

# Computational Fluid Dynamics with the *SU<sup>2</sup>* Code

Brady Metherall                  100516905

Monday April 4, 2016

## List of Figures

1	Possibilities with CFD . . . . .	1
2	Mesh of an Airfoil . . . . .	3
3	Automated Wolfram Script . . . . .	5
4	Turbulent Airfoil Animation, and Laminar Airfoil Streamlines . . . . .	6
5	Static Cylinder . . . . .	8
6	Vortex Shedding Animation . . . . .	9
7	Vortex Shedding at $\frac{1}{4}T$ increments . . . . .	10

## 1 Introduction

CFD joins theory and experiment as third sub-field

CFD is like a wind tunnel but cheaper and faster

reshaping

changing Re, Mach, P ...

portable / remote access

Lorem ipsum dolor sit amet, consectetuer adipiscing elit. Ut purus elit, vestibulum ut, placerat ac, adipiscing vitae, felis. Curabitur dictum gravida mauris. Nam arcu libero, nonummy eget, consectetuer id, vulputate a, magna. Donec vehicula augue eu neque. Pellentesque habitant morbi tristique senectus et netus et malesuada fames ac turpis egestas. Mauris ut leo. Cras viverra metus rhoncus sem. Nulla et lectus vestibulum urna fringilla ultrices. Phasellus eu tellus sit amet tortor gravida placerat. Integer sapien est, iaculis in, pretium quis, viverra ac, nunc. Praesent eget sem vel leo ultrices bibendum. Aenean faucibus. Morbi dolor nulla, malesuada eu, pulvinar at, mollis ac, nulla. Curabitur auctor semper nulla. Donec varius orci eget risus. Duis nibh mi, congue eu, accumsan eleifend, sagittis quis, diam. Duis eget orci sit amet orci dignissim rutrum.

Nam dui ligula, fringilla a, euismod sodales, sollicitudin vel, wisi. Morbi auctor lorem non justo. Nam lacus libero, pretium at, lobortis vitae, ultricies et, tellus. Donec aliquet, tortor sed accumsan bibendum, erat ligula aliquet magna, vitae ornare odio metus a mi. Morbi ac orci et nisl hendrerit mollis. Suspendisse ut massa. Cras nec ante. Pellentesque a nulla. Cum sociis natoque penatibus et magnis dis parturient montes, nascetur ridiculus mus. Aliquam tincidunt urna. Nulla ullamcorper vestibulum turpis. Pellentesque cursus luctus mauris.

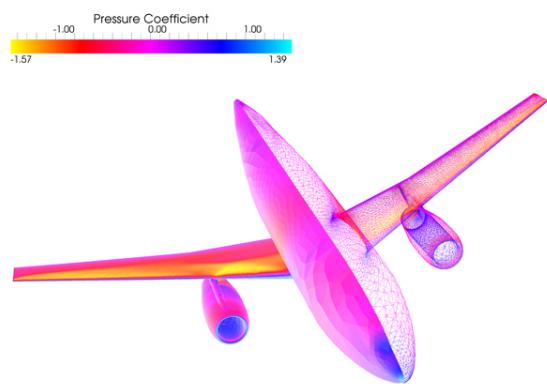


Figure 1: Possibilities with CFD [7]

## 2 Theory

### 2.1 Non-Turbulent Flow Equations

Non-viscous, ideal fluids obey two equations of motion collectively called Euler's equations. The first is the incompressibility condition

$$\nabla \cdot \mathbf{u} = 0 \quad (1)$$

which arises by assuming the volume of a fluid element, as well as it's density is constant. The second equation can be derived from Newton's second law,

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla) \mathbf{u} = -\frac{1}{\rho} \nabla p + \mathbf{g}. \quad (2)$$

The problem with Euler's equations is that they are only applicable for non-viscous fluids, which is not very practical. One of the subtler details is that the force experienced by a fluid element is always normal to it's surface. However, due to the fluid elements interacting, tangential forces arise. Cauchy's equation,

$$\rho \left( \frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla) \mathbf{u} \right) = \nabla \cdot \mathbf{T} + \rho \mathbf{g} \quad (3)$$

adds a stress tensor to handle theses additional forces. If we chose  $\mathbf{T}_{ij} = -p\delta_{ij}$  we simply get (2), as expected, however, if we chose a more complex tensor with a deformation term, such as

$$T_{ij} = -p\delta_{ij} + \mu \left( \frac{\partial u_j}{\partial x_i} + \frac{\partial u_i}{\partial x_j} \right),$$

(3) reduces to

$$\begin{aligned} \rho \left( \frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla) \mathbf{u} \right) &= -\nabla p + \mu \nabla^2 \mathbf{u} + \rho \mathbf{g} \quad \text{or,} \\ \frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla) \mathbf{u} &= -\frac{1}{\rho} \nabla p + \nu \nabla^2 \mathbf{u} + \mathbf{g}, \end{aligned} \quad (4)$$

which along with the incompressibility condition, (1), are the Navier-Stokes equations. Notice that if  $\nu = 0$ , there is no interaction between fluids elements, and this reduces simply to Euler's equations, (2).

