Galaxies

1. Basics & Nomenclature

The basic phenomenology of galaxies is necessary to understand before embarking on their detailed study. In the broadest terms, galaxies are gravitationally bound collections of stars, gas, and dark matter. Their masses range from extremely small, a few 10^6 $M_\odot$ in dark matter, up to around 10^{13} $M_\odot$ in dark matter. Baryonic mass in the form of stars and cold gas is only about 5% of the dark matter mass, smaller than the cosmic mean fraction. The extent of stars and gas ranges from a few tenths of a kpc to 10s of kpc; the extent of the dark matter appears to be much larger. Galaxies the size of the Milky Way have number densities such that their mean separation is about 5 Mpc.

Theoretically, it appears that galaxies form near the centers of dark matter halos and subhalos that form as dark matter collapses gravitationally from the initial density perturbations. To zeroth order galaxy phenomenology can be explained by the relationship of galaxies to their host halos and subhalos.

In the optical, a critical quantity characterizing a galaxy is its luminosity. The luminosity is typically measured from light in an image associated with the galaxy, either by summing light with a circular or elliptical aperture or by fitting the amplitude of a model. Aperture fluxes can be defined in multiple ways. The most robust is the Petrosian definition used by the Sloan Digital Sky Survey. Most aperture fluxes need to be corrected for the effect of the point spread function, especially at small galaxy sizes. At faint fluxes, aperture fluxes defined to contain most of the expected light tend to be noisy.

Model fluxes are higher signal-to-noise and can be constructed to account for the point spread function and any inherent ellipticity in the model. Inaccuracy in the model can on the other hand result in systematic errors in the flux.

The *luminosity function* $\Phi(L)$ of galaxies characterize their luminosity distribution. A simple approximation to this function is the Schechter function:

$$\Phi(L)\mathrm{d}L = \left(\frac{\phi_0}{L_*}\right)\left(\frac{L}{L_*}\right)^{\alpha} \exp\left(L/L_*\right)\mathrm{d}L \tag{1}$$

where L_* is similar to the Milky Way luminosity and $\alpha \sim -1.3$ is the *faint end slope*. A better approximation is a broken power law at the faint end, which gets steeper at lower luminosities (e.g. Blanton et al. 2005). The luminosity function depends on band pass, because the colors of galaxies depend on luminosity. A similar function characterizes the stellar mass function.

Estimates of the luminosity function are based on counting galaxies as a function of luminosity. However, for realistic catalogs we do not have complete samples and all estimates must be corrected for incompleteness. The most common sort of incompleteness is to have a *flux-limited* sample, so that underluminous galaxies cannot be seen over a large volume. However, for color-selected surveys or surveys that probe high redshift the incompleteness can take more complicated forms. The completeness is also a function of galaxy surface brightness. The simplest method to correct for incompleteness is the $1/V_{\rm max}$ *method.* If for each galaxy we can calculate the maximum volume over which it could have still entered the sample, then we can estimate the luminosity function at some L as:

$$\Phi(L)\mathrm{d}L = \sum_i \frac{1}{V_{\rm max}} \tag{2}$$

for all galaxies i in a range $\mathrm{d}L$ around L. However, this technique assumes we have a sample of luminosity L galaxies spanning the full range of properties that might affect their selection. It also tends to be sensitive to the presence of large scale structure in the sample. Note that under some conditions, we can construct *volume-limited* samples which are statistically complete over some range of luminosities and some range of distances, which can be useful.

The *stellar mass function* $\Phi(M_*)$ characterizes their distribution in total stellar mass. Estimates of stellar mass depend on broad-band observations and/or spectroscopic observations combined with models of the underlying stellar population. Broadly speaking, the stellar mass is proportional to the light multiplied by a mass-to-light ratio. The mass-to-light ratio depends on the age, metallicity, dust content, and other properties of the stellar population, with the trend in all cases for the redder populations to have higher mass-to-light ratios. For example, Bell & de Jong (2000) find

$$\log_{10}\left(\frac{M}{L}\right) \approx -0.3 + 1.1(g-r) \tag{3}$$

for the stellar mass-to-light ratio in solar units. Stellar mass is defined as the current stellar mass in objects above the hydrogen burning limit, including remnants such as white dwarfs and neutron stars, but not including any mass returned through winds or supernovae to the interstellar medium.

