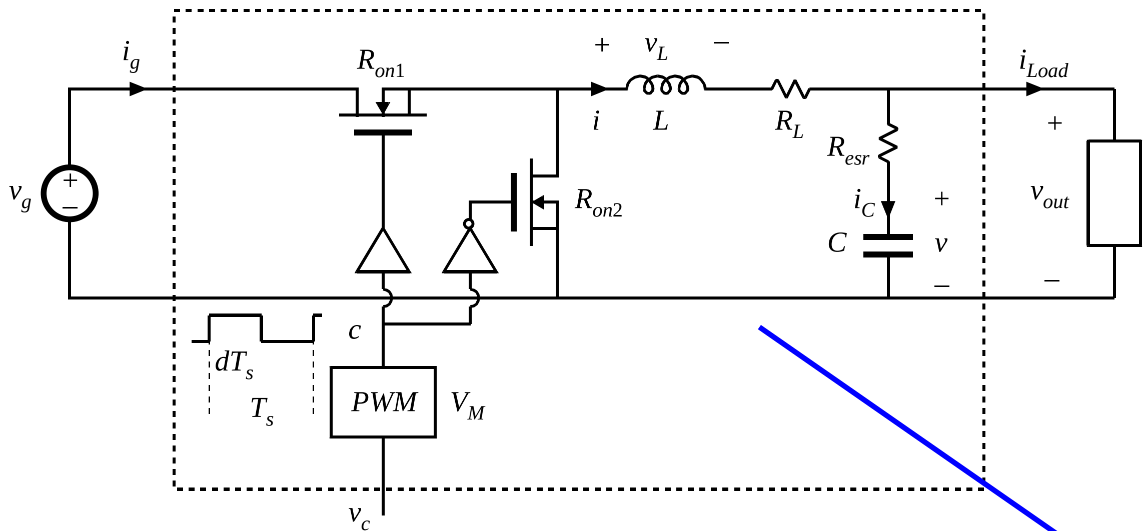
Introduction to Switched-Mode Converter Modeling using MATLAB/Simulink

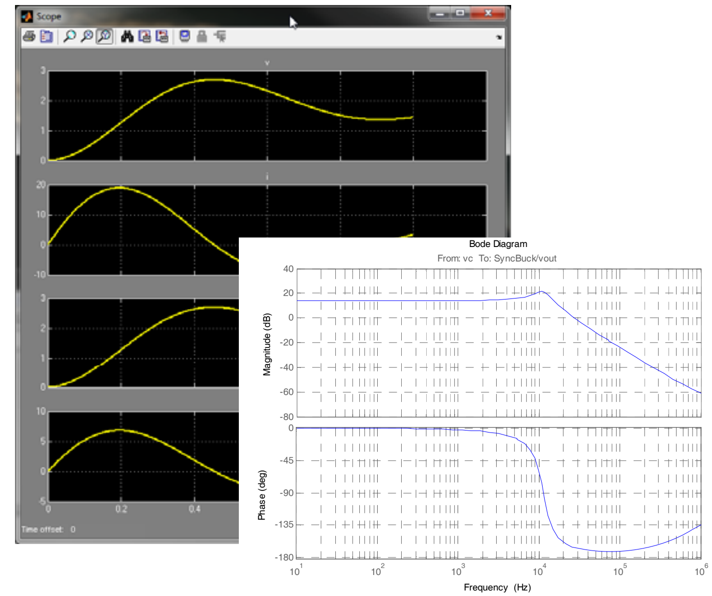
- MATLAB: programming and scripting environment
- Simulink: block-diagram modeling environment inside MATLAB
- Motivation:
  - Powerful environment for system modeling and simulation
  - More sophisticated controller models, analysis and design tools
- But*:
  - Block-diagram based Simulink models, unidirectional signals
  - Not a traditional circuit simulator; specialized physics-based Spice device models or component libraries are not readily available

*Various add-ons to Simulink are available to allow traditional circuit diagram entry and circuit simulations (e.g. SimPowerSystems, PLECS), or to embed Spice within MATLAB/Simulink environment. These add-ons are not required and will not be used in ECEN5807.