### 2.2 Reynolds-Averaged Navier-Stokes (RANS) Equation

The Navier-Stokes equations are notoriously difficult to solve since it is a second order partial differential equation, with three components, and four dependencies each. To make solving the Navier-Stokes equations easier, especially for turbulent flows, we can assume the flow is the superposition of the steady, and turbulent flows, and then take the time average. We can make the following variable substitutions

$$\begin{aligned} \alpha_i &\equiv \bar{\alpha}_i + \alpha'_i, \\ \text{where } \bar{\alpha}_i &= \frac{1}{\tau} \int_0^\tau \alpha_i dt \end{aligned}$$

is the steady time average on some time scale  $\tau$ , and  $\alpha'_i$  is the turbulent perturbations. It follows from this definition that

$$\bar{\alpha}'_i = 0, \quad \frac{\partial \bar{\alpha}_i}{\partial t} = 0, \quad \bar{\alpha}_i = \bar{\alpha}_i, \quad \text{and} \quad \frac{\partial \bar{\alpha}_i}{\partial x_j} = \frac{\partial \bar{\alpha}_i}{\partial x_j}. \quad (5)$$

We can write (4) in the more suggestive component form:

$$\sum_j \frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} = \sum_j -\frac{1}{\rho} \frac{\partial p}{\partial x_i} + \nu \frac{\partial^2 u_i}{\partial x_j^2} + g_i,$$

then, making the appropriate substitutions, taking the time average, and using the properties from (5), we obtain

$$\sum_j \overline{u_j} \frac{\partial \overline{u_i}}{\partial x_j} + \overline{u'_j} \frac{\partial \overline{u'_i}}{\partial x_j} = \sum_j -\frac{1}{\rho} \frac{\partial \overline{p}}{\partial x_i} + \nu \frac{\partial^2 \overline{u_i}}{\partial x_j^2} + \overline{g_i}.$$

Rewriting this in vector form gives the Reynolds-Averaged Navier-Stokes equation,

$$(\overline{\mathbf{u}} \cdot \nabla) \overline{\mathbf{u}} + (\overline{\mathbf{u}' \cdot \nabla}) \overline{\mathbf{u}'} = -\frac{1}{\rho} \nabla \overline{p} + \nu \nabla^2 \overline{\mathbf{u}} + \overline{\mathbf{g}}. \quad (6)$$

### 2.3 Spalart-Allmaras (SA) Turbulent Model

While (6) makes fluid flow predictions easier, a model is still needed for  $\nu$ . According to the Boussinesq hypothesis [4], an increase in the viscosity gives the effect of turbulence, and therefore  $\nu = \bar{\nu} + \nu'$ . One such way to model this altered viscosity and the turbulence, is the Spalart-Allmaras turbulence model. This model determines the turbulent viscosity using the following:

$$\nu' = \hat{\nu} f_{v1}; \quad f_{v1} = \frac{\chi^3}{\chi^3 + c_{v1}^3}; \quad \chi \equiv \frac{\hat{\nu}}{\bar{\nu}}. \quad (7)$$

Assuming the flow is incompressible, and the diffusivity is constant,  $\hat{\nu}$  is obtained by solving the convection-diffusion equation,

$$\frac{\partial \hat{\nu}}{\partial t} = D \nabla^2 \hat{\nu} - \overline{\mathbf{u}} \nabla \hat{\nu},$$

where  $D$  is the diffusivity.

