Inventory

1. Basics

We begin with a basic inventory of what our subject is, and the relevant length and time scales.

The Sun is a not-untypical star in the Milky Way galaxy. It is a hot ball of gas, primarily consisting of H, with about 25% ^{4}He and about 2% higher mass elements, known as *metals*. It produces light through nuclear fusion of H to ^{4}He in its core. Its mass $M_\odot = 2 \times 10^{33}$ g and its radius is $R_\odot = 0.7 \times 10^6$ kc. The distance from the Earth is 1.4×10^8 km = 1 *Astronomical Unit* (AU). The nearest other stars are very far away, typically *parsecs* ($\approx 2 \times 10^5$ AU, or about 3.3 lightyears) away.

Bland-Hawthorn & Gerhard (2016) reviews what we know of the Milky Way galaxy. The Sun orbits the Galaxy in a roughly circular orbit at about 8.5 kpc from the center, with a velocity of ~ 220 km s^{-1}, along with most stars in the Milky Way's disk. The Milky Way visible by eye in the night sky is resolved into many billions of stars, with a total *stellar mass* of about 3×10^{10} $M_\odot$. Like many galaxies, the Milky Way has a *thin disk* (a few 100 pc) thick, a *thick disk* (about 1 kpc thick), a *bulge* and a *bar* in its center, and a *stellar halo* that extends out to about 100 kpc. It has a moderately large black hole (4×10^6 $M_\odot$) in its center, a relatively small *supermassive black hole*. The disk has neutral and molecular gas as well as dust, and outside the disk and surrounding the Milky Way is a diffuse halo of gas. The molecular gas regions are forming young stars only a few million years old. The oldest stars in the galaxy appear to be about 10 billion years old. Orbiting the Milky Way are about 150 globular clusters, which are roughly spherical, very old, bound stellar systems.

The Milky Way is filled with dust consisting of particles of up to a fraction of a micron in size. This dust reddens and extincts light passing through it. Using infrared satellite observations of 100 μm emission from this cold dust, we have estimates of the amount of dust in front of any object outside of our galaxy (Schlegel et al. 1998), and even coarse three-dimensional maps of the dust. All extragalactic observations must be corrected for this effect. Near the Galactic Plane, the dust extinction is extremely high, causing the *Zone of Avoidance* for galaxy maps. The dust hinders study of the Milky Way disk in the optical, making objects fainter and making it difficult to infer their true colors.

The galaxy exists within the deep potential well of a *dark matter* halo, which is only detectable today through its gravitational influence. The rotation velocity stays close to flat at 220 km s^{-1} out to at least 20 kpc. The total dark matter mass appears to be about 10^{12} $M_\odot$ and it extends past 100 kpc. Numerous lines of evidence suggest that the dark matter is not *baryonic* (i.e. not ordinary standard model particles) and interacts with baryonic matter primarily gravitationally.

The nearest galaxies to the Milky Way are its *dwarf galaxy* satellites (McConnachie 2012), the largest of which are the Large and Small MCs (LMC and SMC)[1]. These two satellites are visible from the Southern Hemisphere with the naked eye. They are about 10% and 1% of the Milky Way's luminosity and are about 50 kpc away, and therefore within the Milky Way's dark matter halo. Many dozens of other satellites are known.

The nearest galaxy of comparable size to the Milky Way is Messier 31, also known as M31 or Andromeda (the last name deriving from the constellation it is found in). It is about 800 kpc away, and moving toward the Milky Way, indicating the two are part of a bound group that will eventually merge. M31 is somewhat more luminous then the Milky Way and differs in a number of important details, most obviously having a larger bulge relative to its disk.

There are nearly a billion other detected galaxies, and the census of galaxies suggests there are hundreds of billions total in the observable universe. They have a mean separation of ~ 5 Mpc, but are not uniformly distributed. Instead they exist in dense clusters, connected by filaments and walls, with void regions in between. These large scale structure form from initial primordial density fluctuations through gravitational growth. The galaxies have a range of luminosities, with a characteristic exponential cutoff in number density at high luminosity, called L_* (a bit brighter than the Milky Way's luminosity) and a power law distribution of luminosity below that.

