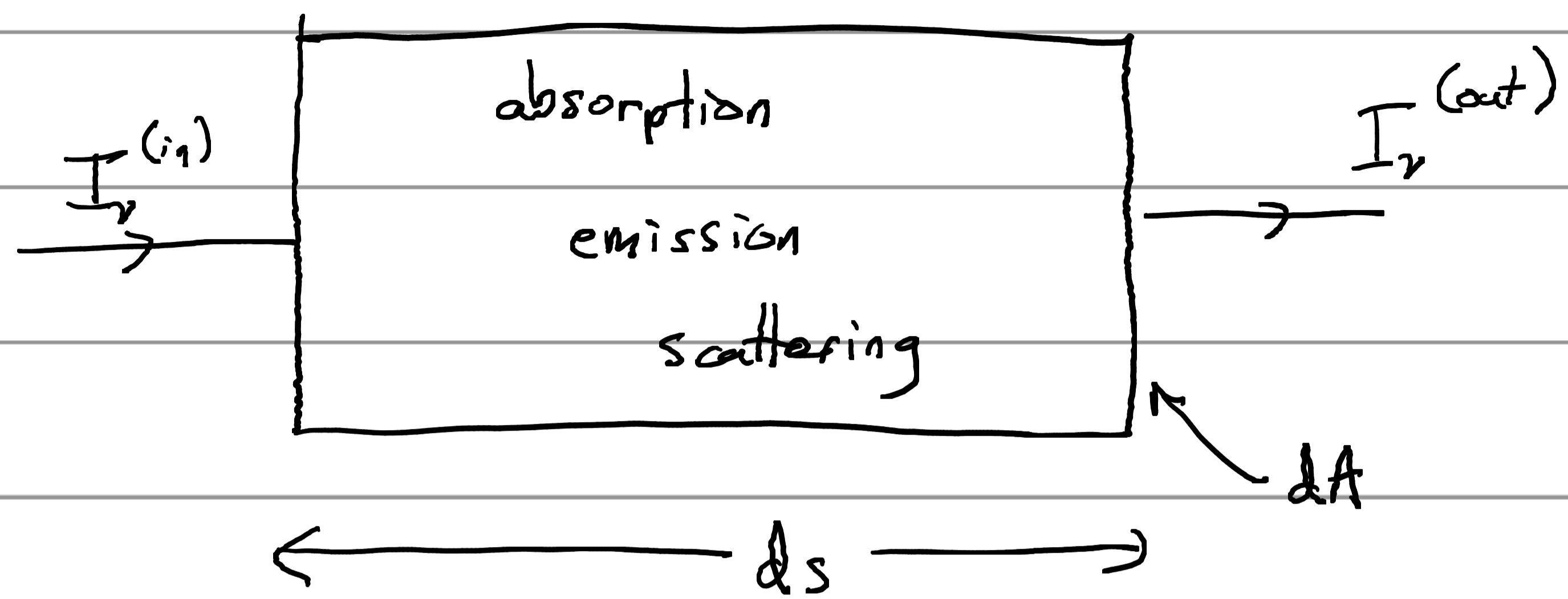


Radiative Transfer Basics - simple macroscopic description



Spontaneous emission along direction of ray:

$$dE_\nu = j_\nu dV d\Omega dt d\nu$$

If emission is isotropic, $j_\nu = \frac{1}{4\pi} P_\nu$ ← total power emitted per unit volume per unit freq.

total power
In $\text{W per unit mass units}$: $E_\nu = \frac{4\pi j_\nu}{\rho}$

$\rho = \text{mass density}$

What is dI_ν for our beam due to spontaneous emission:

$$dI_\nu = I_\nu^{(\text{out})} - I_\nu^{(\text{in})}$$

$$dI_\nu = \frac{dE_\nu}{d\Omega dt d\nu dA} = \frac{j_\nu dV d\Omega dt d\nu}{d\Omega dt d\nu dA}$$

$$= j_\nu \left(\frac{dV}{dA} \right) = j_\nu \underbrace{ds}_{\text{distance traveled}}$$