## 3 Stanford University Unstructured ( $SU^2$ ) Code

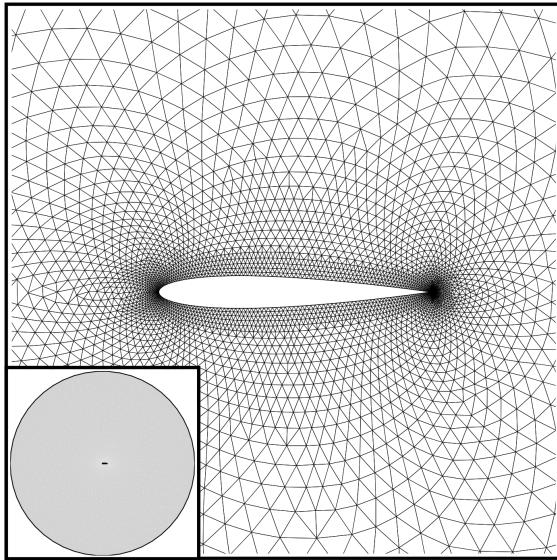


Figure 2: Unstructured mesh of an airfoil [6].

namics software are plagued with such as price, compatibility, and memory [1]. Some of the features of the  $SU^2$  code include, but are not limited to:

- Reynolds-Averaged Navier-Stokes equations (6)
- Spalart-Allmaras Turbulence model (7) ( $c_{v1} = 7.1$  [1] [2])
- Menter Shear Stress Transport model (an alternative to the SA model)
- Compressible flows
- Wave equation, heat equation, and plasma equations
- Optimal shape design (i.e. Supersonic aircraft)
- Dynamic meshes

To conduct a simulation, two main files are required, the configuration file, and the mesh file. All of the parameters such as the fluid speed, Reynolds number, and governing equations are specified in the configuration file. The shape geometry of the problem is stored as an unstructured mesh – hence the name – in the mesh file, an example visualization is in Figure 2. The mesh is unstructured in the sense that there is no algebraic relation between nodes. Instead, the mesh is specified by listing the node coordinates, and the adjacent nodes it is connected to. The density of nodes is greatest at the surface since it is the most complex area, and any errors could quickly propagate through the solution.

Typically a particular node within the mesh will be surrounded by six triangular cells. Using the centroids of the cells, as well as the midpoints of the edges, we can construct a hexagonal region,  $\Omega_i$ , which surrounds the  $i$ th node. Our partial differential equations can be discretized by

$$\int_{\Omega_i} \frac{\partial \mathbf{u}}{\partial t} d\Omega + R_i(\mathbf{u}) = 0$$

where  $R_i(\mathbf{u})$  is the residual. After the spacial components have been discretized, the governing equations can be numerically solved with a variety of methods and optimizations. Furthermore, due to the way the solution is discretized, the mesh can dynamically be updated; this is a key feature in the shape optimization process.

## 4 Scripting and Automation

Throughout this project Wolfram Mathematica 10.1.0 for Linux x86 was used to sift through the vast data files and extract the relevant information to produce the plots. The *SU*<sup>2</sup> code natively writes to the data files in the format used for Tecplot<sup>1</sup>, a visualization and analysis tool for computational fluid dynamics [?]. Since Tecplot was not used, a Wolfram function was needed to convert the data to a form Mathematica could use. Once the data was in a usable form, the `ListDensityPlot` function was used to create the images. The function to tidy the data along with the plotting function were combined into a Wolfram script such as Figure 3, which can be executed from the terminal. To fully automate the image generation, a shell script was written to iterate over each data file.

## 5 Results

Lorem ipsum dolor sit amet, consectetuer adipiscing elit. Ut purus elit, vestibulum ut, placerat ac, adipiscing vitae, felis. Curabitur dictum gravida mauris. Nam arcu libero, nonummy eget, consectetuer id, vulputate a, magna. Donec vehicula augue eu neque. Pellentesque habitant morbi tristique senectus et netus et malesuada fames ac turpis egestas. Mauris ut leo. Cras viverra metus rhoncus sem. Nulla et lectus vestibulum urna fringilla ultrices. Phasellus eu tellus sit amet tortor gravida placerat. Integer sapien est, iaculis in, pretium quis, viverra ac, nunc. Praesent eget sem vel leo ultrices bibendum. Aenean faucibus. Morbi dolor nulla, malesuada eu, pulvinar at, mollis ac, nulla. Curabitur auctor semper nulla. Donec varius orci eget risus.

