# Telescopes

## 1. Basics & Nomenclature

Telescopes are the fundamental tool of astronomers, with a handful of exceptions ($\gamma$-ray detectors, cosmic ray detectors, neutrino detectors, and gravitational wave detectors).

Telescope optics are design to do two things: to collect a lot of light; arrange the collection process to separate the light according to its incoming direction. Detectability of objects depends on how many photons, or equivalently how much energy, is collected. This number is the product of $f_\nu A t \Delta\nu$, where $A$ is the telescope area, $t$ is the length of exposure, and $\Delta\nu$ is the width of detector bandpass.

The two basic types of telescope are *refracting* and *reflecting* telescopes. Refractors transmit the light through transparent media with refractive indices different than air, bending it through Snell's Law. Reflectors are mirrored surfaces, in the optical typically aluminized, whose shapes manipulate beams of light in the desired fashion. Reflecting telescopes virtually always have some refracting elements inside of them.

For astronomical telescopes, they always focus at infinity, meaning they are designed such that parallel rays of light are focused to a single point in the focus of the telescope. Typically, there is some axis of symmetry to the telescope and rays parallel to this axis are *on-axis*, and it is typically here that the telescope is designed to have best focus. *Off-axis* rays may be well focused as well, but as the angle off-axis becomes larger the rays do not all converge. Off-axis rays can be obstructed by either parts of the optics or part of the telescope structure, and generally are more obstructed than on-axis rays; this effect is known as *vignetting*. The usable *field of view* of the telescope is typically determined by the off-axis angle at which the beam is too vignetted or too badly focused to be usable for the desired purpose.

Below is a picture of a lens diagram for a refracting telescope. The lens has a focal length, which is what determines how far beyond the lens a set of parallel, on-axis rays will come to focus. Physically, the lens is bending the light according to Snell's law, and the shape of the front and back surface of the lens as a function of its radius is what conspires to direct all the on-axis light to the focus.

A key parameter in the optical design is the *focal ratio*, or *$f$-ratio*, the ratio of the focal distance $f$ to the aperture diameter $D$. This is usually expressed in the form "$f/N$" (which is not a division), where $N = f/D$. This ratio determines the angular size of the beam that converges on the focal point; in more complex optical configurations, it is this beam width that defines the $f$-ratio. The smaller the $f$-ratio the more the light is being bent; this means that off-axis rays will usually fall out of focus more quickly with off-axis angle without very careful design.

Slightly off-axis light focuses to a slightly offset spot in the focal plane. The relationship between the off-axis angle and physical radius in the focal plane from the on-axis focus (sometimes known as the *plate scale*) is determined by the $f$-ratio. Specifically:

$$\mathrm{d}x \approx f \mathrm{d}\theta \tag{1}$$

so:

$$\frac{\mathrm{d}x}{\mathrm{d}\theta} \approx f = \left(\frac{f}{D}\right) D \left(\frac{10^6\ \mu\mathrm{m}}{\mathrm{m}}\right) v \left(\frac{\mathrm{rad}}{(180/\pi)(3600)\ \mathrm{arcsec}}\right) = \left(\frac{f}{D}\right) \frac{D}{\mathrm{m}} \left(4.85 \frac{\mu\mathrm{m}}{\mathrm{arcsec}}\right) \tag{2}$$

or inversely as

$$\frac{\mathrm{d}\theta}{\mathrm{d}x} \approx \left(\frac{f}{D}\right)^{-1} \frac{\mathrm{m}}{D} \left(0.206 \frac{\mathrm{arcsec}}{\mu\mathrm{m}}\right) \tag{3}$$

This equation holds given the final $f$-ratio of any of the optical systems we describe here. It means the smaller the $f$-ratio, or the smaller the telescope size, the more angular coverage per unit area there is.

