Radiative Processes in Astrophysics / Problem Set #5
Due March 17, 2021

1. The Rosseland mean absorption is the applicable mean absorption throughout most of a star:

$$\alpha_R^{\rm ff} = \frac{\int {\rm d}\nu \partial B_\nu/\partial T}{\int {\rm d}\nu \, (1/\alpha_\nu^{\rm ff}) \, \partial B_\nu/\partial T} \tag{1}$$

   Use the formula for free-free absorption:

$$\alpha_\nu^{\rm ff} \propto n_e n_i \frac{1}{\nu^3} \left[1 - \exp(-h\nu/kT)\right] T^{-1/2} \tag{2}$$

   to derive the dependence of $\alpha_R^{\rm ff}$ on temperature. You do not have to derive the coefficient of the dependence. Note there is an annoying integral you will encounter, whose evaluation you do not have to do explicitly. This dependence is known as *Kramer's Opacity Law* and is relevant in low-to-medium mass stars.

2. Consider a massive cluster of galaxies, with mass $M \sim 10^{15}\ M_\odot$ (total) and radius $R \sim 1$ Mpc. Assume the cluster has the cosmic baryon fraction ($\sim 15\%$) and that most of the gas is distributed in a large spherical region the size of the cluster.

   (a) Estimate the number density of protons and electrons.

   (b) Use the virial theorem relating kinetic energy K and potential energy U ($U = -2K$) to find an order-of-magnitude temperature of the gas (go ahead an assume for this estimate that it is all hydrogen), and the energy in keV of photons at the exponential cutoff of free-free emission.

   (c) Justify the assumption that the gas is fully ionized.

   (d) Compare the optical depth to Thomson scattering in this system to the optical depth to free-free absorption for photons with $h\nu \sim kT$. What is the total effective optical depth? Recall the results we discussed earlier in the semester regarding optical depth in the case of absorption and scattering! Is the system optically thin?

   (e) If the metallicity of the gas is roughly solar (0.02 by mass), how important is the contribution of higher mass ions likely to be to the level of free-free emission? Give just an order of magnitude estimate (and just assume all the metals are oxygen for simplicity).