As a function of redshift, the luminosity function and stellar mass function change. The stellar mass function in general needs to grow with cosmic time, since star formation proceeds within galaxies over time. The luminosity function however tends to decrease with cosmic time, since the rate of star formation decreases and the mass-to-light ratio tends to grow.

The spectral energy distributions of galaxies depend on their luminosity. The simplest characterization of these spectral energy distributions are from the broad-band color. At the highest luminosities, low redshift galaxies tend to be uniformly red, indicating an old stellar population. At the lowest luminosities, they tend to be primarily blue, indicating a young stellar popation, except in those cases where they are satellites of a larger galaxy. At intermediate luminosities (around L_*) both populations exist and the color distribution is said to be *bimodal.* The broad-band colors reflect the underlying spectrum, whose detailed shape can be interpreted in terms of the stellar and gas content of the galaxies.

The internal structure of galaxies also varies. Very broadly speaking, the stellar components of galaxies are:

- *disks*: an extended, flattened, rotating distribution of stars and (usually) gas. Disks often have spiral structure through them, with star formation concentrated on the spiral arms.
- *bulges*: generally puffy, velocity dispersion supported collections of stars near the centers of galaxies, thought to arise through the destruction of merging galaxies.
- *pseudobulges*: generally flatter, more exponential collections of stars near the centers of galaxies, thought to arise through internal processes within disks.
- *bars*: elongated structures near the centers of galaxies, thought to arise through dynamical instabilities of bulges or pseudobulges.
- *stellar halos*: extended distributions of stars, with a power law density profile.

The disks and bulges dominate the stellar mass budget integrated over all galaxies, with the other components each comprising a few percent of the total (e.g., **?**).

Disks tend to have an exponential radial profile when azimuthally averaged:

$$I(r) = I_0 \exp\left(-r/r_e\right) \tag{4}$$

where for the Milky Way $r_e \sim 3$ kpc. When observed at an *inclination* that is $i < 90^\circ$, the two-dimensional image is projected into an ellipse on the sky that can be characterized by an *axis ratio*, which if the disk were transparent and infinitely thin would have a straightforward relationship with the inclination.

Bulges have a range range of radial profiles that can be characterized by the Sérsic profile:

$$I(r) = I_0 \exp\left[-\left(r/r_e\right)^{1/n}\right] \tag{5}$$

where $n \sim 1 - -4$ but can be larger than that. $n = 1$ is the same as an exponential profile, and $n = 4$ is a special profile known as the *de Vaucouleurs* profile. Bulges are typically ellipsoidal or triaxial systems, seen at some angle, so that their axis ratios do not simply translate into an inclination.

The size of a galaxy is often quantified by the *half-light radius* or *effective radius*, which is the radius containing 50% of the estimated total light within a circular or elliptical annulus. Another common measure of size is D_{25}, which is the diameter of the isophote as which the surface brightness in B becomes 25 mag arcsec^{-2}; this definition is not suitable at high redshift because surface brightness is not conserved.

A related measurement is the surface brightness. For example, the *half-light surface brightness* is the mean surface brightness within the half-light radius. The *central surface brightness* is the surface brightness at the center, only measurable if the center is smooth relative to the resolution of the image.

Internal structure has traditionally been characterized by a measure of morphology. Definitions of galaxy morphology vary among investigators, but the broad classifications of elliptical (or E), lenticular (or S0), or spiral (ranging from Sa through Sd) are common.

Elliptical galaxies are red, puffy, and generally velocity dispersion supported. They lie on the red sequence in color space (but S0 and Sa galaxies are there as well). They are very much like bulges unencumbered by disks. They contain small amounts of cold gas, typically about 1% of the stellar mass, and also warm and hot gas. Their Sérsic indices n vary from around 1 at low luminosities (*dwarf ellipticals*, or *dE* galaxies) up to 4 or more for higher luminosities (*giant ellipticals*, sometimes *gE* galaxies). There is a rare class of low luminosity elliptical with $n \sim 4$, known as *compact elliptical*, or *cE*, galaxies.

Lenticular galaxies are red but disk like, though not as thin as true disks. They can be hard to distinguish from ellipticals when seen face on, but commonly have bars or rings, and their stellar distribution commonly has a distinct dropoff in its outer parts (i.e. the exponential profile effectively stops). They also have little gas.

