Stellar Population Synthesis

1. Basics & Nomenclature

The theory of stellar structure and evolution predicts how the luminosity, temperature, and other key parameters defining stars change over time, and how a population of such stars will change over time. The theory of stellar atmospheres and observations of stellar spectra, then tie the stellar parameters to observable spectra and colors of the stellar population. This overall project of *stellar population synthesis* connects a star formation history to its observable results, and can in principle allow observations to be used to infer quantities such as the total stellar mass, mean stellar age, metallicity, and other more detailed parameters.

The ingredients of a stellar population synthesis model start with the stellar initial mass function. The IMF can in principle range from Salpeter (very "bottom-heavy") through to a very "bottom light" distribution. There is little evidence that the IMF is constant in all environments, and some evidence that it does differ in different environments.

Stellar structure and evolution models yield stellar parameters like luminosity and effective surface temperature of the population as a function of time. These results are usually expressed as a series of *isochrones* showing the position of stars in the population as a function of time. Uncertainties in these models arise. Even on the main sequence, three-dimensional effects on stellar evolution, such as convection, rotation, and stellar binarity, are at best approximately included in isochrone modeling. Post main sequence phases suffer from this problem even more greatly. Mass-loss due to winds is uncertain and affects the final fate of the stars. Modeling of thermally-pulsating asymptotic giant branch (TP-AGB) stars is very uncertain (partly due to winds but also modeling uncertainties and a paucity of individual stars for calibration). The TP-AGB stars cause particular uncertainty for NIR observations of intermediate age populations. The dependence of the temperatures of horizontal branch stars on stellar parameters is also uncertain (again likely due to winds).

Stellar atmosphere theory and stellar libraries yield spectra as a function of these parameters. The theoretical models can be computed for any stellar parameters in a uniform fashion, but do not always include all the relevant atomic and molecular lines, or correct consideration of microturbulent velocities, convection, departures from local thermodynamic equilibrium, or the spherical geometry. Empirical libraries, on the other hand, tend to be heterogeneous in their observational properties, suffer from spectrophotometric calibration issues, and sparsely cover the necessary space of temperature, surface gravity, metallicity, and abundance space. Again, the major uncertainties arise for rare populations, and particularly post main sequence populations, but also metal poor and/or α-rich populations.

A population with a single age and abundance is known as a *simple stellar population* (SSP).

Its spectrum can be expressed as:

$$f(t, Z) \int_{m_{\rm low}}^{m_{\rm high}(t)} {\rm d}M \Phi(M) f_{\rm star}(t, M, Z) \tag{1}$$

We often need to model a more complicated star formation and chemical evolution history (sometimes using the post main sequence evolution in the models to self-consistently predict the chemical enrichment of the ISM). Any such history may be written as a sum of SSPs, and the resulting spectrum is the corresponding sum of the individual SSP spectra.

In many galaxies, the effects of dust on the spectrum cannot be ignored. Dust is mixed into the interstellar medium gas and it attenuates the light in a wavelength dependent manner, preferentially scattering blue light and transmitting red light. In modeling emergent spectra from galaxies, it appears necessary to account for the dust's distribution, which is preferentially around star forming, gas-rich regions.

Broad trends from stellar population synthesis are:

- Stellar populations start blue, with spectra that do not have features useful for measuring metallicity.
- Stellar populations evolve redward after the cessation of star formation, during which time their Balmer absorption features become weaker (on time scales of a few 100 million years) and their 4000 Å break grows stronger (on about a billion year time scale).
- When they are red, their spectra are similar to K stars, since they are dominated by K giants; these spectra are metallicity-sensitive, but with a degeneracy with age.
- At early times, the populations have low mass to light ratios; as the stellar population ages, it fades and the mass to light ratio increases up to higher than solar values.
- Low mass stars contribute a large fraction of the mass but a small fraction of the light, which means that considerable mass can reside in old stellar populations (if there is a younger population to mask them) or in low mass stars in the IMF (if there are massive stars to mask them).

Stellar population synthesis methods are commonly used to infer galactic star formation histories and other physical properties. In broad strokes, the galactic properties are parametrized, and then parameters are varied to fit some set of observable quantities. There is considerable variation among investigators as to the quantities used and the parametrizations.

The *stellar mass* is almost always a quantity of interest. It is defined as the mass currently in stars and stellar remnants. Of the total mass in stars that have formed, only of order half the mass will remain as stars or remnants after a few billion years, and about a quarter of that is in the (extremely low luminosity) remnants.