---

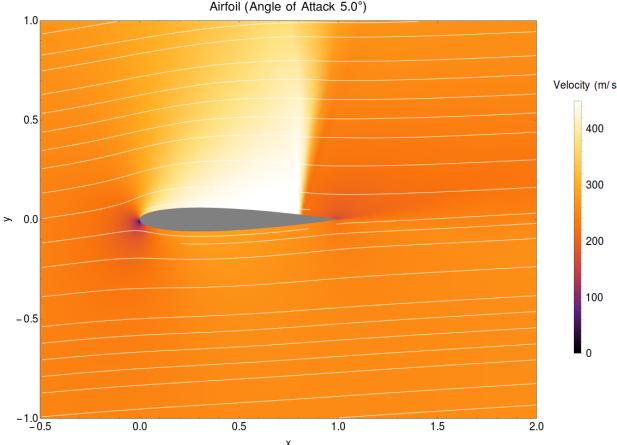
<sup>1</sup><http://www.tecplot.com/>

```

1  #!/usr/local/bin/WolframScript -script
2
3  (*Change the current directory*)
4  SetDirectory["/home;brady/SU2/CFD/Results/Pitching_Airfoil_Turb"];
5  Print[ToString[$CommandLine[[4]]]]; (*To check the progress*)
6
7  CSV = "surface_flow_0" <> ToString[$CommandLine[[4]]] <> ".csv";
8  DAT = "flow_0" <> ToString[$CommandLine[[4]]] <> ".dat";
9  PNG = "Airfoil_Turb" <> ToString[$CommandLine[[4]]] <> ".png";
10
11 (*Set the plot limits, colour function, and the legend style*)
12 xyzlimits = {{-0.5, 2}, {-1, 1}, {0, 450}};
13 colfunc = ColorData["SunsetColors"][[xyzlimits[[3, 2]]]] &;
14 leg = BarLegend[{colfunc, xyzlimits[[3]]}, LegendLabel → "Velocity (m/s)",
15     LegendMarkerSize → 500];
16
17 (*Draw a gray ploygon using the points of the surface_flow.csv*)
18 shape = Graphics[{Gray, Polygon[Import[CSV][[2;;-1, {2, 3}]]]}];
19
20 (*Clean the data so it's in a usable form*)
21 (*Import the data file, and remove the preamble (three lines)*)
22 datafile = Import[DAT][[4 ;; -1]];
23
24 (*There is four seemingly random numbers per line for several lines at*)
25 (*the end, this ignores those lines*)
26 Do[If[Dimensions[datafile[[i]]][[1]] == 4,
27     {CleanData = datafile[[1 ;; i - 1]], Break[]}],
28     {i, 1, Dimensions[datafile][[1]]}];
29 (*Only the first 5 columns are needed: x, y, \[\[Rho], \[\[Rho]u, \[\[Rho]v*)
30 Data = CleanData[[All, 1 ;; 5]];
31
32 (*Declare and fill array for the velocity*)
33 velocity = {};
34 Do[AppendTo[velocity, {Data[[i, 1]], Data[[i, 2]],
35     Sqrt[(Data[[i, 4]]/Data[[i, 3]])^2 + (Data[[i, 5]]/Data[[i, 3]])^2]}],
36     {i, 1, Length[Data]}]
37
38 velplot = ListDensityPlot[velocity, ColorFunction → colfunc,
39     PlotRange → xyzlimits, AspectRatio → Automatic,
40     LabelStyle → {Black, FontSize → 18}, PlotLegends → leg,
41     ColorFunctionScaling → False, Frame → True,
42     FrameLabel → {"x", "y"}, PlotLabel → "Pitching Airfoil",
43     ImageSize → Full];
44
45 contplot = ListContourPlot[velocity, PlotRange → xyzlimits,
46     ContourShading → None, Contours → {200, 250, 300, 350}];
47
48 SetDirectory["/home;brady/SU2/CFD/TeX/Airfoil_Animation_Turb"];
49 Export[PNG, Show[velplot, contplot, shape]]

```

Figure 3: The Wolfram script used to generate Figure 4b. Syntax highlighting modified from [5].



(a) Airfoil no turbulence.

(b) Pitching airfoil, with velocity contours.

Figure 4: Airfoil things.

Duis nibh mi, congue eu, accumsan eleifend, sagittis quis, diam. Duis eget orci sit amet orci dignissim rutrum.

Nam dui ligula, fringilla a, euismod sodales, sollicitudin vel, wisi. Morbi auctor lorem non justo. Nam lacus libero, pretium at, lobortis vitae, ultricies et, tellus. Donec aliquet, tortor sed accumsan bibendum, erat ligula aliquet magna, vitae ornare odio metus a mi. Morbi ac orci et nisl hendrerit mollis. Suspendisse ut massa. Cras nec ante. Pellentesque a nulla. Cum sociis natoque penatibus et magnis dis parturient montes, nascetur ridiculus mus. Aliquam tincidunt urna. Nulla ullamcorper vestibulum turpis. Pellentesque cursus luctus mauris.

