Computational Fluid Dynamics
using the SU^2 Code

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Introduction

- A new discipline of fluid dynamics has emerged, computational fluid dynamics – essentially a perfect wind tunnel.
- Advantages include cost, speed, ease of use, and range of operation.
- Computational fluid dynamics has the ability to provide a broader range of data, as well as simulate impractical situations to test in a wind tunnel.

Laminar Flow Equations

- Incompressibility Condition: $\nabla \cdot \mathbf{u} = 0$
- Euler's Equation: $\dfrac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla)\,\mathbf{u} = -\dfrac{1}{\rho}\nabla p + \mathbf{g}$
- Cauchy's Equation: $\rho\left(\dfrac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla)\,\mathbf{u}\right) = \nabla \cdot \mathbf{T} + \rho\mathbf{g}$
- Stress Tensor: $T_{ij} = -p\delta_{ij} + \mu\left(\dfrac{\partial u_j}{\partial x_i} + \dfrac{\partial u_i}{\partial x_j}\right)$
- Navier-Stokes Equation:

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla)\,\mathbf{u} = -\frac{1}{\rho}\nabla p + \nu\nabla^2\mathbf{u} + \mathbf{g}$$

Reynolds-Averaged Navier-Stokes (RANS)

To make solving the Navier-Stokes equations easier, especially for turbulent flows, we can assume the flow is the superposition of the laminar, and turbulent flows, and then take the time average.

$$\alpha_i \equiv \overline{\alpha_i} + \alpha_i',$$

$$\text{where } \overline{\alpha_i} = \frac{1}{\tau}\int_0^\tau \alpha_i dt$$

$$\overline{\alpha_i'} = 0, \quad \frac{\partial \overline{\alpha_i}}{\partial t} = 0, \quad \overline{\overline{\alpha_i}} = \overline{\alpha_i}, \quad \text{and} \quad \overline{\frac{\partial \alpha_i}{\partial x_i}} = \frac{\partial \overline{\alpha_i}}{\partial x_i}$$

Making the appropriate substitutions, taking the time average, and using the properties, we obtain the Reynolds-Averaged Navier-Stokes equation,

$$\left(\overline{\mathbf{u}} \cdot \nabla\right) \overline{\mathbf{u}} + \overline{\left(\mathbf{u}' \cdot \nabla\right) \mathbf{u}'} = -\frac{1}{\rho}\nabla\overline{p} + \nu\nabla^2\overline{\mathbf{u}} + \overline{\mathbf{g}}$$

Spalart-Allmaras Turbulence

- A model is still needed for ν
- According to the Boussinesq hypothesis, an increase in the viscosity gives the effect of turbulence
- Therefore $\nu = \overline{\nu} + \nu'$
- One such way to model the turbulence, is the Spalart-Allmaras turbulence model

$$\nu' = \hat{\nu} f_{v1}; \qquad f_{v1} = \frac{\chi^3}{\chi^3 + c_{v1}^3}; \qquad \chi \equiv \frac{\hat{\nu}}{\overline{\nu}}.$$

Assuming the flow is incompressible, and the diffusivity is constant, $\hat{\nu}$ is obtained by solving the convection-diffusion equation.

SU^2 Code Development

- The Stanford University Unstructured (SU^2) code is a relatively new, open source, constrained partial differential equation solver, optimized for computational fluid dynamics problems.
- Written mostly in C++.
- The capabilities have since been extended by many teams of developers from around the world.

SU^2 Code Capabilities

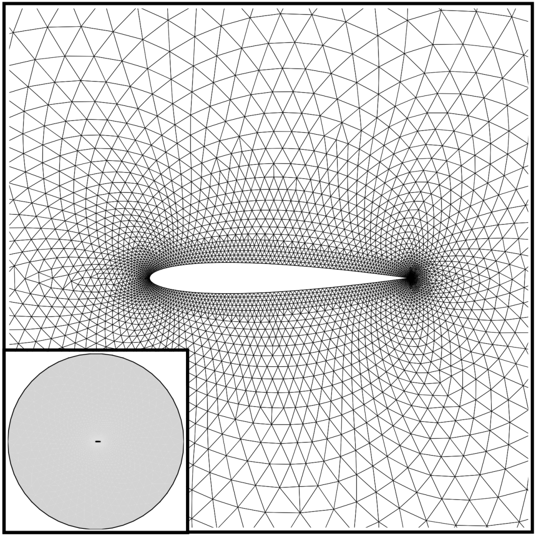
Some of the features of the SU^2 code include, but are not limited to:

- Reynolds-Averaged Navier-Stokes equations
- Spalart-Allmaras turbulence model ($c_{v1} = 7.1$)
- Menter Shear Stress Transport model (an alternative to the SA model)
- Compressible flows
- Wave equation, heat equation, and plasma equations
- Optimal shape design (i.e. Supersonic aircraft)
- Dynamic meshes

SU^2 Code File Types

- All of the parameters such as the fluid speed, Reynolds number, and governing equations are specified in the configuration file.
- The shape geometry of the problem is stored as an unstructured mesh – hence the name – in the mesh file.
- The mesh is unstructured in the sense that there is no algebraic relation between nodes.

