SIMULATING A POSITRON CONVERTER IN BMAD

Authors, Cornell University, Ithaca, NY, 14850, USA

Abstract

A numerical model for the behaviour of positron converters has been developed. This model simulates the fundamental physics of Bremsstrahlung radiation using the Geant4 toolkit, and produces a probabilistic model of the positrons produced in the converter. This model emulates the physics of the converter much faster than the Geant4 routines, making it suitable for use within *Bmad*. The model predicts the distribution of positions and momenta for outgoing positrons over a range of incoming electron energies and target thicknesses. Preliminary support for positron polarization tracking is also discussed. This model enables optimizations to accelerator lattices involving converters that were previously impossible in *Bmad*, which could allow increased positron yield for accelerators using a converter as their positron source.

INTRODUCTION

Electron-positron accelerators require a source of positrons. The standard method of obtaining positrons for use in an accelerator is through the use of a positron converter. This is typically a slab of heavy metal, such as tungsten, located in a linear branch of the accelerator. The converter is bombarded with electrons with energies of order ~ 100 MeV. The electrons emit photons via Bremsstrahlung, which in turn decay to e^+e^- pairs:

$$e^- + Z \longrightarrow e^- + Z + \gamma \longrightarrow e^- + Z + e^- + e^+.$$

The electrons are then quickly filtered off with a magnet, effectively "convertering" a beam of incoming electrons into a beam of outgoing positrons.

There is some discussion in the literature regarding the Bremsstrahlung energy spectrum in various materials, as well as the energy spectra of the e^+e^- secondaries. However, there is no closed form, analytical solution that describes the kinematics of the emitted e^+ in terms of the kinematics of the incoming e^- and the physical properties of the converter.

Previous attempts have also been made at developing numerical models of a positron converter. However, these results have been highly specialized, and have been completely standalone solutions. There has also been a recent demand for modelling the transfer of spin from the incoming beam of electrons to the outgoing beam of positrons.

In response, the authors have developed a positron converter model for use with the *Bmad* accelerator toolkit. Our model is designed to be as flexible as possible, accomadating any converter material and thickness. The model can even be extended to support other kinds of incoming and outgoing particles, and is not limited to taking electrons in and sending positrons out. The model is also fast to use within *Bmad*, with the expensive physics calculations performed only once as the converter model is being developed.

This converter model enables *Bmad* users to optimize the design of lattices that contain positron converters. This was not previously possible, as there was no way to simulate the effects of a converter within *Bmad*.

THE CONVERTER MODEL

The physical processes that occur in the positron converter to produce the Bremsstrahlung photons and subsequent positrons are governed by quantum electrodynamics. The details of these interactions are not critical when only the aggregate behavior of the converter is of interest. Therefore, the converter's behavior is modelled with a probability distribution, which describes the overall distribution of the produced positrons in terms of the target and incoming electron's properties.

Coordinate System

Consider an electron incident on the upstream face of a positron converter of thickness T, with momentum p_-c perpendicular to the face of the converter as depicted in Figure 1. Positrons produced in the converter emerge from the downstream face with some radial displacement r and at an angle θ relative to the horizontal, as depicted in Figure 2. The direction of the radial displacement defines for each outgoing positron an $\hat{\mathbf{x}}$ and $\hat{\mathbf{y}}$ axis, with $\hat{\mathbf{x}}$ taken in the same direction as $\mathbf{r}$, and $\hat{\mathbf{y}}$ taken so that $(\hat{\mathbf{x}}, \hat{\mathbf{y}}, \mathbf{s})$ is a right-handed orthogonal coordinate system.

