Stellar Evolution

1. Basics & Nomenclature

Stars are formed from interstellar gas, through gravitational collapse within molecular clouds. A spectrum of objects are formed from below the hydrogen burning limit of 0.08 $M_\odot$ up to 100–200 $M_\odot$. Stars spend most of their lifetime on a *main sequence*, burning hydrogen in their cores. Depending on their mass they then proceed through a sequence of post-main sequence phases and leave remnants in the form of white dwarfs, neutron stars, or black holes (or in some cases no permanent remnant). In any system of stars, their history is encoded in their distribution of luminosities and colors, which can be measured directly in resolved stellar populations or inferred from spectra and/or broad band imaging. Stellar evolution processes are responsible for most of the elements higher mass than helium.

The *stellar initial mass function*, or *IMF*, defines the spectrum of initial masses of stars. This spectrum is difficult to determine observationally, because almost all systems we observe have been altered dynamically or by stellar evolution. For many decades, the standard was the *Salpeter IMF*:

$$\Phi(M) \propto M^{-2.35}. \tag{1}$$

This form leads to a large fraction of mass between 0.08 and 0.5 $M_\odot$. However, evidence from local systems implies that in many cases, the true IMF has a flatter slope at lower masses (Bastian et al. 2010. This difference is significant because low mass stars emit very little luminosity, so their presence is very difficult to directly detect, and therefore inferences of total mass in stars depend strongly on the assumptions of how many low mass stars are in the system.

The first and usually longest phase of stellar evolution is core hydrogen burning on the main sequence. The cores are typically millions to billions of Kelvin and fully ionized. Hydrogen is burned to helium through two major processes, the *p-p chain* at low masses and the *CNO cycle* at high masses ($M > 2M_\odot$). Each process yields one ^{4}He nucleus with a mass of $3.96m_p$ from 4 protons; the one percent difference yields the energy for stellar luminosity. Numerous other nuclear processes are occurring simultaneously that contribute to the luminosity (and to the neutrino output). The nuclear processes depend on the tail of nuclei in the Maxwell-Boltzman distribution that are sufficiently high energy to tunnel through the Coulomb repulsion of the nuclei. They are therefore strongly temperature sensitive.

Stellar structure is controlled by the following equations:

- Mass conservation:
$$\frac{\mathrm{d}M}{\mathrm{d}r} = 4\pi r^2 \rho \tag{2}$$
- Energy conservation:
$$\frac{\mathrm{d}L}{\mathrm{d}r} = 4\pi r^2 \rho \epsilon \tag{3}$$

- Hydrostatics:
$$\frac{\mathrm{d}P}{\mathrm{d}r} = -\frac{GM(r)\rho}{r^2} \tag{4}$$
- Energy transport, which comes in the form of radiative transfer:
$$\frac{\mathrm{d}T}{\mathrm{d}r} = \frac{L(r)\kappa(r)\rho(r)}{16\pi r^2 caT^3} \tag{5}$$
or in the form of convection.

$\kappa(r)$ is the opacity defined as the cross-section to absorption per unit mass (so is $1/\rho l$, where l is the mean free path of a photon). Opacity is a critical parameter, as it strongly affects the structure of the star, and therefore its size. The higher the metallicity, the higher the opacity, the larger the star, and therefore the lower the surface temperature at a given luminosity.

The most important contributions to opacity inside the bulk of the star come from Thomson scattering, free-free absorption, and bound-free absorption. The latter two effects scale according to *Kramer's Law*, which scales as follows:

$$\kappa \propto Z\rho T^{-7/2} \tag{6}$$

The scaling results from just the consideration of the effects of brehstrahhlung on a Planck spectrum, and the density of free electrons. Because of the temperature dependence, at high temperatures ($T > 10^6$ K) Thomson scattering dominates, for which κ is constant. There are two other major sources of opacity, bound-bound absorption, which is subdominant over most of the star, and H^- absorption, which is only possible in the outer layers.

In the exercises, we will show that these equations imply a scaling of luminosity with mass of approximately $L \propto M^4$ and with surface temperature of $L \propto T_s^8$ on the main sequence. The former relationship implies that the stellar lifetimes on the main sequence scale as M^{-3}.

