Light II: emission and propagation

1. Basics & Nomenclature

In thermal equilibrium, a photon distribution has the Planck spectrum, with a volume energy density per unit frequency as follows:

$$u_\nu = \frac{8\pi\nu^2}{c^3} \frac{h\nu}{\exp(-h\nu/kT) - 1} \tag{1}$$

Typical units are erg Hz^{-1} cm^{-3}. The peak of u_ν or u_λ is simply related to T according to Wien's Law:

$$h\nu_{\rm max} = 2.8kT \qquad \lambda_{\rm max}T = 2.9 \text{ mmK} \tag{2}$$

The specific intensity of the Planck radiation field is:

$$I_\nu = \frac{u_\nu c}{4\pi}. \tag{3}$$

The flux density through a flat surface is then:

$$f_\nu = \pi I_\nu \tag{4}$$

and the total flux is (integrating over ν):

$$f = \frac{2\pi^5 k^4}{15c^2h^3} T^4 = \sigma T^4 \tag{5}$$

where this equation uses the definition of the Stefan-Boltzmann constant σ.

At low frequency this is a power-law distribution, the Rayleigh-Jeans tail:

$$u_\nu \approx \frac{8\pi kT\nu^2}{c^3} \tag{6}$$

and the specific intensity is:

$$I_\nu \approx \frac{2\nu^2}{c^2} kT \tag{7}$$

This fact motivates the definition used in radio astronomy of a *brightness temperature* T_B to characterize a measured specific intensity, which is calculated from a measured I_ν using Equation 7 (whether or not the source is thermal, which can lead to the brightness temperature having values that would be unreasonable if it were a real temperature).

Under many conditions, the photon distribution is not in thermal equilibrium, and therefore differs from the Planck spectrum. Such spectra reveal details about the specific physical interactions the photons are undergoing that yield important clues about the conditions of the emitting material.

Lines, or sharp features in the spectrum, can be created due to discrete energy level differences between atomic or molecular states. *Emission lines* occur when photons are released (and escape the medium) from a downwards transition. *Absorption lines* occur when photons coming toward the observer incite upwards transitions in intervening material. We usually speak of these lines as separate from the *continuum* spectrum, so that emission is in addition to the continuum and absorption is absorbing the continuum; however what we mean by "continuum" varies somewhat depending on context. The interpretation of these features of spectra comprise a large portion of optical astrophysics, and in this section we discuss only a couple of examples to introduce nomenclature.

Lines can be quantified in several ways:

- Both emission and absorption lines have some intrinsic width, which can be expressed as a full width half maximum (FWHM) or otherwise. Physically, this width can come from a combination of the intrinsic transition width, pressure-induced width, and Doppler velocity width (due to thermal or other motions).

- Emission lines have a total flux or luminosity that can be associated with them by subtracting an estimate of the continuum and integrating over wavelength or frequency (e.g., with units erg s^{-1} for luminosity). There is an equivalent quantity for absorption lines (the flux or luminosity of the continuum that is absorbed) but this is rarely referred to.

- Both emission and absorption lines have an *equivalent width* (EW), which is usually expressed in units of wavelength, and is the flux emitted or absorbed divided by a continuum flux density estimate (e.g. f_λ) at the location of the line. Sometimes we use the convention that positive EW indicates absorption and negative indicates emission; sometimes the opposite.

Atomic transition lines often have few-eV-scale energies (i.e. are near the optical), because these are typical numbers for many lower n transitions; however highly ionized heavy atoms can have much higher energy transitions, and the upper n levels of any atom can get almost arbitrarily close in energy, leading to radio emission lines (for example, high-n recombination lines of hydrogen). Molecular disassociation energies are lower, and therefore the transitions tend to be lower energy, with vibrational transitions around 1 μm (i.e. 1 eV) and rotational transitions in the mm or cm range.

An illustrative example is the atomic transition sequences of Hydrogen. These transitions are determined by the well-known Bohr sequence:

$$E_n = -\frac{e^4 m_e}{2\hbar^2}\frac{1}{n^2} \tag{8}$$

The transitions between these states and important in stellar atmospheres (typically in absorption) and in interstellar medium emission (typically in emission). The lowest state is $E_0 = -13.6$ eV,

Table 1: Wavelengths (vacuum $\AA$) of hydrogen transitions between n and m

Series	Lower state (n)	α ($m = n + 1$)	β ($m = n + 2$)	γ ($m = n + 3$)	δ ($m = n + 4$)
Lyman (Ly)	1	1216	1026	973	950
Balmer (H)	2	6564	4861	4340	4102
Paschen (Pa)	3	18756	12822	10941	10052

which corresponds to a photon of 912 Å; higher frequency photons will ionize H. The transitions between states are classified according to the lower state as follows:

A relatively special case is neutral hydrogen, which has a *hyperfine* transition, which means it is a transition in the spin of the electron relative to the proton. The transition is at 21 cm, or about 1.4 GHz, and a long time constant. It is the best tracer of neutral hydrogen, with many applications in astrophysics and cosmology.