There are three major limitations of refracting telescopes:

- The index of refraction of light in lens materials varies with wavelength. This means that the focal length depends on wavelength as well, and only one wavelength can be in perfect focus. This effect is known as *chromatic aberration* and is a limitation of refracting telescopes (and affects reflecting telescopes with refracting optics inside them). It is somewhat mitigated with large $f$-ratios, which minimize the angular deflections.
- Mitigating chromatic aberration and maintaining focus off-axis both drive the telescope design to reasonably high $f$-ratios. With a refractor, this inevitably lengthens the telescope structure in ways that can be avoided with reflecting telescope. A very long telescope necessitates a large dome, a more difficult engineering problem to manipulate the telescope, and more difficulty in minimizing flexure (bending) in the system.
- The practical size limit for refracting optical elements in the optical is about 1 meter. It is extremely hard to create precise optical elements larger than that size at finite cost.

These considerations drove astronomers in the late 1800s and early 1900s to move to reflecting telescopes for large telescope applications.

The principles of reflecting telescopes to first order can be understood through the mathematics of conic sections: parabolae, ellipses, and hyperbolae. Reflecting surfaces in telescopes are close approximations to axisymmetric surfaces of revolution whose radial shape is defined by these conic sections. Conic section can be defined in terms of their two *foci*. One of their mathematical properties is that at any point on the curve, the normal to the curve bisects the directions to the foci. This mathematical property has the consequence that a ray of light emitted from the direction of one focus will be reflected from the surface along the direction defined by the other focus.

Astronomical observations are almost invariably of extremely distant objects, the light rays from which are parallel to each other. This corresponds to a *focus at infinity*. A conic section with one focus at infinity and one at finite distance is a parabola. Therefore, if a parabolic mirror is aligned with these rays, they will be reflected towards the other focus of the parabola. A simple design for a telescope therefore is to put a detector at the focus of a parabolic mirror, referred to as the *primary.* A slight variant known as the Newtonian design is to direct this *prime focus* with a flat pickoff mirror.

More complex designs are possible with multiple curved surfaces. The general principle behind these designs are to use the fact that conic sections like ellipsoids and hyperboloids have two foci. A *secondary mirror* is designed to have one focus coincide with the prime focus; the reflective surface then will refocus the light to its other focus. A third (*tertiary*) mirror is used in some designs (for example in the Large Synoptic Survey Telescope).

There are multiple purposes of the extra surfaces. They allow the focus to be redirected to a mechanically more convenient spot than the primary. They allow freedom to manipulate the $f$-ratio of the beam. For wide-angle observations, the design of the primary, secondary, and other mirror shapes can be slightly perturbed from the conic section ideals to allow off-axis light to remain in better focus (at the cost of focus on-axis). In operation, that they are smaller than the primary makes them more easily adjustable in real time to perform focus changes and tip/tilt corrections for image motion.

Many reflecting telescope systems have *correctors*, which are refractive elements near their focus used to reduce distortions across the field of view.

The diffraction-limited point spread function of a telescope is related to its aperture. The "pupil" of the system is the image of the aperture including any obstructions in the optical path, for a plane wave arriving at the telescope aperture. In detail, optical imperfections will also lead to phase differences as a function of position in the pupil. Thus the "complex pupil function" $Ae^{i\phi}$ gives the throughput $A$ and phase shift $\phi$ of each point in the aperture. Huygen's Principle of diffraction means the plane wave is focused onto a spot with an intensity function that is the real component of the Fourier transform of the complex pupil function.

For the simplest aperture, a circular one with no phase shifts, the width of the diffraction-limited point spread function is related to the aperture diameter $D$. Specifically:

$$I(x) \propto \left(\frac{J_1(x)}{x}\right)^2, \tag{4}$$

where $J_1$ is the first-order Bessel function and $x = \pi D\theta/\lambda$. Here $\theta$ is the equivalent angle from the center of the spot.

Most of these general considerations hold for telescopes outside the optical. However, the designs of these telescopes differ in detail. From the UV through the mid-infrared they are usually quite similar to the optical. High energy photons do not reflect easily off aluminized surfaces, or near the normal to the surface. X-ray observations therefore use gold-coated surfaces under "grazing incidence," with nested annular hyperboloid surfaces. Radio telescopes simply require conductive surfaces and much lower tolerances due to their longer wavelength, meaning they can be built at fixed construction cost to much larger aperture than optical telescopes. To avoid interference, radio telescopes also typically use asymmetric sections of a paraboloid.

