vdash x : \tau}$$

$$\textsc{typ-abs}\quad \frac{\Delta, x : \tau_1 \vdash e : \tau_2}{\Delta \vdash \lambda x.\, e : \tau_1 \to \tau_2}$$

$$\textsc{typ-app}\quad \frac{\Delta \vdash e_1 : \tau_1 \to \tau_2 \qquad \Delta \vdash e_2 : \tau_1}{\Delta \vdash e_1\, e_2 : \tau_2}$$

$$\textsc{typ-pair}\quad \frac{\Delta \vdash e_1 : \tau_1 \qquad \Delta \vdash e_2 : \tau_2}{\Delta \vdash \langle e_1, e_2 \rangle : \tau_1 \times \tau_2}$$

$$\textsc{typ-capp}\quad \frac{\Delta \vdash e : \tau \qquad c \vdash \tau \rhd \tau'}{\Delta \vdash c\, e : \tau'}$$

$$\textsc{typ-rcd}\quad \frac{\Delta \vdash e : \tau}{\Delta \vdash \{l = e\} : \{l : \tau\}}$$

$$\textsc{typ-proj}\quad \frac{\Delta \vdash e : \{l : \tau\}}{\Delta \vdash e.l : \tau}$$

$\boxed{c \vdash \tau_1 \rhd \tau_2}$ *(Coercion typing)*

$$\textsc{cotyp-refl}\quad \frac{}{\mathsf{id} \vdash \tau \rhd \tau}$$

$$\textsc{cotyp-trans}\quad \frac{c_1 \vdash \tau_2 \rhd \tau_3 \qquad c_2 \vdash \tau_1 \rhd \tau_2}{c_1 \circ c_2 \vdash \tau_1 \rhd \tau_3}$$

$$\textsc{cotyp-top}\quad \frac{}{\mathsf{top} \vdash \tau \rhd \langle\rangle}$$

$$\textsc{cotyp-topArr}\quad \frac{}{\mathsf{top}_{\to} \vdash \langle\rangle \rhd \langle\rangle \to \langle\rangle}$$

$$\textsc{cotyp-topRcd}\quad \frac{}{\mathsf{top}_{\{l\}} \vdash \langle\rangle \rhd \{l : \langle\rangle\}}$$

$$\textsc{cotyp-arr}\quad \frac{c_1 \vdash \tau_1' \rhd \tau_1 \qquad c_2 \vdash \tau_2 \rhd \tau_2'}{c_1 \to c_2 \vdash \tau_1 \to \tau_2 \rhd \tau_1' \to \tau_2'}$$

$$\textsc{cotyp-pair}\quad \frac{c_1 \vdash \tau_1 \rhd \tau_2 \qquad c_2 \vdash \tau_1 \rhd \tau_3}{\langle c_1, c_2 \rangle \vdash \tau_1 \rhd \tau_2 \times \tau_3}$$

$$\textsc{cotyp-projl}\quad \frac{}{\pi_1 \vdash \tau_1 \times \tau_2 \rhd \tau_1}$$

$$\textsc{cotyp-projr}\quad \frac{}{\pi_2 \vdash \tau_1 \times \tau_2 \rhd \tau_2}$$

$$\textsc{cotyp-rcd}\quad \frac{c \vdash \tau_1 \rhd \tau_2}{\{l : c\} \vdash \{l : \tau_1\} \rhd \{l : \tau_2\}}$$

$$\textsc{cotyp-distRcd}\quad \frac{}{\mathsf{dist}_{\{l\}} \vdash \{l : \tau_1\} \times \{l : \tau_2\} \rhd \{l : \tau_1 \times \tau_2\}}$$

$$\textsc{cotyp-distArr}\quad \frac{}{\mathsf{dist}_{\rightarrow} \vdash (\tau_1 \rightarrow \tau_2) \times (\tau_1 \rightarrow \tau_3) \rhd \tau_1 \rightarrow \tau_2 \times \tau_3}$$

$\boxed{e \longrightarrow e'}$ *(Small-step reduction)*

$$\textsc{step-id}\quad \frac{}{\mathsf{id}\ v \longrightarrow v} \qquad \textsc{step-trans}\quad \frac{}{(c_1 \circ c_2)\ v \longrightarrow c_1\ (c_2\ v)} \qquad \textsc{step-top}\quad \frac{}{\mathsf{top}\ v \longrightarrow \langle\rangle} \qquad \textsc{step-topArr}\quad \frac{}{(\mathsf{top}_{\rightarrow}\ \langle\rangle)\ \langle\rangle \longrightarrow \langle\rangle}$$

$$\textsc{step-topRcd}\quad \frac{}{\mathsf{top}_{\{l\}}\ \langle\rangle \longrightarrow \{l = \langle\rangle\}} \qquad \textsc{step-arr}\quad \frac{}{((c_1 \rightarrow c_2)\ v_1)\ v_2 \longrightarrow c_2\ (v_1\ (c_1\ v_2))}$$

$$\textsc{step-pair}\quad \frac{}{\langle c_1, c_2\rangle\ v \longrightarrow \langle c_1\ v, c_2\ v\rangle} \qquad \textsc{step-distArr}\quad \frac{}{(\mathsf{dist}_{\rightarrow}\ \langle v_1, v_2\rangle)\ v_3 \longrightarrow \langle v_1\ v_3, v_2\ v_3\rangle}$$

$$\textsc{step-distRcd}\quad \frac{}{\mathsf{dist}_{\{l\}}\ \langle\{l = v_1\}, \{l = v_2\}\rangle \longrightarrow \{l = \langle v_1, v_2\rangle\}} \qquad \textsc{step-projl}\quad \frac{}{\pi_1\ \langle v_1, v_2\rangle \longrightarrow v_1} \qquad \textsc{step-projr}\quad \frac{}{\pi_2\ \langle v_1, v_2\rangle \longrightarrow v_2}$$

$$\textsc{step-crcd}\quad \frac{}{\{l : c\}\ \{l = v\} \longrightarrow \{l = c\ v\}} \qquad \textsc{step-beta}\quad \frac{}{(\lambda x.\ e)\ v \longrightarrow e[x \mapsto v]} \qquad \textsc{step-projRcd}\quad \frac{}{\{l = v\}.l \longrightarrow v}$$

$$\textsc{step-app1}\quad \frac{e_1 \longrightarrow e_1'}{e_1\ e_2 \longrightarrow e_1'\ e_2} \qquad \textsc{step-app2}\quad \frac{e_2 \longrightarrow e_2'}{v_1\ e_2 \longrightarrow v_1\ e_2'} \qquad \textsc{step-pair1}\quad \frac{e_1 \longrightarrow e_1'}{\langle e_1, e_2\rangle \longrightarrow \langle e_1', e_2\rangle} \qquad \textsc{step-pair2}\quad \frac{e_2 \longrightarrow e_2'}{\langle v_1, e_2\rangle \longrightarrow \langle v_1, e_2'\rangle}$$

$$\textsc{step-capp}\quad \frac{e \longrightarrow e'}{c\ e \longrightarrow c\ e'} \qquad \textsc{step-rcd1}\quad \frac{e \longrightarrow e'}{\{l = e\} \longrightarrow \{l = e'\}} \qquad \textsc{step-rcd2}\quad \frac{e \longrightarrow e'}{e.l \longrightarrow e'.l}$$