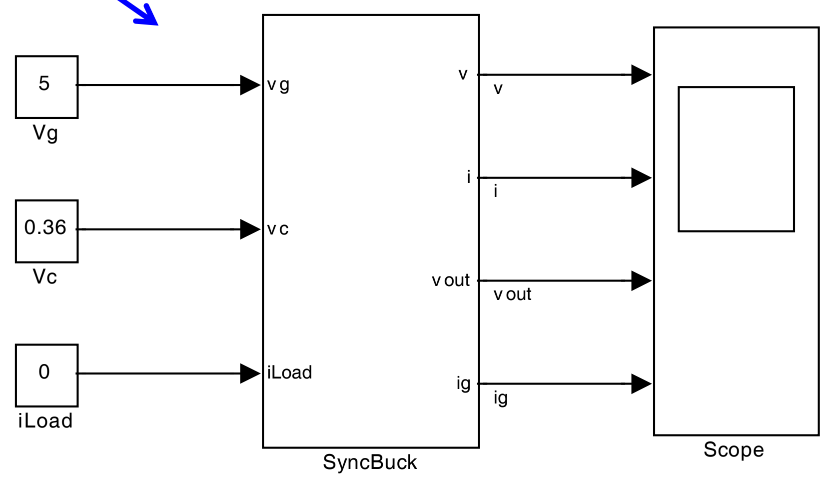
Introduction through an example

Synchronous buck converter





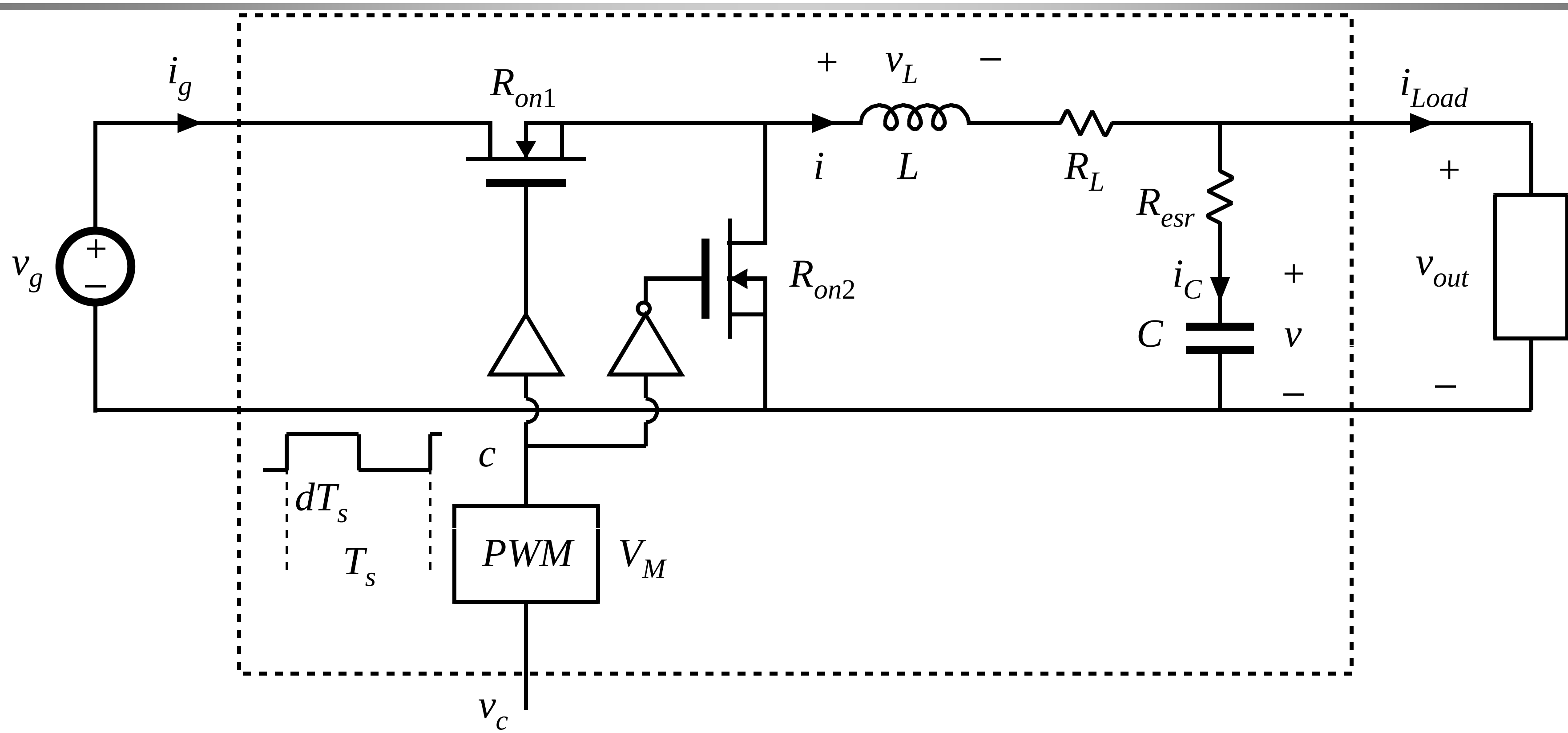
- Switching model
- Averaged model
- Small-signal linearization and frequency responses



See MATLAB/Simulink page on the course website (“Materials” page) for complete step-by-step details, and to download the example files

