Images

1. Basics & Nomenclature

Two-dimensional array-based detectors at the focal planes of telescopes produce images — two-dimension rasters of values. Here we consider the basic properties such images with an emphasis on the sorts of issues affecting ultraviolet, optical, and infrared imaging instruments. Many of the principles hold for other sorts of data sets.

These images can be thought of as noisy samplings of an *image function* which is the convolution of the image coming from space with a response function known as the *point spread function* (PSF). The PSF can have contributions from the Earth's atmosphere, the telescope's physical and geometric optics, and the detector.

The width of the PSF is usually characterized by its full-width half-maximum (FWHM). An important property of an imaging system is its sampling density relative to the FWHM. If this density is high enough, typically greater than 2 pixels per FWHM, the PSF will be close to *Nyquist sampled*, meaning that if the image were noiseless it would preserve all of the information in the image function. The exercises describe the origin of this criterion, which is related to the true Nyquist sampling criterion of a particular band-limited function (one with constant Fourier amplitudes out to some wavenumber). Typically instruments do not truly Nyquist sample the image function, but in certain circumstances it can be possible.

Images taken outside the Galactic Plane which are not extraordinarily deep usually allow one to separate the flux into individual objects, which are generally classifiable as *point sources* or *extended sources*. The point sources are those whose images are consistent with the PSF. With ordinary imaging capability, stars are point sources. Galaxies are often extended sources, with FWHM larger than the PSF FWHM. High redshift, luminous quasars usually appear as point sources, though with high contrast imaging from space their extended host galaxies can be detected.

The image locations can be quantified by their x and y pixel values. The center of each object is typically determined by the mode of the light distribution, or sometimes by a weighted centroid of the flux distribution (which is less precise but for extended sources may be more desirable at times). The relationship between these x and y pixel values and the RA and Dec locations they correspond to is called the *astrometric solution*. Usually the astrometric solution is expressed in terms of a World Coordinate System (Greisen & Calabretta 2002) description of the image, which provides the metadata encoding an approximate projection of the image and distortions in that projection.

The raw image is usually obtained in some uncalibrated form. In terms of the specific intensity of the image $I_\nu(\alpha, \delta)$ coming from outside the Earth's atmosphere convolved with the instrumental resolution, the raw image in $\mathrm{DN}(x, y)$ in pixel coordinates may be written:

$$\mathrm{DN}(x, y, t) = \mathrm{DN}_{\mathrm{back}}(x, y, t) + \int \frac{\mathrm{d}\nu}{\nu} Q(\nu, \mathrm{alt}, \mathrm{az}, x, y, t) \left[I_\nu(\nu, \alpha, \delta) + S_\nu(\nu, \mathrm{alt}, \mathrm{az}, t)\right] \quad (1)$$

Q is a general response function written to depend on the pixels detecting the objects, the local sky coordinates (though it might have a more general form in some cases), frequency, and time. S_ν is background, generally due to sky emission, and written in terms of altitude and azimuth, to suggest the important axes, but also in terms of time (e.g. where the Moon is). $\mathrm{DN}_{\mathrm{back}}$ is the instrumental background.

Often the response Q is at least roughly separable as follows

$$Q(\nu) = R(\nu) A(\mathrm{alt}, \mathrm{az}, \nu) F(x, y) \quad (2)$$

where $R(\nu)$ is the instrumental bandpass, A is the atmospheric throughput, and F is the flat-field. R is often defined to incorporate the dependence of the atmosphere on wavelength at some nominal altitude, so that A would express only the differences from that. F is often defined to include the determinant of the Jacobian between (α, δ) and (x, y) which would otherwise need to appear and which primarily depends on the instrument. F is usually defined to have a mean around unity.

A final common approximation is that the functions are sufficiently separable or the dependence on angle is sufficient small over the field of view of the image that we can write:

$$\mathrm{DN}(x, y, t) = \mathrm{DN}_{\mathrm{back}}(x, y, t) + \bar{A} F(x, y) \left[\mu(\alpha, \delta) + \mu_S(\mathrm{alt}, \mathrm{az}, t)\right] \quad (3)$$

Here, $\bar{A}$ is the throughput of the instrument and atmosphere averaged over wavelength. We separate $\mathrm{DN}_{\mathrm{back}}$ and μ_S because they are from different physical sources. For example, in a CCD, $\mathrm{DN}_{\mathrm{back}}$ is due to dark current and and bias in the device, whereas μ_S is emission from the sky itself.

The process of *photometric calibration* is the conversion of the observed DN to μ. In its most basic form, calibration requires subtracting the instrumental background, scaling the remaining flux by the overall factor $1/\bar{A}$, dividing by the flat field, and then subtracting the sky. Typically, the instrumental background is determined through bias frames taken with the shutter shut. The flat field is determined by observing either the sky in twilight or by observing a screen (sometimes just the dome wall) illuminated such that it mimics light uniformly entering the telescope aperture.

It is somewhat less standard how μ_S and $\bar{A}$ are determined. In most cases, determining either one requires some detection and measurement of objects in the image to occur, and their determination is usually somewhat iterative.

For μ_s, usually one takes an initial stab at the background level (e.g. just a median), detecting the bright objects, and then redetermining the background and its variation across the image when excluding "detected" pixels; one can then iterate this procedure. Typically this procedure subtracts not just the atmospheric sky emission, but also other sources such as zodiacal light, and also some fraction of the light from detected stars and galaxies. Whether these other sources should or should not be subtracted depends on what is wanted out of the image.

