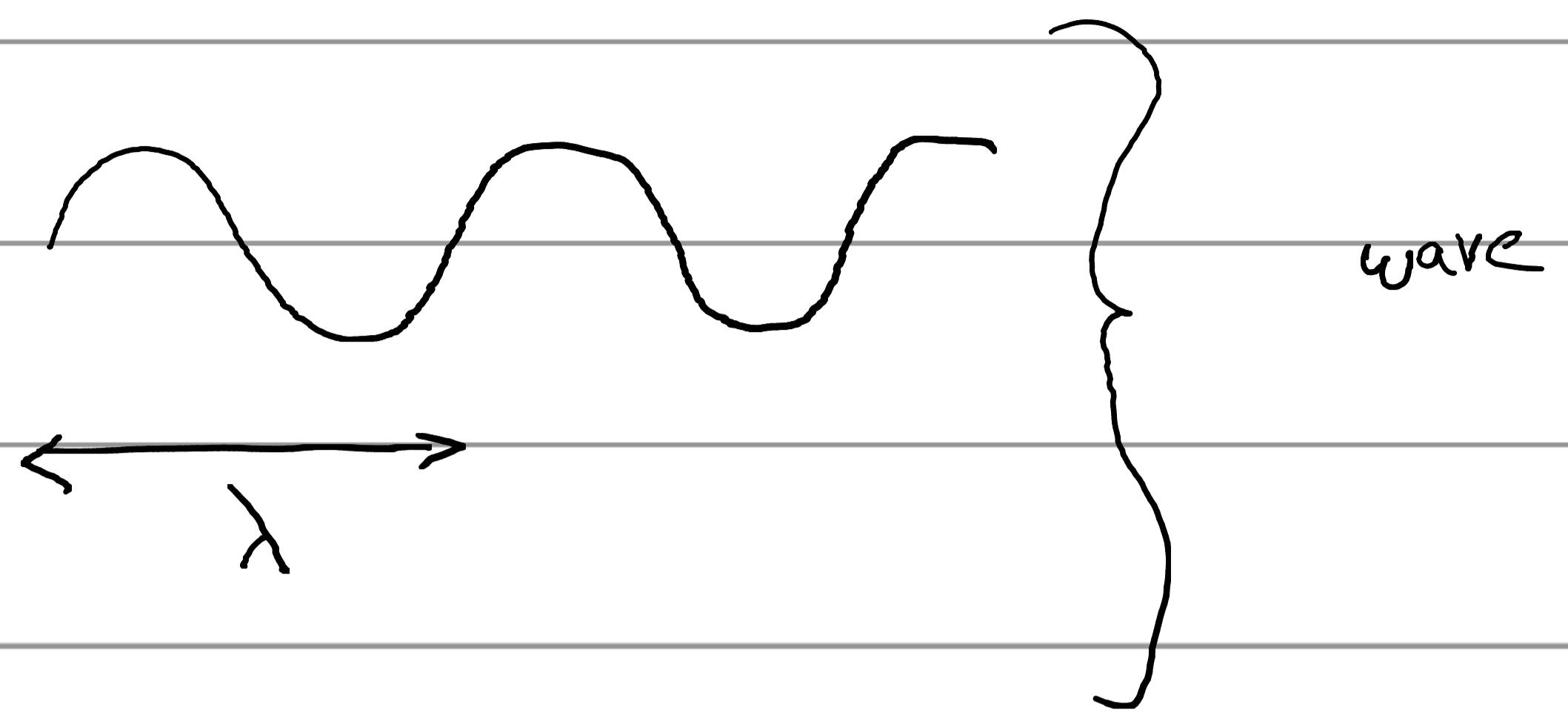


The Electromagnetic Spectrum

$$\lambda\nu = c$$

$$c = 2.99792 \times 10^{10} \frac{\text{cm}}{\text{s}}$$



$$E = h\nu$$

$$h = 6.63 \times 10^{-27} \text{ erg s}$$

↑
Planck's constant

} particle
(photon)