Simulink model: syncbuck_OL.mdl

Synchronous Buck Converter

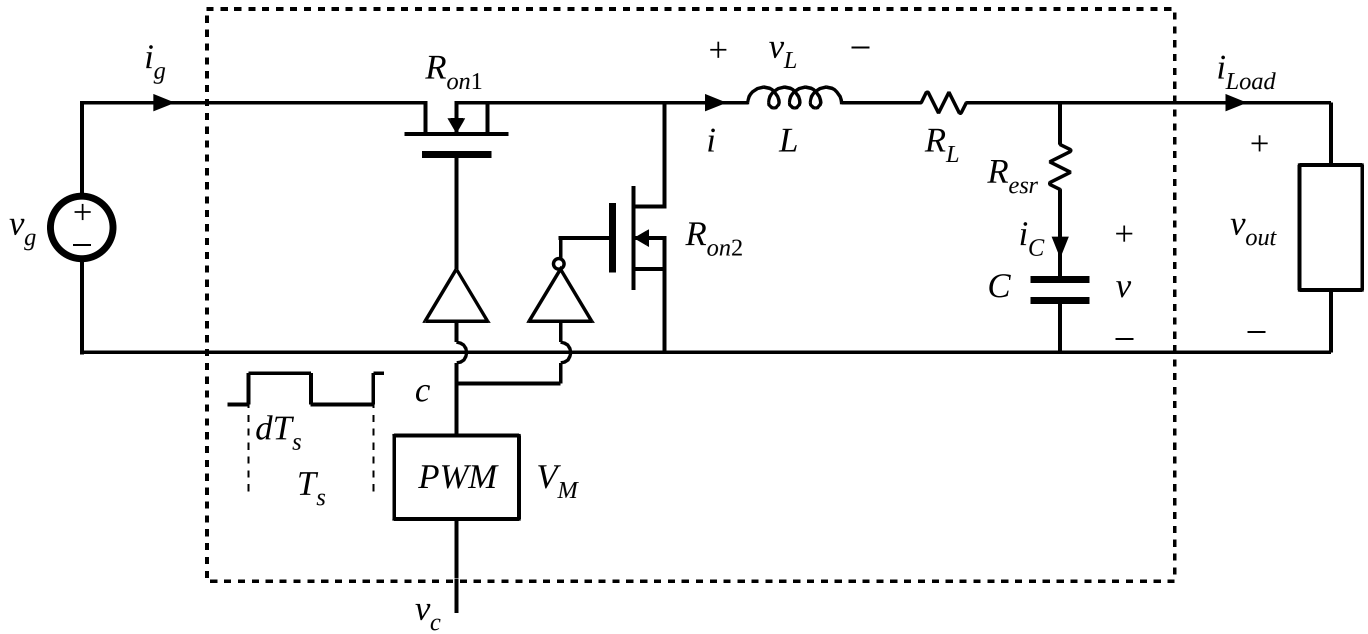


Inputs: v_g, i_{Load}, v_c

Outputs: v_{out}, i_g

State variables: v, i

Converter state equations



State equations

$$v_L = L\frac{di}{dt} = \begin{cases} v_g - (R_{on1} + R_L)i - v_{out} & (c=1) \\ -(R_{on2} + R_L)i - v_{out} & (c=0) \end{cases}$$