These galaxies come in a number of varieties. Hubble (1936) classified galaxies with comparable luminosities to the Milky Way from *early-type* or *elliptical* galaxies, through *late-type* or *spiral* galaxies. Elliptical galaxies old, red, and puffy. Spiral galaxies are younger, bluer, and have cold thin disks. An apparently intermediate variety of *lenticular* or *S0* galaxies have disks like spiral galaxies, but are puffier and do not have spiral structure. There are *irregular* galaxies of various types. Dwarf galaxies tend to deviate from Hubble's system in detail, as do distant galaxies observed as they were when the universe was younger.

The dynamics of stellar systems, revealed through Doppler shifts or proper motions, reveals their matter density. A characteristc mass can be derived through the *virial relation*, which dimensionally is:

$$v^2 \sim \frac{GM}{R} \tag{1}$$

for characteristic velocity v, mass M, and radius R. Stellar dynamics theory yields a *virial theorem*, which defines more precisely what we mean by these characteristic quanties. The virial theorem can be quantified as $U = -2K$, where U is the total potential energy and K is the total kinetic energy.

As one looks at galaxies of greater and greater distance, one finds that they are receding with a velocity $v = H_0 d$, where $H_0 \sim 70$ km s^{-1} Mpc^{-1} (Freedman & Madore 2010). The line-of-sight velocities are determined through their Doppler shift, the recession can also be quantified by the

[1]Known commonly still as the Magellanic Clouds, a name falling into disfavor because Magellan did not discover, characterize, or ever even remark upon the Clouds, and furthermore was not a terrific person.

redshift z relating $\lambda_{\rm obs} = (1+z)\lambda_{\rm em}$. At low redshifts, we can relate velocity and redshift with $v = cz$. The Hubble recession is what we mean when we say the universe is expanding. A rough calculation of the age of the universe from this expansion yields 14 billion years, which is remarkably close to the right answer.

Essentially every luminous galaxy has a supermassive black hole at its center. Larger galaxies tend to have larger black holes, ranging up to about $10^9\ M_\odot$. These black holes presumably grew through accretion. During episodes of accretion, these black holes can become much more luminous than their host galaxies. Accreting black holes are referred to as *active galactic nuclei* (AGN), and the most luminous ones are referred to as *quasars.* These quasars were most common about 10 billion years ago, corresponding to redshifts of $z \sim$ 2–3. They can be used as backlights upon which gas aborption signatures are imprinted, and thus reveal the gas distribution throughout the universe.

The relative abundance of different atomic elements in the Universe is known. By mass, the hydrogren fraction is $X \sim 0.75$, the helium fraction is about $Y \sim 0.24$, and the fraction of other elements is $Z \sim 0.01$, with about half of that in oxygen. The elements more massive than helium are collectively known in astronomy as *metals* and their fraction in any system is known as the *metallicity* of the system, even though only roughly half of those elements would be referred to by a chemist as a metal or metalloid. Metallicity is usually quantified on a log scale relative to solar metallicity, with notation:

$$[Z/H] = \log_{10} \frac{N_Z/N_H}{N_{Z,\odot}/N_{H,\odot}}, \tag{2}$$

with similar definitions for individual elemental abundances (e.g. [Fe/H], [O/H], etc.). For the Sun, $Z \approx 0.02$, but note that the elemental abundances of the Sun are still subject to periodic revision.