## 5.1 Airfoil

Quisque ullamcorper placerat ipsum. Cras nibh. Morbi vel justo vitae lacus tincidunt ultrices. Lorem ipsum dolor sit amet, consectetur adipiscing elit. In hac habitasse platea dictumst. Integer tempus convallis augue. Etiam facilisis. Nunc elementum fermentum wisi. Aenean placerat. Ut imperdiet, enim sed gravida sollicitudin, felis odio placerat quam, ac pulvinar elit purus eget enim. Nunc vitae tortor. Proin tempus nibh sit amet nisl. Vivamus quis tortor vitae risus porta vehicula.

Fusce mauris. Vestibulum luctus nibh at lectus. Sed bibendum, nulla a faucibus semper, leo velit ultricies tellus, ac venenatis arcu wisi vel nisl. Vestibulum diam. Aliquam pellentesque, augue quis sagittis posuere, turpis lacus congue quam, in hendrerit risus eros eget felis. Maecenas eget erat in sapien mattis porttitor. Vestibulum porttitor. Nulla facilisi. Sed a turpis eu lacus commodo facilisis. Morbi fringilla, wisi in dignissim interdum, justo lectus sagittis dui, et vehicula libero dui cursus dui. Mauris tempor ligula sed lacus. Duis cursus enim ut augue. Cras ac magna. Cras nulla. Nulla egestas. Curabitur a leo. Quisque egestas wisi eget nunc. Nam feugiat lacus vel est. Curabitur consectetur.

Suspendisse vel felis. Ut lorem lorem, interdum eu, tincidunt sit amet, laoreet vitae, arcu. Aenean faucibus pede eu ante. Praesent enim elit, rutrum at, molestie non, nonummy vel, nisl. Ut lectus eros, malesuada sit amet, fermentum eu, sodales cursus, magna. Donec eu purus. Quisque vehicula, urna sed ultricies auctor, pede lorem egestas dui, et convallis elit erat sed nulla. Donec luctus. Curabitur et nunc. Aliquam dolor odio, commodo pretium, ultricies non, pharetra in, velit. Integer arcu est, nonummy in, fermentum faucibus, egestas vel, odio.

Sed commodo posuere pede. Mauris ut est. Ut quis purus. Sed ac odio. Sed vehicula hendrerit sem. Duis non odio. Morbi ut dui. Sed accumsan risus eget odio. In hac habitasse platea dictumst. Pellentesque non elit. Fusce sed justo eu urna porta tincidunt. Mauris felis odio, sollicitudin sed, volutpat a, ornare ac, erat. Morbi quis dolor. Donec pellentesque, erat ac sagittis semper, nunc dui lobortis purus, quis congue purus metus ultricies tellus. Proin et quam. Class aptent taciti sociosqu ad litora torquent per conubia nostra, per inceptos hymenaeos. Praesent sapien turpis, fermentum vel, eleifend faucibus, vehicula eu, lacus.

## 5.2 Steady Flow Around a Static Cylinder

Lorem ipsum dolor sit amet, consectetur adipiscing elit. Ut purus elit, vestibulum ut, placerat ac, adipiscing vitae, felis. Curabitur dictum gravida mauris. Nam arcu libero, nonummy eget, consectetur id, vulputate a, magna. Donec vehicula augue eu neque. Pellentesque habitant morbi tristique senectus et netus et malesuada fames ac turpis egestas. Mauris ut leo. Cras viverra metus rhoncus sem. Nulla et lectus vestibulum urna fringilla ultrices. Phasellus eu tellus sit amet tortor gravida placerat. Integer sapien est, iaculis in, pretium quis, viverra ac, nunc. Praesent eget sem vel leo ultrices bibendum. Aenean faucibus. Morbi dolor nulla, malesuada eu, pulvinar at, mollis ac, nulla. Curabitur auctor semper nulla. Donec varius orci eget risus. Duis nibh mi, congue eu, accumsan eleifend, sagittis quis, diam. Duis eget orci sit amet orci dignissim rutrum.

Nam dui ligula, fringilla a, euismod sodales, sollicitudin vel, wisi. Morbi auctor lorem non justo. Nam lacus libero, pretium at, lobortis vitae, ultricies et, tellus. Donec aliquet, tortor sed accumsan bibendum, erat ligula aliquet magna, vitae ornare odio metus a mi. Morbi ac orci et nisl hendrerit mollis. Suspendisse ut massa. Cras nec ante. Pellentesque a nulla. Cum sociis natoque penatibus et magnis dis parturient montes, nascetur ridiculus mus. Aliquam tincidunt urna. Nulla ullamcorper vestibulum turpis. Pellentesque cursus luctus mauris.

## 5.3 Turbulent Cylinder – Vortex Shedding

The equation (2) is interesting.