After the main sequence, stellar evolution depends on the mass of the star. At high luminosities (canonically above 8 $M_\odot$), stars have short main sequence lifetimes (10s of Myrs) and thereafter undergo a series of nuclear burning phases in their cores: He, C, Si, and so on. Each burning phase is shorter than the last. Shell burning is occurring at the same time. Once Fe and Ni form in the core, energy cannot be further released, and the core collapses. The result in many and potentially all cases is a core-collapse supernova. The Fe and Ni produced in the core is disintegrated in this process and most of that mass becomes part of the neutron star or black hole that forms at the center. However, the elements remaining from burning in the regions outside the core, which are rich in α elements like O, Mg, and so on, can be returned to the interstellar medium.

At lower masses, stars instead start burning hydrogen in a shell around the inert helium core. This shell burning yields tremendous luminosity and also induces the outer layers of the stars to expand up to AU or greater size. The result is a red giant. The red giant evolves up the red giant branch, increasing in luminosity until the tip of the red giant branch. The color of the red giant branch is largely set by the *Hayashi limit*, which determines how low a temperature the atmosphere can become and still satisfy energy transport constraints. In the red giant phase, stellar winds can be active.

The maximum luminosity at the tip of the red giant branch is set by the onset of helium burning. For stellar masses of $M > 2M_\odot$, the core is nondegenerate and expands, leading the envelope to contract and the star to become blue; some loops in the color-magnitude diagram can occur. For lower stellar masses, there is a thermal runaway process called the *helium flash* and the stellar structure readjusts, with the stars ending up on the *horizontal branch*, with a luminosity determined by the core mass (usually around 0.5 $M_\odot$) and the temperature determined by how much envelope was lost through winds.

The net result of stellar evolution is that at early times the stellar population is mostly on the main sequence and it is dominated by the bluest stars. At late times it is dominated by the red giants, which have recently (within $\sim$ a Gyr) left the main sequence.

Stars are normally classified according to their MK system: OBAFGKM, which is in order of decreasing temperature. Subclassifications exist (O1, O2, ..., O9, B1, ...). Hotter stars are referred to as "early type" and cooler stars are referred to as "late type." These classifications are according to their spectra and the ordering was originally based on the spectral phenomenology, which is why the current nomenclature appears somewhat random.

Broadly speaking O and B stars have few lines, with He II lines in O stars, He I lines in B stars, and weak Balmer lines in both. A stars have strong Balmer lines. Balmer lines become weaker again for later type stars. In F and G stars, lines of other neutral atoms become important. Particularly in G stars the Ca II H and K lines (right below 4000 Å) appear. In G, K, and particularly M stars, molecular lines become important as molecules like CH, CN, and TiO become able to survive in the cooler atmospheres.

In galactic systems, the consequence of stellar evolution is that the stellar continuum of young systems is dominated by hot stars, yielding a blue spectrum with few clues to metallicity, whereas the stellar continuum of old systems is dominated by cooler, old stars, with red spectra that depend on metallicity (due to variations in the abundances and their effects on the stellar opacity and therefore temperature).

2. Commentary

The basic consequences of stellar evolution are well-established, particularly on the main sequence. But the post-main sequence phases are not well-constrained in terms of (for example) the temperatures to expect for horizontal branch stars and the numbers and temperatures of AGB stars (and how dust obscured they should be), among other uncertainties. Furthermore, the prediction of stellar atmosphere emission spectra for stars in a given phase and metallicity is not perfect, nor well constrained by data in all regimes. In addition, many stars are in binary systems (perhaps the majority of high mass stars) and the extent to which this matters for the interpretation of stellar populations is not known.

3. Key References

- *Nucleosynthesis and Chemical Evolution of Galaxies, Pagel (2009)*; this textbook gives a good introduction to the aspects of stellar evolution relevant to galaxies.

4. Important numbers

- $M_\odot = 1.989 \times 10^{30}$ kg
- $R_\odot = 6.955 \times 10^{8}$ m
- $T_\odot$(surface) $= 5500$ K
- $T_\odot$(core) $= 1.5 \times 10^{7}$ K
- $L_\odot = 3.828 \times 10^{33}$ erg s^{-1}

5. Order-of-magnitude Exercises

1. Argue why higher mass stars produce more of their energy through the CNO cycle than lower mass stars do.

2. Detailed stellar evolution calculations predict a main sequence lifetime for the Sun of 10 billion years. What fraction of the total hydrogen in the Sun needs to be converted to helium to provide this lifetime?

