Radiative Processes in Astrophysics / Problem Set #6
Due March 31, 2021

1. Rybicki & Lightman problem 4.3. Feel free to use the solutions to guide your work, but it is worth working this problem through fully!

2. The essential features of the synchrotron spectrum can be derived from the fact that for electrons with high energy ($\gamma \gg 1$) the "pulse shape" seen as the beamed emission sweeps by is defined by a function that depends on angle θ only in the combination $\gamma\theta$. This has the consequence that no matter the width of the pulse in time Δt_A, the shape is always the same. That means the spectrum of the emission is characterized purely by some critical frequency $\nu_c \sim 1/\Delta t_A \propto \gamma^2$, which has a number of important consequences described in class.

   The scattered power can be written as:

$$\frac{\mathrm{d}P}{\mathrm{d}\Omega} \propto \frac{1}{(1-\beta\cos\theta)^4}\left[1 - \frac{\sin^2\theta\cos^2\phi}{\gamma^2\,(1-\beta\cos\theta)^2}\right] \tag{1}$$

   Show in the limit that $\gamma \gg 1$ and for angles "in the beam", i.e. $\theta < 1/\gamma$ and $1/\gamma \ll 1$, that this power is a function of θ only through the combination $\gamma\theta$.