Another important process, especially in the radio and X-ray domain, is *bremsstrahlung*, the radiation due to the acceleration of charges. An astrophysical plasma emits *thermal bremsstrahlung* due to Coulomb accelerations of the electrons against each other. Under most conditions, at wavelengths less than about 1 m (frequencies greater than 1 GHz), this emission occurs under optically thin conditions. At low enough frequencies this process is optically thick and thus thermal (since $h\nu \ll kT$ in practice, $f_\nu \propto \nu^2$). At higher frequency but still at $h\nu < kT$ (typically in the cm-radio regime), the emission is optically thin. In this regime, bremsstrahlung has a very flat spectrum, and $f_\nu \propto \nu^{-0.1}$. Above $h\nu > kT$, the Boltzmann cutoff leads to $f_\nu \propto \exp(-h\nu/kT)$. The calculation of thermal bremsstrahlung is somewhat complex.

In plasmas with significant magnetic fields, electrons spiraling around the magnetic fields yield synchrotron radiation due to their acceleration. The distribution of electron energies determines the shape of the resulting spectrum, which can often be approximated as a power law $f_\nu \propto \nu^{-\alpha}$, where α can range from 0 to over 2.

A final highly significant effect in the propagation of light across space is due to the effect of interstellar dust. Interstellar dust typically consists of silica grains and some carbonaceous grains, plus a small admixture of organic molecules like polycyclic aromatic hydrocarbons (Weingartner & Draine 2001; Draine 2003). The grains are typically less than a few tenths of a micron in size. Because of this, radio and infrared frequencies are not affected by dust very much. In general, the amount of extinction is wavelength dependent, with bluer frequencies experiencing more absorption and scattering; the dependence varies depending on the nature of the dust but is very approximately $A(\lambda) \propto \lambda^{-1}$, where $A(\lambda)$ is in units of magnitudes. In the ultraviolet through near infrared this causes *reddening* of the light. Meanwhile, at the very highest frequencies (X-ray and γ-ray) the photons pass through the dust (in fact, extreme UV and X-ray radiation can destroy dust). The Milky Way dust imposes a screen of extinction and reddening that must be corrected for all extragalactic observations, and for most observations within the Galaxy (Schlegel et al. 1998).

As light from cosmic sources comes toward us, it can be modified in other ways than the ones we just discussed, for example through refraction in astrophysical plasma, by intervening radiation fields, or through other processes.

2. Key References

- *Allen's Astrophysical Quantities*, Cox (2000), Chapter 5

3. Order-of-magnitude Exercises

1. As you can see when looking outside during the day, the Sun is neither very blue nor very red. Assuming it emits approximately as a blackbody, estimate the temperature of its surface.

   If we assume its peak output is near the middle of our eye's sensitivity range, around $\lambda \sim 5500$ Å, we can use Wien's Law to show that the temperature of the emitting surface is:

   $$T \sim \frac{2.9 \text{ mm K}}{5.5 \times 10^{-4} \text{ mm}} \sim 5300 \text{ K}, \tag{9}$$

   which is surprisingly close to the right answer.

2. Estimate the approximate temperature of a radiation field that will provide a substantial flux of photons to ionize hydrogen.

   The energy required to ionize a hydrogen atom is 13.6 eV. This corresponds to a photon of wavelength $\lambda = 912$ Å. Now, from Wien's Law, we can estimate the temperature of a gas whose emission would peak at this wavelength.

   $$\lambda_{\text{max}} T = 2.9 \text{ mm K} \tag{10}$$
   $$T \approx 32000 \text{ K} \tag{11}$$

   Below this temperature, the number of photons available to ionize hydrogen will rapidly decline due to the exponential tail of the Planck energy distribution.

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3. If you have an optical spectrograph with $R \sim 4000$, for what line-of-sight velocity dispersion is the intrinsic width of the line equal to the width due to the resolution? We will learn later that depending on signal-to-noise ratio, velocities much smaller than the resolution are hard to measure.

4. Galaxy clusters emit thermal bremstrahlung at energies $\nu > 1$ keV. What is the temperature necessary to do this?

5. The center of the Milky Way is very heavily extincted: by about 30 magnitudes in the V band. Approximately how much is that in the near-infrared K band?

   The effective wavelength of V is 5500 Å, and of K is about 2.2 μm, or about 4 times longer. If the extinction curve scales as λ^{-1}, then the magnitude difference will be around 7.5 magnitudes. This fact means that observations of the Galactic center, completely impossible in the optical, are possible in the infrared. In fact, the extinction curve is somewhat steeper so the benefit is even larger than we estimate here.