   If time in the main sequence is T, and the luminosity of the Sun is L, roughly the total energy spent is:

   $$E = TL \tag{7}$$

   And since this energy primarily comes from converting the hydrogen to helium, with the loss of one percent mass,

   $$\begin{aligned} (.01)m_c c^2 &= TL \\ m_c &= \frac{TL}{.01c^2} \end{aligned} \tag{8}$$

   where m_c is the mass converted in the reaction. For 10 Gyr and $L \sim 4 \times 10^{33}$ erg s^{-1}, a total of 1.3×10^{32} g of hydrogen is converted to helium. The mass of the Sun is 2×10^{33} g, of which 75%, or 1.5×10^{33} g, is hydrogen at the beginning of the process. Thus, about 10% of the hydrogen in the Sun is converted to helium over its lifetime. Note that the helium ash created increases the mass fraction from 0.25 to around 0.35, but that ash is in the core so not apparent in spectra, which reflect the stellar atmospheric abundances. It also will not be returned to the interstellar medium, since in the Sun's later phases it will be burnt mostly into carbon and oxygen.

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3. Estimate the scaling relations between luminosity, mass, and surface temperature on the main sequence from the equations of stellar structure. Assume that the central temperature T_c is approximately constant (it is more like $T \propto M^{-1/2}$ in reality). Calculate the scaling separately for high mass stars, assuming their opacity is dominated by Thomson scattering, and low mass stars, assuming their opacity is dominated by Kramer's Law.

Using dimensional analysis, the energy transport equation implies the relations:

$$\begin{aligned} \frac{T_c}{R} &\propto \frac{L\rho}{R^2 T_c^3} \quad \text{high mass} \\ \frac{T_c}{R} &\propto \frac{L\rho^2}{R^2 T_c^{7.5}} \quad \text{low mass} \end{aligned} \tag{9}$$

and we can rearrange these:

$$\begin{aligned} L &\propto \frac{T_c^4 R}{\rho} \propto \frac{T_c^4 R^4}{M} \quad \text{high mass} \\ L &\propto \frac{T_c^{8.5} R}{\rho^2} \propto \frac{T_c^{8.5} R^7}{M^2} \quad \text{low mass} \end{aligned} \tag{10}$$

The hydrostatic equation implies:

$$P \propto \frac{M^2}{R^4} \tag{11}$$

and we can convert the ideal gas law ($P = nkT$) to:

$$P \propto \frac{MT_C}{R^3}. \tag{12}$$

Equating the two expressions, balancing temperature and density against gravity leads to:

$$T_c \propto \frac{M}{R} \tag{13}$$

or $M \propto R$. Then plugging into the energy transport equation:

$$\begin{aligned} L &\propto M^3 \quad \text{high mass} \\ L &\propto M^5 \quad \text{low mass} \end{aligned} \tag{14}$$

Now we can use the Stefan-Boltzmann law to relate the surface temperature to the luminosity:

$$L \propto R^2 T_s^4 \tag{15}$$

If we just average the high and low mass exponents and assert $L \propto M^4$, it is simple to show that

$$L \propto T_s^8, \tag{16}$$

explaining the extremely strong dependence of luminosity on surface temperature in the Hertzsprung-Russell diagram. Note that using T_c that varies with mass changes these exponents somewhat but only weakly. Of course getting things right in detail requires detailed numerical calculations.

6. Numerics and Data Exercises

1. Using Gaia, for a relatively nearby open cluster, plot the HR diagram around it (do not use the parallaxes — just leave use the apparent magnitudes). Can you understand its published age based on what you see in the diagram? How can you determine what in the diagram is from background or foreground stars and what is from the open cluster itself?

2. Using Gaia, for a relatively nearby globular cluster, do the same.

3. Using Gaia and its high signal-to-noise ratio parallaxes, make a plot of the HR diagram locally. What can you conclude about the star formation history around us in the Milky Way.

4. Identify a luminous galaxy on the red sequence at low redshift (say $z < 0.03$) from the SDSS spectroscopic survey. Then find a star (in a low reddening region) with similar $g - r$ colors that has a spectrum. Compare the spectra in the rest frame.

REFERENCES

Bastian, N., Covey, K. R., & Meyer, M. R. 2010, ARA&A, 48, 339

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This preprint was prepared with the AAS LaTeX macros v5.0.