Radiative Processes in Astrophysics / Problem Set #4
Due March 10, 2021

1. The cosmic microwave background at $z \sim 1100$ is the last scattering surface of the photons from the hot ionized gas that filled the universe before that time. Before recombination, these photons Thomson scatter efficiently off the electons in the ionized gas, and are kept in thermal equilibrium at about $T \sim 3000$ K at that epoch. As hydrogen atoms recombine at $z \sim 1100$, the gas becomes transparent to most of these photons, which then travel towards us.

   (a) Using the known cosmic baryon density, estimate the mean free path (in physical units) of a photon to Thomson scattering when the ionization fraction is 0.5.

   (b) The photons reaching us are those emitted exactly normal to the surface defined by the recombination epoch. These photons are the result of scattering from the gas surrounding the point in question. If the temperature of the CMB were uniform, do you expect the light reaching us to be polarized, and if why or why not?

   (c) Considering a local patch of the reionization surface, sketch a pattern of temperature fluctuations on the surface that would yield a net polarization.

2. Consider the dependence of the "equivalent width" associated with an absorption line with a Voigt profile, on the optical depth at line center. Assumed the absorbed continuum is flat in f_λ. You may treat $\mathrm{d}\lambda/\lambda \propto \mathrm{d}\nu/\nu$ in the region of the line. The equivalent width is the integral of the absorbed light in f_λ divided by the flux density of the continuum in f_λ.

   (a) For $\tau \ll 1$, approximate the dependence of equivalent width on τ.

   (b) For $\tau \gg 1$, neglecting the Lorentzian term (i.e. $\Gamma = 0$), approximate the dependence of equivalent width on τ.

   (c) For $\tau \gg 1$, neglecting the Doppler term (i.e. $\sigma = 0$), approximate the dependence of equivalent width on τ.

   (d) Assume Λ is the classical damping width, and the velocity dispersion $\sigma = 10$ km s^{-1}. Based on the scalings you just calculated, sketch (the log of) equivalent width vs τ.