Disk galaxies can be blue or red, but generally consist of thin disks of gas and stars (a few 100 pc thick). The gas is typically a mix of ionized, neutral, and molecular gas, the latter occurring in molecular clouds. Much ionized gas surrounds star forming regions that are often embedded in the molecular clouds, while warm ionized gas does exist elsewhere in the disks.

The centers of disk galaxies can consist of a range of interesting structures, including bars, bulges, and pseudobulges. High luminosity galaxies tend to have large bulges, and low luminosity galaxies weak ones. Spirals have a morphology classication that ranges from Sa to Sd, but there are different choices in different catalogs as to what the classification indicates. They generally are meant to capture the pitch angle of the spiral arms, their organizational structure, and the prominence of the bulge. The Reference Catalog classifications of de Vaucouleurs tend to emphasize the bulge, whereas the Revised Shapley-Ames Catalog of Sandage and Tammann depends more strongly on the spiral arms.

A common quantitative measure of morphology is the bulge-to-total ratio B/T, which is the result of fitting a bulge and disk model to a galaxy. By definition, this quantity ranges from zero to one, and pure disk galaxies lie at zero and elliptical galaxies lie at one. A model independent, related quantity is the *concentration*, defined by r_{90}/r_{50}, the 90% to 50% radius ratio.

The dark matter content of galaxies is revealed by their dynamics and in some cases by gravitational lensing. Elliptical galaxies are supported by a combination of rotation and velocity dispersion; interpreting the dynamical masses of elliptical galaxies is complex but achievable. The best evidence to date suggests that massive elliptical galaxies have high mass-to-light ratios, which indicates either a non-standard IMF that changes with stellar mass, or the presence of dark matter. Disk galaxies are primarily supported by rotation. Particularly in their outer parts, the gas dynamics is relatively straightforward to interpret and unambiguous signatures of dark matter are present.

Two major scaling relations quantify the relationship between luminosity and dynamics: The *fundamental plane* of giant elliptical galaxies, the relationship between size, luminosity, and velocity dispersion, which takes the approximate form:

$$r_e \propto \sigma^\alpha I_e^{-\beta} \tag{6}$$

which implies a plane in the three-dimensional space of the logarithms of these quantities. The exercises below explore the implications of this relationship (Bender et al. 1992).

The *Tully-Fisher law* disk galaxies near L_* relates their luminosities and their peak circular velocities, as measured with optical or radio observations (Tully & Fisher 1977). Since the colors of disk galaxies depend on luminosity, the slope of this relationship depends on the band that the luminosity is measured in. In the I-band, the relationship is equivalent to $L \propto v^{3.1}$, and has a scatter of 0.2–0.4 dex (depending on the sample used). At low luminosities, the velocity is independent of L (but is dependent on the total baryonic mass).

Luminous galaxies all seem to have a supermassive black holes at their centers, with evidence for the black holes from active galactic nucleus emission (called *Seyfert galaxies*) but also from dynamical signatures through stellar dynamics, reverberation mapping, and central masers.

2. Commentary

The description here relies on our best understanding of underlying physical properties, but these properties are imperfectly measured, especially with regards to inferred stellar populations. In the stellar population notes, the uncertainties are more fully explored.

Unusual galaxies exist in the universe: there are ellipticals with gas disks of with typically small but sometimes large masses, polar ring galaxies with gas disks or rings perpendicular to the stellar disk, galaxies with strong central star bursts, and other unusual cases. Of growing interest are the populations of very low surface surface brightness galaxies (not detectable by large surveys like SDSS) and ultra-compact dwarf galaxies (indistinguishable from stars by large surveys); it is unclear yet how common these cases are overall relative to more "typical" galaxies.