The most ambitious approaches attempt constraints on the full star formation history and fit to a detailed observed spectrum. A proper analysis must marginalize over stellar population uncertainties and dust effects and over spectrophotometric calibration uncertainties. In addition, the presence of interstellar and AGN emission lines needs to be accounted for, either by excluding their wavelengths from the fit or including the lines explicitly.

Other approaches simplify the problem, usually in an attempt to regularize the model space in ways thought to be reasonable, or in an attempt to mitigate observational uncertainties (without performing an explicit marginalization).

The classic *Lick index* analysis technique, commonly used for elliptical galaxies since the 1980s, employs both approaches. Lick indices are defined for 21 features between 4000–6500 Å. They are a form of *equivalent width.* Equivalent widths measure:

$$\mathrm{EW} = \frac{f_{\mathrm{feature}}}{f_{\lambda,\mathrm{continuum}}}, \tag{2}$$

the total absorption or emission of a feature (f_{feature}, in units of flux) relative to some local definition of the continuum upon which the feature is imprinted ($f_{\mathrm{continuum}}$, in units of flux density). Usually, the flux density in question is in f_λ units and the equivalent width is expressed in units of Angstroms (thus the terminology "width," which does not in all cases imply a measure of actual width in wavelength space). Often, the continuum level is measured using side bands slightly blueward and redward of the feature.

Equivalent widths, and specifically Lick indices, have some attractive qualities. Because they are relatively local measurements in wavelength, they are insensitive to dust and to spectrophotometric error. In the case of Lick indices, the index measurements were performed on stars with well-calibrated properties, allowing the corresponding measurements in galaxies to be tied directly to the stellar models.

However, equivalent widths involve defining a continuum, which can be ambiguous in regions with other features. In addition, for Lick indices specifically, the stellar observations were taken at relatively low resolution, which needs to be accounted for in studying low velocity dispersion galaxies and higher resolution spectra.

The Lick indices were chosen by hand for their sensitivity to specific atomic features. They are in the blue regime (between 4000 and 5300 Angstroms), due to the technology available at the time of their original definition. The Lick index analysis proceeds by measuring a set of indices associated with the Balmer absorption features (measuring the fraction of young stars), Fe features (which arise in a complex suite of lines), and the Mg b feature. The Balmer and Fe features require some correction for emission line contamination. The indices are then compared to a simple stellar population model characterized by its age, metallicity, and α-enhancement. Since galaxies do not have simple stellar populations, the resulting parameters at best express some weighted average of the underlying star formation history; in the case of the standard Lick analysis, the ages are always smaller than the mass-weighted age estimate. Other uncertainties are known to affect the existing Lick analyses, to do with the stellar population modeling accuracy; most significantly, old stellar populations contain an uncertain number of blue horizontal branch stars, which confuse the interpretation of the Balmer absorption features.

Between full spectral analysis and Lick index analysis lies a large space of possible analyses that has been explored by many investigators. Some important facts revealed by these analyses are as follows:

- Age and metallicity both have similar effects on spectral features, both broad band and for individual lines. Older ages lead to a greater dominance of the red giant branch, and thus redder colors and stronger absorption features. High metallicity leads to lower effective temperatures of stars, and also strong absorption at fixed temperature, both effects also producing redder colors and stronger absorption features.

- Stellar mass-to-light ratios depend to a surprising degree just on the color of the stellar population; greater age, greater metallicity, and greater dust all affect the broad band flux by making it redder and fainter (relative to mass), in similar proportions to each other.

- A definition $D_n(4000)$ for the 4000 Åbreak is informative as to stellar population age (though it carries metallicity dependence as well).

Beyond metallicity, stellar population analyses can be sensitive to individual abundances as well. Most commonly, these are expressed in term of [α/Fe], the α-element abundance relative to iron. In contrast to observations of the interstellar medium, typical observations of stellar populations are most sensitive to Mg rather than the more abundant oxygen, due to the lack of atomic oxygen lines in stars in the optical. In the Milky Way, it is known that oxygen and magnesium do not, however, always vary together, so Mg as a proxy for other α elements is not perfect.

A special case for the use of stellar populations is the inference of recent star formation, as directly traced by O and B stars. Ultraviolet emission can be used to trace the star formation rate, preferably far UV ($\sim$ 1500 Å), though near near UV ($\sim$ 2500 Å) or even u-band is occasionally used. Such measures need to be dust-corrected. In the UV, this is often done using the FUV to NUV ratio (since in principle this ratio is fixed for hot stars). If optical imaging is available, one can estimate reddening using stellar population analysis of the optical bands. If infrared imaging is available, the IR flux directly traces the extincted light, modulo the geometric anisotropy of the UV extinction. In this last case, a weighted sum of the UV and infrared luminosity can yield a reliable star formation rate estimate. The mid-IR is thought to be more reliable in this regard, since the colder dust in the far-IR can be heater by old stellar populations.