We will find it useful to characterize a photon's energy

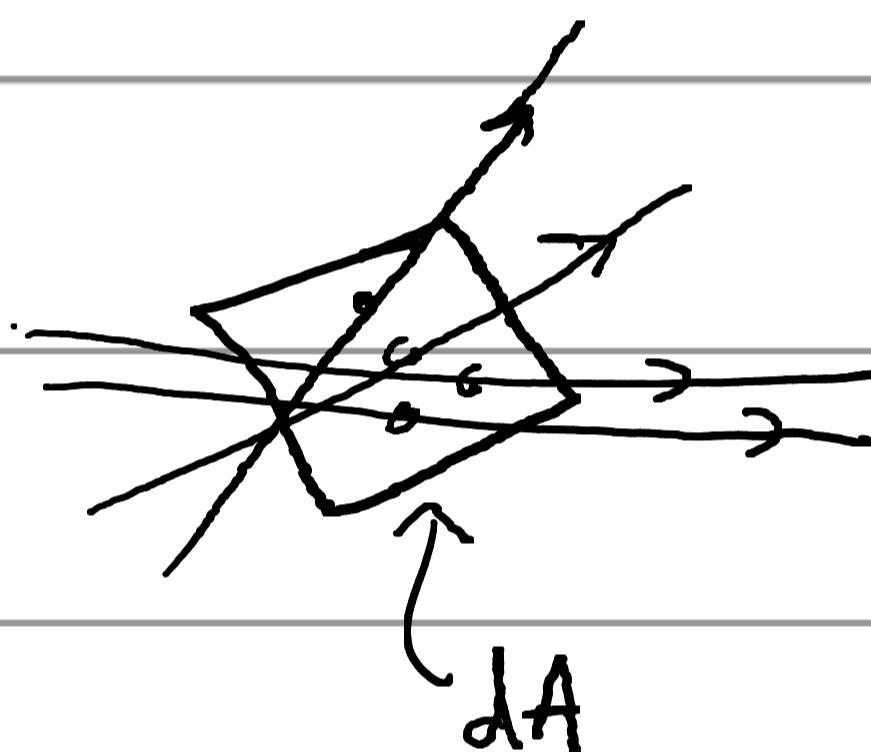
in terms of a temperature $T \sim E/k \sim h^2/k$

where $h = 1.38 \times 10^{-16} \text{ erg K}^{-1}$,

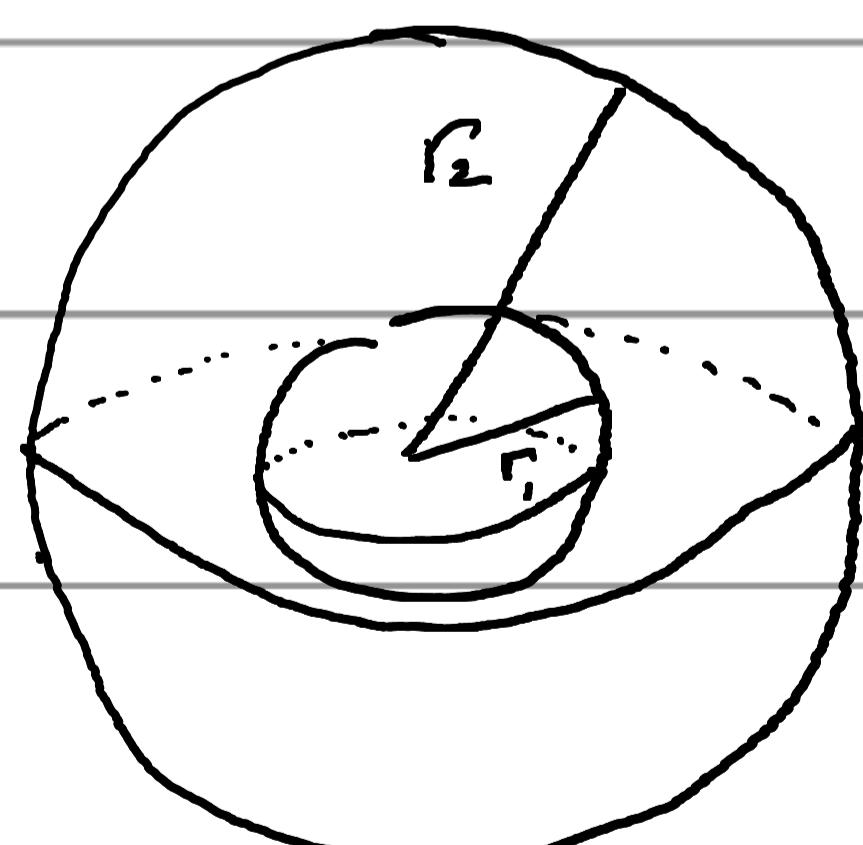
Radiative Flux

When considering propagation of light in systems whose size is $\gg \lambda$, then light travels in straight lines in homogeneous media. We can characterize it with an energy flux:

$$F = \text{erg s}^{-1} \text{ cm}^{-2} = \frac{dE}{dA dt}$$



For an isotropic source energy conservation requires:



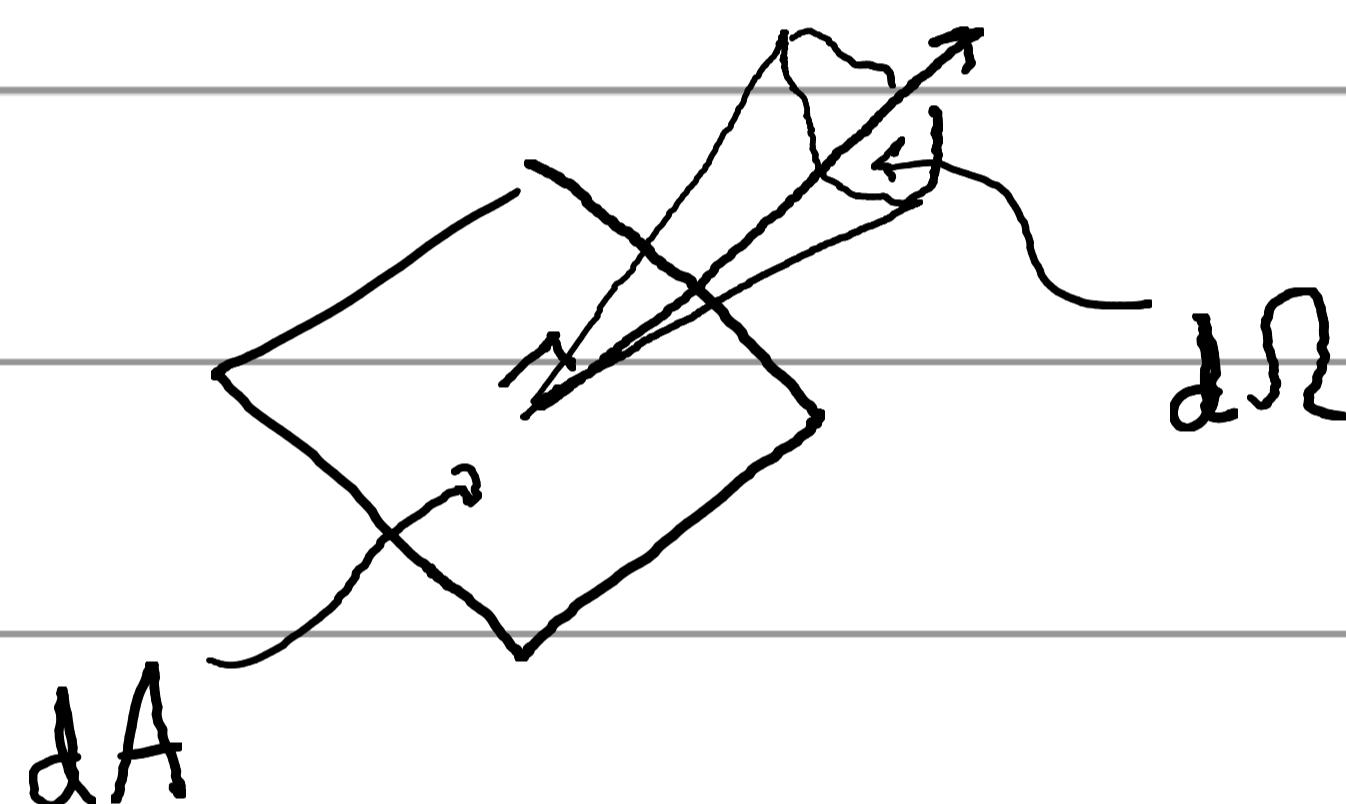
$$F_1(4\pi r_1^2) = F_2(4\pi r_2^2)$$

$$\therefore F \propto \frac{1}{r^2}$$

In fact: $F = \frac{L}{4\pi r^2}$ where $L = \frac{dE}{dt}$

Specific Intensity

"Flux" is defined in terms of all light rays passing through a surface. But we are often interested in radiation from a particular direction. So we consider rays with $d\Omega$ of that ray passing through an area dA normal to it!



and define the specific intensity:

$$I_\nu = \frac{dE}{dA dt d\Omega d\nu} \quad \leftarrow \begin{matrix} \text{also per unit} \\ \text{frequency} \end{matrix}$$

