Chemical Evolution

1. Basics

Chemical evolution models attempt to track the metallicity (or in the more general case, abundance) enrichment over time. They rely on nucleosynthetic yield predictions and in some cases inflow, outflow, and mixing scenarios. In the most sophisticated cases, they are incorporated into full hydrodynamic models for galaxy formation. However, the simplest forms of chemical evolution models can also be edifying.

1.1. One-zone model

The simplest version of chemical evolution is a *one-zone* model of metal enrichment. Start with a mass M_g in gas and form stars at a rate $\dot{M}_*$. Define m_Z as the *yield*; the mass in metals created for each mass of star formation. Define the *recycling rate* r where $r \sim 0.5$, the fraction of gas that is returned to the ISM. Define M_* as the actual mass of stars (accounting for the fact that some of the mass gets recycled). Define M_Z as the mass in metals in the gas. Now you assume instantaneous and well-mixed recycling inside the one zone in the model, and that no gas or stars enter or leave your zone (this last approximation is what is meant by *closed box*).

Then the changes of these variables over some time $\mathrm{d}t$ are:

$$\begin{aligned}
\mathrm{d}M_* &= (1-r)\dot{M}_*\mathrm{d}t \\
\mathrm{d}M_g &= -\mathrm{d}M_* = (1-r)\dot{M}_*\mathrm{d}t \\
\mathrm{d}M_Z &= m_Z\dot{M}_*\mathrm{d}t - Z\mathrm{d}M_* \\
&= \left(\frac{m_Z}{1-r} - Z\right)\mathrm{d}M_*
\end{aligned} \tag{1}$$

We can show from these equations that

$$Z(t) = -\frac{m_Z}{1-r}\ln f_{\mathrm{gas}}. \tag{2}$$

m_Z is typically known as the *net yield*, and is the metals produced per unit of stars formed. The more common quoted quantity is the *absolute yield*:

$$p = \frac{m_Z}{1-r} \tag{3}$$

which is the metals produced per unit star/stellar remnant mass. Finally, the quantity:

$$y = -\frac{Z(t)}{\ln f_{\mathrm{gas}}} \tag{4}$$

is actually an observable, sometimes known as the *effective yield*. It is equal to the yield in the closed box case.

The gas is increasing in metallicity monotonically, so the metallicity and time map to each other. This means we can determine the fraction of stars of a particular metallicity as:

$$M_*(< Z(t)) = M_*(t) = M_g(0) - M_g(t) = M_g(0)\left[1 - \exp\left(-Z(t)/p\right)\right] \tag{5}$$

We can show that as $f_g \to 0$, the mean metallicity of the stars $\langle Z \rangle \to p$.

You can also calculate the fraction of stars with some factor α fewer metals than the current gas:

$$\frac{M_*(< \alpha Z)}{M_*(< Z)} = \frac{1 - \exp(-\alpha Z/p)}{1 - \exp(-Z/p)} = \frac{1 - f_g^{\alpha}}{1 - f_g} \tag{6}$$

For $\alpha \sim 0.3$ and (appropriate to the solar radius) $f_g \sim 0.1$, this fraction is 50%. So you expect *many* low metallicity stars in the solar neighborhood with a closed box model. However, in reality the number of low metallicity stars is rather small — the metallicity distribution is quite narrow. Because this was determined early from metallicity distributions of G dwarfs, this issue is known as the *G-dwarf problem.* There are numerous potential solutions to this problem, which involve breaking the closed box assumption of course.

We can consider the differential distribution:

$$\frac{\mathrm{d}F}{\mathrm{d}\log\alpha Z} = \alpha \ln 10 \frac{-f_g^{\alpha-1}}{1 - f_g} \tag{7}$$

1.2. Extreme infall one-zone model

Another interesting case can be easily done analytically, one in which inflowing primordial abundance gas exactly replaces the gas lost due to star formation. The set of relevant equations is:

$$\begin{aligned}
\mathrm{d}M_* &= (1-r)\dot{M}_*\mathrm{d}t \\
\mathrm{d}M_g &= 0 \\
\mathrm{d}M_Z &= m_Z\dot{M}_*\mathrm{d}t - Z\mathrm{d}M_* \\
&= \left(\frac{m_Z}{1-r} - Z\right)\mathrm{d}M_*
\end{aligned} \tag{8}$$

In this case, the metallicity of the gas approaches the absolute yield p. The mean metallicity of the stars will remain below the metallicity of the gas but also approach the yield p.

