Inventory

1. Basics

We begin with a basic inventory of what our subject is, and the relevant length and time scales.

The Sun is a not-untypical star in the Milky Way galaxy. It is a hot ball of gas, primarily consisting of H, with about 25% ^{4}He and about 2% higher mass elements, known as *metals*. It produces light through nuclear fusion of H to ^{4}He in its core. Its mass $M_\odot = 2 \times 10^{33}$ g and its radius is $R_\odot = 0.7 \times 10^6$ kc. The distance from the Earth is 1.4×10^8 km = 1 *Astronomical Unit* (AU). The nearest other stars are very far away, typically *parsecs* ($\approx 2 \times 10^5$ AU, or about 3.3 lightyears) away.

The Sun orbits the Milky Way galaxy in a roughly circular orbit at about 8.5 kpc from the center, with a velocity of ~ 220 km s^{-1}, along with most stars in the Milky Way's disk. The Milky Way visible by eye in the night sky is resolved into many billions of stars, with a total *stellar mass* of about 3×10^{10} $M_\odot$. Like many galaxies, the Milky Way has a *thin disk* (a few 100 pc) thick, a *thick disk* (about 1 kpc thick), a *bulge* and a *bar* in its center, and a *stellar halo* that extends out to about 100 kpc. It has a moderately large black hole (4×10^6 $M_\odot$) in its center, a relatively small *supermassive black hole*. The disk has neutral and molecular gas as well as dust, and outside the disk and surrounding the Milky Way is a diffuse halo of gas. The molecular gas regions are forming young stars only a few million years old. The oldest stars in the galaxy appear to be about 10 billion years old. Orbiting the Milky Way are about 150 globular clusters, which are roughly spherical, very old, bound stellar systems.

The galaxy exists within the deep potential well of a *dark matter* halo, which is only detectable today through its gravitational influence. The rotation velocity stays close to flat at 220 km s^{-1} out to at least 20 kpc. The total dark matter mass appears to be about 10^{12} $M_\odot$ and it extends past 100 kpc. Numerous lines of evidence suggest that the dark matter is not *baryonic* (i.e. not ordinary standard model particles) and interacts with baryonic matter primarily gravitationally.

The nearest galaxies to the Milky Way are its *dwarf galaxy* satellites, the largest of which are the Large and Small Magellanic Clouds (LMC and SMC). These two satellites are visible from the Southern Hemisphere with the naked eye. They are about 10% and 1% of the Milky Way's luminosity and are about 50 kpc away, and therefore within the Milky Way's dark matter halo. Many dozens of other satellites are known.

The nearest galaxy of comparable size to the Milky Way is Messier 31, also known as M31 or Andromeda. It is about 800 kpc away, and moving toward the Milky Way, indicating the two are part of a bound group that will eventually merge. M31 is somewhat more luminous then the Milky Way and differs in a number of important details, most obviously having a larger bulge relative to its disk.

There are nearly a billion other detected galaxies, and the census of galaxies many suggests there are hundreds of billions total in the observable universe. They have a mean separation of a few Mpc, but are not uniformly distributed. Instead they exist in dense clusters, connected by filaments and walls, with void regions in between. These large scale structure form from initial primordial density fluctuations through gravitational growth. The galaxies have a range of luminosities, with a characteristic exponential cutoff in number density at high luminosity, called L_* (a bit brighter than the Milky Way's luminosity) and a power law distribution of luminosity below that.

These galaxies come in a number of varieties. Hubble classified galaxies with comparable luminosities to the Milky Way from *early-type* or *elliptical* galaxies, through *late-type* or *spiral* galaxies. Elliptical galaxies old, red, and puffy. Spiral galaxies are younger, bluer, and have cold thin disks. An apparently intermediate variety of *lenticular* or *S0* galaxies have disks like spiral galaxies, but are puffier and do not have spiral structure. There are *irregular* galaxies of various types. Dwarf galaxies tend to deviate from Hubble's system in detail, as do distant galaxies observed as they were when the universe was younger.

As one looks at galaxies of greater and greater distance, one finds that they are receding with a velocity $v = H_0 d$, where $H_0 \sim 70 km\ \mathrm{s}^{-1}\ \mathrm{Mpc}^{-1}$. The line-of-sight velocities are determined through their Doppler shift, the recession can also be quantified by the *redshift* z relating $\lambda_{\rm obs} = (1+z)\lambda_{\rm em}$. At low redshifts, we can relate velocity and redshift with $v = cz$. The Hubble recession is what we mean when we say the universe is expanding. A rough calculation of the age of the universe from this expansion yields 14 billion years, which is remarkably close to the right answer.

Essentially every luminous galaxy has a supermassive black hole at its center. Larger galaxies tend to have larger black holes, ranging up to about $10^9\ M_\odot$. These black holes presumably grew through accretion. During episodes of accretion, these black holes can become much more luminous than their host galaxies. Accreting black holes are referred to as *active galactic nuclei* (AGN), and the most luminous ones are referred to as *quasars*. These quasars were most common about 10 billion years ago, corresponding to redshifts of $z \sim 2$–3. They can be used as backlights upon which gas aborption signatures are imprinted, and thus reveal the gas distribution throughout the universe.