2. Important numbers

- 1 eV $= 1.602 \times 10^{-12}$ erg
- $c = 2.99792 \times 10^8$ m s^{-1}
- $G = 6.6738 \times 10^{-11}$ m^3 kg^{-1}s^{-2}
- $h = 2\pi\hbar = 6.626 \times 10^{-27}$ erg Hz^{-1} $= 4.136 \times 10^{-15}$ eV Hz^{-1}
- $k_B = 1.3806503 \times 10^{-23}$ m^2 kg s^{-2} K^{-1} $= 8.617 \times 10^{-5}$ eV K^{-1}
- $m_p = 1.6726 \times 10^{-27}$ kg
- $m_n = 1.6749 \times 10^{-27}$ kg
- $m_e = 9.1049 \times 10^{-31}$ kg

- $M_{\rm Earth} = 5.974 \times 10^{24}$ kg
- $M_\odot = 1.989 \times 10^{30}$ kg
- $R_\odot = 6.955 \times 10^{8}$ m
- $T_\odot = 5500$ K
- $L_\odot = 3.828 \times 10^{33}$ erg s^{-1}
- $L_* \sim 10^{10} L_\odot$
- 1 AU $= 1.496 \times 10^{11}$ m
- lightyear $= 9.461 \times 10^{15}$ m
- parsec $= 3.086 \times 10^{16}$ m
- year $= 3.156 \times 10^{7}$ s
- $H_0 \approx 70$ km s^{-1} Mpc^{-1}

3. Key References

- *Extragalactic Astronomy and Cosmology: An Introduction, Schneider (2015)*

4. Order-of-magnitude Exercises

1. Estimate the mean distance between stars in the Milky Way disk in units of the solar radius. Are stellar collisions likely to be particularly common?

   The disk extends to at least 8.5 kpc radius, is has about 10^{10} stars, and is a few hundred pc thick. The mean density of stars is therefore:

$$n \sim \frac{10^{10}}{\pi(300\ \mathrm{pc})(10\ \mathrm{kpc})^2} \sim \frac{10^{10}}{10^{11}\ \mathrm{pc}^3} \sim 0.1\ \mathrm{pc}^{-3} \tag{3}$$

   This means the mean distance between stars is:

$$d \sim n^{-1/3} \sim 2\ \mathrm{pc} \sim 2\ \mathrm{pc} \times \frac{3 \times 10^{16}\ \mathrm{m}}{1\ \mathrm{pc}} \times \frac{R_\odot}{7 \times 10^{8}\mathrm{m}} \sim 10^{8} R_\odot \tag{4}$$

   These distances are very large relative to stellar radii. We can go further and ask for a relative velocity of ~ 200 km s^{-1} (an overestimate) what over 10 billion years is the probability that any two stars will collide.

$$p \;=\; n(\pi R_\odot^2)vt = d^{-3}(\pi R_\odot^2)vt$$

$$
\begin{aligned}
&\sim \pi(10^{-24}R_\odot^{-3})R_\odot^2\left(3\times10^{-4}R_\odot\mathrm{s}^{-1}\right)\left(3\times10^{17}\mathrm{s}\right)\\
&\sim 3\times10^{-10}
\end{aligned}
\tag{5}
$$

Thus, the chances for any individual star to actually collide with another is very small. Even accounting for the fact that there 10^{10} in the Milky Way indicates that the total rate of encounters is of order unity per 10 billion years.

2. Use dimensional analysis to derive the virial relationship among characteristic size, mass, and velocity in an equilibrium gravitating system.

   A gravitating system has a characteristic radius R, mass M, and velocity v of its orbiters. Newton's laws introduce the constant G, with units m^3 kg^{-1} s^{-2}. The relationship between the four relevant quantities can be expressed as:

$$
v \propto G^\alpha M^\beta R^\gamma \tag{6}
$$

   To match the mass, length, and time units respectively:

$$
\begin{aligned}
0 &= -\alpha+\beta\\
1 &= 3\alpha+\gamma\\
-1 &= -2\alpha
\end{aligned}
\tag{7}
$$

   Therefore, $\alpha = 1/2$, and therefore $\gamma = -1/2$ and $\beta = 1/2$. If we square both sides of the relationship we find:

$$
v^2 \propto \frac{GM}{R}, \tag{8}
$$

   which up to a factor of order unity is the virial relation.