Example Mesh File



Scripting and Automation

- Wolfram Mathematica was used to sift through the data files and extract the relevant information to produce the plots.
- A Wolfram function was needed to convert the data to a form Mathematica could use.
- This function along with the plotting function were combined into a Wolfram script which can be executed from the terminal.

Airfoil

Computational Fluid Dynamics

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Introduction

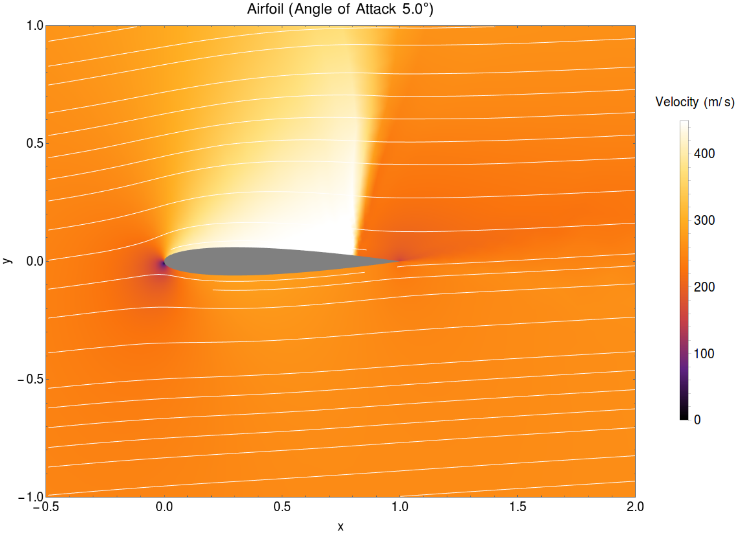
Theory

SU^2 Code

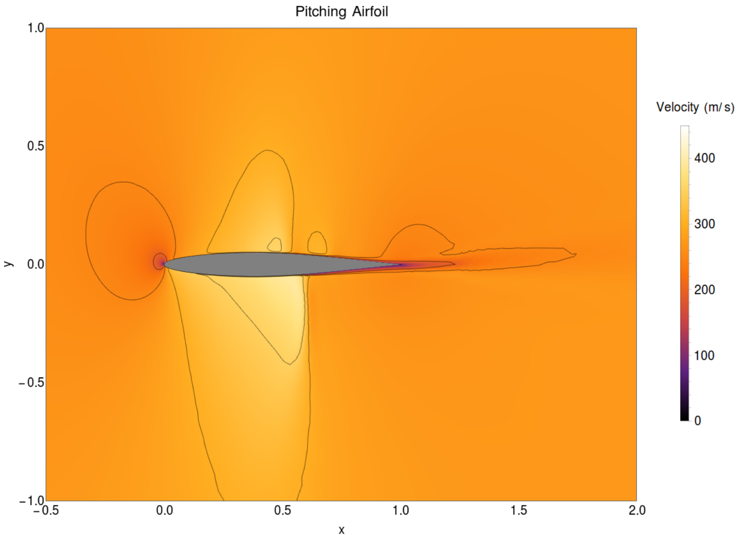
Scripting and Automation

Results

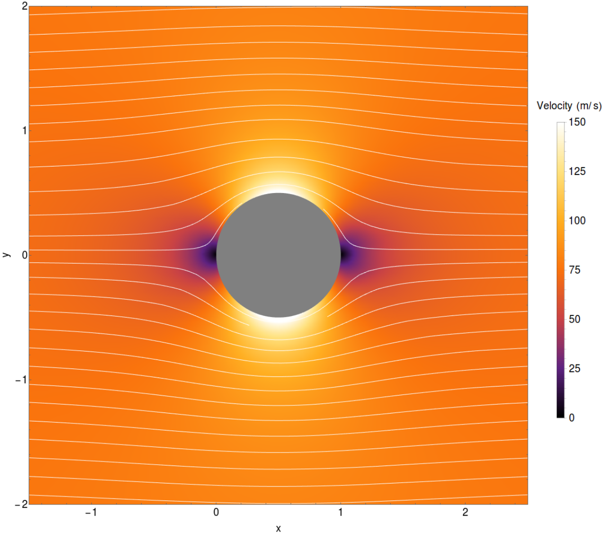
Conclusion



Pitching Airfoil

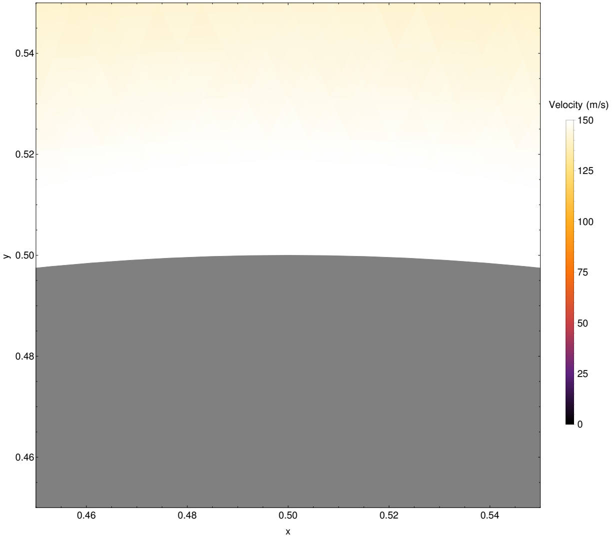