With spectra, star formation averaged over longer time scales is possible. For example, star formation over a few hundred million year time scales is traced by the Balmer lines. A very special cases arises for K+A, sometimes called E+A, or post-starburst, galaxies. Such galaxies show few traces of star formation on million year time scales (as traced usually by Hα interstellar emission), but do show strong star formation on a few hundred million year timescales (as traced by Balmer absorption). These galaxies likely experienced a large burst of star formation a few hundred million years ago, which is now over.

Interpretation of star formation rates, especially but not only at low values, requires care. Old stellar populations do emit some UV light, depending on their population of horizontal branch stars; in practice, measuring specific star formation rates below about 10^{-11} yr^{-1} is not possible. Both ultraviolet and near-infrared emission can be powered by AGN as well, which near the centers of galaxies can be a confusing factor as well.

There are a handful of cases for which stellar population histories can be derived directly from resolved stellar populations. Stellar clusters in the Milky Way are one case. To a certain extent, the population of stars with parallaxes in the Milky Way can also be used in this way. Some nearby galaxies (M31, M33, and others) have resolved stellar populations from space, and these cases can be studied from the isochrones. Since they also have broad band and spectroscopic measurements similar to more distant galaxies, they can provide a calibration between the two approaches.

2. Commentary

In principle, stellar population analysis requires marginalization over all of the uncertain parameters that characterize the stellar population modeling. These uncertainties are:

- Stellar IMF
- TP-AGB phases
- Blue stragglers
- Extreme horizontal branch stars
- Effects of convection on evolution
- Mass-loss through winds
- Effects of binarity on evolution
- Stellar atmospheres models & libraries
- Dust and dust geometry

3. Key References

- *Old stellar populations. 5: Absorption feature indices for the complete Lick/IDS sample of stars*, Worthey et al. (1994)
- *Modeling the Panchromatic Spectral Energy Distributions of Galaxies*, Conroy (2013)

4. Important numbers

- $M_\odot = 1.989 \times 10^{30}$ kg
- $R_\odot = 6.955 \times 10^8$ m
- $T_\odot$(surface) = 5500 K
- $T_\odot$(core) = 1.5×10^7 K
- $L_\odot = 3.828 \times 10^{33}$ erg s^{-1}

5. Order-of-magnitude Exercises

1. Estimate the ratio between star formation rate and FUV flux around $\lambda \sim 1200$ Å. Assume that the luminosity in the FUV band of an O-star is $\sim 10^{34}$ erg s^{-1} Å^{-1} at that wavelength (a very approximate number for the O2 star BI 237 in the Large Magellanic Cloud, according to the atlas of Smith 2012).

2. Estimate the difference in mass-to-light ratio in the optical between a 200 million year old and 10 billion year old stellar population. Assume that the solar mass stars (which dominate the flux of the older population) spend about 10% of their time at the tip of the red giant branch and that the distribution of stars along the red giant branch is constant per unit magnitude.

   At 200 Myr in age, the stars in stellar population stars are mostly in main sequence. The luminosity is dominated by late-type B stars of about 5 $M_\odot$, whereas the mass is dominated by low mass stars. The mass-to-light ratio may be calculated as:

$$\frac{M}{L} = \frac{\int_{0.08M_\odot}^{5M_\odot} \mathrm{d}M M \Phi(M)}{\int_{0.08M_\odot}^{5M_\odot} \mathrm{d}M L(M) \Phi(M)} \tag{3}$$

   where $\Phi(M)$ is the initial stellar mass function. If we assume $(L/L_\odot) = (M/M_\odot)^4$ and $\Phi \propto M^{-2.35}$, then in solar units:

$$\frac{M}{L} = \frac{\int_{0.08}^{5} \mathrm{d}x x^{-1.35}}{\int_{0.08}^{5} \mathrm{d}x x^{1.65}} \tag{4}$$

which evaluates to $M/L \sim 0.2$.

