HOWTO g06: Synthesis of planetary topographies

You will learn how to synthesize a planetary topography. In fact, the same approach can be applied to any surface spherical harmonic synthesis of the form

$$f(\varphi, \lambda) = \sum_{n=0}^{n_{\max}} \sum_{m=0}^{n} \left(\bar{C}_{nm} \cos(m\,\lambda) + \bar{S}_{nm} \sin(m\lambda)\right) \bar{P}_{nm}(\sin\varphi),$$

where $\bar{C}_{nm}$ and $\bar{S}_{nm}$ are 4π-fully-normalized (real) surface spherical harmonic coefficients of the function f, n and m are spherical harmonic degree and order, respectively, $\bar{P}_{nm}(\sin\varphi)$ are the 4π-fully-normalized (real) associated Legendre functions of the first-kind, and, finally, φ and λ are the spherical latitude and longitude, respectively. This means GrafLab can synthesize a wide range of (real) functions given on a sphere.

All the GrafLab input parameters are explained in ../docs/graflab.md.

```
clear; clc; init_checker();
```

Synthesis of the Earth's topography

We need to perform the basic *surface* spherical harmonic synthesis shown above. This can be achieved with the *surface* synthesis of the gravitational potential, see the equation for "V" in https://blazejbucha.com/graflab/Definition_of_functionals_of_the_geopotential_used_in_GrafLab_software.pdf.

The trick is that we have to set "GM = 1.0", "R = 1.0" and the radius of the evaluation points to "r = 1.0". Obviously, we have to do the synthesis on the unit sphere, so "crd = 1". Finally, we set "quantity" to "11" (see ../docs/graflab.md).

Define the GrafLab inputs.

```
GM                  = 1.0;   % Important
R                   = 1.0;   % Important
nmin                = 0;
nmax                = 360;
ellipsoid           = 1;
GGM_path            = '../data/input/DTM2006.mat';
crd                 = 1;   % Important
point_type          = 0;
lat_grd_min         = -90.0;
lat_grd_step        =   1.0;
lat_grd_max         =  90.0;
lon_grd_min         =   0.0;
lon_grd_step        = lat_grd_step;
lon_grd_max         = 360.0;
h_grd               =   0.0; % Note that the synthesis is here done at a grid,
                             % so "h_grd" needs to be set to a height above the
                             % sphere with the radius "R", hence "0.0" (see
                             % <../docs/graflab.md ../docs/graflab.md>).  In this
                             % way, the radius of the evaluation points "r" will
                             % be "1.0".  If you do the synthesis at scattered
                             % points, you should set "h_sctr" to "1.0".
out_path            = '../data/output/howto-g06-topography';
```

```matlab
quantity_or_error = 0;
quantity          = 11;  % Gravitational potential; in this case, however,
                         % we synthesize the Earth's topography
fnALFs            = 1;
export_data_txt   = 1;
export_report     = 1;
export_data_mat   = 1;
display_data      = 2;
graphic_format    = 6;
colormap          = 1;
number_of_colors  = 60;
dpi               = 300;
status_bar        = 1;
```

Do the synthesis

```matlab
out = GrafLab('OK', ...
    GM, ...
    R, ...
    nmin, ...
    nmax, ...
    ellipsoid, ...
    GGM_path, ...
    crd, ...
    point_type, ...
    lat_grd_min, ...
    lat_grd_step, ...
    lat_grd_max, ...
    lon_grd_min, ...
    lon_grd_step, ...
    lon_grd_max, ...
    h_grd, ...
    [], ...
    [], ...
    [], ...
    [], ...
    out_path, ...
    quantity_or_error, ...
    quantity, ...
    fnALFs, ...
    [], ...
    export_data_txt, ...
    export_report, ...
    export_data_mat, ...
    display_data, ...
    graphic_format, ...
    colormap, ...
    number_of_colors, ...
    dpi, ...
    status_bar);
```

You may now take a look at the output files.

```
fprintf("The ""%s*_Gravitational_potential.png"" file shows the " + ...
        "synthesized topography.\n", out_path);
```

Note that GrafLab thinks it computed the gravitational potential. It has no idea (how could it?) that we actually computed the Earth's topography. This is why the plot, the report file, the file name, etc. still report the gravitational potential.