2. Important numbers

- $c = 2.99792 \times 10^8\ \mathrm{m\ s}^{-1}$
- $G = 6.6738 \times 10^{-11}\ \mathrm{m}^3\ \mathrm{kg}^{-1}\mathrm{s}^{-2}$
- $m_p = 1.6726 \times 10^{-27}\ \mathrm{kg}$
- $m_n = 1.6749 \times 10^{-27}\ \mathrm{kg}$
- $m_e = 9.1049 \times 10^{-31}\ \mathrm{kg}$

- $M_{\rm Earth} = 5.974 \times 10^{24}$ kg
- $M_\odot = 1.989 \times 10^{30}$ kg
- $R_\odot = 6.955 \times 10^8$ m
- $T_\odot = 5500$ K
- $L_\odot = 3.828 \times 10^{33}$ erg s^{-1}
- $L_* \sim 10^{10} L_\odot$
- 1 AU $= 1.496 \times 10^{11}$ m
- lightyear $= 9.461 \times 10^{15}$ m
- parsec $= 3.086 \times 10^{16}$ m
- year $= 3.156 \times 10^{16}$ s

3. Key References

- *Extragalactic Astronomy and Cosmology: An Introduction, Schneider (2015)*

4. Order-of-magnitude Exercises

1. Estimate the mean distance between stars in the Milky Way disk in units of the solar radius. Are stellar collisions likely to be particularly common?

   The disk extends to at least 8.5 kpc radius, is has about 10^{10} stars, and is a few hundred pc thick. The mean density of stars is therefore:

$$n \sim \frac{10^{10}}{\pi(300\ \text{pc})(10\ \text{kpc})^2} \sim \frac{10^{10}}{10^{11}\ \text{pc}^3} \sim 0.1\ \text{pc}^{-3} \tag{1}$$

   This means the mean distance between stars is:

$$d \sim n^{-1/3} \sim 2\ \text{pc} \sim 2\ \text{pc} \times \frac{3 \times 10^{16}\ \text{m}}{1\ \text{pc}} \times \frac{R_\odot}{7 \times 10^8 \text{m}} \sim 10^8 R_\odot \tag{2}$$

   These distances are very large relative to stellar radii. We can go further and ask for a relative velocity of ~ 200 km s^{-1} (an overestimate) what over 10 billion years is the probability that any two stars will collide.

$$\begin{aligned} p &= n(\pi R_\odot^2) vt = d^{-3}(\pi R_\odot^2) vt \\ &\sim \pi(10^{-24} R_\odot^{-3}) R_\odot^2 \left(3 \times 10^{-4} R_\odot \text{s}^{-1}\right) \left(3 \times 10^{17} \text{s}\right) \end{aligned}$$

$$\sim \quad 3 \times 10^{-10} \tag{3}$$

Thus, the chances for any individual star to actually collide with another is very small. Even accounting for the fact that there 10^{10} in the Milky Way indicates that the total rate of encounters is of order unity per 10 billion years.

2. Estimate the approximate dynamical mass interior to the Sun.

Assuming a circular orbit at 8.5 kpc of 220 km s^{-1}, we can use the force law:

$$a = \frac{GM}{r^2} = \frac{v^2}{r} \tag{4}$$

to infer:

$$\begin{aligned} M &= \frac{v^2 r}{G} = \frac{(2.2 \times 10^5 \text{ m s}^{-1})^2 (8500 \times 3 \times 10^{16} \text{ m})}{6.7 \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{s}^{-2}} \\ &\sim 2 \times 10^{41} \text{ kg} \sim 10^{11} M_\odot \end{aligned} \tag{5}$$

This is somewhat more than the total mass inferred in stars, the real evidence for dark matter comes when you consider that the rotation curve remains constant ("flat") to much larger radii, continuing to add to the discrepancy.

3. Use the Hubble Law to estimate the age of the Universe.

We use the standard estimate of time from distance velocity:

$$t = \frac{d}{v} = \frac{d}{H_0 d} = H_0^{-1} \tag{6}$$

That is, there is a time in the past at which all the galaxies were apparently at a single point. This is time of the Big Bang, and the *Hubble time* is:

$$H_0^{-1} = \left(70 \frac{\text{km s}^{-1}}{\text{Mpc}}\right)^{-1} = \frac{3 \times 10^{22} \text{ m}}{7 \times 10^4 \text{ m s}^{-1}} \sim 4.3 \times 10^{17} \text{ s} \sim 1.4 \times 10^{10} \text{ yr} \sim 14 \text{ billion years} \tag{7}$$

This estimate assumes that the galaxies have been traveling at constant velocity. However, in reality the mass density of the universe causes deceleration at early times, and at late times there is an unexplained acceleration called "dark energy." At the current time, these two effects tend to cancel and the above estimate of the universe's age is correct to better than 5%.

REFERENCES

Schneider, P. 2015, Extragalactic Astronomy and Cosmology: An Introduction

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