## 2. Commentary

Astrophysics and astronomy courses typically focus on the optical aspects of telescopes and the nature of their detectors. The engineering aspects of telescopes, both in hardware and in software, are not well covered in the astrophysics or in the engineering disciplines.

## 3. Key References

- *Design and Construction of Large Telescopes*, Bely (2003)

## 4. Order-of-magnitude Exercises

1. By considering two on-axis light rays each hitting each side of a telescope of diameter $D$ and interfering near the focal plane, estimate the diffraction limit in terms of the wavelength of light $\lambda$ and diameter $D$.

   By Huygens' Principle, the two rays can be treated as each producing a spherically outgoing wave from their location on the aperture. If the two rays are in phase when they hit the perfectly constructed mirror, they will constructively interfere exactly at focus. Near but slightly off focus in the focal plane, they will not exactly interfere because the distances from the mirror for each ray will be slightly different, so their phases at this location will not be the same. The size of the central image can be characterized by the position of the first null — i.e. when the phase difference of the two rays becomes $\pi$, so that they destructively interfere.

   The path length from either aperture location to perfect focus is:

$$d = \sqrt{f^2 + \left(\frac{D}{2}\right)^2}, \tag{5}$$

   where $f$ is the focal length. For some $\delta$ in distance away from perfect focus, the path length for one ray will be:

$$\begin{aligned} d_L &\approx \sqrt{f^2 + \left(\frac{D}{2} + \delta\right)^2} \\ &\approx \sqrt{d^2 + D\delta} \\ &\approx d\left(1 + \frac{D\delta}{2d}\right), \end{aligned} \tag{6}$$

and the other ray will have a slightly shorter path by the same amount:

$$d_R \approx d\left(1 - \frac{D\delta}{2d}\right). \tag{7}$$

The difference in angle that $\delta$ corresponds to relative to incoming rays on the sky will be $\alpha \approx \delta/d$. That is, the difference in focal plane distance $\delta$ corresponds to a difference in $\alpha$ on sky for perfect geometric options. So we can write the difference in path length in units relative to sky:

$$\Delta d = d_L - d_R = D\alpha. \tag{8}$$

These two waves will destructively interfere when $\Delta d = \lambda/2$. This will be when:

$$\alpha = \frac{\lambda}{2D} \tag{9}$$

This of course is smaller than the first null for an Airy function, because the Airy function involves the interference of *all* of the waves coming from the surface of the mirror, many of which have much smaller separations. But this argument shows how $\lambda/D$ enters the problem.

2. The LSST telescope in Cerro Pachon has a diameter of $\sim 8$ m and is $f/1.4$ at its focus. To sample with at least two pixels per full-width half maximum (e.g. the Nyquist critical sampling) for the best atmospheric seeing in Chile it might encounter (about 0.5 arcsec FWHM), what must the detector pixel size be?

3. How accurate does the surface of a telescope mirror have to be in: the $X$-rays? the optical? the radio?

## 5. Analytic Exercises

1. The Cassegraine design and Gregorian design both involve a curved secondary redirecting the focus along the original primary focal axis. In the Cassegraine, the secondary is before the prime focus. In the Gregorian it is after. What are the shapes of each of these secondary choices: ellipsoid, paraboloid, or hyperboloid? Draw the mirrors and the paths for on-axis light in the two cases.

2. In the two above cases, there usually needs to be a hole in the primary mirror (and the secondary would obstruct that area anyway). What is the analytic form of the diffraction limited point spread function for an aperture like this?

## 6. Numerics and Data Exercises

1. Calculate the ideal point spread function for a Cassegraine-type design with four struts to hold the secondary creating an extra obstruction. Compare to an actual color image from

the Hubble Space Telescope and comment on where the diffraction-related features in that image come from.

2. Add random small scale phases shifts to your ideal aperture. What is the effect of this? How large do these shifts need to be before they strongly affect the point spread function?

## REFERENCES

Bely, P. Y. 2003, The Design and Construction of Large Optical Telescopes