$$i_C = C\frac{dv}{dt} = i - i_{Load}$$

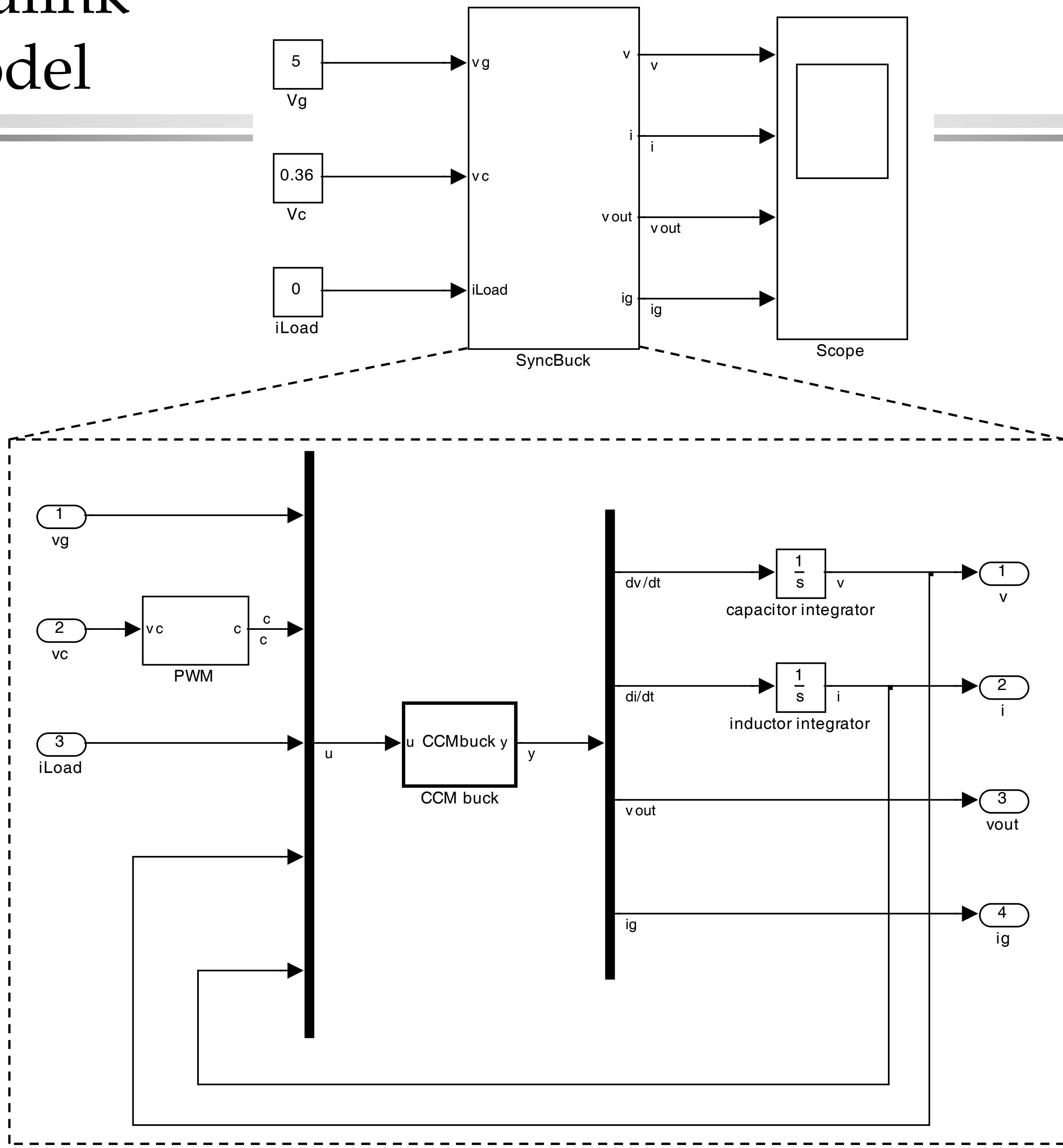
Output equations

$$i_g = \begin{cases} i & (c=1) \\ 0 & (c=0) \end{cases}$$

$$v_{out} = v + R_{esr}(i - i_{Load})$$

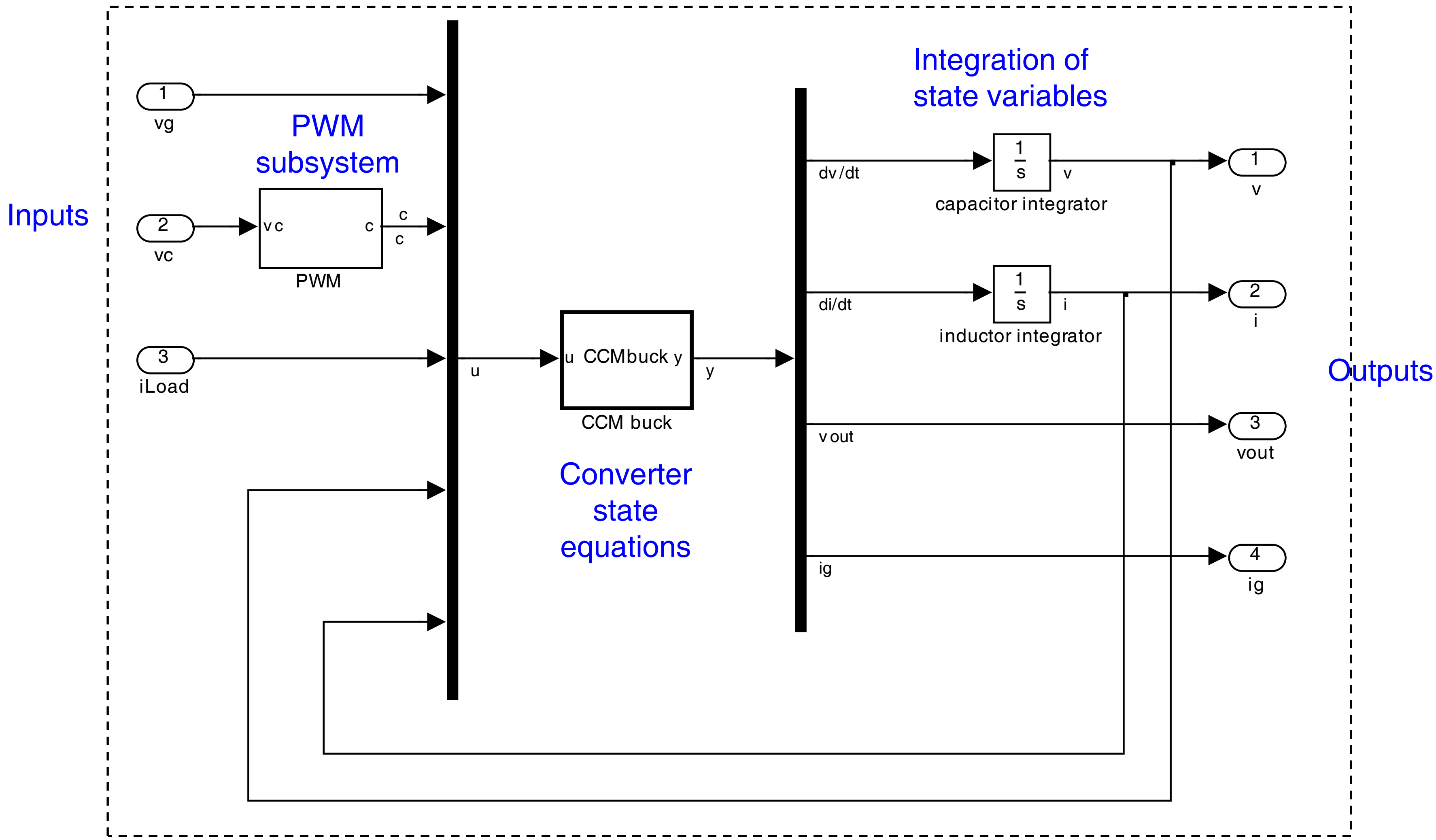
Simulink model

SyncBuck subsystem block

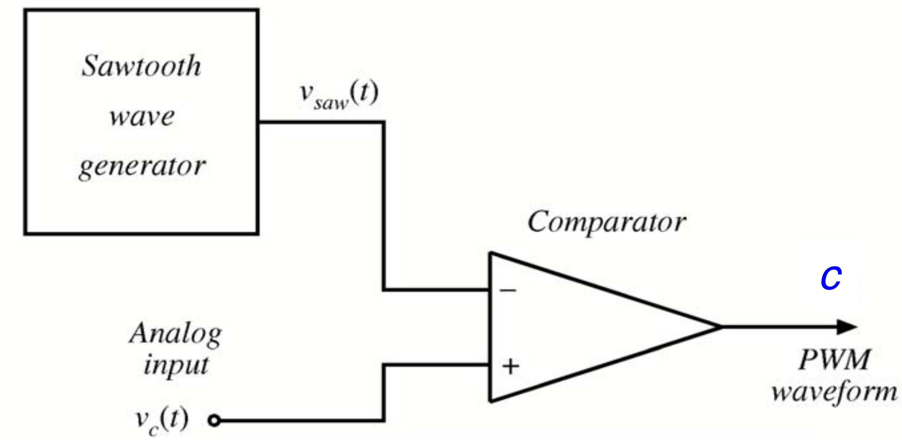