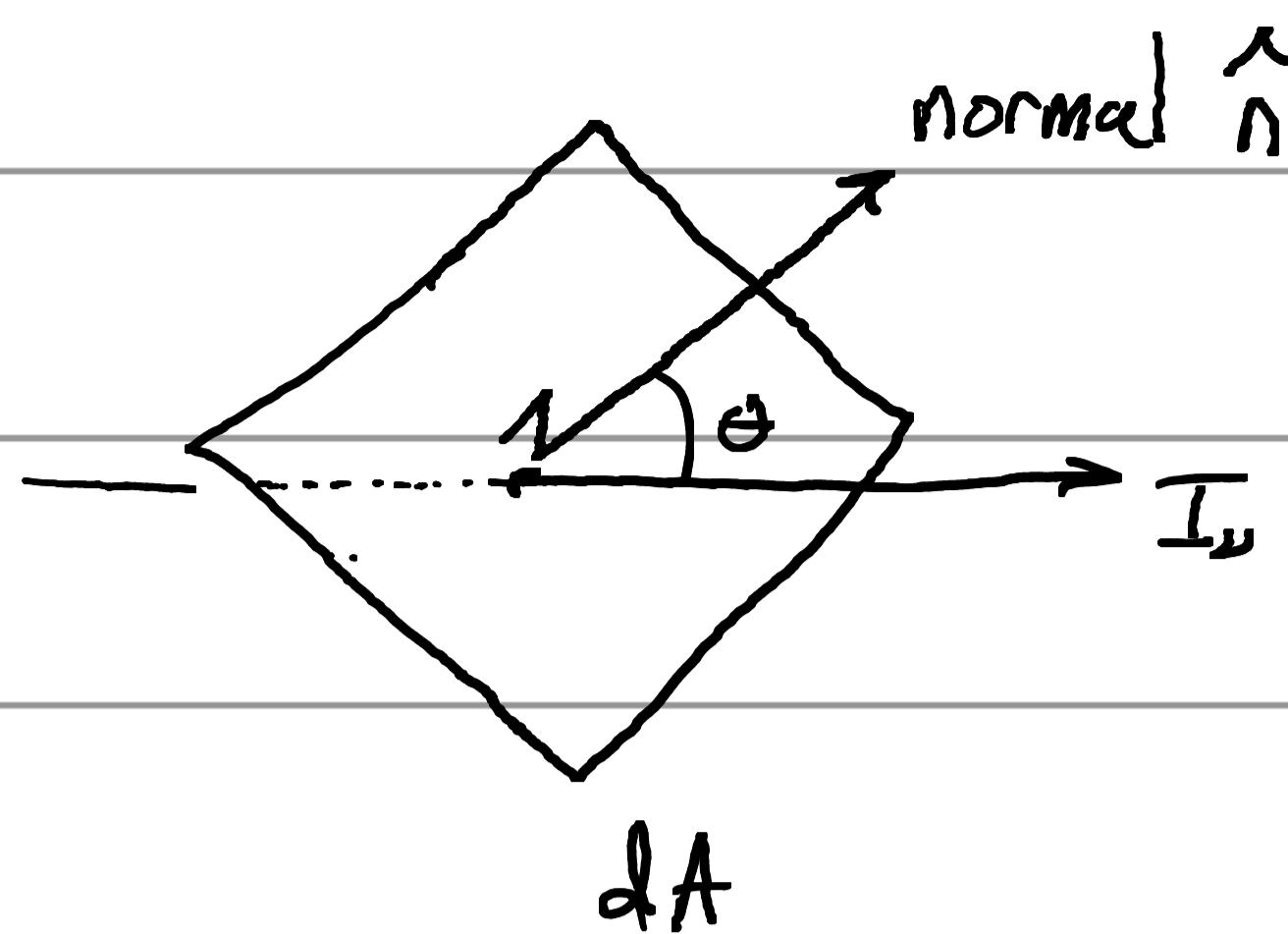
in units $\text{erg s}^{-1} \text{cm}^{-2} \text{ster}^{-1} \text{Hz}^{-1}$

Or alternatively: I_λ in $\text{erg s}^{-1} \text{cm}^{-2} \text{ster}^{-1} \text{\AA}^{-1}$

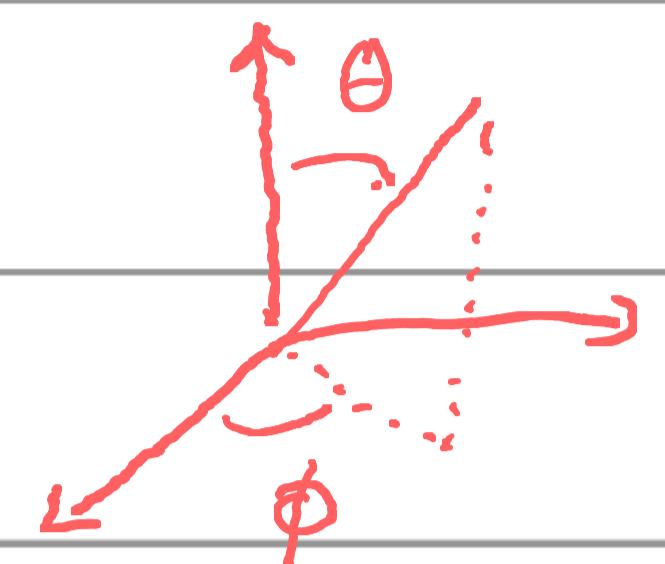
where: $I_\lambda |d\lambda| = I_\nu |d\nu| \rightarrow I_\lambda = \left| \frac{d\nu}{d\lambda} \right| I_\nu = \frac{c}{\lambda^2} I_\nu = \frac{\nu^2}{c} I_\nu$

Net Flux

Consider again:



$$d\Omega = \sin \theta d\theta d\phi$$



$$dF_v = I_v \cos \theta d\Omega \leftarrow \text{for a given direction}$$

$$F_v = \int d\Omega \cos \theta I_v \leftarrow \text{integrated over all directions}$$

