Computational Fluid Dynamics

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Introduction

Non-Turbulent Flow

- Incompressibility Condition: $\nabla \cdot \mathbf{u} = 0$
- Euler's Equations: $\dfrac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla)\,\mathbf{u} = -\dfrac{1}{\rho}\nabla p + \mathbf{g}$
- Cauchy's Equation: $\rho\left(\dfrac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla)\,\mathbf{u}\right) = \nabla \cdot \mathbf{T} + \rho\mathbf{g}$
- Stress Tensor: $\mathbf{T}_{ij} = -p\delta_{ij} + \mu\left(\dfrac{\partial u_j}{\partial x_i} + \dfrac{\partial u_i}{\partial x_j}\right)$
- Navier-Stokes Equation:

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla)\,\mathbf{u} = -\frac{1}{\rho}\nabla p + \nu\nabla^2\mathbf{u} + \mathbf{g}$$

Reynolds-Averaged Navier-Stokes

To make solving the Navier-Stokes equations easier, especially for turbulent flows, we can assume the flow is the superposition of the steady, and turbulent flows, and then take the time average.

$$\alpha_i \equiv \overline{\alpha_i} + \alpha_i',$$
$$\text{where } \overline{\alpha_i} = \frac{1}{\tau}\int_0^\tau \alpha_i dt$$

$$\overline{\alpha_i'} = 0, \quad \frac{\partial \overline{\alpha_i}}{\partial t} = 0, \quad \overline{\overline{\alpha_i}} = \overline{\alpha_i}, \quad \text{and} \quad \overline{\frac{\partial \alpha_i}{\partial x_i}} = \frac{\partial \overline{\alpha_i}}{\partial x_i}$$

Making the appropriate substitutions, taking the time average, and using the properties we obtain the Reynolds-Averaged Navier-Stokes equation,

$$\left(\overline{\mathbf{u}} \cdot \nabla\right)\overline{\mathbf{u}} + \overline{\left(\mathbf{u}' \cdot \nabla\right)\mathbf{u}'} = -\frac{1}{\rho}\nabla\overline{p} + \nu\nabla^2\overline{\mathbf{u}} + \overline{\mathbf{g}}$$

Spalart-Allmaras Turbulence

- A model is still needed for ν
- According to the Boussinesq hypothesis, an increase in the viscosity gives the effect of turbulence
- Therefore $\nu = \overline{\nu} + \nu'$
- One such way to model the turbulence, is the Spalart-Allmaras turbulence model

$$\nu' = \hat{\nu} f_{v1}; \qquad f_{v1} = \frac{\chi^3}{\chi^3 + c_{v1}^3}; \qquad \chi \equiv \frac{\hat{\nu}}{\overline{\nu}}.$$

Assuming the flow is incompressible, and the diffusivity is constant, $\hat{\nu}$ is obtained by solving the convection-diffusion equation,

$$\frac{\partial \hat{\nu}}{\partial t} = D\nabla^2\hat{\nu} - \overline{\mathbf{u}}\nabla\hat{\nu},$$

where D is the diffusivity.

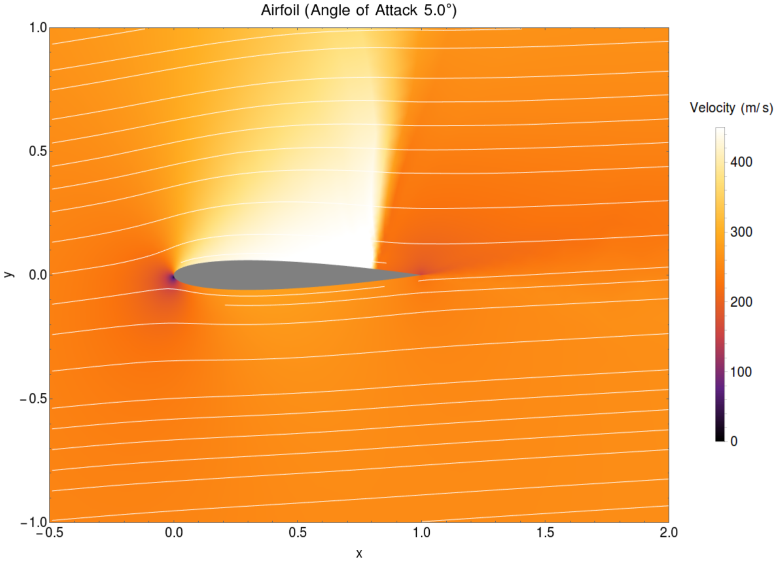
SU^2 Code

Mesh and Numerics

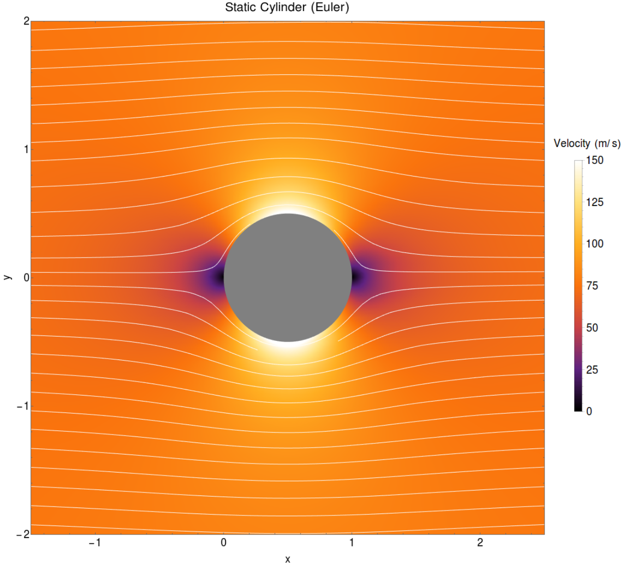
Scripting and Automation

- Wolfram Mathematica 10.1.0 for Linux x86 was used to sift through the vast data files and extract the relevant information to produce the plots
- A Wolfram function was needed to convert the data to a form Mathematica could use
- Mathematica's `ListDensityPlot` function was used to create the images
- The function to tidy the data along with the plotting function were combined into a Wolfram script which can be executed from the terminal
- To fully automate the image generation, a shell script was written to iterate over each data file