SyncBuck subsystem block internals

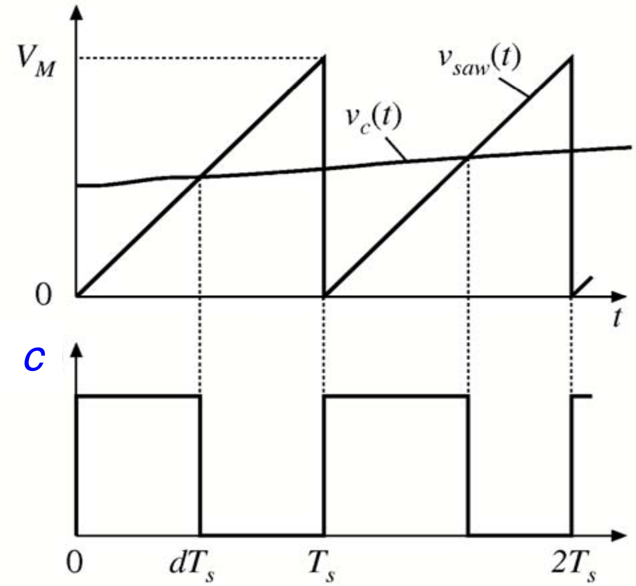
Synchronous buck (SyncBuck) subsystem

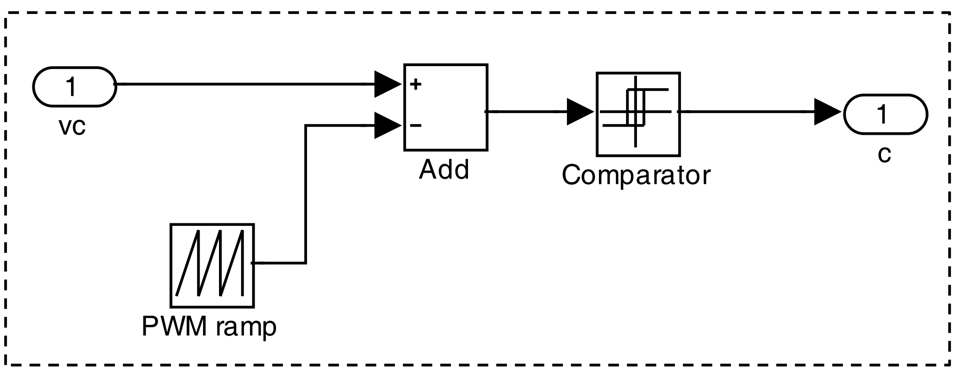


PWM operation and model



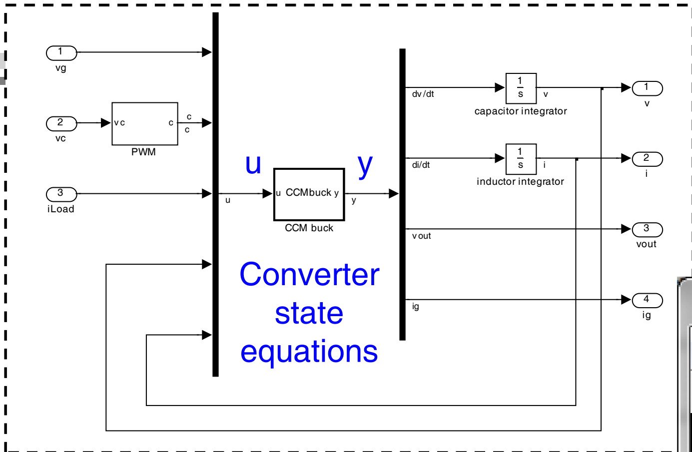
$$d(t) = \frac{v_c(t)}{V_M} \qquad for\ 0 \le v_c(t) \le V_M$$