4. Analytic Exercises

1. Prove Wien's law, Equation 2.

   We begin with the Planck spectrum:

$$u(\nu) = \frac{8\pi h}{c^3}\frac{\nu^3}{\exp[h\nu/kT]-1}$$

   To find the max of the energy density with respect to ν, I will differentiate with respect to ν, and then set equal to 0 and solve for ν.

$$\begin{aligned}
\frac{du}{d\nu} &= \frac{8\pi h}{c^3}\left(\frac{3\nu^2}{\exp[h\nu/kT]-1} - \frac{\nu^3}{(\exp[h\nu/kT]-1)^2}\exp[h\nu/kT]\left(\frac{h}{kT}\right)\right) \\
\frac{du}{d\nu} &= \frac{8\pi h}{c^3}\frac{3\nu^2(\exp[h\nu/kT]-1) - \nu^3\frac{h}{kT}\exp[h\nu/kT]}{(\exp[h\nu/kT]-1)^2} \\
0 &= \frac{8\pi h}{c^3}\frac{3\nu^2(\exp[h\nu/kT]-1) - \nu^3\frac{h}{kT}\exp[h\nu/kT]}{(\exp[h\nu/kT]-1)^2} \\
0 &= 3\nu^2(\exp[h\nu/kT]-1) - \nu^3\frac{h}{kT}\exp[h\nu/kT] \\
0 &= 3(\exp[h\nu/kT]-1) - \nu\frac{h}{kT}\exp[h\nu/kT]
\end{aligned}$$

   Now let $x = \frac{h\nu}{kT}$.

$$0 = 3(\exp[x]-1) - x\exp[x]$$

   This root-finding problem cannot be further reduced analytically. Newton's method as implemented by `scipy` in Python can be applied, which converges to $x = 2.82$. Thus:

$$2.8kT = h\nu$$

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2. The Bohr sequence is a consequence of Schroedinger's equation applied to the electron. However, the original Bohr model was based on the classical picture of an electron orbiting the

proton due to Coulomb attraction. In this context, the electron is actually orbiting the center of mass of the proton-electron system, but this two-body problem can be reduced to a central force problem, with the true electron mass being replaced by the *reduced mass*. Given this understanding, how should the spectrum of deuterium compare to the spectrum of hydrogen?

We assume that we are only concerned with Coulomb forces, we approximate $m_p \approx m_n$, and we still treat the nucleus of deuterium as a point particle, just of a large mass particle. Then we can compare the reduced mass of the electron for hydrogen:

$$\mu_{e,H} = \frac{m_p m_e}{m_p + m_e} \tag{12}$$

to that of deuterium:

$$\mu_{e,D} = \frac{2m_p m_e}{2m_p + m_e} \tag{13}$$

Then:

$$\frac{\mu_{e,D}}{\mu_{e,H}} = \frac{2(m_p + m_e)}{2m_p + m_e} = \frac{(1 + m_e/m_p)}{1 + m_e/2m_p} \approx 1 + \frac{m_e}{m_p} - \frac{1}{2}\frac{m_e}{m_p} = 1 + \frac{1}{2}\frac{m_e}{m_p} \tag{14}$$

The reduced mass is what should appear in the Bohr formula for the energy levels of the electron:

$$E_n = -\frac{\mu e^4}{2\hbar^2}\frac{1}{n^2} \tag{15}$$

Since $m_e/m_p \sim 5 \times 10^{-3}$, all of the energy levels get deeper by a factor $\sim 2.5 \times 10^{-4}$, and all of the transitions will be different in energy by that factor. The photons are then correspondingly shorter in wavelength (or higher in frequency). For example, the Lyman-α transition that in vacuum is normally 1215.7 Åbecomes 1215.4 Å. Note that this is *much* bigger than the fine structure splitting of this line.

There is observational relevance of this shift, since high resolution measurements of Lyman-α in absorption at high redshift allows a measurement of the deuterium-to-hydrogen abundance ratio, which can be tied to Big Bang Nucleosynthesis, and is part of the evidence that the baryonic density is much lower than Ω_m (Burles & Tytler 1998).

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5. Numerics and Data Exercises

1. Download an optical spectrum of an A star. Identify all Balmer absorption lines that are apparent in that spectrum.

2. Download an optical spectrum of a star forming galaxy. Identify all Balmer emission lines that are apparent in the spectrum. Zooming in on Hα or $H\beta$, visually compare the Balmer absorption (in the stellar continuum) to the emission.

3. Download an optical spectrum of a luminous quasar at redshift $z \sim 2.5$. Identify the Lyα emission line and estimate its full width half maximum in Å. Assuming the width is dominated by Doppler motions, what is that width in km s^{-1}. You should also be able to see absorption lines blueward of Lyα; this is Lyα absorption by intervening gas that is therefore less redshifted.

4. Schlegel et al. (1998) published an estimate of Galactic dust extinction in our galaxy. Download optical spectra from SDSS of four stars with similar temperatures and metallicities over a range of Galactic reddening values. Utilize the extinction model of Cardelli et al. (1989) and try to explain the broad-band differences between stellar spectra. Can you explain any discrepancies with the SFD prediction?

5. Access the dust map of Schlegel et al. (1998) and plot the $E(B-V)$ reddening as a function of Galactic Coordinates.

6. Find the SDSS optical spectra and images for the two galaxies UGC 10227 (a typical-looking disk galaxy observed at high inclination) and MCG -01-53-020 (a typical-looking disk galaxy observed at low inclination). A major difference in observing galaxies at these inclinations is the resulting amount of dust extinction. For a standard reddening law, how much extinction due you need to explain the first galaxy spectrum as a reddened version of the second?

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