Now, for the 10 Gyr stellar population, solar mass stars are just entering the red giant branch. The luminosity of solar mass stars in the red giant phase is $L_{\mathrm{RGB}} \sim 10^3 L_\odot$. Solar mass stars spend 10% of the main sequence lifetime on the RGB, meaning that the set of stars there will be those that leave the main sequence within about 10% of the total lifetime (1 Gyr). Let us call the range of masses this corresponds to $\Delta M(t_\odot)$. We can write the mass to light ratio as:

$$\frac{M}{L} = \frac{\int_{0.08M_\odot}^{M_\odot} \mathrm{d}M M \Phi(M)}{\Phi(M_\odot)\Delta M(t_\odot)\langle L_{\mathrm{RGB}}\rangle} \tag{5}$$

We can determine $\Delta_M(t_\odot)$ as follows, by relating the range in time to a range in mass:

$$\Delta M(t) \sim \Delta t \left|\frac{\mathrm{d}M}{\mathrm{d}t}\right| \tag{6}$$

and differentiating the relationship

$$\left(\frac{t}{t_\odot}\right) = \left(\frac{M}{M_\odot}\right)^3 \tag{7}$$

to show

$$\left|\frac{\mathrm{d}M}{\mathrm{d}t}\right| = \frac{M_\odot}{3t_\odot}\left(\frac{M}{M_\odot}\right)^4 \tag{8}$$

and so:

$$\Delta M(t) \sim \frac{1}{3} M_\odot \frac{\Delta t}{t_\odot}\left(\frac{M}{M_\odot}\right)^4 \tag{9}$$

Then we can calculate:

$$\langle L_{\mathrm{RGB}}\rangle = \frac{\int_{L_\odot}^{10^3 L_\odot} \mathrm{d}L N(L) L}{\int_{L_\odot}^{10^3 L_\odot} \mathrm{d}L N(L)} \tag{10}$$

where $N(L) \propto 1/L$ (constant per unit magnitude), so:

$$\langle L_{\mathrm{RGB}}\rangle = \frac{\int_{L_\odot}^{10^3 L_\odot} \mathrm{d}L}{\int_{L_\odot}^{10^3 L_\odot} \mathrm{d}L L^{-1}} \sim \frac{1000}{3\ln 10} L_\odot \sim 140 L_\odot \tag{11}$$

Plugging in and using solar units:

$$\frac{M}{L} = \frac{\int_{0.08}^{1} \mathrm{d}x x^{-1.35}}{(1/3)(140)(0.1)} \sim 1 \tag{12}$$

In fact, this is an underestimate; for old populations the RGB tends to be more heavily populated per unit magnitude near its base, which increases the mass-to-light ratio by a factor of a few.

An important point is that although the old population is dominated by solar mass stars, and also has a mass-to-light ratio similar to solar values, this is the consequence of the balance between the numerator, dominated by the lower stellar masses, and the denominator, dominated by the post-main sequence lifetime and luminosity distribution. Thus the rough agreement in mass-to-light ratio is a coincidence.

3. Using a Salpeter initial mass function, and assuming that $L \propto M^4$ determines both the luminosity and lifetimes of the stars, calculate how the stellar luminosity varies as a function of time expected at early times (e.g. ~ 100 Myr), before the red giant branch population of stars becomes important. Do not worry about the absolute value of the luminosity, just its time dependence.

   The lifetime of a star with mass m can be estimated by comparing to the lifetime of the Sun, $\tau_\odot \approx 10^{10}\, yr$, as $\tau \approx \frac{m}{L}\tau_\odot$, where m and L are the mass and luminosity of a star in solar units. Assuming a mass-to-light ratio of $L \propto m^4$, we can rewrite this equation in terms of mass only:

$$\tau \approx \frac{1}{m^3}\tau_\odot \tag{13}$$

   Solving this equation for m sets the upper limit on the masses of stars in a population at time τ. Assuming a Salpeter IMF, which asserts that the number of stars in a stellar population with mass between m and $m + dm$ is proportional to $m^{-2.35}$, we can determine the time dependence of a stellar population's luminosity. To do this, we also assume that each star has constant luminosity until the end of its lifetime, at which point it has no luminosity. Then the total luminosity is just the integral of the stars

$$L(t) \propto \int_{0.08}^{\left(\frac{10^{10}}{t}\right)^{\frac{1}{3}}} m^{-2.35}\, m^4\, dm \tag{14}$$

   The term corresponding to the lower limit on the integral is negligibly small ($\sim 10^{-4}$) and so we get

$$L(t) \propto t^{-0.883} \tag{15}$$

   This dependence should hold at early times, when post-main sequence stars are not dominating the luminosity. As the population ages past a billion years, the luminosity is dominated by red giant branch stars and this dependence changes (slowing down generally).