3. Estimate the approximate dynamical mass interior to the Sun.

   Assuming a circular orbit at 8.5 kpc of 220 km s^{-1}, we can use the force law:

$$
a = \frac{GM}{r^2} = \frac{v^2}{r} \tag{9}
$$

   to infer:

$$
\begin{aligned}
M &= \frac{v^2 r}{G} = \frac{(2.2\times10^5\ \mathrm{m\ s^{-1}})^2(8500\times3\times10^{16}\ \mathrm{m})}{6.7\times10^{-11}\ \mathrm{m^3\ kg^{-1}s^{-2}}}\\
&\sim 2\times10^{41}\ \mathrm{kg}\sim10^{11}M_\odot
\end{aligned}
\tag{10}
$$

   This is only somewhat more than the total mass inferred in stars. The real evidence for dark matter comes when you consider that the rotation curve remains constant ("flat") to much larger radii, continuing to add to the discrepancy.

4. Use the Hubble Law to estimate the age of the Universe.

We use the standard estimate of time from distance velocity:

$$t = \frac{d}{v} = \frac{d}{H_0 d} = H_0^{-1} \tag{11}$$

That is, there is a time in the past at which all the galaxies were apparently at a single point. This is time of the Big Bang, and the *Hubble time* is:

$$H_0^{-1} = \left(70 \frac{\mathrm{km\ s^{-1}}}{\mathrm{Mpc}}\right)^{-1} = \frac{3 \times 10^{22}\ \mathrm{m}}{7 \times 10^4\ \mathrm{m\ s^{-1}}} \sim 4.3 \times 10^{17}\ \mathrm{s} \sim 1.4 \times 10^{10}\ \mathrm{yr} \sim 14\ \text{billion years} \tag{12}$$

This estimate assumes that the galaxies have been traveling at constant velocity. However, in reality the mass density of the universe causes deceleration at early times, and at late times there is an unexplained acceleration called "dark energy." At the current time, these two effects tend to cancel and the above estimate of the universe's age is correct to better than 5%.

5. Estimate the cosmic mass density in g cm^{-3} of stars and of dark matter due to galaxies like the Milky Way, using the mean separation quoted above. Compare to the closure density in General Relativity of $\rho_c = 3H_0^2/8\pi G$.

   If the mass of one galaxies in stars or dark matter is denoted M, and their mean separation denoted d, the density for baryons or dark matter will be:

$$\rho \sim \frac{M}{d^3} \tag{13}$$

   For stars:

$$\begin{aligned} \rho &\sim \frac{3 \times 10^{10} M_\odot}{(5\ \mathrm{Mpc})^3} \\ &\sim \frac{6 \times 10^{43}\ \mathrm{g}}{(1.5 \times 10^{25}\ \mathrm{cm})^3} \\ &\sim \frac{6 \times 10^{43}\ \mathrm{g}}{3.4 \times 10^{75}\ \mathrm{cm}^3} \\ &\sim 2 \times 10^{-32}\ \mathrm{gcm}^{-3} \end{aligned} \tag{14}$$

   And for $M \sim 10^{12} M_\odot$ for the dark matter we find:

$$\rho \sim 6 \times 10^{-31}\ \mathrm{gcm}^{-3}. \tag{15}$$

   Meanwhile, using the calculation in the previous prohlem that $H_0 \sim (4.3 \times 10^{17}\ \mathrm{s})^{-1}$:

$$\begin{aligned} \rho_c &\sim \frac{3}{8\pi\,(4.3 \times 10^{17}\ \mathrm{s})^2\,(6.67 \times 10^{-8}\ \mathrm{cm^3\ g^{-1}\ s^{-2}})} \\ &\sim 10^{-29}\ \mathrm{g\ cm^{-3}} \end{aligned} \tag{16}$$

So galaxies like the Milky Way are not enough to close the universe, by a fairly long shot (about a factor of 15–20). Something like this was pointed out as early as the early 1970s (Gott et al. (1974)). Of course this was not nearly enough evidence to conclude that the universe is open or Λ-dominated—the mass could have been hiding in very faint galaxies (which it turns out do have much higher total mass-to-light ratios) or in the outer regions of galaxies. This increases the crude estimate above by a factor of about 5, we now know, not enough for the universe's mass density to be critical.

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