We can calculate the metallicity distribution of the stars at a given time t:

$$F = \frac{M_*(< \alpha Z(t))}{M_*(< Z(t))} = -\frac{M_g}{M_*}\ln\left[1 - \alpha(1 - \exp\left(-M_*/M_g\right))\right] \tag{9}$$

Taken the logarithmic derivative we find:

$$\frac{\mathrm{d}F}{\mathrm{d}\log\alpha Z} = \alpha\ln(10)\frac{1}{s}\frac{1 - \exp(-s)}{1 - \alpha\left(1 - \exp(-s)\right)} \tag{10}$$

where $s = M_*/M_g$. As s increases, we see the distribution become extremely peaked at the maximum metallicity.

This case is a pretty extreme. We expect over time for the inflowing gas to be declining. But it shows how a constant inflow of *pristine* gas can generate a very peaked distribution in stellar metallicity, more similar to what is observed locally.

1.3. Outflows

Another basic process is outflow. Outflows have been observed through absorption studies, wherein metals in the outflowing gas absorb and yield clear indicators of the outflow. Outflows can be characterized by a *mass-loading parameter* η as the ratio of the outflow rate to the increase in stellar mass.

If the outflowing gas has the current mean gas metallicity then we can show:

$$Z(t) = -\frac{1}{1+\eta}\frac{m_Z}{1-r}\ln f_{\rm gas} \tag{11}$$

This case thus behaves like a closed box but with a lower effective yield.

The outflows actually can be metal-enhanced, since much of the metals are produced by or near the same stars and supernovae that are producing the wind. This enhancement will reduce the amount of outflow necessary to reach a particular effective yield.

1.4. More realistic models

These illustrative models are not realistic enough to compare with real data, or to extract the full set of information available in the observations. Multizone models are necessary because we now recognize radial migration of stars and winds of gas which may fall back down onto the galaxy, mixing the enrichment within a single galaxy. Obviously more realistic models of infall and outflow are necessary. These two issues are in principle most realistically addressed in the context of a full hydrodynamic simulations.

Non-instantaneous recycling is an important issue, most notably because of the differing delays between the α-rich enrichment of core-collapse supernovae and the iron-peak-rich enrichment of Type Ia supernovae. The core collapse supernovae occur within a few million years, but the Type Ia supernovae are spread out over a broad delay period.

discuss Weinberg results

1.5. Typical yield values

1.6. Observational results

mass-metallicity, effective yield

alpha and ellipticals

within MW

2. Key References

- Weinberg paper
- Pagel

3. Order-of-magnitude Exercises

1. The overall metallicity of baryons in the universe is about $Z = 0.01$. What does that imply for the total production of stars?

4. Analytic Exercises

1. Prove Equation (2), the gas metallicity as a function of time for the closed box model, for constant net yield m_Z and recycling rate r.

   The change in Z is:

$$\begin{aligned} \mathrm{d}Z &= \mathrm{d}\left(\frac{M_Z}{M_g}\right) = \frac{1}{M_g}\mathrm{d}M_Z - \frac{M_Z}{M_g^2}\mathrm{d}M_g \\ &= \frac{1}{M_g}\left(\frac{m_Z}{1-r} - Z\mathrm{d}M_* - \frac{M_Z}{M_g}\mathrm{d}M_g\right) \\ &= \frac{1}{M_g}\left(\frac{m_Z}{1-r}\mathrm{d}M_* - Z\mathrm{d}M_* - \frac{M_Z}{M_g}\mathrm{d}M_g\right) \\ &= -\frac{\mathrm{d}M_g}{M_g}\left(\frac{m_Z}{1-r}\right) \end{aligned} \tag{12}$$

   For constant m_Z and r, and if the gas starts out with $Z = 0$, then:

$$Z(t) = -\frac{m_Z}{1-r}\ln\left(\frac{M_g(t)}{M_g(t=0)}\right) \tag{13}$$

   Since no gas is added or subtracted this can be written as:

$$Z(t) = -\frac{m_Z}{1-r}\ln f_{\mathrm{gas}} \tag{14}$$

2. Show that for the closed box model, the mean metallicity of the stars $\langle Z\rangle$ approaches the effectve yield p.

