Gravitational Lensing

1. Basics & Nomenclature

Under general relativity, in the presence of mass light is bent by the curvature of spacetime. On astronomical scales this can cause the phenomenon of *gravitational lensing*.

Understanding lensing begins with the point mass case. It can be shown that a photon traveling by a point mass, with an impact parameter r, is in the small deflection limit deflected by an angle:

$$\theta_D = \frac{4GM}{rc^2} \tag{1}$$

This differs by a factor of two from the equivalent Newtonian calculation. An important feature of lensing is that it is achromatic; i.e., independent of wavelength.

Figure **??** describes the symmetric point lens case and defines the distances involved. In an analog to the optical thin lens approximation, we define the *source plane* and the *lens plane*. In the perfectly aligned case the observer sees the source as a ring surrounding the lens; perfect alignment means an offset substantially than the source size. A characteristic quantity of a lens is radius of this ring, which is the *Einstein angle*:

$$\theta_E = \sqrt{\frac{4GM}{c^2}}\sqrt{\frac{D_{LS}}{D_L D_S}} \tag{2}$$

which can be related to the *Einstein radius* in the lens plane $r_E = D_L \theta_E$.

Figure **??** describes the offset point lens case. If the source is a point source, this will result in two magnified (and one highly demagnified) images for the observer. The condition on the source angle:

$$\beta < \theta_E \tag{3}$$

defines the *strong lensing* regime. In this regime, the two images appear near the Einstein ring location. If the source is extended instead of point-like, it can appear highly distorted in the strong lensing case.

The opposite case is known as the *weak lensing* regime.

In either case, the distortion of lensing has an effect on the apparent brightness of the object. The total magnification can be defined as the increase in the solid angle of the image. This solid angle increase occurs even if our instrumentation still cannot detect the extended nature of the image. Because surface brightness (more technically specific intensity) is conserved in general relativity this magnification leads to an increase in total flux density.

2. Important numbers

3. Key References

- *Binney & Tremaine* Cox (2000), Chapter 5

4. Order-of-magnitude Exercises

1. Typical Einstein angles for stars, galaxies, clusters.
2. Typical shear values
3. Estimate probability of microlensing in galaxy

5. Analytic Exercises

1. GR calculation of lensing offset
2. Show Einstein angle
3. Calculate offset from Einstein angle for strong lensing
4. Calculate offset from source angle for weak lensing
5. Calculate magnification

6. Numerics and Data Exercises

1. Specific strong lenses

REFERENCES

Cox, A. N. 2000, Allen's astrophysical quantities

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