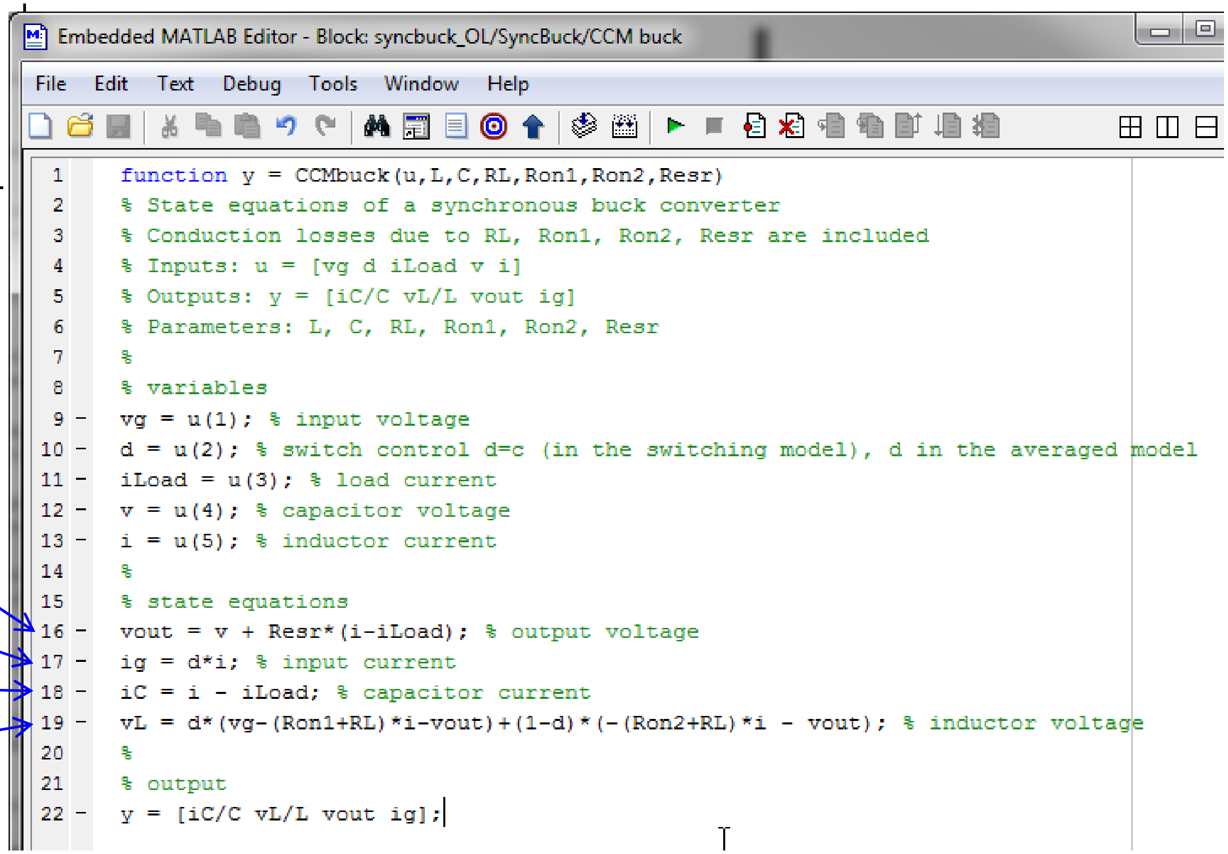


Simulink PWM model

Converter state equations: embedded MATLAB script



u = inputs = [vg c iLoad v i]
y = outputs = [ic/C vL/L vout ig]




```
function y = CCMbuck(u,L,C,RL,Ron1,Ron2,Resr)
% State equations of a synchronous buck converter
% Conduction losses due to RL, Ron1, Ron2, Resr are included
% Inputs: u = [vg d iLoad v i]
% Outputs: y = [iC/C vL/L vout ig]
% Parameters: L, C, RL, Ron1, Ron2, Resr
%
% variables
vg = u(1); % input voltage
d = u(2); % switch control d=c (in the switching model), d in the averaged model
iLoad = u(3); % load current
v = u(4); % capacitor voltage
i = u(5); % inductor current
%
% state equations
vout = v + Resr*(i-iLoad); % output voltage
ig = d*i; % input current
iC = i - iLoad; % capacitor current
vL = d*(vg-(Ron1+RL)*i-vout)+(1-d)*(-(Ron2+RL)*i - vout); % inductor voltage
%
% output
y = [iC/C vL/L vout ig];
```

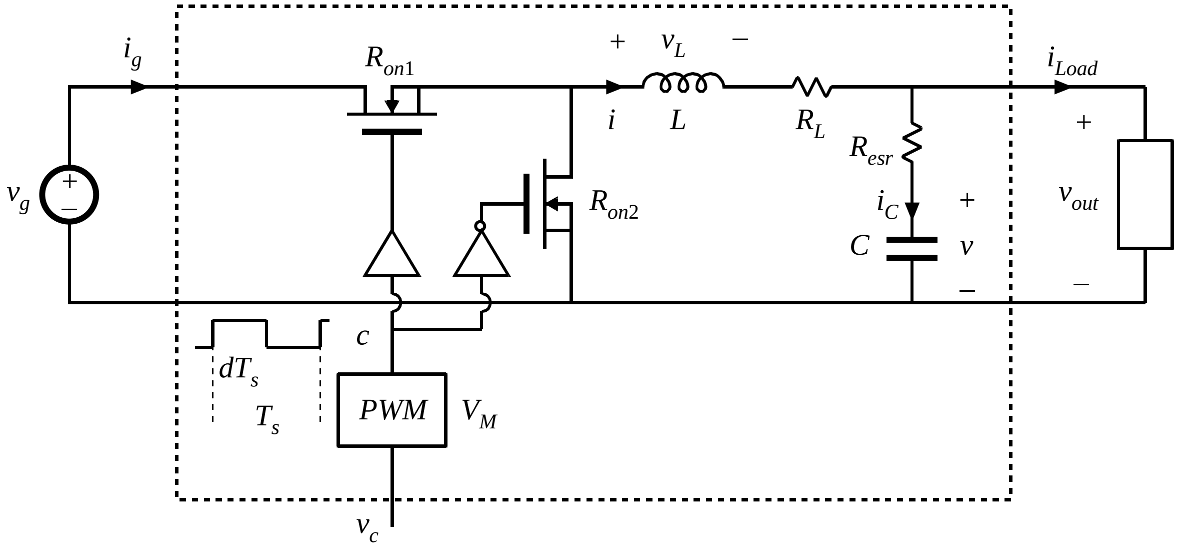
$$v_{out} = v + R_{esr}\left(i - i_{Load}\right)$$