3. Key References

- *Physical Properties and Environments of Nearby Galaxies*, Blanton & Moustakas (2009)
- *A Concise Reference to (Projected) Sérsic Quantities*, Graham & Driver (2005)

4. Order-of-magnitude Exercises

1. Using the numbers in the text, estimate Ω_b and Ω_m.

2. Estimate the stellar mass surface density for a Milky Way-like disk galaxy.

5. Analytic Exercises

1. The fundamental plane (Equation 6) can be reexpressed in quantities with a clearer physical interpretation. The usual form is called κ-space (Bender et al. 1992):

$$\begin{aligned} \kappa_1 &\equiv \left[\log_{10}\sigma^2 + \log_{10} r_e\right]/\sqrt{2} \\ \kappa_2 &\equiv \left[\log_{10}\sigma^2 + 2\log_{10} I_e - \log_{10} r_e\right]/\sqrt{6} \\ \kappa_3 &\equiv \left[\log_{10}\sigma^2 - \log_{10} I_e - \log_{10} r_e\right]/\sqrt{3} \end{aligned} \tag{7}$$

where σ is in km s^{-1}, r_e is in kpc, and I_e is in $L_\odot$ kpc^{-2}.

(a) Show that κ_1 is proportional to the logarithm of the virial mass.

Starting from the virial relation, assuming all elliptical galaxies are homologous:

$$M \propto \sigma^2 r_e \tag{8}$$

where σ is the velocity dispersion at radius r_e and M is the mass inside that radius, taking the logarithm of both sides, we find:

$$\log_{10}(M) \propto \log_{10}(\sigma^2) + \log_{10}(r_e) \propto \kappa_1 \tag{9}$$

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(b) Show that κ_3 is proportional to the logarithm of the virial mass to stellar light ratio.

Since luminosity is proportional to surface brightness times radius squared, we can write the scaling relation of the mass-to-light ratio as

$$\frac{M}{L} \propto \frac{M}{I_e r_e^2} \propto \frac{\sigma^2}{I_e r_e} \tag{10}$$

where we have again invoked the virial theorem. Applying the logarithm to both sides we find:

$$\log_{10}\frac{M}{L} \propto \log_{10}(\sigma^2) - \log_{10}(r_e) - \log_{10}(I_e) \propto \kappa_3 \tag{11}$$

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(c) Bender et al. (1992) and others find:

$$\kappa_3 \approx 0.15\kappa_1 + 0.36. \tag{12}$$

How does the mass-to-light ratio depend on virial mass (or alternatively, on luminosity)?

Using the relation given by Bender et al. (1992) with the relation given above, we find:

$$\frac{M}{L} \propto 10^{\kappa_3} \propto 10^{0.15\kappa_1 + 0.36} \propto 10^{0.15\log_{10} M + 0.36} \propto M^{0.15}. \tag{13}$$

Alternatively, through some simply algebra we can write this relation in terms of luminosity:

$$\frac{M}{L} \propto L^{0.176} \tag{14}$$

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(d) Elliptical galaxies fall along the red sequence, which depends on absolute magnitude as:

$$g - r \approx \text{constant} - 0.03 M_r \tag{15}$$

This trend implies that the stellar mass to light ratio is a function of luminosity. Assuming that Equation 3 is constant as a function of luminosity, and a constant virial-to-stellar mass ratio, what would this trend imply for the virial mass-to-light ratio dependence on luminosity? Comment.

Bell & de Jong (2000) give a relation for the stellar mass-to-light ratio

$$\log_{10}(\frac{M_*}{L}) \approx 1.1(g - r) - 0.3 \tag{16}$$

and thus:

$$\frac{M_*}{L} \propto 10^{1.1(g-r)} \propto 10^{-1.1(0.03 M_r)} \tag{17}$$

In terms of the luminosity instead of absolute magnitude:

$$\frac{M}{L} \propto L^{0.0825} \tag{18}$$

Comparing this result to the result from part (c), we see that assuming a constant virial-to-stellar mass ratio does not explain the observed scaling of the virial mass-to-light ratio with luminosity in the fundamental plane. In particular, under this assumption, the virial mass-to-light ratios of giant elliptical galaxies would increase more slowly with increasing luminosity than they do from applying the virial theorem to the results of Bender et al. (1992). This discrepancy would be resolved if the virial-to-stellar mass ratio for these galaxies scaled with luminosity as

$$\frac{M_{virial}}{M_*} \propto L^{0.094} \tag{19}$$

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2. The Tully-Fisher relation can be approximated under similarly simple assumptions. Assume (a) a constant dynamical-to-stellar mass ratio, (b) that the stellar mass-to-light ratio scales with the colors as in Equation 3, with a color dependence on absolute magnitude of:

$$g - r \approx \text{constant} - 0.06 M_r \tag{20}$$

(again from Blanton et al. 2003a), and (c) that the observed dependence of surface brightness on luminosity for exponential-profile galaxies found in Blanton et al. (2003a) of $I \propto L^{1/4}$ in the i-band (see Figure 14 and Table 1). Under these conditions and using the virial relation, how does luminosity depend on circular velocity?