↳ distance traveled

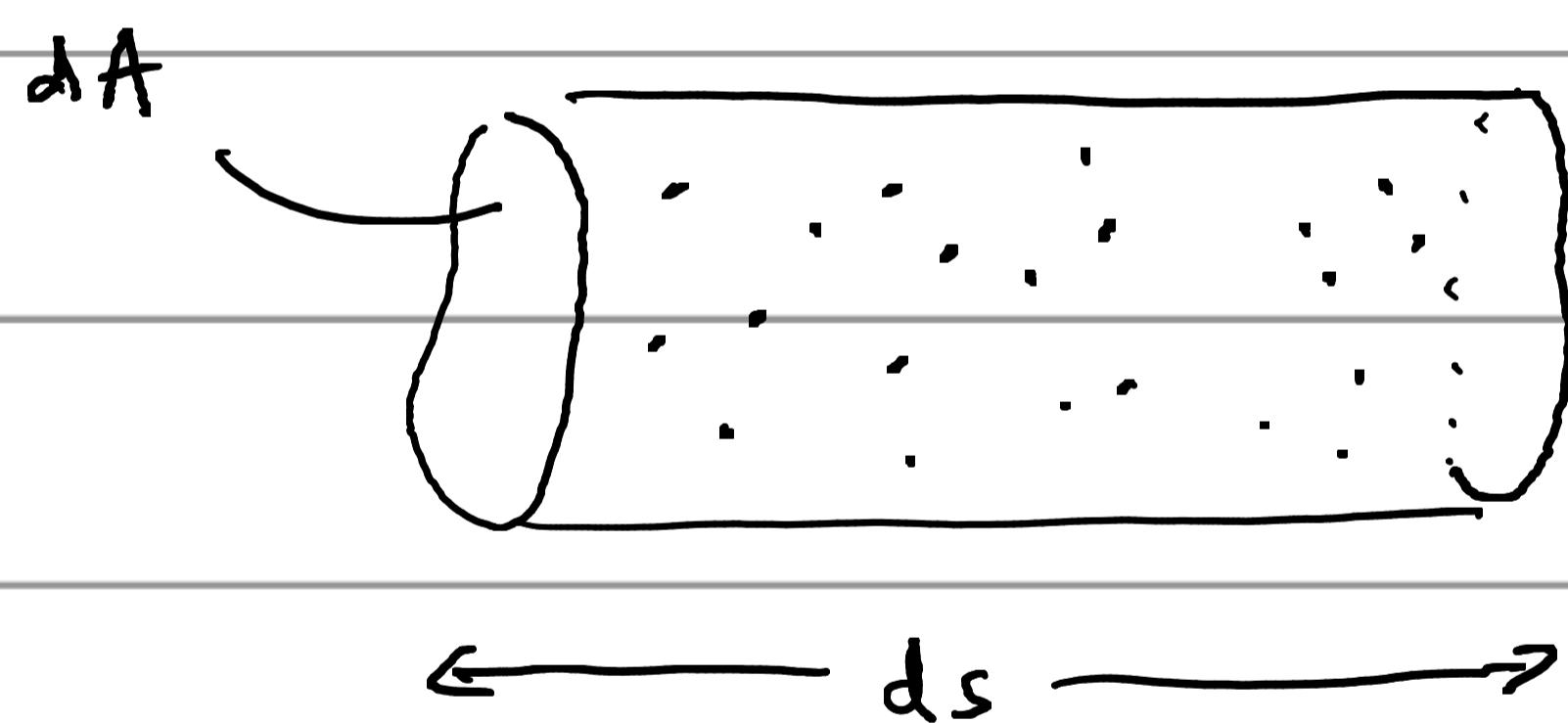
Absorption

Define absorpton coefficient α_ν :

$$dI_\nu = -\alpha_\nu I_\nu ds \quad \alpha_\nu \text{ in } \text{cm}^{-1} \text{ units}$$