Net momentum flux

$$\text{For a single photon } p = \frac{E}{c}$$

Therefore:

$$dp_v = \frac{dF_v}{c} \cos \theta \quad \text{momentum normal to } dA$$

$$P_v = \frac{1}{c} \int d\Omega I_v \cos^2 \theta \quad \left(\frac{g \text{ cm}}{\delta^2} \right) \cdot \text{cm}^{-2} \cdot \text{Hz}^{-1}$$



one factor for flux of photons
a second for projection $\hat{q} \cdot \hat{n}$

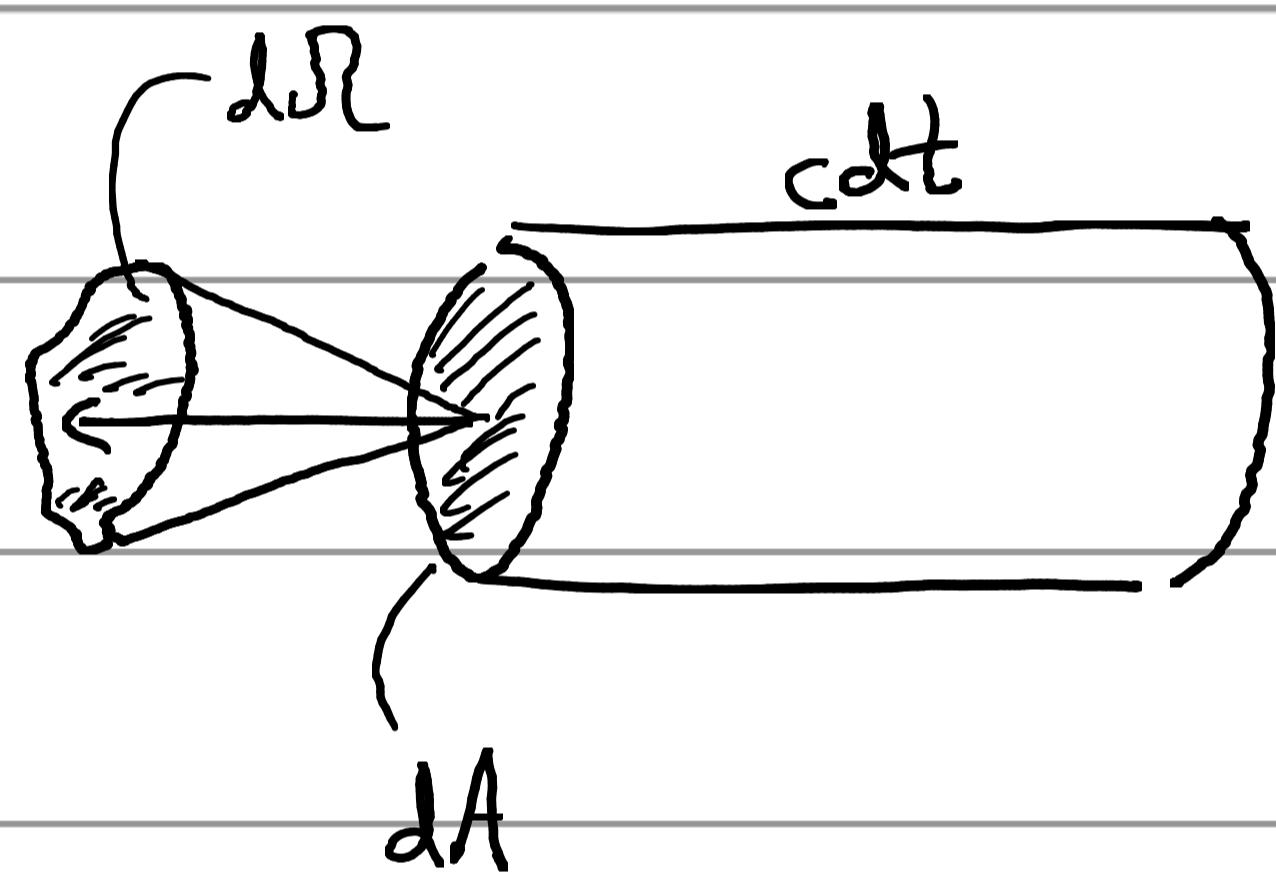
Radiative Energy Density

$$u_\nu = \frac{dE}{dV d\nu}$$

To relate this to other quantities consider

$$u_\nu(\theta, \phi) = \frac{dE}{dV d\nu d\Omega} = \frac{dE}{c dA dt d\nu d\Omega} = \frac{I_\nu}{c}$$

" I_ν



← consider dV like this

$$\text{then } dV = cdt dA$$

Therefore:

$$u_\nu = \int d\Omega u_\nu(\theta, \phi) = \frac{1}{c} \int d\Omega I_\nu(\theta, \phi)$$

We can define the mean intensity $J_\nu = \frac{1}{4\pi} \int d\Omega I_\nu(\theta, \phi)$

