Radiative Processes in Astrophysics / Problem Set #3
Due March 3, 2021

Choose **ONE** of the two problems below to solve.

1. The effect of a polarization filter or a quarter-wave plate, or other device affecting the polarization of light is often expressed in terms of a 4-by-4 "Mueller matrix" **M** defined such that:

$$\begin{pmatrix} I_{\rm out} \\ Q_{\rm out} \\ U_{\rm out} \\ V_{\rm out} \end{pmatrix} = \mathbf{M} \cdot \begin{pmatrix} I_{\rm in} \\ Q_{\rm in} \\ U_{\rm in} \\ V_{\rm in} \end{pmatrix} \tag{1}$$

   (a) Imagine a polarization filter that admits only linearly polarized light along an axis, taken to be θ with respect to the x-axis. What is the Mueller matrix for this filter?

   (b) Now imagine you use this filter on the observations of some object and use a CCD to measure the intensity $I_{\rm out}$. Assuming $V = 0$ (almost always true in astrophysical applications), what is the minimum number of measurements with different choices of θ that you have to do to measure the polarization fraction Π?

   (c) Most (all?) polarimetry observations use more than that minimum number and much cleverer techniques to observe an object through various polarization filter simultaneously. Comment on why that is a good idea (beyond any extra total signal-to-noise you get from more observations).

2. Consider an interstellar medium with a constant dust density, so that the absorption and scattering factors α_ν and σ_ν are constant. Assume the scattering is isotropic and coherent.

   (a) If from the Earth (i.e. from within this interstellar medium) I observe the spectrum of a star at some distance d (using a narrow aperture including just the light coming from exactly the direction of the star). By what factor is the flux I measure at frequency ν affected by the intervening dust?

   (b) Now imagine I observe the total spectrum of a distant galaxy from outside, using an aperture that covers all of the light coming from the galaxy. Assume the galaxy is spherical and the dust, the stars are intermixed randomly, and for simplicity they are all the same type of star as in part (a). If $\alpha_\nu = 0$ but σ_ν is significant, how is the spectrum affected by the presence of the dust?

(c) If $\sigma_\nu = 0$ but α_ν is significant, how does the effect of dust on the spectrum differ from the effect on the single star in part (a)?

(d) How do you expect the spectrum of the galaxy to differ qualitatively from the spectrum of the individual star?