Radiative Processes in Astrophysics / Problem Set #9
Due May 13, 2021

1. Stars form in molecular clouds as the cooling of gas leads to fragmentation of cold cores and ultimately collapse into gravitational bound objects. A major sources of cooling is CO rotational line emission. You may wonder why it is CO and not the much more abundant H_2, and it turns out there are good reasons.

   (a) Estimate the energies and frequencies associated with the first rotational transition and the first vibrational transition of H_2. To make your life easier later, write down their scaling with the reduced mass of the molecule. Remember that $\Delta J = \pm 1$ are forbidden.

   (b) What are the populations for the two upper states relative to the ground state if the populations are in thermodynamic equilibrium with the gas, for example through collisions?

   (c) The Einstein A coefficient for the J 1—0 transition in H_2 is $\sim 3 \times 10^{-11}$ s^{-1} and for the v 1—0 transition is $\sim 2.5 \times 10^{-7}$ s^{-1}. Assume you are considering a cloud which is optically thin in these lines; then transitions from these upper states yield photons that escape the cloud and therefore cool the gas. Assuming the gas has a temperature $\sim$ 10K, which kind of transition (from the first vibrational or first rotational state) will dominate the cooling?

   (d) How long will it take for the gas to cool to $\sim$ 5K, to an order of magnitude? Assume all the gas is in molecular hydrogen form.

   (e) The Einstein A coefficient for the J 1—0 transition in CO is $\sim 2 \times 10^{-7}$ s^{-1}, and the typical (number) ratio of CO to H_2 is 6×10^{-5}. How does the presence of CO change the cooling time? Assume only the first excited rotational state matters (this will underestimate the cooling rate by a factor two or so).

2. Consider Figure 1, which shows a spectrum which includes absorption from warm CO gas in our galaxy. Remember that CO's ground state is Σ, i.e. has no net orbital angular momentum of the electrons, it has no $\Delta J = 0$ transitions during a rotational-vibrational transition.

(a) Argue on the basis of the energies of these absorption lines that they involve vibrational transitions.

(b) The missing Q branch location at the center locates the energy difference between two adjacent vibrational states (with $\Delta J = 0$. Assuming the Einstein A coefficients do not depend very much on the particular rotational states involved, use the figure to make a very rough estimate of the temperature of the CO gas.

3. Dust grains in the interstellar medium are heated by starlight and cool down through thermal emission. They have some absorption efficiency $Q_{\rm a}(\nu)$ that defines how well they absorb at a given wavelength, which is also the efficiency with which they emit.

   (a) If the energy density of starlight is written as $u_{\nu *}(\nu)$, and the dust grains are spherical with a radius a, write the formula for the heating rate of a dust grain. This should involve an integral over ν.

   (b) What is the cooling rate due to thermal emission from the surface of the dust grain?

   (c) If you assume that $Q_{\rm a}(\nu) = 1$ (a constant), and $u_* = 10^{-12}$ erg cm^{-3} (a typical value expected in the interstellar medium) then find the equilibrium temperature of the grains.

   (d) In fact, $Q_{\rm a}(\nu) \propto \nu^\beta$, where $\beta \sim 1$–2 (actually it is more complicated than a power law). Do you expect the actual grain temperature to be larger or smaller than predicted in the previous question?

   (e) Potentially collisions with gas particles could also heat the grains to the gas temperature (which is typically higher than the dust temperature—they are out of equilibrium for reasons that will become clear in this problem!). For a number density n of some gas species at temperature $T_{\rm gas}$ estimate the rate of collisions with a dust grain of radius a. The average energy exchange should be $\sim k(T_{\rm gas} - T_{\rm dust})$ for a collision that is at least somewhat inelastic. For the u_* given above, $T_{\rm gas} = 100$K, and $n = 10$ cm^{-3}, what is the relative contribution of collisional and radiative heating? Explain why this means that the dust is not in thermodynamic equilibrium with the gas.

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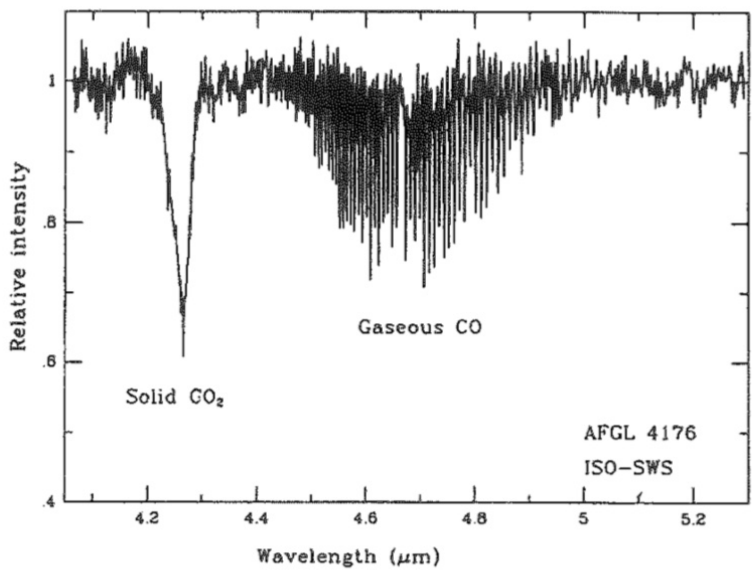

Fig. 10.9. Spectra obtained towards the massive young stellar object AFGL 4176 using the Infrared Space Observatory (ISO). The strong, largely structureless band absorption at 4.27 μm is due to solid CO_2, whereas the characteristic vibration–rotation P- and R-branch structure between 4.4 and 4.9 μm is due to the presence of warm, gaseous CO along the line of sight. [Reproduced from E.F. van Dishoeck, in *The Molecular Astrophysics of Stars and Galaxies*, eds. T.W. Hartquist and D.A. Williams (Clarendon Press, Oxford, 1998).]

Figure 1: From Tennyson, *Astronomical Spectroscopy*