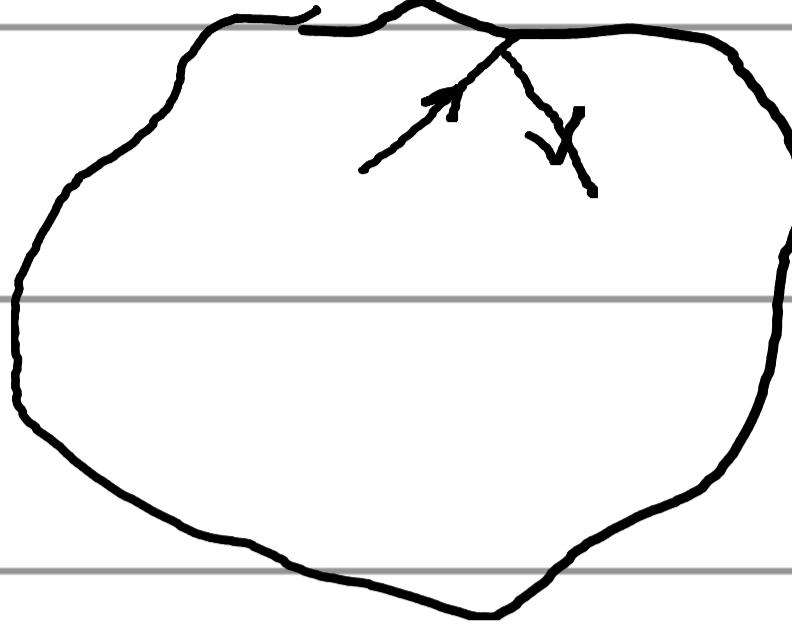
and then:

$$u_\nu = \frac{4\pi}{c} J_\nu$$

The total energy density is: $u = \int d\nu u_\nu$

Radiation Pressure

Imagine a reflecting enclosure:



upon reflection
momentum exchange
 \perp to surface is
twice momentum
flux

$$P_\nu = \frac{2}{c} \int_{2\pi} d\Omega I_\nu \cos^2 \theta \quad \text{i.e. integrated over half-sphere}$$

If radiation field is isotropic then $I_\nu = J_\nu$

$$P = \int d\nu P_\nu = \int d\nu \frac{2}{c} \int d\Omega I_\nu \cos^2 \theta$$

$$= \frac{2}{c} \underbrace{\int d\nu J_\nu}_{\text{constant}} \underbrace{\int d\Omega \cos^2 \theta}_{\text{constant}}$$

$$\frac{4c}{4\pi}$$

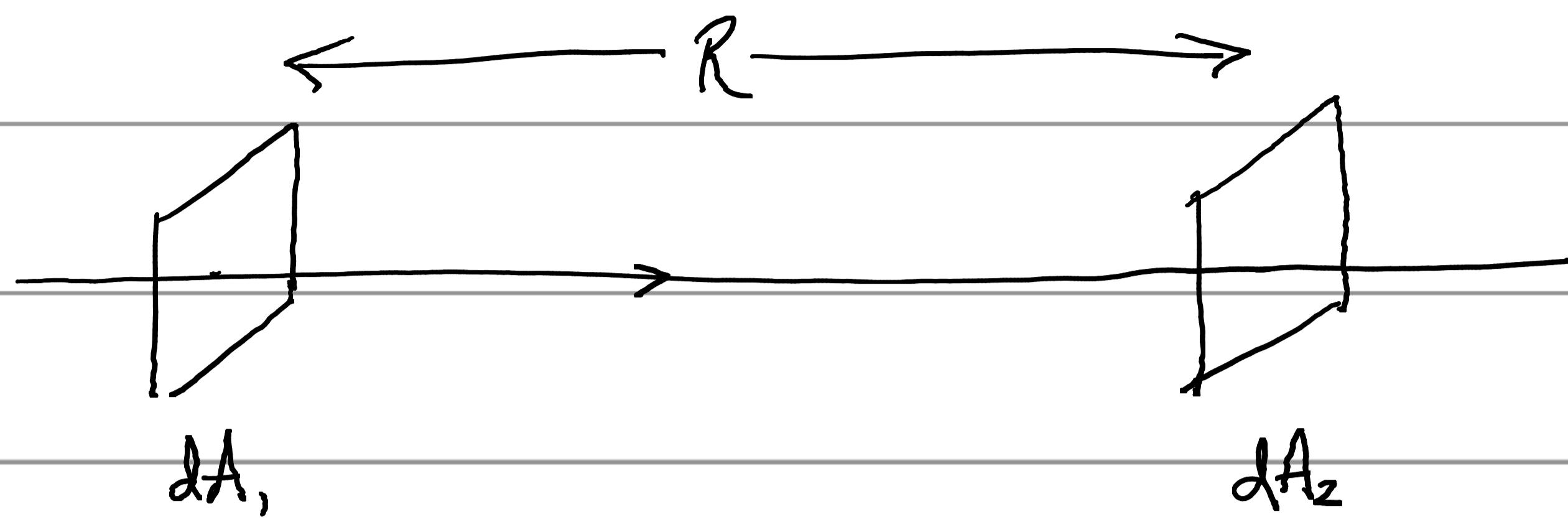
$$\int_0^{2\pi} d\phi \int_0^{\pi} d\theta \sin \theta \cos^2 \theta$$

