Mapping Méléard and Roelly (1992) to Week et. al.

Description	Méléard and Roelly 92	Week et al
Arbitrary finite measure	$\mu \in M_F(\mathbb{R}^d)$	NA
Bounded function	$f \in C_b(\mathbb{R}^d)$	$f \in C_b(\mathbb{R})$
Non-negative continuous integrable function	NA	$\nu(\cdot, t) \in C_1^+(\mathbb{R})$
"Spatial" location	$x \in \mathbb{R}^d$	$x \in \mathbb{R} \dots$ interpreted as trait value
Infinitesimal generator of spatial movement	$A(\mu) \dots$ satisfies (H_1) and (H_2) below	$\frac{\mu}{2}\Delta \dots$ Laplace operator with diffusion coefficient $\mu/2$
Superprocess	$X_t \in M_F(\mathbb{R}^d)$ for $t \in [0, \infty)$	NA
Density process	NA	$\nu(x, t)$
Duality bracket	$\langle X_t, f \rangle \dots$ $X_t \in M_F(\mathbb{R}^d),\ f \in C_b(\mathbb{R}^d)$	$\int_{\mathbb{R}} \nu(x,t) f(x) dx \dots$ $\nu(\cdot, t) \in C_1^+(\mathbb{R}),\ f \in C_b^2(\mathbb{R})$
Malthusian parameter	$b(\mu)(x) \dots\ b(\mu) \in C_b(\mathbb{R}^d)$ for $\mu \in M_F(\mathbb{R}^d)$	$m\big(\nu(x,t), x\big) \dots$ bounded above, but not below
Stochastic parameter	$c(\mu)(x) \dots\ c(\mu) \in C_b(\mathbb{R}^d)$ for $\mu \in M_F(\mathbb{R}^d)$	$V > 0 \dots$ constant
Reproductive law	$p_k(x, \mu)$	NA
Mean of reproductive law	$m(x, \mu) = \sum_{k \in \mathbb{N}} k p_k(x, \mu) \dots$ $\sup_{x,\mu} \lvert m(x, \mu) \rvert < \infty$	$\mathscr{W}(\nu, x) \dots$ $\sup_{\nu,x} \mathscr{W}(\nu, x) < \infty$ introduced in appendix
Unnamed	$v^2(x, \mu) = \sum_{k \in \mathbb{N}} (k-1)^2 p_k(x, \mu)$	NA
Death rate	$\lambda(x, \mu)$	we assume constant "lifetimes"

Notes

- (H_1): For all $\mu \in M_F(\mathbb{R}^d)$ and all f in the domain of $A(\mu)$, there is a constant $K > 0$ such that $\|A(\mu)f\|_\infty \leq K\langle \mu, 1\rangle$.
- (H_2): For all f in the domain of $A(\mu)$, the map $\mu \to \langle \mu, A(\mu)f\rangle$ is continuous.
- In Week et al, spatial movement is interpreted as mutation. Hence, durations of spatial movement are not interpreted as lifetimes. Instead, we assume discrete non-overlapping generations for the branching diffusion process.
- In Champagnat, Ferrière and Méléard (2006) the condition $\sup_{x,\mu} |m(x,\mu)| < \infty$ has been weakened to $\sup_{x,\mu} m(x,\mu) < \infty$ and $\ln m(x,\mu) \geq C(1 + \mu(\mathbb{R}^d))$ for some constant $K > 0$ which they refer to as being linear or "more than linear".