# Random deep neural networks are biased towards simple functions

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## The generalization problem

- Deep neural networks do not overfit despite # weights >> # training points
- Conjecture [Valle Pérez et al., ICLR 2019]: deep neural networks are biases towards simple functions, weights highly redundant
- Experiments on bit string classifiers generated by deep neural networks with randomly initialized weights: classifiers with low Lempel-Ziv complexity occur with higher probability

## Deep neural networks as Gaussian processes

(Google Brain)

- Random weights with normal iid Gaussian distribution
- Limit of infinite width of hidden layers
- Generated function $\phi$ is Gaussian process: for any $x_1$, …, $x_k$ in $R^n$, $\phi(x_1)$, …, $\phi(x_k)$ have correlated Gaussian distribution

## Binary classifiers of bit strings

- Encode strings of $n$ bits in $R^n$ as $\{-1,1\}^n$
- Input $x$ classified as sign($\phi(x)$)
- Hamming distance $h(x,y)$ = # of different bits
- Uniformly random classifier: for any $x$ in $\{-1,1\}^n$ with high probability there exists $y$ with different classification at $h(x,y) = 1$
- Same properties for classifiers generated by random deep neural networks?

## Main results

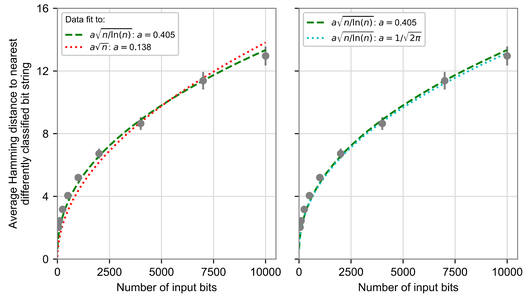
- For any input string $x$ of $n$ bits, the average Hamming distance of the closest bit string with a different classification is

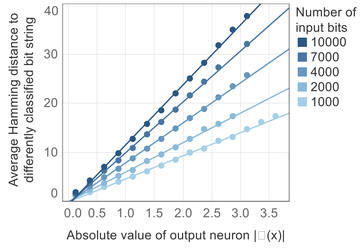
$$\sqrt{\frac{n}{2\pi \ln n}}$$

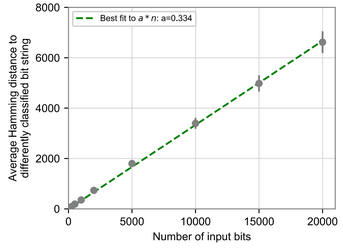
and grows linearly with $\phi(x)$ as $\frac{|\phi(x)|}{2}\sqrt{\frac{n}{\ln n}}$

- For any input bit string $x$ of $n$ bits, the minimum number of random bit flips required to change the classification scales linearly with $n$

## Experiments on random DNNs (2 hidden layers)

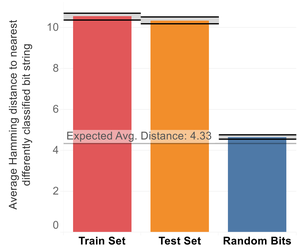


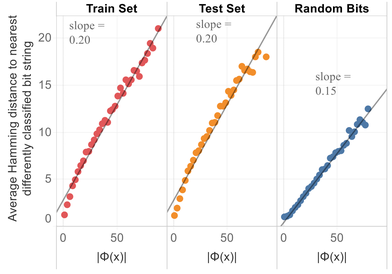




## Experiments on MNIST (2 hidden layers)

- Robustness survives training




- Robustness correlated with generalization


## Conclusions

- Binary classifiers of bit strings generated by random deep neural networks are biased towards simple functions
- Experiments: simplicity bias survives after training
- Analytical proof for trained DNNs??