Makes some sense to define in fractional sense relative

to I_ν . A microscopic model that shows this is to consider a number density n of obscuring particles of cross-section σ_ν . Per path length ds the energy absorbed is:



$$dE_\nu = - \underbrace{\left(\sigma_\nu n dA ds \right)}_{\text{area blocked}} I_\nu d\Omega d\nu dt$$

Then:

$$dI_\nu = \frac{dE_\nu}{dA d\Omega d\nu dt} = - \underbrace{\left(\sigma_\nu n \right)}_{\alpha_\nu} I_\nu ds$$

Or $K_\nu = \frac{\alpha_\nu}{P}$ ("opacity")

This picture, in addition to being simplified,
depends on:

$$\sigma_\nu^{1/2} \ll n^{-1/3}$$

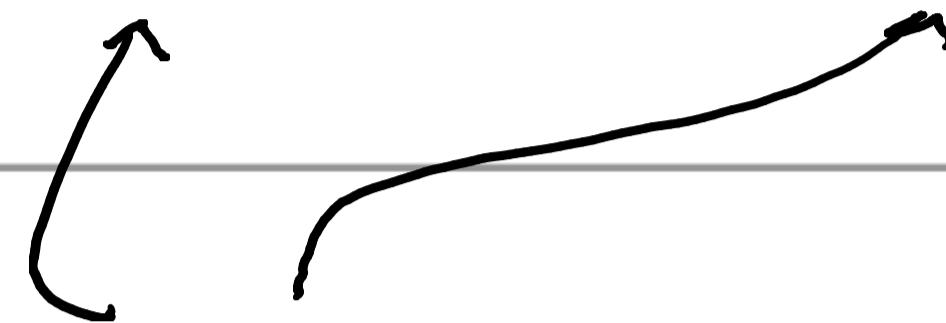
Stimulated Emission

Quantum mechanical interactions can lead to
stimulated emission. This response is $\propto I_\nu$.

In this macroscopic picture, we will treat
this as "negative" absorption.

Radiative Transfer Equation

$$\frac{dI_\nu}{ds} = -\alpha_\nu I_\nu + j_\nu$$



forms to be determined by
microscopic properties

This form omits scattering effects, which complicates
this by mixing angles; we will look at that later.

Let us look at some basic solutions:

Emission only: $\alpha_\nu = 0$

$$\frac{dI_\nu}{ds} = j_\nu \rightarrow I_\nu(s) = I_\nu(s_0) + \int_{s_0}^s ds' j_\nu(s')$$

Absorption only: $j_\nu = 0$

$$\frac{dI_\nu}{ds} = -\alpha_\nu I_\nu$$

$$\int_{s_0}^s ds' \frac{dI_\nu}{I_\nu} = - \int_{s_0}^s ds' \alpha_\nu$$

$$[\ln I_\nu]_{s_0}^s = - \int_{s_0}^s ds' \alpha_\nu$$

$$I_\nu(s) = I_\nu(s_0) \exp \left[- \int_{s_0}^s ds' \alpha_\nu \right]$$

Based on this solution we define the "optical depth":

$$\tau_\nu = \int_{s_0}^s ds' \alpha_\nu(s') \quad \text{and thus } d\tau_\nu = \alpha_\nu ds$$