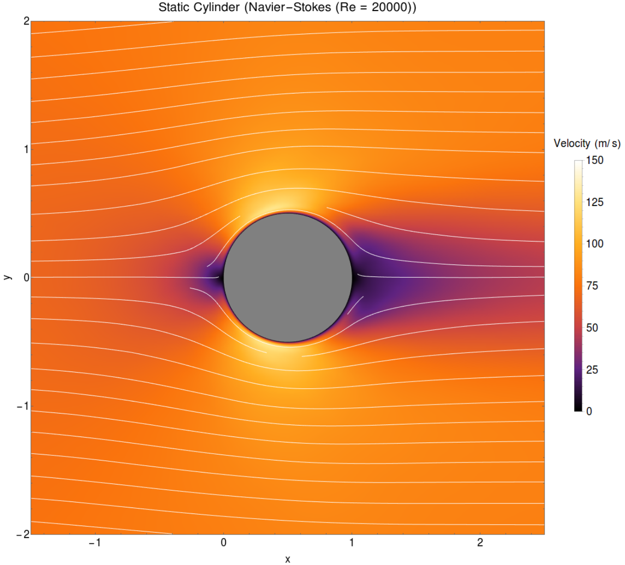
Airfoil


Airfoil (Angle of Attack 5.0°)
1.0
0.5
0.0
−0.5
−1.0
y
−0.5
0.0
0.5
1.0
1.5
2.0
x
Velocity (m/s)
400
300
200
100
0

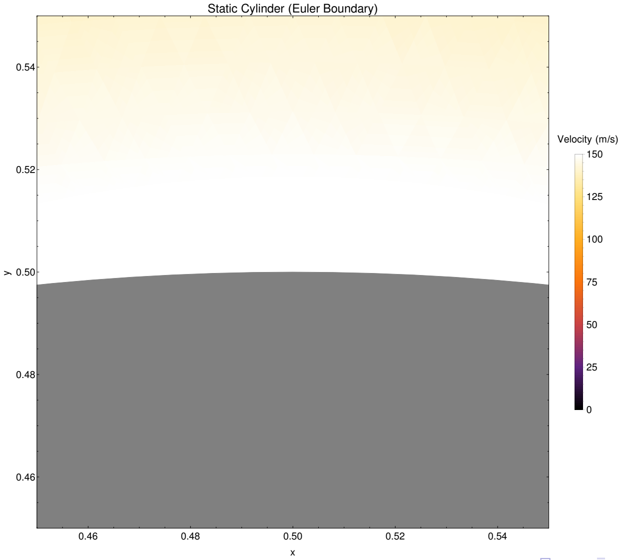
Static Cylinder
Euler



Static Cylinder
Navier-Stokes

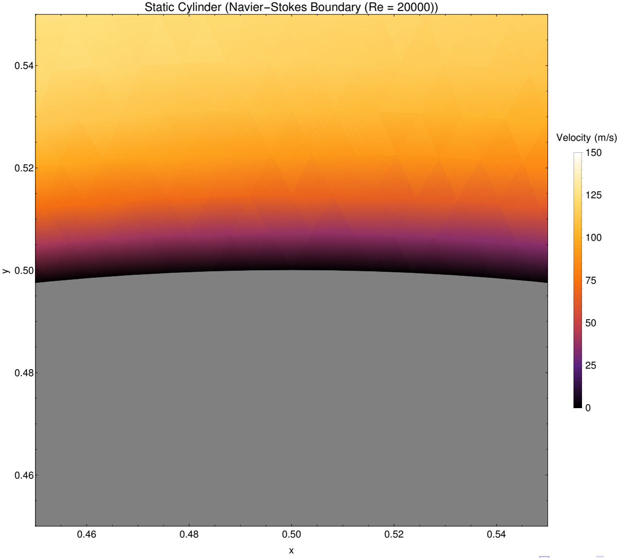


Static Cylinder
Euler Boundary

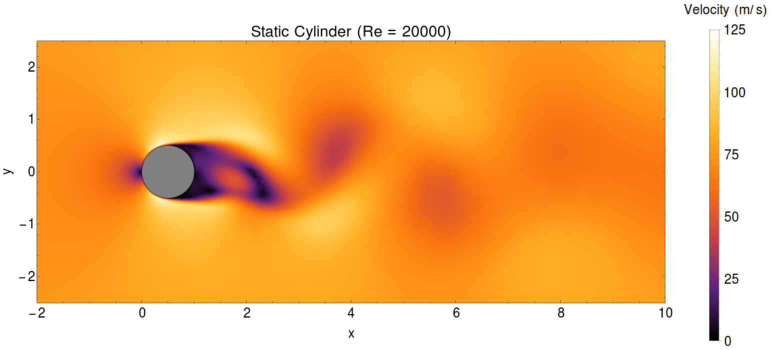


Static Cylinder

Navier-Stokes Boundary



Vortex Shedding



Conclusion