Area and error calculations for the HEK

Cis Verbeeck

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1 Event_C1Error, Event_C2Error (in helio-projective cartesian coordinates)

$$\text{Event\_C1Error} = W \cdot \frac{180^\circ}{\pi D_\odot} \cdot \Big(1 + \frac{\Delta D_\odot}{D_\odot}|x - x_\text{center}|\Big),$$

$$\text{Event\_C2Error} = W \cdot \frac{180^\circ}{\pi D_\odot} \cdot \Big(1 + \frac{\Delta D_\odot}{D_\odot}|y - y_\text{center}|\Big),$$

where (x, y) are pixel coordinates, $D_\odot$ is the distance observer-Sun in Mm, $\Delta D_\odot$ is its error estimate in Mm, and W is the *number of Mm in 1 pixel length*. From `http://en.wikipedia.org/wiki/Astronomical_unit`, we find

$$D_\odot \simeq 149\,597.871\,\text{Mm}.$$

Using the Earth orbit's eccentricity of $e \simeq 0.0167$, the yearly maximal error $\Delta D_\odot \simeq 149\,597.871 \times 0.0167\,\text{Mm}$. Note that $\frac{\Delta D_\odot}{D_\odot} \simeq 0.0167$.

2 Area_Raw (in Mm^2)

This is the area as it is seen in the image, hence projected from a 3-D ball onto a 2-D disk.

$$\text{Area\_Raw} = \text{Event\_Npixels} \cdot (\frac{R_0}{R})^2,$$

where R_0 and R are the radius of the Sun expressed in Mm respectively pixels. In [1], we find $R_0 = 695.508$ Mm and $\Delta R_0 = 0.026$ Mm. Note that R can be found in the FITS header of the image, and $\Delta R = 2$ (pixels).

3 Area_Uncert (in Mm^2)

$$\text{Area\_Uncert} = (\frac{R_0}{R})^2 \cdot \Big((\text{number of pixels in contour}) + 2 \cdot \text{Event\_Npixels} \cdot (\frac{\Delta R_0}{R_0} + \frac{\Delta R}{R}) \Big).$$

4 Area_AtDiskCenter (in Mm^2)

This is the deprojected area, i.e., the area on the 3-D ball.

$$\text{Area\_AtDiskCenter} = (\frac{R_0}{R})^2 \cdot \sum_{\text{pixels } (x_i, y_i)} \frac{R}{\sqrt{R^2 - (x_i - x_{\text{center}})^2 - (y_i - y_{\text{center}})^2}}.$$

Note that Area_AtDiskCenter will go to infinity as the pixel goes toward the eastern or western limb. In order to avoid this, I propose to adopt Paul Higgins's approach to limit the correction factor (i.e., the terms of the sum) to 16, which corresponds to a LOS angle of $\simeq 86°$. For AR containing a pixel with a corresponding term exceeding 16, we don't provide Area_AtDiskCenter.

5 Area_AtDiskCenterUncert (in Mm^2)

$$\begin{gathered}\text{Area\_AtDiskCenterUncert} = (\tfrac{R_0}{R})^2 \cdot \textstyle\sum_{\text{pixels } (x_i, y_i) \in \text{boundary AR}} \frac{R}{\sqrt{R^2 - (x_i - x_{\text{center}})^2 - (y_i - y_{\text{center}})^2}} \\ + (\tfrac{R_0^2}{R}) \cdot \textstyle\sum_{\text{pixels } (x_i, y_i) \in \text{AR}} \\ \Big(\frac{|x_i - x_{\text{center}}| + |y_i - y_{\text{center}}| + 2 \frac{\Delta R_0}{R_0} \cdot \big(R^2 - (x_i - x_{\text{center}})^2 - (y_i - y_{\text{center}})^2 \big) + \frac{\Delta R}{R} \cdot \big(2R^2 - (x_i - x_{\text{center}})^2 - (y_i - y_{\text{center}})^2 \big)}{\big(R^2 - (x_i - x_{\text{center}})^2 - (y_i - y_{\text{center}})^2 \big)^{1.5}} \Big).\end{gathered}$$

References

[1] T.M. Brown, J. Christensen-Dalsgaard, "Accurate determination of the solar photosphere radius", *ApJ*, 500:L195–L198, 1998.