# Distance Ladder

## 1. Basics & Nomenclature

Determination of distance is a fundamental problem in astronomy, to which is owed numerous breakthroughs. The distance to nearby galaxies has been established through a calibrated *distance ladder*, refined over the past century.

The distance ladder begins with the measurement of *parallax*, which is the apparent motion of sufficiently nearby stars due to the motion of the Earth around the Sun. Precise measurements of stars over time in a fixed reference frame yield *proper motions* and parallax-based distances. Most recently, the Gaia satellite has provided a large sample of stars with measured parallaxes reaching across our Galaxy.

To reach distances outside our galaxy, we must rely on less direct measurements of distance. Usually this involves finding objects or phenomena that have pairs of observables with a known relation but with different dependences on distance.

Many of these techniques fall into the *standard candle* category: a luminosity is estimated by some means, and a flux is measured, and the distance is inferred from the inverse-square law $f = L/4\pi D^2$. The most prominent standard candles are:

- *RR Lyrae* variable stars. These stars are relatively low mass horizontal branch stars. They vary due to the $\kappa$ mechanism, which is an instability associated with the reaction of the atmospheric opacity to changes in pressure and density that causes oscillations in stellar size and luminosity. They have periods of order a day. They are prominent in globular clusters but can also be found throughout the Milky Way. Historically they have been most useful for determining distances within the Galaxy. However, the advent of Gaia and the James Webb Space Telescope opens up the possibility of observing these stars in nearby galaxies.

- *Cepheid* variable stars. These stars are high mass and luminous. They also vary due to the $\kappa$ mechanism but have longer periods. Historically, they have provided one of the most direct links between local distance scales and distant galaxies.

- *Tip of the Red Giant Branch* stars. The red giant branch is commences when core hydrogen burning ends; at this point, shell hydrogen burning commences and the star becomes rapidly brighter and much larger. At a certain point the helium ash at the core ignites, moving the star to its next phase, the smaller and less luminous horizontal branch. The core ignites rapidly at a specific stage in evolution, so that at a given metallicity red giant stars all end this stage at the same luminosity. This leads to a rapid fall-off in the luminosity distribution of RGB stars and a clear tip of the red giant branch that can be used as a standard candle.

- *Type Ia supernovae.* These supernovae result from the nuclear detonation of white dwarfs,

probably due to accretion from or collision with a binary companion. The Phillips relation connects the time scale of the supernova light curve to the luminosity of the supernova. These sources were used to provide the first definitive evidence for the existence of cosmic acceleration.

Historically, a number of other distance measurements have also been used that could be categorized as standard candle distances. Some methods have relied on other supernova types. Other methods have generally capitalized on scaling relations of galaxies, such as the Tully-Fisher relation for spirals relating luminosity and circular velocity, and the Fundamental Plane for ellipticals relating luminosity, size, and velocity dispersion. The difficulty in implementing these methods in a way that is free of systematic errors generally has led to the use of supernovae for the most recent measurements of the distance ladder.

Several standard candle techniques prove useful, or may soon do so. I will discuss eclipsing binaries, masers, and gravitational waves:

- Some binary stars, like Algol, eclipse each other during their orbit. Their light curve alone yields a period $P$ and their radii relative to the orbit size, $r_1/a$ and $r_2/a$ for any given eccentricity and phase of eclipse. If both their spectra can also be measured, this can yield velocities, which yields enough information to determine their total mass $M$, eccentricity $e$, and the line-of-sight semimajor axis of the orbit $a \sin i$. The mass and semimajor axis are related by:
$$M = \frac{4\pi^2 a^3}{GP^2}. \tag{1}$$
For eclipsing binaries $i \approx 90°$. Thus, from gravitational and geometric considerations, you can determine the sizes of the stars. Spectra can yield constraints on their surface temperatures. The sizes and temperatures determines the luminosities, which combined with fluxes determines the distance.

- Under certain conditions the supermassive black holes at the centers of galaxies may contain $H_2O$ masers. Under even more special conditions, these systems can be sufficiently clean that one may measure velocities and accelerations over time of clouds in circular orbits, which yields a physical distance. Combined , this combination yields the mass of the black hole and the distance. Only one clean enough system is known, about 8 Mpc away in NGC 4258.

