Radiative Processes in Astrophysics / Problem Set #8
Due May 3, 2021

1. In Shengqi Yang's PhD defense last week she discussed the IR lines within the ground configuration of OIII, which is $1s^2 2s^2 2p^2$. For the IR transitions within the ^{3}P term, she considered, and the plots of the OIII transitions within this configuration always show, just the 52 μm line (between $J = 2$ and $J = 1$) and the 88 μm line (between $J = 1$ and $J = 0$). But there also is a potential transition between $J = 2$ and $J = 0$. Determine what type of transition that third one is, i.e. electric dipole, electric quadrupole, or magnetic dipole, and explain why in terms of the selection rules. Show how the spacing between the three transitions results from Landé's interval rule for spin-orbit interactions. If you want to see why the third line is usually omitted you can look it up using the line lists on NIST, and you will see why Shengqi is safe in ignoring the transition with current technology.

2. Also in the same presentation by Dr. Yang (as she is now!) she discussed the variation of the line ratio between 52 μm and 88 μm as a function of electron density, showing a transition occurring somewhere in the range $n_e \sim 10^2$–10^3 cm^{-3}. It is this variation in the line ratio that allows a constraint on n_e.

   (a) Explain qualitatively what is going on—i.e. why is there a special density at which this line ratio is likely to change? How will that density depend on the q and A values among the three levels?

   (b) Dr. Wang presented constraints on metallicity and electron density; but she did not talk about the temperature of the gas (i.e. the electron temperature). Argue why the relative populations of the levels of the ^{3}P term will not depend on temperature, justifying why she didn't talk about it. (Remember OIII only exists in gas that is ionized, either collisionally or photoionized).

   (c) Write the equations for the balance between the three energy states, in terms of the number densities n_0, n_1, and n_2, ad in terms of q_{ij} and A_{ij}. You should end up with a homogeneous linear system of three equations. I'm not asking you to solve the system fully (though it can be done). Also, you can leave in factors like q_{02} and q_{20}; i.e. you don't need to use the relationship between those two rates imposed by detailed balance considerations.

(d) Instead, let's think about the low density limit, when $n_e \to 0$. Use the equations for n_2 and n_1 in order to find two equations, one for n_2/n_0 and one for n_1/n_0, in terms of each other. Use the assumption that n_e is small to find approximations for n_2/n_0 and n_1/n_0 to first order in n_e (it should be a very simple formula for each!). Finally, determine what the relative line flux will be between the $J = 2$ to $J = 1$ vs. the $J = 1$ ro $J = 0$ transition under these conditions.

(e) In the high density limit, we can assume the levels will be kept in thermodynamic equilibrium with the electrons. Determine the relative line flux in this case (remember part (b), note that the multiplicities of each state will come into the calculation, and also leave things in terms of q and A when necessary!).