The Bell & de Jong (2000) dependence of stellar mass-to-light ratio color can be combined with the dependence of color on absolute magnitude:

$$\begin{aligned} \log_{10}\left(\frac{M_*}{L}\right) &\approx -0.3 + 1.1(g-r) \\ &\approx \mathrm{constant} - 0.066 M_r \end{aligned} \tag{21}$$

or equivalently:

$$\begin{aligned} \left(\frac{M_*}{L}\right) &\propto L^{(-0.066)(-2.5)} \\ \left(\frac{M_*}{L}\right) &\propto L^{0.17} \\ M_* &\propto L^{1.17} \end{aligned} \tag{22}$$

The stellar radius (the scale out to which the circular velocity is probed by an optical rotation curve) is related to the surface brightness and luminosity as:

$$L \propto I_* r_*^2 \propto L^{1/4} r_*^2, \tag{23}$$

where we have (somewhat shakily) used i-band relationship between I and L. Rearranging we find:

$$r_* \propto L^{3/8} \tag{24}$$

If the dynamical-to-stellar mass ratio is constant then the virial relation gives:

$$M_* \propto L^{1.17} \propto v^2 r_* \propto v^2 L^{3/8} \tag{25}$$

or equivalently:

$$L \propto v^{2.5} \tag{26}$$

This obviously is a rough approximation because the relationship between the dynamical and stellar mass depends a lot on the radius at which you define it.

6. Numerics and Data Exercises

1. Download a catalog SDSS Legacy Survey galaxies in the Main Sample between redshifts $0.01 < z < 0.2$. Estimate the shape of the luminosity function in the r-band using the $1/V_{\max}$ method. In calculating $V_{\max}$ and luminosities, ignore both K-corrections and evolution of the population; a precise estimate would require estimating both, but in the r-band it is approximately the case that these effects cancel. Also account only for the redshift selection effects, and for the angular selection effects simply assume an effective area of 7500 square degrees.

2. Using a Schechter function approximation to the luminosity function in the optical (e.g. from the r-band as found in Blanton et al. (2003b)) compare to the predicted mass function of halos (e.g. from Appendix C of Tinker et al. 2008). Specifically, find and plot the (zero-scatter) relationship between L and $M_{\rm halo}$ that satisfies the abundance-matching requirement:

$$\Phi\left(> L(M_{\rm halo})\right) = \Phi\left(> M_{\rm halo}\right). \tag{27}$$

3. Download the catalog of galaxies from the NASA Sloan Atlas between redshifts $0.01 < z < 0.05$. Plot their absolute magnitude vs. color. Select several galaxies along the red and blue sequences, and download and show their color images. For the same galaxies, download from the SDSS database their spectra. Zoom in on the Hα region and the 4000 Åregion. Comment on the major differences between the blue and red galaxies.

4. With the NASA Sloan Atlas, for red and blue galaxies separately, plot the distribution of their axis ratios b/a. Show the color images for several high and low axis ratio cases. Choose galaxies (red or blue) with $n < 2$ and plot their color vs. axis ratio — can you explain the correlation that you see?

5. For five red sequence galaxies from the NASA Sloan Atlas with between about 10^9–10^{11} $M_\odot$, download their spectra. Choose bright galaxies (e.g. $m_r < 16$) to maximize signal-to-noise ratio. Plot the spectra around the region of the Mg b signature, which is an important absorption line in red giant stars (which dominate the light of red galaxies with old stellar populations). Normalize the spectra so you can compare the widths of the absorption line. If you assume the line is intrinsically narrow, can you determine the velocity dispersion of the stars in the galaxy?

6. Install the `sdss-marvin` package to get MaNGA data within Python. Search for MaNGA IFU observations of several nearly edge-on galaxies on the blue sequence between 10^9–10^{11} $M_\odot$. Plot the rotation curve for each one just by plotting the velocity as a function of position along the major axis. How does the maximum rotation velocity depend on stellar mass?

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