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**Abstract** The text of your abstract. 150 – 250 words.

**Keywords** key · dictionary · word ·

**Mathematics Subject Classification (2000)** MSC code 1 · MSC code 2 ·

## 1 Introduction

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## 2 Background

In this section we first review two well-established techniques commonly used in sustainable fishery management. These are the maximum sustainable yield (MSY) and the constant escapement (CE) approaches. After this, deep reinforcement learning is briefly reviewed

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## 2.1 Fishery management

## 2.2 Deep reinforcement learning

## 3 Dynamical models used

In this section we present three models of increasing complexity which plausibly describe the population dynamics of a marine ecosystem. These models will form the test beds for the comparison between classical fishery management strategies and DRL.

### 3.1 A one-dimensional tipping point model.

Consider a population  $V$  whose dynamics is given by

$$\frac{dV}{dt} = rV(1 - V/K) - \frac{\beta HV^2}{V_0^2 + V^2}. \quad (1)$$

This model has been used in [2] to describe a grazing ecosystem, where a species  $V$  of vegetation is harvested by a constant herbivore population  $H$ .

In (1), a population  $V$  grows logistically with rate  $r$  up to carrying capacity  $K$ . This is expressed by the first term in the equation,

$$L(V \mid r, K) := rV(1 - V/K).$$

Moreover,  $V$  is predated on by a (constant) population  $H$ , as can be seen from the negative term

$$F(V, H \mid \beta, V_0) := \frac{\beta HV^2}{V_0^2 + V^2}$$

which saturates to  $\beta H$  as  $V \rightarrow \infty$ , and whose half maximum is  $V_0$ , i.e.  $F(V = V_0, H; \beta, V_0) = \beta H/2$ .

Ref. [2] studies the fixed points of (1) in order to show that in certain parameter regimes, its dynamics can undergo a *catastrophe*. A catastrophe is a sudden change in the state of the system from one stable state to another—often, the final state is ecologically detrimental, possibly associated with extinction or near-extinction events.

Fig. ?? shows the stable  $V$  populations for differing values of  $H$ . Here one sees that as  $H \rightarrow T_2$ , the top stable state is annihilated with the unstable fixed point. This way, if, e.g.  $H$  were to slowly drift until  $H = T_2$ , the system would collapse to the low stable state, leading to a near extinction of  $V$ .

## 4 Results

## 5 Discussion

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$$a^2 + b^2 = c^2 \quad (2)$$

## References

1. R. Mislevy, in *Educational Assessment*, ed. by R.L. Brennan (American Council on Education and Praeger Publishers, 2006), chap. 8
2. R.M. May, Thresholds and breakpoints in ecosystems with a multiplicity of stable states, *Nature* **269**(5628), 471 (1977)