Lorem ipsum dolor sit amet, consectetur adipiscing elit. Ut purus elit, vestibulum ut, placerat ac, adipiscing vitae, felis. Curabitur dictum gravida mauris. Nam arcu libero, nonummy eget, consectetur id, vulputate a, magna. Donec vehicula augue eu neque. Pellentesque habitant morbi tristique senectus et netus et malesuada fames ac turpis egestas. Mauris ut leo. Cras viverra metus rhoncus sem. Nulla et lectus vestibulum urna fringilla ultrices. Phasellus eu tellus sit amet tortor gravida placerat. Integer sapien est, iaculis in, pretium quis, viverra ac, nunc. Praesent eget sem vel leo ultrices bibendum. Aenean faucibus. Morbi dolor nulla, malesuada eu, pulvinar at, mollis ac, nulla. Curabitur auctor semper nulla. Donec varius orci eget risus. Duis nibh mi, congue eu, accumsan eleifend, sagittis quis, diam. Duis eget orci sit amet orci dignissim rutrum.

Nam dui ligula, fringilla a, euismod sodales, sollicitudin vel, wisi. Morbi auctor lorem non justo. Nam lacus libero, pretium at, lobortis vitae, ultricies et, tellus. Donec aliquet, tortor sed accumsan bibendum, erat ligula aliquet magna, vitae ornare odio metus a mi. Morbi ac orci et nisl hendrerit mollis. Suspendisse ut massa. Cras nec ante. Pellentesque a nulla. Cum sociis natoque penatibus et magnis dis parturient montes, nascetur ridiculus mus. Aliquam tincidunt urna. Nulla ullamcorper vestibulum turpis. Pellentesque cursus luctus mauris.

## 6 Conclusion

Fusce mauris. Vestibulum luctus nibh at lectus. Sed bibendum, nulla a faucibus semper, leo velit ultricies tellus, ac venenatis arcu wisi vel nisl. Vestibulum diam. Aliquam pellentesque, augue quis sagittis posuere, turpis lacus congue quam, in hendrerit risus eros eget felis. Maecenas eget erat in sapien mattis porttitor. Vestibulum porttitor. Nulla facilisi. Sed a turpis eu lacus commodo facilisis. Morbi fringilla, wisi in dignissim interdum, justo lectus sagittis dui, et vehicula libero dui cursus dui. Mauris tempor ligula sed lacus. Duis cursus enim ut augue. Cras ac magna. Cras nulla. Nulla egestas. Curabitur a leo. Quisque egestas wisi eget nunc. Nam feugiat lacus vel est. Curabitur consectetur.

Suspendisse vel felis. Ut lorem lorem, interdum eu, tincidunt sit amet, laoreet vitae, arcu. Aenean faucibus pede eu ante. Praesent enim elit, rutrum at, molestie non, nonummy vel, nisl. Ut lectus eros, malesuada sit amet, fermentum eu, sodales cursus, magna. Donec eu purus. Quisque vehicula, urna sed ultricies auctor, pede lorem egestas dui, et convallis elit erat sed nulla. Donec luctus. Curabitur et nunc. Aliquam dolor odio, commodo pretium, ultricies non, pharetra in, velit. Integer arcu est, nonummy in, fermentum faucibus, egestas vel, odio.

Sed commodo posuere pede. Mauris ut est. Ut quis purus. Sed ac odio. Sed vehicula hendrerit sem. Duis non odio. Morbi ut dui. Sed accumsan risus eget odio. In hac habitasse platea dictumst. Pellentesque non

