

## S2 Appendix: Conditional sampling from NHPPPs

Algorithm B is a direct modification of the order statistics algorithm (Algorithm 6 in the main text) to sample conditional on observing  $m$  events in  $(a, b]$ . (The modification is in line 1.)

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**Algorithm B** Modified order statistics algorithm for sampling at least  $m$  events from an NHPPP given  $\Lambda(t), \Lambda^{-1}(z)$ .

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[h!]

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Require:  $\Lambda(t), \Lambda^{-1}(z), t \in (a, b]$ 
1:  $N \leftarrow N \sim \text{TruncatedPoisson}_{N \geq m}(\Lambda(b) - \Lambda(a))$  ▷  $\Lambda^{-1}(z)$  possibly numerically
2:  $t \leftarrow a$  ▷  $(m - 1)$ -truncated Poisson
3:  $\mathcal{Z} \leftarrow \emptyset$  ▷  $\mathcal{Z}$  is an ordered set
4: if  $N > 0$  then
5:   for  $i \in [N]$  do:
6:      $U_i \leftarrow U_i \sim \text{Uniform}(0, 1)$  ▷ Generate order statistics
7:      $\mathcal{Z} \leftarrow \mathcal{Z} \cup \{\Lambda^{-1}(\Lambda(a) + U_i(\Lambda(b) - \Lambda(a)))\}$ 
8:   end for
9:    $\mathcal{Z} \leftarrow \text{sort}(\mathcal{Z})$ 
10: end if
11: return  $\mathcal{Z}$  ▷ Up to  $k$  earliest points: return  $\{Z_{(i)} \mid i \leq k, Z_{(i)} \in \mathcal{Z}\}$ 

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To sample exactly  $m$  points, change line 1 of Algorithm B to

$$N \leftarrow m.$$

To sample up to  $k$  earliest points, replace line 11 with in Algorithm B with

**return**  $\{Z_{(i)} \mid i \leq k, Z_{(i)} \in \mathcal{Z}\}.$