By symmetry, θ must be uniformly distributed from 0 to 2π. Therefore, the kinematic properties of the produced positrons which must be modelled are its radial displacement and outgoing momentum. The outgoing momentum $\mathbf{p}_+c$ is described in terms of its magnitude, p_+c, and its slopes along the $\hat{\mathbf{x}}$ and $\hat{\mathbf{y}}$ directions:

$$\frac{dx}{ds} = \frac{p_x}{p_s} \tag{1}$$

$$\frac{dy}{ds} = \frac{p_y}{p_s}. \tag{2}$$

Probability Distributions

Positrons produced in the converter are described by the distribution

$$P\left(p_+c, r, \frac{dx}{ds}, \frac{dy}{ds}\right) \tag{3}$$

which indicates the probability that an outgoing positron will have a given momentum and radial displacement. P is normalized to account for the efficiency of positron production:

$$\int P\left(p_+c, r, \frac{dx}{ds}, \frac{dy}{ds}\right)\, d(p_+c)\, dr\, d\!\left(\frac{dx}{ds}\right) d\!\left(\frac{dy}{ds}\right) = \frac{N_+}{N_-}, \tag{4}$$

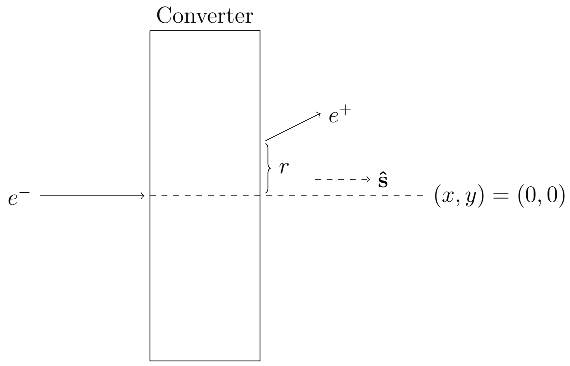

Figure 1: Positron converter coordinate system (side view).

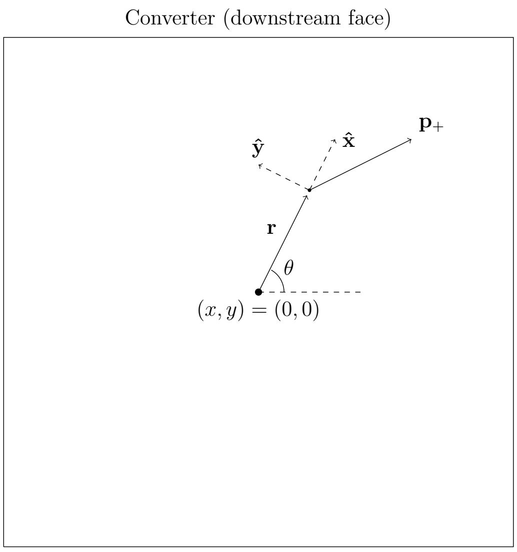

Figure 2: Positron converter coordinate system (view of the downstream face).

where N_+ is the total number of positrons produced, and N_- is the number of electrons incident upon the converter. To make the problem easier to grapple with, P is decomposed into two subdistributions,

$$P\left(p_+c, r, \frac{dx}{ds}, \frac{dy}{ds}\right) = P_1\left(p_+c, r\right) P_2\left(\frac{dx}{ds}, \frac{dy}{ds}; p_+c, r\right) \tag{5}$$

which are normalized to N_+/N_- and 1 respectively. Note that P_2 depends on p_+c and r, so that the shape of P_2 varies with p_+c and r.

Using Geant[1], a large number of electrons incident upon the converter are simulated, and the produced positrons and their kinematics are recorded. The produced positrons are binned into a two-dimensional histogram by their p_+c and r values. This histogram gives an approximation of P_1. For P_2, fits are performed to $\frac{dx}{ds}$ and $\frac{dy}{ds}$ for the positrons in each (p_+c, r) bin. These fits yeild $P_2\left(\frac{dx}{ds}, \frac{dy}{ds}; p_+c, r\right)$. Examples of the P_1 and P_2 distributions obtained from the Geant simulation are shown in Figures 3 and 4 respectively.