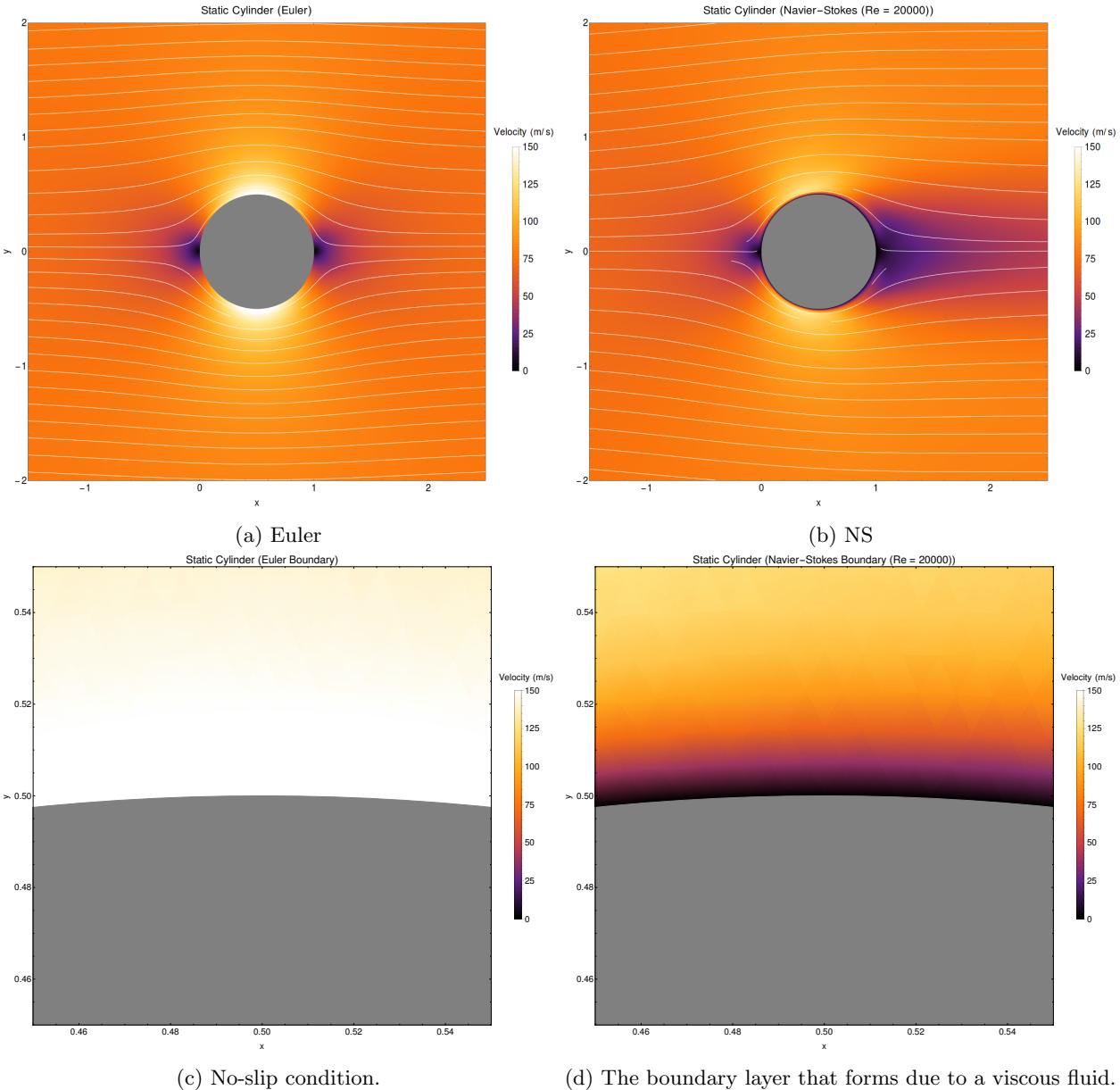


Figure 5: Static Cylinders



Figure 6: Neat! The full video can be found at <https://youtu.be/Zh5lWMpsCJg>

elit. Fusce sed justo eu urna porta tincidunt. Mauris felis odio, sollicitudin sed, volutpat a, ornare ac, erat. Morbi quis dolor. Donec pellentesque, erat ac sagittis semper, nunc dui lobortis purus, quis congue purus metus ultricies tellus. Proin et quam. Class aptent taciti sociosqu ad litora torquent per conubia nostra, per inceptos hymenaeos. Praesent sapien turpis, fermentum vel, eleifend faucibus, vehicula eu, lacus.