   We write:

$$\begin{aligned}\langle Z\rangle &= \frac{1}{M_*}\int_0^{M_*} \mathrm{d}M'_* Z(M'_*) \\ &= \frac{1}{M_*}\int_0^{M_*} \mathrm{d}M'_* \frac{m_Z}{1-r}\ln\left[\frac{M'_*+M_g}{M_g}\right] \end{aligned} \tag{15}$$

   We can replace $\mathrm{d}M'_*$ with $-\mathrm{d}M_g = -\mathrm{d}f_g(M_*+M_g)$ and find:

$$\begin{aligned}\langle Z\rangle &= \frac{M_*+M_g}{M_*}\int_1^{f_g} \mathrm{d}f'_g \frac{m_Z}{1-r}\ln f_g \\ &= \frac{M_*+M_g}{M_*}\frac{m_Z}{1-r}\left[f'_g\ln f'_g - f'_g\right]_1^{f_g} \\ &= \frac{1}{1-f_g}\frac{m_Z}{1-r}\left[f_g\ln f_g - f_g + 1\right] \\ &= \frac{m_Z}{1-r}\left[1+\frac{f_g\ln f_g}{1-f_g}\right] \end{aligned} \tag{16}$$

   As $f_g \to 0$, $\langle Z\rangle \to p$. So the mean metallicity of the stars becomes just the absolute yield.

3. Show that for the extreme infall model, the gas metallicity and the mean stellar metallicity approach the absolute yield.

   The change in Z is:

$$\begin{aligned}\mathrm{d}Z &= \mathrm{d}\left(\frac{M_Z}{M_g}\right) = \frac{1}{M_g}\mathrm{d}M_Z \\ &= \frac{1}{M_g}\left(\frac{m_Z}{1-r} - Z\right)\mathrm{d}M_* \\ \frac{\mathrm{d}Z}{\mathrm{d}M_*} &= \frac{1}{M_g}\left(\frac{m_Z}{1-r} - Z\right) \end{aligned} \tag{17}$$

   If the initial Z is zero, this is solved by:

$$Z = \frac{m_Z}{1-r}\left[1-\exp(-M_*/M_g)\right] \tag{18}$$

   because

$$\begin{aligned}\frac{\mathrm{d}Z}{\mathrm{d}M_*} &= \frac{1}{M_g}\frac{m_Z}{1-r}\exp(-M_*/M_g) \\ &= \frac{1}{M_g}\frac{m_Z}{1-r}\exp(-M_*/M_g) - \frac{Z}{M_g} + \frac{1}{M_g}\frac{m_Z}{1-r}\left[1-\exp(-M_*/M_g)\right] \\ &= \frac{1}{M_g}\frac{m_Z}{1-r} - \frac{Z}{M_g} = \frac{1}{M_g}\left(\frac{m_Z}{1-r} - Z\right) \end{aligned} \tag{19}$$

   So the gas metallicity approaches the yield.

The mean metallicity of the stars will be below this. We can calculate it as:

$$\begin{aligned}
\frac{\langle Z\rangle}{p} &= \frac{1}{M_*}\int_0^{M_*} \mathrm{d}M'_*\left[1-\exp(-M'_*/M_g)\right] \\
&= \frac{1}{M_*}\left[M'_* + M_g \exp(-M'_*/M_g\right]_0^{M_*} \\
&= \frac{1}{M_*}\left[M_* + \exp(-M_*/M_g) - M_g\right] \\
&= 1 + \frac{M_g}{M_*}\left[\exp(-M_*/M_g) - 1\right] \qquad (20)
\end{aligned}$$

At small M_*/M_g this tends towards 0, as it should. At large M_*/M_g it tends toward unity, approaching it from below.

4. For the unenriched outflow case, show that the gas metallicity increases in the same manner as for a closed box but with a lower effective yield.

   We can write:

$$\begin{aligned}
\mathrm{d}M_* &= (1-r)\dot{M}_*\mathrm{d}t \\
\mathrm{d}M_g &= -\mathrm{d}M_* - \eta\mathrm{d}M_* \\
\mathrm{d}M_Z &= m_Z\dot{M}_*\mathrm{d}t - Z\mathrm{d}M_* - Z\eta\mathrm{d}M_* \\
&= \left(\frac{m_Z}{1-r} - (1+\eta)Z\right)\mathrm{d}M_* \qquad (21)
\end{aligned}$$

   In this case I get to just replace $Z \rightarrow (1+\eta)Z$ in the solutions for the simple closed box:

$$\begin{aligned}
(1+\eta)Z(t) &= -\frac{m_Z}{1-r}\ln f_{\mathrm{gas}} \\
Z(t) &= -\frac{1}{1+\eta}\frac{m_Z}{1-r}\ln f_{\mathrm{gas}} \qquad (22)
\end{aligned}$$

   This means this case behaves just like a closed box, except with a lower effective yield.

5. Numerics and Data Exercises

1. Mass-metallicity in SDSS
2. Effective yield in SDSS
3. Model alpha enrichment vs duration of SF