For $\bar{A}$, the concept is to use detected objects in the field whose μ is known from a catalog of standards to calibrate the entire image. Sometimes the objects used for calibration are not in the same field but are objects observed close in time to the field of interest with the same instrument; in such cases the difference in airmass of the field and the standard field needs to be accounted for. Often in these cases the standards are not known in exactly the same filters as the observations, and these color-dependent effects must be taken into account. Sometimes there are no standards as such, but instead a suite of overlapping observations can be calibrated onto a self-consistent scale (e.g., Finkbeiner et al. 2015).

Once an image is calibrated, the stars, galaxies, or other objects may be measured for their fluxes and other properties. The fluxes may be measured through fixed apertures, or one may fit models to the data and infer fluxes from those models.

2. Commentary

The description above of calibration procedures is roughly accurate, but is more illustrative than anything. There are many different techniques in use, driven by the nature of the instrument and observations as well as individual tastes. In part because each individual experiment is so different, astronomers (including myself) tend to learn about this subject in the context of a particular set of observations rather than from a generalized perspective. Keep that in mind when people (including me!) are telling you how this process works!

At the highest precision, the calibration effects described above are not truly separable. For example, the astrometric solution depends on the definition of the PSF, because that determines what exactly you consider the location of each object, and the flat-fielding, which can also affect the centroiding.

A more subtle example is that the flux definitions cannot truly be separated from the calibration. The calibration must assume some effective aperture for the fluxes of the calibrating sources, but some flux will leak outside the aperture, in ways that can vary across the images used in the calibration. These aperture effects need to be accounted for percent level calibration accuracy.

3. Key References

- *Design and Construction of Large Telescopes*, Bely (2003)
- *Astrophysical Techniques*, Kitchin (2009)

4. Analytic Exercises

1. *Origin of the Nyquist criterion.* The Nyquist sampling criterion is based on the concept of a "band limit."

   (a) Show that if the 2D Fourier transform $f(\vec{k})$ of the point spread function has zero power higher than the band limit $k_{\rm max}$, then it can be perfectly described using a discrete Fourier transform that extends only up to that $k_{\rm max}$. What is the necessary configuration-space sampling for the discrete Fourier transform?

   (b) Assume $f(\vec{k}) =$ constant below the band limit, and zero above it. What is the resulting PSF $f(\vec{x})$?

   (c) Under what circumstances might the criterion in part (b) hold, given what you know about how telescopes work?

   (d) What is the FWHM of $f(\vec{x})$ in units of the sampling? This sets the Nyquist criterion for sampling.

2. *Interpolation.* Imagine starting with an image sampled on a rectangular grid. Interpolation is the process of inferring the value of an image in between the given sampled points of the image. The case of Nyquist sampled, band limited images motivates a particular method of interpolation.

   (a) Explain how linear interpolation can be recast as constructing a model of the image using a set of basis functions, or "kernels," centered on the original grid points.

   (b) Explain why a (noiseless) Nyquist sampled, band limited image contains all the information necessary for perfect interpolation.

   (c) If you interpolate perfectly in that situation, what is the effective kernel you are using? Note that this is called *sinc interpolation.*

   (d) Explain why using exactly that interpolation kernel might be problematic.

3. Following the methods of Section 2 of Vakili & Hogg (2016), show how the best possible centroiding accuracy depends on the FWHM and on the total signal-to-noise ratio (S/N) of a Gaussian point source.

5. Numerics and Data Exercises

1. *Interpolation.* Create an image of a critically sampled Gaussian PSF centered at the middle of a pixel. Create another image of the same PSF, but with its center offset some fraction of a pixel from the first case. We will test how well different types of interpolation work by trying to shift the first image using interpolation and comparing it to the second image.

(a) Use linear interpolation to try to shift the first image so the PSF has the same center as in the second image. Compare the absolute and fractional differences (pixel-by-pixel) between first image shifted and the second image.

(b) Perform the same test with sinc interpolation.

(c) Perform the same test with a “damped” version of sinc interpolation, which multiplies the kernel by a broad Gaussian (say, a few FWHM broad).

2. CCD images from SDSS are available as corrected frames. Those images have valid WCS headers associated with them. Using the tools from `astropy`, take RA and Dec values from objects in a randomly chosen field (get these from the `photoObj` table in CAS) and overplot their locations on the CCD r-band image.

3. This problem tests the measurement of image centroids.

   (a) Write a piece of code to generate a fake, critically-sampled image of a double Gaussian, with a center that isn't necessarily at the center of a pixel. For the second Gaussian, use $A_2 = 0.1A_1$ and $\sigma_2 = 2.47\sigma_1$, where A indicates the value at the center of the Gaussian. This choice is an approximate description of the atmospheric PSF (Jim Gunn, private communication).

   (b) Write a routine to find the light-weighted centroid of the image. Start by using the maximum pixel value, and use your knowledge of the PSF FWHM to calculate the center based on the light within 3 FWHM, and iterate to convergence.

   (c) Write a routine to find the mode of the image. Start by using the maximum pixel value, but then use the 3×3 grid of pixels in the center to perform a quadratic interpolation to find the peak.

   (d) Now add noise to the images, and use a Monte Carlo test to evaluate how the precision of each estimate depends on the total S/N within 3 FWHM.

4. It is common to use the measured signal from a Poisson process to estimate the noise. Use a Monte Carlo technique to estimate the bias that this causes as a function of the expectation value $\bar{N}$ for the Poisson process.

REFERENCES

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