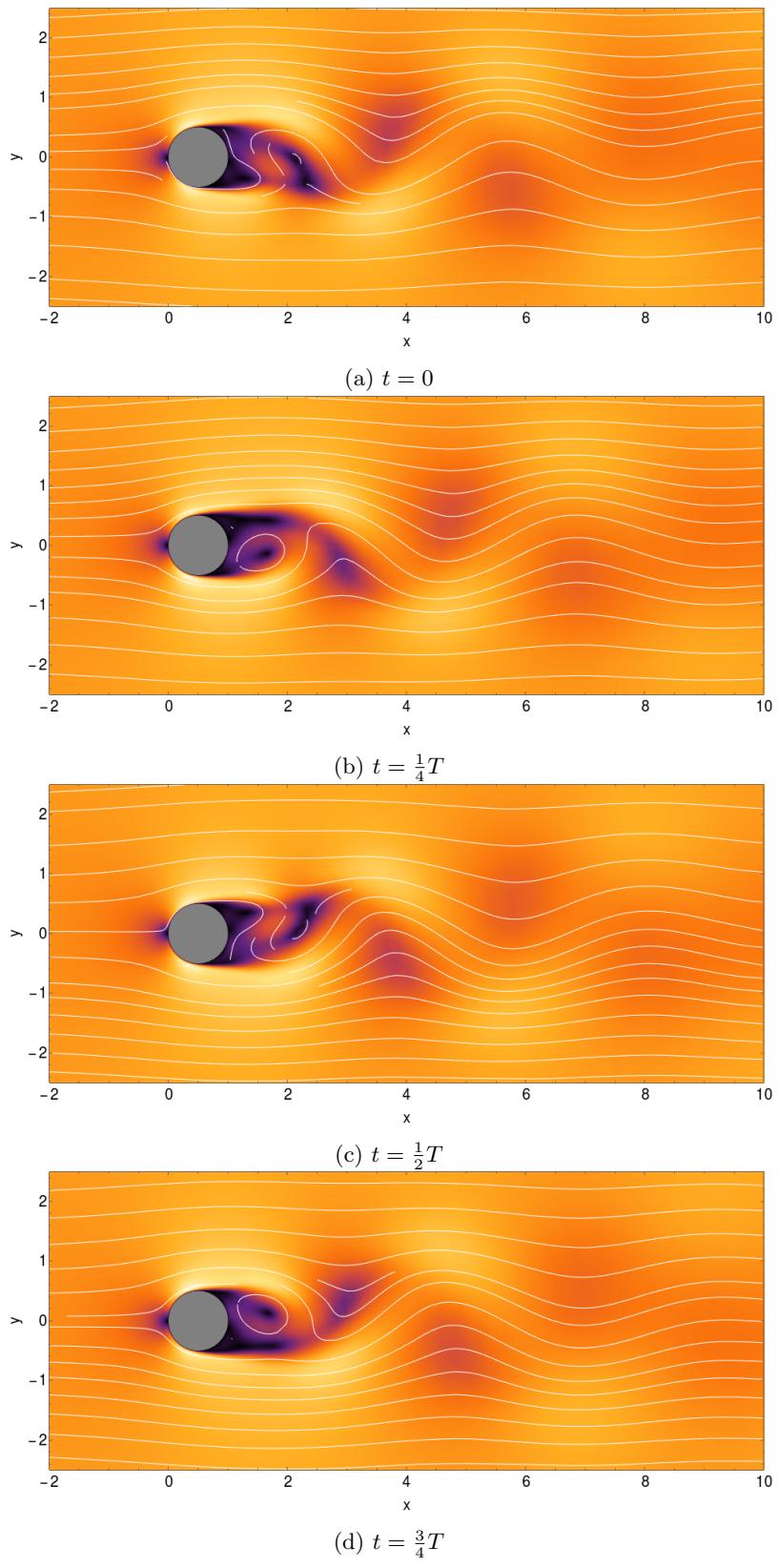


Figure 7: Vortex shedding!

## References

- [1] Francisco Palacios, Juan Alonso, Karthikeyan Duraisamy, Michael Colonna, Jason Hicken, Aniket Aranake, Alejandro Campos, Sean Copeland, Thomas Economou, Amrita Lonkar, Trent Lukaczyk, and Thomas Taylor, *Stanford University Unstructured (SU<sup>2</sup>): An open-source integrated computational environment for multi-physics simulation and design*, 51st AIAA Aerospace Sciences Meeting including the New Horizons Forum and Aerospace Exposition, 2013.
- [2] Francisco Palacios, Thomas D. Economou, Aniket Aranake, Sean R. Copeland, Amrita K. Lonkar, Trent W. Lukaczyk, David E. Manosalvas, Kedar R. Naik, Santiago Padron, Brendan Tracey, Anil Variyar, and Juan J. Alonso, *Stanford University Unstructured (SU<sup>2</sup>): Open-source Analysis and Design Technology for Turbulent Flows*, 52nd Aerospace Sciences Meeting. National Harbor, Maryland, 2014.
- [3] John D. Anderson, Jr, *Computational Fluid Dynamics*, McGraw-Hill, Inc., 1995.
- [4] D.C. Wilcox, *Turbulence Modeling for CFD*. 2nd Ed., DCWIndustries, Inc., 1998.
- [5] Belisarius (<http://mathematica.stackexchange.com/users/29581/belisarius>), Highlighting Mathematica code in L<sup>A</sup>T<sub>E</sub>Xdocument, URL: <http://mathematica.stackexchange.com/a/83811>
- [6] <https://github.com/su2code/SU2/wiki/Quick-Start>
- [7] <http://news.stanford.edu/news/2012/january/aero-engineering-software-012412.html>