Light II: emission and propagation

1. Basics & Nomenclature

In thermal equilibrium, a photon distribution has the Planck spectrum, with a volume energy density per unit frequency as follows:

$$u_\nu = \frac{8\pi\nu^2}{c^3}\frac{h\nu}{\exp(-h\nu/kT) - 1} \tag{1}$$

Typical units are erg Hz^{-1} cm^{-3}. At low frequency this is a power-law distribution, the Rayleigh-Jeans tail:

$$u_\nu \approx \frac{8\pi kT\nu^2}{c^3} \tag{2}$$

The peak of u_ν or u_λ is simply related to T:

$$h\nu_{\mathrm{max}} = 2.8kT \qquad \lambda_{\mathrm{max}}T = 2.9 \text{ mmK} \tag{3}$$

This leads to a specific intensity of this radiation field:

$$I_\nu = \frac{u_\nu c}{4\pi} = \frac{2\nu^2}{c^2} \tag{4}$$

The flux density through a flat surface is then:

$$f_\nu = \pi I_\nu \tag{5}$$

and the total flux is:

$$f = \frac{2\pi^5 k^4}{15c^2h^3}T^4 = \sigma T^4 \tag{6}$$

where this question uses the definition of the Stefan-Boltzmann constant σ.

Under many conditions, the photon distribution is not in thermal equilibrium, and therefore differs from the Planck spectrum. Such spectra reveal details about the specific physical interactions the photons are undergoing that yield important clues about the conditions of the emitting material.

Lines, or sharp features in the spectrum, can be created due to discrete energy level differences between atomic or molecular states. *Emission lines* occur when photons are released (and escape the medium) from a downwards transition. *Absorption lines* occur when photons coming toward the observer incite upwards transitions in intervening material. We usually speak of these lines as separate from the *continuum* spectrum, so that emission is in addition to the continuum and absorption is absorbing the continuum; however what we mean by "continuum" varies somewhat depending on context. The interpretation of these features of spectra comprise a large portion of optical astrophysics, and in this section we discuss only a couple of examples to introduce nomenclature.

Lines can be quantified in several ways:

- Both emission and absorption lines have some intrinsic width, which can be expressed as a full width half maximum (FWHM) or otherwise. Physically, this width can come from a combination of the intrinsic transition width, pressure-induced width, and Doppler velocity width (due to thermal or other motions).

- Emission lines have a total flux or luminosity that can be associated with them by subtracting an estimate of the continuum and integrating over wavelength or frequency (e.g., with units erg s^{-1} for luminosity). There is an equivalent quantity for absorption lines (the flux or luminosity of the continuum that is absorbed) but this is rarely referred to.

- Both emission and absorption lines have an *equivalent width* (EW), which is usually expressed in units of wavelength, and is the flux emitted or absorbed divided by a continuum flux density estimate (e.g. f_λ) at the location of the line. Sometimes we use the convention that positive EW indicates absorption and negative indicates emission; sometimes the opposite.

An illustrative example is the atomic transition sequences of Hydrogen. These transitions are determined by the well-known Bohr sequence:

$$E_n = -\frac{e^4 m_e}{2\hbar^2}\frac{1}{n^2} \tag{7}$$

The transitions between these states and important in stellar atmospheres (typically in absorption) and in interstellar medium emission (typically in emission). The lowest state is $E_0 = -13.6$ eV, which corresponds to a photon of 912 Å; higher frequency photons will ionize H. The transitions between states are classified according to the lower state as follows:

Table 1: Wavelengths (vacuum $\AA$) of hydrogen transitions between n and m

Series	Lower state (n)	α ($m = n + 1$)	β ($m = n + 2$)	γ ($m = n + 3$)	δ ($m = n + 4$)
Lyman (Ly)	0	1216	1026	973	950
Balmer (H)	1	1216	1026	973	950
Paschen (Pa)	2	1216	1026	973	950

Another important process, especially in the radio and X-ray domain, is *bremsstrahlung*, the radiation due to the acceleration of charges. An astrophysical plasma emits *thermal bremsstrahlung* due to Coulomb accelerations of the electrons against each other. Under most conditions, at wavelengths less than about 1 m (frequencies greater than 1 GHz), this emission occurs under optically thin conditions. At low enough frequencies this process is optically thick and thus thermal (since $h\nu \ll kT$ in practice, $f_\nu \propto \nu^2$). At higher frequency but still at $h\nu < kT$ (typically in the cm-radio regime), the emission is optically thin. Free-free absorption at these wavelengths leads to $f_\nu \propto \nu^{-0.1}$. Above $h\nu > kT$, the Boltzmann cutoff leads to $f_\nu \propto \exp(-h\nu/kT)$. The calculation of thermal bremsstrahlung is somewhat complex.

In plasmas with significant magnetic fields, electrons spiraling around the magnetic fields yield synchrotron radiation due to their acceleration. The distribution of electron energies determines the shape of the resulting spectrum, which can often be approximated as a power law $f_\nu \propto \nu^{-\alpha}$, where α can range from 0 to over 2.

A final highly significant effect in the propagation of light across space is due to the effect of interstellar dust. Interstellar dust typically consists of silica grains and some carbonaceous grains, plus a small admixture of organic molecules like polycyclic aromatic hydrocarbons. The grains are typically less than a few tenths of a micron in size. Because of this, radio and infrared frequencies are not affected by dust very much. In general, the amount of extinction is wavelength dependent, with bluer frequencies experiencing more absorption and scattering; the dependence varies depending on the nature of the dust but is very approximately $\sim \lambda^{-1}$. In the ultraviolet through near infrared this causes "reddening" of the light. Meanwhile, at the very highest frequencies (X-ray and γ-ray) the photons pass through the dust (in fact, extreme UV and X-ray radiation can destroy dust).