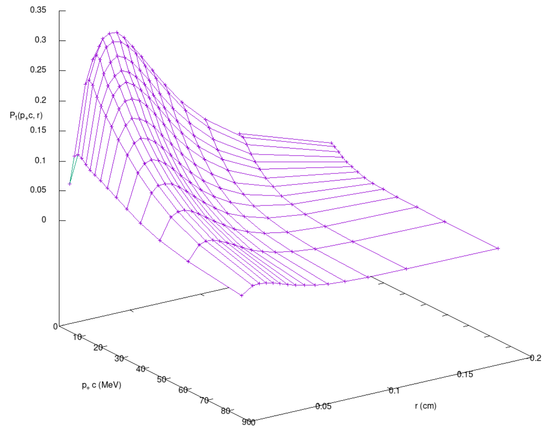

Figure 3: $P_1(p_+c, r)$ for incoming electrons with $p_-c = 300$ MeV and a tungsten target of thickness $T = 6.35$ mm.

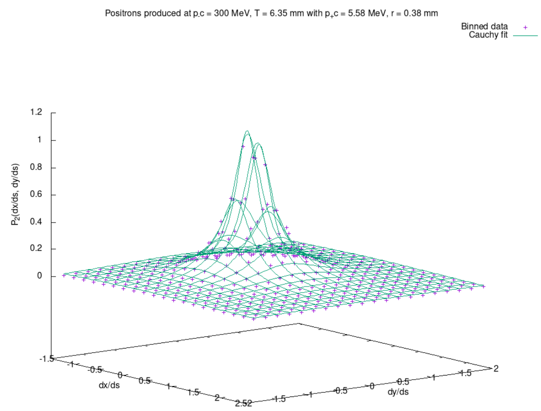

Figure 4: $P_2\left(\frac{dx}{ds}, \frac{dy}{ds}; p_+c, r\right)$ for incoming electrons with $p_-c = 300$ MeV and a tungsten target of thickness $T = 6.35$ mm, and outgoing positrons with $p_+c = 5.58$ MeV and $r = 0.38$ mm. The purple points indicate data obtained directly from the Geant simulation, while the green curve shows the fit to the data.

SPIN TRACKING

Polarization transfer from the incoming electrons to the outgoing positrons has also been modelled. Any incoming polarization $\mathbf{S}_-$ may be specified, and histograms describing S_x, S_y, and S_z of the produced positrons as functions of p_+c and r are produced. The authors have found that only the longitudinal polarization of the incoming electrons is ever transferred to the produced positrons; the produced positrons always have S_x and S_y essentially zero regardless of the incoming electron polarization. As an example, Figure 5 illustrates S_z as a function of p_+c and r for incoming electron with momentum $p_-c = 300$ MeV.

Recently, the PEPPo collaboration has published experimental results of polarized positron production from a polarized electron beam via Bremsstrahlung, to which the spin

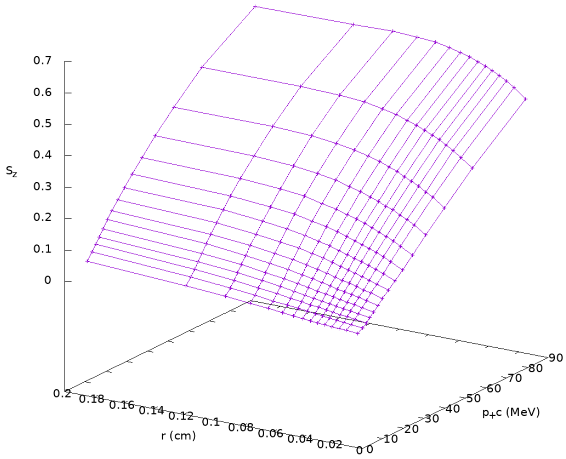

Figure 5: Positron S_z as a function of p_+c and r for incoming electrons with $p_-c = 300$ MeV and a tungsten target of thickness $T = 6.35$ mm.