- Colliding compact objects whose gravitational waves are detected can yield constraints on distance from the gravitational waveform, because the total mass of the system is related to both the waveform frequency and the total energy. If the redshift is known from an electromagnetic counterpart, then the rest-frame waveform frequency can be determined, and the object can be placed on the redshift-distance relation.

A major application of these methods is to compare the redshifts and distances of local galaxies.

This comparison led Hubble to the Hubble Law:

$$v = H_0 d \tag{2}$$

where $v$ is velocity, $H_0$ is the Hubble constant, and $d$ is the distance. Gravitational attraction causes galaxies to move toward each other with respect to this flow, with *peculiar velocities* of 100s of km s$^{-1}$. For example, M31 is moving toward us at 400 km s$^{-1}$, and large clusters, such as Virgo and Coma, have internal motions and thus peculiar velocities of 1000s of km s$^{-1}$. Thus, determinations of the Hubble constant need to use galaxies sufficiently far to reduce this effect. Alternatively, they need to estimate and correct for these peculiar velocities, using the density field of galaxies itself to estimate the magnitudes and directions of the expected velocities.

The Hubble Law is a fundamental measurement of the nature of the

universe we are in, but it also is a tool to map the universe on larger scales, since measuring Doppler shifts is relatively easy compared to galactic distances.

## 2. Commentary

Briefly, the most precise current estimates of the local Hubble constant come from the Riess et al. (2016) group. These use a number of Milky Way Cepheids with parallax distances, along with eclipsing binaries and NGC 4258, to anchor extragalactic Cepheids in galaxies with SNe Ia, and use those to anchor a much larger SN Ia data set. As of 2018, the results for $H_0$ from these analyses tend to be considerably higher (several sigma) than those inferred from cosmic microwave background and large scale structure measurements, using standard cosmological models to extrapolate to $z = 0$. The cause of this discrepancy is not known.

The current ladder to a precise $H_0$ is strongly dependent on the Cepheid step. There are several important issues with the interpretation of the Cepheid distance scale, the most prominent of which is that the period-luminosity relation is metallicity dependent. The LMC has a much lower metallicity than the other galaxies in the analysis, and so if improperly modelled the metallicity dependence can cause a bias.

Beaton et al. (2016) present an alternative path using different indicators (RR Lyrae and TRGB) to calibrate the SN Ia, both of which tend to trace an older, more metal poor population even within metal rich galaxies. This path may soon become competitive. This second indepedent measurement will be critical in assessing the existing discrepancy.

The detection of much more distant masing systems suitable for an analysis similar to that of NGC 4258, or a sufficient number of gravitational wave sources, could provide a "one step" alternative to the distance ladder for the purpose of determining the cosmological constant.

## 3. Key References

- *A 2.4% Determination of the Local Value of the Hubble Constant*, Riess et al. (2016)
- *The Carnegie-Chicago Hubble Program. I. An Independent Approach to the Extragalactic Distance Scale Using Only Population II Distance Indicators*, Beaton et al. (2016)

## 4. Order-of-magnitude Exercises

1. Estimate age of the universe from the Hubble constant assuming constant expansion.

   Hubble's law states $v = H_0 d$, where $H_0 \approx 70$ km s$^{-1}$ Mpc$^{-1}$. If we assume constant expansion (no acceleration or deceleration), the age of universe will be:

$$T = \frac{d}{v} = \frac{1}{H_0} \approx \frac{\text{Mpc}}{70 \text{ km s}^{-1}} = \frac{10^6 * 3.086 * 10^{13} \text{ km}}{70 \text{ km}} \text{s} \approx 4.4 * 10^{17} \text{ s} \approx 13.9 \text{ Gyr}. \tag{3}$$

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2. What is the maximum parallax at 1 pc, 10 pc, 1 kpc, and 1 Mpc? Assuming you can get good positions to with a few hundredths of a FWHM, what size telescopes do you need so that you can measure parallax at these distances?

   The definition of the maximum parallax p is:

$$p = \frac{1 \text{ AU}}{d} = \frac{1''}{d/1 \text{ pc}}. \tag{4}$$

   Thus if $d$ equals 1 pc, 10 pc, 1 kpc, 1 Mpc, the parallax will be $1''$, $0.1''$, $10^{-3}\,''$, $10^{-6}\,''$, respectively.