Laminar Flow Around a Static Cylinder
Euler

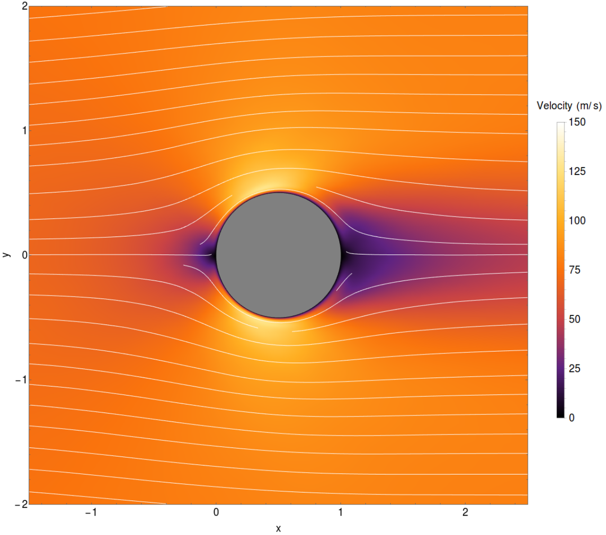


Laminar Flow Around a Static Cylinder

Euler Boundary

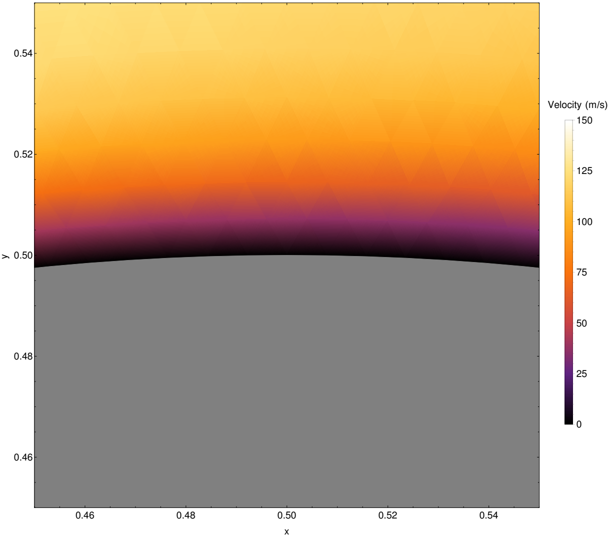


Laminar Flow Around a Static Cylinder
Navier-Stokes



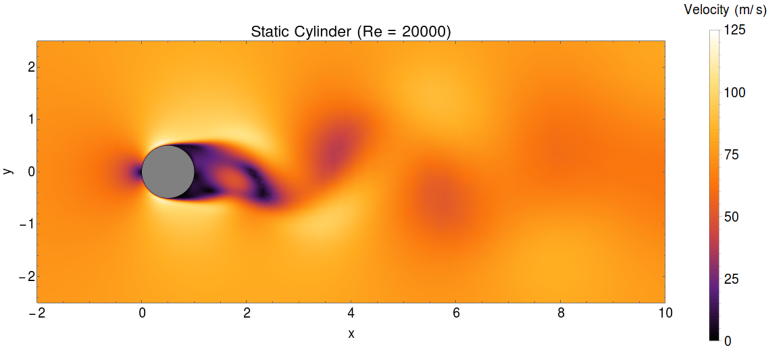
Laminar Flow Around a Static Cylinder

Navier-Stokes Boundary



Turbulent Flow Around a Static Cylinder

Vortex Shedding



Conclusion

- The preliminary results in this project are in agreement with the theoretical predictions.
- Although only a small set of its capabilities were used in this project, SU^2 is a viable computational fluid dynamics simulator.
- Computational fluid dynamics is now a critical discipline of fluid dynamics.