"Optically thick" $\rightarrow \tau_\nu \gg 1$

"Optically thin" $\rightarrow \tau_\nu \ll 1$

We can rewrite the radiative transfer equation as:

$$\frac{1}{\alpha_\nu} \frac{dI_\nu}{ds} = -I_\nu + \frac{j_\nu}{\alpha_\nu}$$

defining $S_\nu = \frac{j_\nu}{\alpha_\nu}$ as the "source function"

$$\frac{dI_\nu}{d\tilde{\tau}_\nu} = -I_\nu + S_\nu$$

The solution to this can be written as:

$$I_\nu(\tilde{\tau}_\nu) = I_\nu(0) e^{-\tilde{\tau}_\nu} + \int_0^{\tilde{\tau}_\nu} d\tilde{\tau}'_\nu e^{-(\tilde{\tau}_\nu - \tilde{\tau}'_\nu)} S_\nu$$

attenuation of
incoming ray

source function
as attenuated
according to
optical depth

Now consider $S_\nu = \text{constant}$

Then

$$I_\nu(\tilde{\tau}_\nu) = I_\nu(0) e^{-\tilde{\tau}_\nu} + S_\nu \left[e^{-(\tilde{\tau}_\nu - \tilde{\tau}'_\nu)} \right]^{\tilde{\tau}_\nu}_0$$

$$= I_\nu(0) e^{-\tilde{\tau}_\nu} + S_\nu \left(1 - e^{-\tilde{\tau}_\nu} \right)$$

$$= S_\nu + e^{-\tilde{\tau}_\nu} (I_\nu(0) - S_\nu)$$

$$I_\nu(\tau_\nu) = S_\nu + e^{-\tau_\nu} (I_\nu(0) - S_\nu)$$

$$\text{For } \tau_\nu \rightarrow \infty, I_\nu(\tau_\nu) \rightarrow S_\nu$$

Generally, the source function S_ν will be

approached at higher & higher optical depths.

Mean free path

Calculate mean optical depth reached by rays:

$$\begin{aligned} \langle \tau_\nu \rangle &= \int_0^\infty d\tau_\nu e^{-\tau_\nu} \tau_\nu = -\frac{\partial}{\partial \beta} \left[\int_0^\infty d\tau_\nu e^{-\beta \tau_\nu} \right]_{\alpha=1} \\ &= -\frac{\partial}{\partial \beta} \left[-\frac{1}{\beta} e^{-\beta \tau_\nu} \Big|_0^\infty \right]_{\beta=1} = \frac{\partial}{\partial \beta} \left[\frac{1}{\beta} (0 - 1) \right]_{\beta=1} \\ &= -\frac{\partial}{\partial \beta} \frac{1}{\beta} \Big|_{\beta=1} = \frac{1}{\beta^2} \Big|_{\beta=1} = 1 \end{aligned}$$

In a homogeneous medium we can write:

$$\langle \tau_\nu \rangle = \alpha_\nu (\text{m.f.p.})_\nu = 1 \Rightarrow$$

$$\text{mean free path} = \frac{1}{\alpha_\nu} = \frac{1}{n \sigma_\nu} \quad (\text{in terms of toy microscopic model})$$

Radiation Force

When matter absorbs radiation, the energy and momentum must go somewhere. Here we consider the momentum imparted.

The net radiation flux vector is:

$$\vec{F}_\nu = \int d\Omega \hat{n} I_\nu(\theta, \phi)$$

why $\cos\theta$ not $\cos^2\theta$
as in rad. pressure?

because keeping
as net vector
not \perp to
some specific
 dA

This corresponds to a net momentum $\frac{\vec{F}_\nu}{c}$,

Then we see:

$$\frac{d\vec{p}_\nu}{dA dt d\nu} = \frac{\vec{F}_\nu}{c} \alpha_\nu ds$$

not a pressure,
per unit volume

assumes
isotropic
absorption
and
reemission

$$\frac{d\vec{p}_\nu}{dt dV d\nu} = \frac{1}{c} \alpha_\nu \vec{F}_\nu$$

$$\text{or } \frac{d\vec{p}}{dt dV} = \frac{1}{c} \int d\nu \alpha_\nu \vec{F}_\nu$$

$$\text{or } \frac{d\vec{p}}{dt dm} = \frac{1}{c} \int d\nu K_\nu \vec{F}_\nu$$

higher $\alpha, K \rightarrow$ more momentum

where $K_\nu = \frac{\epsilon_\nu}{p}$