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4. What is the difference between the mass-to-light ratios of two 10-billion-year-old stellar populations, one with a Salpeter IMF and one with an IMF which is Salpeter above $M = 0.7M_\odot$ but has $\Phi(M) \propto$ constant at lower masses?

   In this case we can avoid the calculation of the relative luminosities of these two populations. For a 10-billion-year-old population, star with mass $M_\odot$ has just entered the post-main sequence phase. So for two populations with the same number of solar mass stars, the population of red giants is the same for both populations. Since the red giant populations dominate the luminosity for an old stellar population, the luminosities of these two populations will be nearly identical.

So the difference in mass-to-light ratio can be calculated as the difference in total mass for two populations with the same number of solar mass stars.

For the Salpeter case, using a normalization constant A:

$$\begin{aligned} M &= A \int_{0.08M_\odot}^{M_\odot} \mathrm{d}M' M' \Phi(M') \\ &= A M_\odot \int_{0.08}^{1} \mathrm{d}x x^{-1.35} \\ &= -\frac{AM_\odot}{0.35} x^{-0.35}\Big|_{0.08}^{1} \approx 4.1 A M_\odot \end{aligned} \tag{16}$$

For the non-Salpeter case, using the same normalization to match $\Phi(M)$ at $M = M_\odot$:

$$\begin{aligned} M &= A \int_{0.08M_\odot}^{M_\odot} \mathrm{d}M' M' \Phi(M') \\ &= A M_\odot \left[\int_{0.7}^{1} \mathrm{d}x\, x^{-1.35} + \int_{0.08}^{0.7} \mathrm{d}x\, x\right] \\ &= A M_\odot \left[-\frac{1}{0.35} x^{-0.35}\Big|_{0.7}^{1.0} + \frac{1}{2} x^2\Big|_{0.08}^{0.7}\right] \\ &\approx (0.38 + 0.48) A M_\odot \approx 0.86 A M_\odot \end{aligned} \tag{17}$$

So for the same luminosity (dominated by the red giants) the non-Salpeter case has a mass to light ratio about 1/5 of the Salpeter case. This emphasizes that the bulk of the "stellar mass" that is inferred from stellar population analyses is usually actually hardly contributing to the light, and could be removed from the stellar population without much affecting the spectrum. Therefore the stellar mass values are very dependent on the assumed stellar IMF. Note that I've picked a pretty extreme non-Salpeter case—a Kroupa (2001) or Chabrier (2003) IMF, which reflect our best estimates of the IMF in the Milky Way, is more like $\Phi(M) \propto M^{-1}$ at low masses rather than a constant.

6. Numerics and Data Exercises

1. There is a Python interface to the Flexible Stellar Population Synthesis (FSPS; Conroy et al. 2009) software, which is one of several libraries that exist for this purpose. Install this package and (using the scant available documentation!) run it to produce spectra for dust-free populations with a range of ages and metallicities. Specifically, do 25 models, with five metallicities and five ages spaced logarithmically between $Z/Z_\odot = 0.01$ and $Z/Z_\odot = 2$, and age $= 10^8$ years and age $= 10^{10}$ years. Then create the same set of models with a modest amount of dust reddening (say around $A_V \sim 0.5$). Plot the spectra for comparison with each other in the UV through near-IR range. Warning: this software is incredibly slow.

2. For the same set of models as the first question, use FSPS to project the spectra onto the u and r filters. Then plot the mass-to-light ratio in the r-band as a function of $u - r$ color for all models. Also plot $g - r$ versus age. In each plot, distinguish between the no-dust and dust cases.

3. For the same set of models as the first question, use D_n4000, and plot it versus age, again distinguishing between the no-dust and dust cases.

4. For the same set of models as the first question, calculate the Lick Mg b and the $\langle$Fe$\rangle$ indices (the latter being the mean of Fe5270 and Fe5335). Plot the mapping of this set of models into the Lick index space as two grids (one for each dust case).

REFERENCES

Chabrier, G. 2003, PASP, 115, 763

Conroy, C. 2013, Annual Review of Astronomy and Astrophysics, 51, 393

Conroy, C., Gunn, J. E., & White, M. 2009, ApJ, 699, 486

Kroupa, P. 2001, MNRAS, 322, 231

Smith, M. A. 2012, ApJS, 202, 17

Worthey, G., Faber, S. M., Gonzalez, J. J., & Burstein, D. 1994, ApJS, 94, 687