   If we can get good positions to $\sim 10^{-2}$ FWHM, suppose the telescope has size $D$ and measures light at a wavelength $\lambda$, we find we can measure parallaxes of:

$$p \sim \frac{\lambda}{D} \times 10^{-2} \tag{5}$$

   Recalling that $1'' = \pi/180/3600$ rad $\approx 4.84 * 10^{-6}$ rad, then:

$$D \sim 10^{-2} \times \frac{\lambda}{p} \approx \frac{\lambda(d/1 \text{ pc})}{484 \times 10^{-6}} \approx \left(\frac{\lambda}{484 \text{ nm}}\right)\left(\frac{d}{1 \text{ pc}}\right) 10^{-3} \text{ m} \tag{6}$$

   For instance, if the telescope is optical telescope, $\lambda \sim 500$ nm, the size $D$ of the telescope would need to be $\sim 1$ km in order to have the resolution to measure parallax.

   This diameter requirement only accounts for the diffraction limit and not other considerations. For example, although the diffraction limit of a $\sim$ mm-sized telescope is in principle sufficient

to measure parallax at 1 pc, obviously that is impractical because you cannot gather enough photons in a reasonable time to perform the measurement! Conversely, although a baseline $D \sim 1$ km is necessary based on the diffraction limit to measure parallax at 1 Mpc, you may not need a km-diameter worth of light-gathering power for that measurement. E.g. using an interferometer fed by two large but possible-to-build optical telescopes, separated by a km, would be enough (though also challenging and expensive to do!!).

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3. If the typical peculiar velocity of a galaxy is $\sim 300$ km s$^{-1}$, at what redshift does its contribution to the distance error drop below 5%?

4. Based on their luminosities, what are the relative distances to which you should be able to determine distances to RR Lyrae, TRGB stars, Cepheids, and SN Type Ia?

   Let us assume there is a minimum flux required for the object to be detected and measured at some given precision. For this fixed minimum flux, the distance to some object is proportional to $L^{1/2}$, where $L$ is the luminosity of the object. Roughly, the typical luminosity of RR Lyrae stars is $10^2 L_{\rm sun}$, of TRGB stars is $10^4 L_{\rm sun}$, of Cepheids is $10^2$–$10^4 L_{\rm sun}$, and of SN Type Ia is $10^{10} L_{\rm sun}$. Therefore the distances (relative to RR Lyrae) to which we should be able to determine distance with these standard candles is 1, 10, 1–10, and $10^4$ for RR Lyrae, TRGB stars, Cepheids, and SN Type Ia, respectively. This explains why RR Lyrae are the most difficult to use for extragalactic distances, why SN Type Ia are necessary in order to reach distances which can measure the Hubble flow well, and why the Cepheid and TRGB methods are in between.

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## 5. Numerics and Data Exercises

1. Allen Sandage used to use brightest galaxies in clusters as a standard candle. Download redmapper, a modern catalog of galaxy clusters from SDSS. Select the brightest cluster galaxies for massive clusters. Based on looking at these galaxies within a narrow range of redshift, are they reasonable standard candles? Show a Hubble diagram assuming that they are and compare to a standard Planck cosmology.

2. Clementini et al. (2018) publish a catalog of RR Lyrae variables from Gaia DR2. Muraveva et al. (2018) publish a calibration of the RR Lyrae period-luminosity-metallicity (PLZ) relation. Use the RR Lyrae stars in the catalog and the published PLZ relation to estimate the ratio of the distance to the Sculptor dwarf galaxy to the distance to the Messier 62 globular cluuster.

## REFERENCES

Beaton, R. L., Freedman, W. L., Madore, B. F., et al. 2016, ApJ, 832, 210

Clementini, G., Ripepi, V., Molinaro, R., et al. 2018, ArXiv e-prints, arXiv:1805.02079

Muraveva, T., Delgado, H. E., Clementini, G., Sarro, L. M., & Garofalo, A. 2018, MNRAS, 481, 1195

Riess, A. G., Macri, L. M., Hoffmann, S. L., et al. 2016, ApJ, 826, 56

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