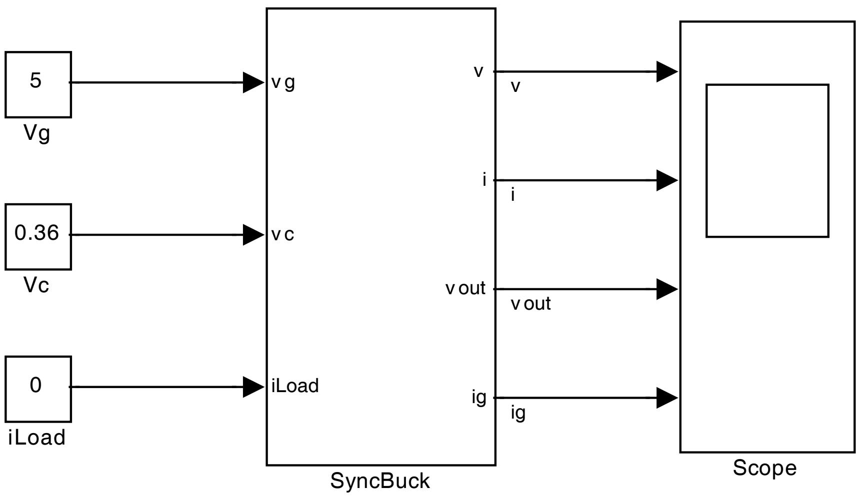
$$i_g = \begin{cases} i & (c=1) \\ 0 & (c=0) \end{cases}$$

$$i_C = C\frac{dv}{dt} = i - i_{Load}$$

$$v_L = L\frac{di}{dt} = \begin{cases} v_g - \left(R_{on1} + R_L\right)i - v_{out} & (c=1) \\ -\left(R_{on2} + R_L\right)i - v_{out} & (c=0) \end{cases}$$