$$2\pi$$

$$- \frac{1}{3} \cos^3 \theta \Big|_0^{\pi/2} = \frac{1}{3}$$

$$\boxed{P = \frac{1}{3} u}$$

Conservation of Specific Intensity along Ray Paths



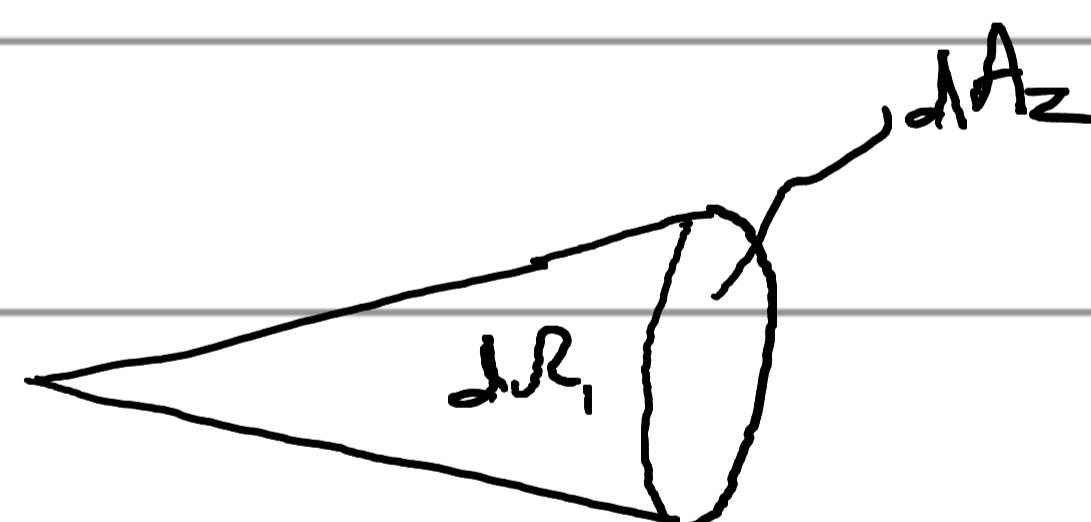
$$I_{\nu_1} = \frac{dE_1}{dA_1 dt d\Omega_1 d\nu}$$

$$I_{\nu_2} = \frac{dE_2}{dA_2 dt d\Omega_2 d\nu}$$

Construct dA_1 & dA_2 such that all rays through dA_1 pass through dA_2 and vice-versa (possible for the differential range of angles $d\Omega_1$ & $d\Omega_2$).

Then : $dE_1 = dE_2$

$$d\Omega_1 = \frac{dA_2}{R^2}$$



$$d\Omega_2 = \frac{dA_1}{R^2}$$

$$d\nu_1 = d\nu_2 \quad \rightarrow \quad I_{\nu_1} = \frac{dE}{dA_1 dA_2 dt d\nu} = I_{\nu_2}$$

A special case of Liouville's theorem.

In a general relativistic context this still holds

except γ can change. One can show:

$$I_\nu \gamma^{-3} = \text{constant}$$

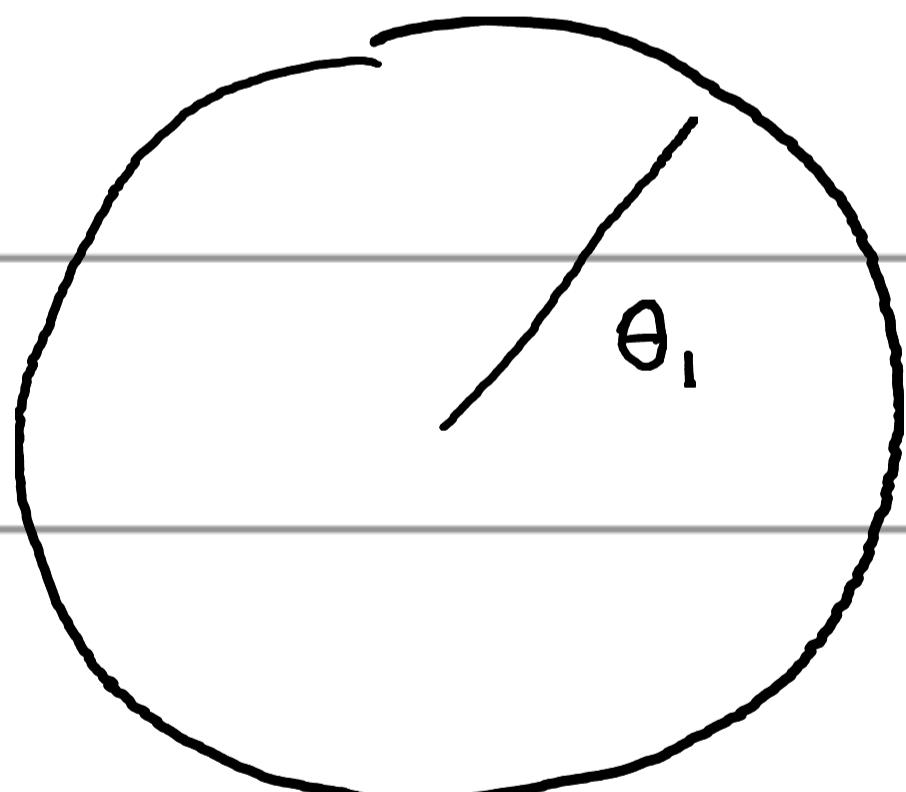
E.g. if there is emission at high z , $\gamma_0 = \frac{\gamma_e}{(1+z)}$

and this reduces the observed specific intensity.

