# Stellar Evolution

## 1. Basics & Nomenclature

Stars are formed from interstellar gas, through gravitational collapse within molecular clouds. A spectrum of objects are formed from below the hydrogen burning limit of 0.08 $M_\odot$ up to 100–200 $M_\odot$. Stars spend most of their lifetime on a *main sequence*, burning hydrogen in their cores. Depending on their mass they then proceed through a sequence of post-main sequence phases and leave remnants in the form of white dwarfs, neutron stars, or black holes (or in some cases no permanent remnant). In any system of stars, their history is encoded in their distribution of luminosities and colors, which can be measured directly in resolved stellar populations or inferred from spectra and/or broad band imaging. Stellar evolution processes are responsible for most of the elements higher mass than helium.

The *stellar initial mass function*, or *IMF*, defines the spectrum of initial masses of stars. This spectrum is difficult to determine observationally, because almost all systems we observe have been altered dynamically or by stellar evolution. For many decades, the standard was the *Salpeter IMF*:

$$\Phi(M) \propto M^{-2.35}. \tag{1}$$

This form leads to a large fraction of mass between 0.08 and 0.5 $M_\odot$. However, evidence from local systems implies that in many cases, the true IMF has a flatter slope at lower masses. This difference is significant because low mass stars emit very little luminosity, so their presence is very difficult to directly detect, and therefore inferences of total mass in stars depend strongly on the assumptions of how many low mass stars are in the system.

The first and usually longest phase of stellar evolution is core hydrogen burning on the main sequence. The cores are typically millions to billions of Kelvin and fully ionized. Hydrogen is burned to helium through two major processes, the *p-p chain* at low masses and the *CNO cycle* at high masses ($M > 2M_\odot$). Each process yields one $^4He$ nucleus with a mass of $3.96m_p$ from 4 protons; the one percent difference yields the energy for stellar luminosity. Numerous other nuclear processes are occurring simultaneously that contribute to the luminosity (and to the neutrino output). The nuclear processes depend on the tail of nuclei in the Maxwell-Boltzman distribution that are sufficiently high energy to tunnel through the Coulomb repulsion of the nuclei. They are therefore strongly temperature sensitive.

Stellar structure is controlled by the following equations:

- Mass conservation:

$$\frac{\mathrm{d}M}{\mathrm{d}r} = 4\pi r^2 \rho \tag{2}$$

- Energy conservation:

$$\frac{\mathrm{d}L}{\mathrm{d}r} = 4\pi r^2 \rho \epsilon \tag{3}$$

- Hydrostatics:

$$\frac{\mathrm{d}P}{\mathrm{d}r} = -\frac{GM(r)\rho}{r^2} \tag{4}$$

- Energy transport, which comes in the form of radiative transfer:

$$\frac{\mathrm{d}T}{\mathrm{d}r} = \frac{L(r)\kappa(r)\rho(r)}{16\pi r^2 c a T^3} \tag{5}$$

  or in the form of convection.

$\kappa(r)$ is a critical parameter, as it strongly affects the structure of the star, and therefore its size. The higher the metallicity, the higher the opacity, the larger the star, and therefore the lower the surface temperature at a given luminosity.

In the exercises, we will show that these equations imply a scaling of luminosity with mass of approximately $L \propto M^4$ and with surface temperature of $L \propto T_s^8$ on the main sequence. The former relationship implies that the stellar lifetimes on the main sequence scale as $M^{-3}$.

After the main sequence, stellar evolution depends on the mass of the star. At high luminosities (canonically above 8 $M_\odot$), stars have short main sequence lifetimes (10s of Myrs) and thereafter undergo a series of nuclear burning phases in their cores: He, C, Si, and so on. Each burning phase is shorter than the last. Shell burning is occurring at the same time. Once Fe and Ni form in the core, energy cannot be further released, and the core collapses. The result in many and potentially all cases is a core-collapse supernova. The Fe and Ni produced in the core is disintegrated in this process and most of that mass becomes part of the neutron star or black hole that forms at the center. However, the elements remaining from burning in the regions outside the core, which are rich in $\alpha$ elements like O, Mg, and so on, can be returned to the interstellar medium.

At lower masses, stars instead start burning hydrogen in a shell around the inert helium core. This shell burning yields tremendous luminosity and also induces the outer layers of the stars to expand up to AU or greater size. The result is a red giant. **get helium flash right here and further phases**.

The net result of stellar evolution is that at early times the stellar population is mostly on the main sequence and it is dominated by the bluest stars. At late times it is dominated by the red giants, which have recently (within $\sim$ a Gyr) left the main sequence.

Stars are normally classified according to their MK system: OBAFGKM, which is in order of decreasing temperature. Subclassifications exist (O1, O2, ..., O9, B1, ...). Hotter stars are referred to as "early type" and cooler stars are referred to as "late type." These classifications are according to their spectra and the ordering was originally based on the spectral phenomenology, which is why the current nomenclature appears somewhat random.

Broadly speaking O and B stars have few lines, with He II lines in O stars, He I lines in B stars, and weak Balmer lines in both. A stars have strong Balmer lines. Balmer lines become weaker again for later type stars. In F and G stars, lines of other neutral atoms become important. Particularly in G stars the Ca II H and K lines (right below 4000 Å) appear. In G, K, and particularly M stars, molecular lines become important as molecules like CH, CN, and TiO become able to survive in the cooler atmospheres.

In galactic systems, the consequence of stellar evolution is that the stellar continuum of young systems is dominated by hot stars, yielding a blue spectrum with few clues to metallicity, whereas the stellar continuum of old systems is dominated by cooler, old stars, with red spectra that depend on metallicity (due to variations in the abundances and their effects on the stellar opacity and therefore temperature).

## 2. Commentary

## 3. Key References

- *Blanton & Moustakas*
- *Graham paper on Sersic*

Gunn et al. (2006)

## 4. Important numbers

- $M_\odot = 1.989 \times 10^{30}$ kg
- $R_\odot = 6.955 \times 10^8$ m
- $T_\odot$(surface) = 5500 K
- $T_\odot$(core) = 1.5 $times 10^7$ K
- $L_\odot = 3.828 \times 10^{33}$ erg s$^{-1}$

## 5. Order-of-magnitude Exercises

1. Argue why higher mass stars produce more of their energy through the CNO cycle than lower mass stars do.

2. Detailed stellar evolution calculations predict a main sequence lifetime for the Sun of 10 billion years. What fraction of the total hydrogen in the Sun needs to be converted to helium to provide this lifetime?

## 6. Analytic Exercises

1. Under Salpeter, calculate luminosity as a function of time

## 7. Numerics and Data Exercises

1. HR diagram for an open cluster
2. HR diagram for a globular cluster
3. HR diagram locally
4. Compare K spectrum to E spectrum
5. Compare O/B spectrum to blue galaxy

## REFERENCES

Gunn, J. E., et al. 2006, AJ, 131, 2332

This preprint was prepared with the AAS LaTeX macros v5.2.