As light from cosmic sources comes toward us, it can be scattered or absorbed in a number of ways other than atomic and molecular transitions, for example by interstellar dust, by plasma, by intervening radiation fields, or through other processes. We will discuss these as appropriate later.

2. Key References

- *Allen's Astrophysical Quantities*, Cox (2000), Chapter 5

3. Order-of-magnitude Exercises

1. As you can see when looking outside during the day, the Sun is neither very blue nor very red. Assuming it emits approximately as a blackbody, estimate the temperature of its surface.

2. Estimate the approximate temperature of a radiation field that will provide a substantial flux of photons to ionize hydrogen.

3. If you have a spectrograph with $R \sim 4000$, for what line-of-sight velocity dispersion is the intrinsic width of the line equal to the width due to the resolution? [We will learn later that depending on signal-to-noise ratio, velocities much smaller than the resolution are hard to measure.]

4. Galaxy clusters emit thermal bremstrahlung at energies $\nu > 1$ keV. What is the temperature necessary to do this?

5. The center of the Milky Way is very heavily extincted: by about 30 magnitudes in the V band. Approximately how much is that in the near-infrared K band?

   The magnitude difference of 30 corresponds to a factor of 10^{12} in luminosity. Using Equation **??**, this translates to 10^6 in distance. So Vega-like stars are visible (in principle) to about 8 Mpc.

6. Estimate the number of photons per second that enter your eye per second in visible light (4000–7000 Å) from a star with magnitude ~ 6 (about the faintest visible at a dark site). Assume a nighttime pupil diameter of 5 mm.

   In this wavelength range, Vega and AB magnitudes are about the same at the precision necessary here, so we don't have to worry about which version we are dealing with. So:

$$f_\nu \sim (3631 \text{ Jy})10^{-0.4m} \sim 14 \text{ Jy} = 1.4 \times 10^{-22} \text{ erg cm}^{-2} \text{ s}^{-1} \text{ Hz}^{-1} \tag{8}$$

   The flux density in the visible should be $f_\nu \Delta\nu$ where:

$$\Delta\nu = c\left(\frac{1}{4000 \text{ Å}} - \frac{1}{7000 \text{ Å}}\right) \sim 320 \text{ THz} \tag{9}$$

   And thus $f \sim 4 \times 10^{-8}$ erg cm^{-2} s^{-1}. Each photon has an energy (assuming $\lambda = 5500$ Å):

$$E = h\nu = (6.62 \times 10^{-27} \text{ erg Hz}^{-1})(550 \text{ THz}) \sim 4 \times 10^{-12} \text{ erg} \tag{10}$$

   So the flux of photons is:

$$\frac{\dot{N}}{A} = \frac{f_\nu \Delta\nu}{h\nu} \sim 10^4 \text{ s}^{-1} \text{ cm}^{-2} \tag{11}$$

   If $A \sim \pi r^2 \sim 0.2$ cm^2 then $\dot{N} \sim 2000$ s^{-1}.

7. How much does surface brightness dimming change the magnitudes per square arcsecond for a galaxy at redshift $z \sim 1$?

   The specific intensity is reduced by $(1+z)^4$. In magnitudes this is:

$$\Delta m = 2.5 \log_{10}(1+z)^4 = 10 \log_{10}(1+z) \sim 3 \text{ mag} \tag{12}$$

4. Analytic Exercises

1. For a Gaussian line spread function with a standard deviation σ, what is the FWHM? The FWHM is determined by:

$$\Delta\lambda = 2\ln(0.5) \tag{13}$$

2. Prove Equation **??**, based on the fact that photon density in phase space is conserved.

3. Based on Equation **??**, how is the angular diameter distance related to the luminosity distance?

5. Numerics and Data Exercises

1. Retrieve a spectrum of a star, a quasar, and a galaxy from the Sloan Digital Sky Survey. Plot each of them. These spectra are given in f_λ (per-Å) units. Convert one them to f_ν (per-Hertz) and plot it. Smooth one of them in f_λ with a Gaussian corresponding to $R \sim 100$ and plot it.

2. Plot D_L versus z based on the equations found in Hogg (1999), for a flat ΛCDM cosmology with $\Omega_m = 0.3$ and $H_0 = 70$ km s^{-1} Mpc^{-1}. Determine where the difference in inferred luminosity of an object would reach 1%.

3. Download the filter curve for the SDSS g and r bands. Calculate the observed g and r band magnitudes corresponding to a galaxy spectrum (say for some galaxy with $z < 0.1$). Note that this won't necessarily be the same as the magnitudes measured from the images, since the spectra are taken through 2- or 3-arcsec diameter fibers. Calculate the rest-frame $g - r$ color, and also what the K-correction would be for galaxies with this SED in the r-band between about $z \sim 0$ and $z \sim 0.25$. Download a sample of galaxies between about $z \sim 0$ and 0.25. Plot their $g - r$ colors versus redshift, together with the predicted colors of the galaxy you have a spectrum of.

4. **Would be nice to have radio, X-ray, other examples**

REFERENCES

Cox, A. N. 2000, Allen's astrophysical quantities

Hogg, D. W. 1999, astro-ph/9905116

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