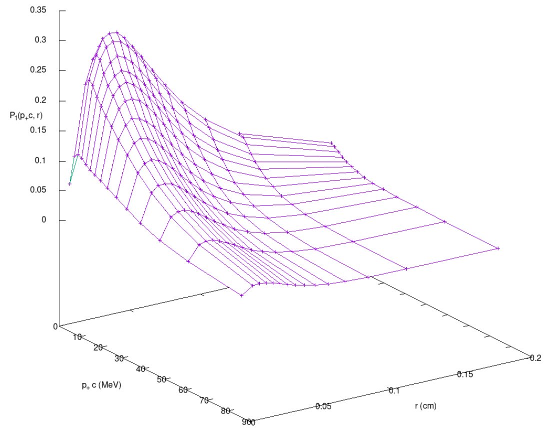

Figure 6: Placeholder figure for comparison between Geant and Bmad.

tracking results from this simulation can be compared. This simulation yields polarization transfer efficiencies significantly higher than those observed experimentally for outgoing positrons with p_+c less than 5 MeV. In contrast, the simulation's results for higher momentum positrons agrees closely with experiment. The discrepancy is likely due to approximations made by the Geant library in computing polarization transfer during the pair production step. While this disagreement shows that the spin tracking components of this simulation should not be relied on for accuracy at present, all of the infrastructure is in place to handle spin tracking. When more accurate spin tracking methods become available in Geant, this simulation, and the *Bmad* converter element, will be ready to use their results.

THE BMAD CONVERTER ELEMENT

The converter simulation and fitting programs aggregate their results into a single output file. This output file provides *Bmad* with all of the binned data and fit parameters necessary to emulate the behaviour of the converter.

A converter element has been added to *Bmad* which uses the model developed here to simulate a positron converter. Previously, *Bmad* lattices could be constructed to model the elements upstream or downstream of a converter. Now, the converter element can be used to connect these lattices together, allowing for the simulation of an entire linac end-to-end.

Since the converter simulation program samples a discrete set of incoming electron energies and target thicknesses, *Bmad* performs interpolation to determine the correct behaviour of the converter for intermediate values. For each incoming electron, *Bmad* randomly pulls values of p_+c, r, $\frac{dx}{ds}$, and $\frac{dy}{ds}$ from the interpolated probability distributions. The likelihood of producing a positron from the incoming electron is encoded in the weighting of the outgoing particle, a system which *Bmad* uses counting fractions of particles.

Agreement between direct output from Geant4 and emulated output from *Bmad* is quite good, with the distribution of particles from *Bmad* coming within a few percent of the Geant distributions. This is illustrated in Figure 6. This close agreement indicates that little accuracy is sacrificed in using the aggregate model of the converter. Moreover, *Bmad* simulates the positron converter approximately Y times faster than the direct Geant4 physics simulations.

CONCLUSION

This method of modeling the positron converter provides simulation results that are in close agreement to Geant4's direct simulation of Bremsstrahlung radiation and pair production. A small sacrifice in accuracy is made in return for a significant increase in simulation speed, making this method appropriate for use in lattice simulation and optimization software such as *Bmad*.

One of the primary motivators for this work was the desire to fully model the CESR linac lattice. Work is currently underway to do so using the model developed here. A first goal in this endeavour is to assess the agreement between the *Bmad* model of the linac as a whole the linac's experimentally observed beam. Once good agreement is established, *Bmad* and Tao can be used to optimize the lattice design, including quadrupole and solenoid tuning. Other future applications could include the optimization of element positioning on the linac beam line.

ACKNOWLEDGMENTS

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REFERENCES

[1] S. Agostinelli *et al.*, "Geant4—a simulation toolkit," *Nuclear Instruments and Methods in Physics Research Section A: Accelerators, Spectrometers, Detectors and Associated Equipment*, vol. 506, no. 3, pp. 250–303, 2003, ISSN: 0168-9002. DOI: `https://doi.org/10.1016/S0168-9002(03)01368-8`.

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