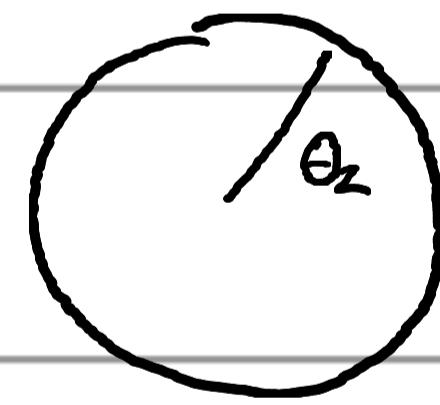
Note also that $I_\nu = \text{constant}$ makes sense given that

$F_\nu \propto 1/r^2$. Imagine a disk of size D , with constant I_ν :

at r_1



at r_2



$$\Omega_1 = \pi \theta_1^2 = \pi \left(\frac{D}{r_1}\right)^2$$

$$\Omega_2 = \pi \theta_2^2 = \pi \left(\frac{D}{r_2}\right)^2$$

$$f_1 = \frac{I_\nu \pi D^2}{r_1^2}$$

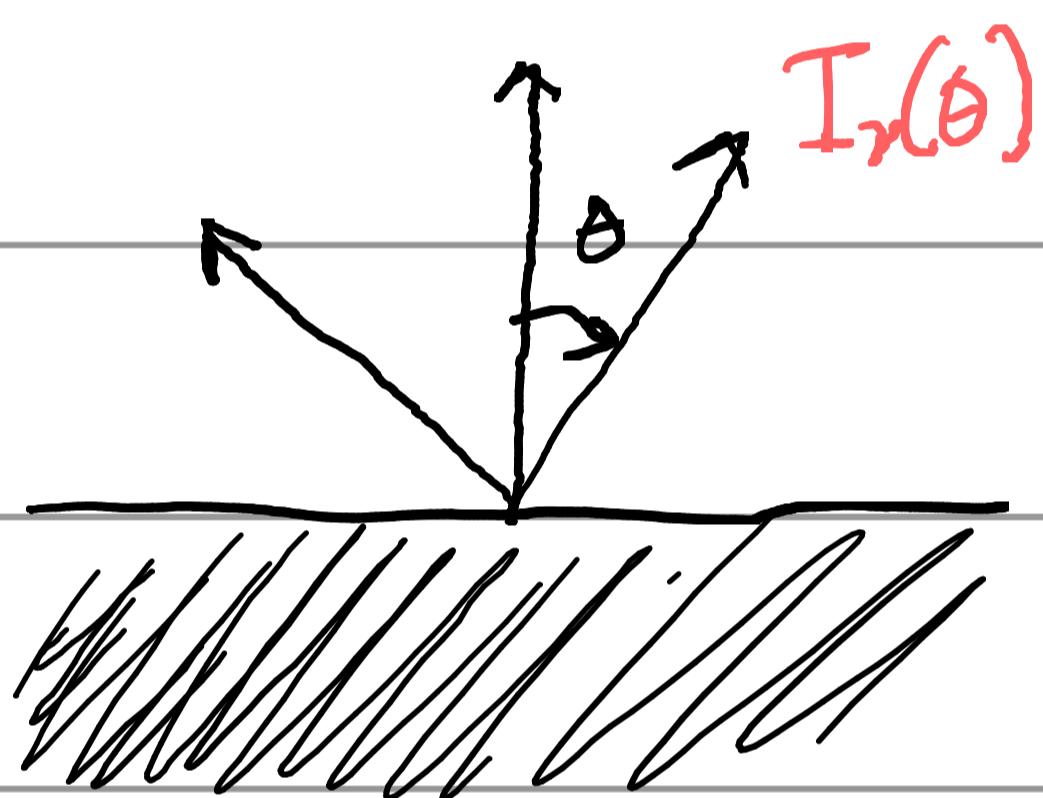
$$f_2 = \frac{I_\nu \pi D^2}{r_2^2}$$

$$\text{i.e. } f \propto \frac{1}{r^2}$$

Uniformly Bright Sphere

Now instead of a disk imagine a sphere for which all rays have the same I_r . We will see later that this applies to a perfect blackbody.

Consider the flux at its surface:



$$F = \int_{2\pi} d\Omega \cos\theta I = I \int_0^{2\pi} d\phi \int_0^{\pi/2} d\theta \sin\theta \cos\theta = 2\pi I \left[-\frac{1}{2} \cos^2\theta \right]_0^{\pi/2}$$

$$F = \pi I$$