Numerical example

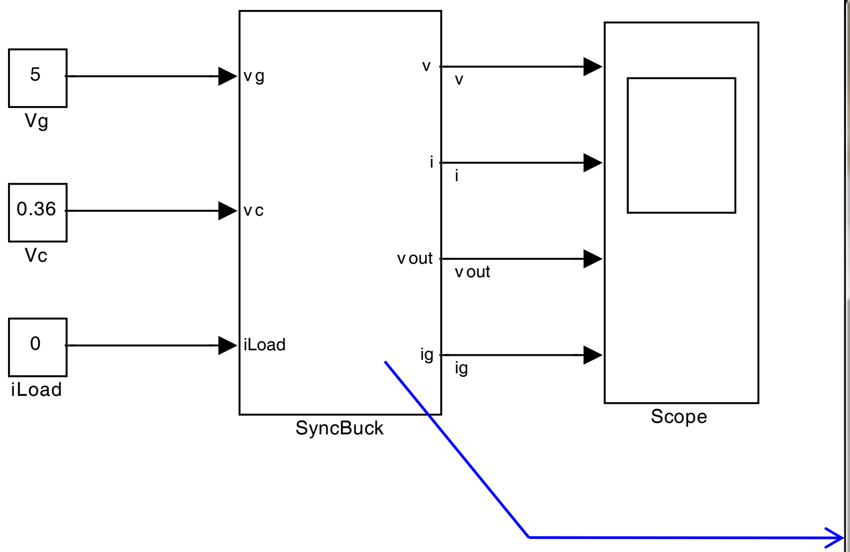


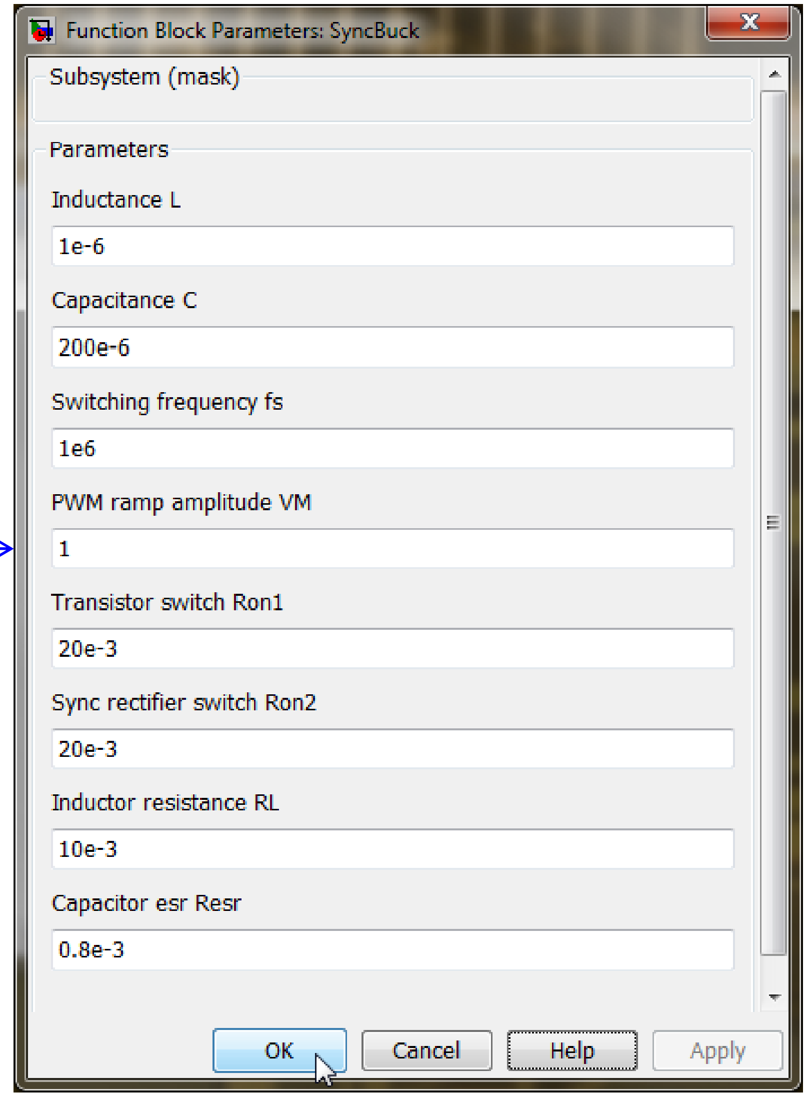


Simulink model: syncbuck_OL.mdl

- Switching frequency: f_s = 1MHz
- $I_{out} = 0$
- $V_g = 5$ V
- $L = 1$ μH
- $R_L = 10$ mΩ
- $R_{on1} = R_{on2} = 20$ mΩ
- $C = 200$ μF
- $R_{esr} = 0.8$ mΩ
- PWM ramp amplitude $V_M = 1$ V
- $V_c = 0.36,\ D = 0.36$

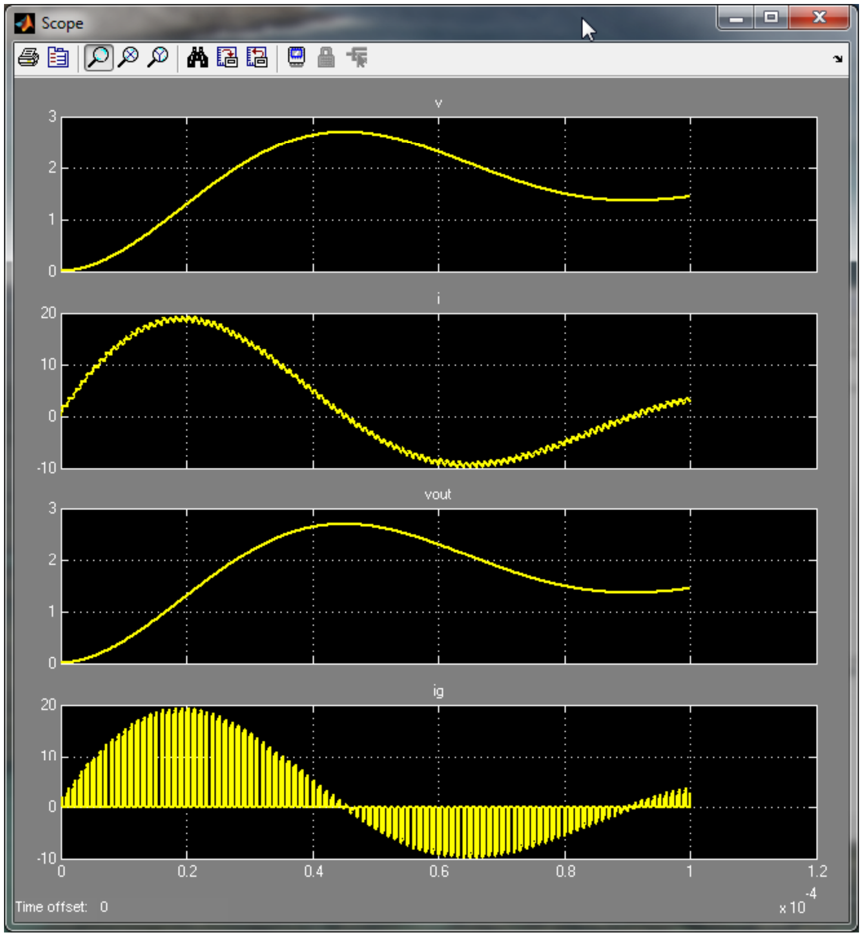
Numerical example: synchronous buck converter model



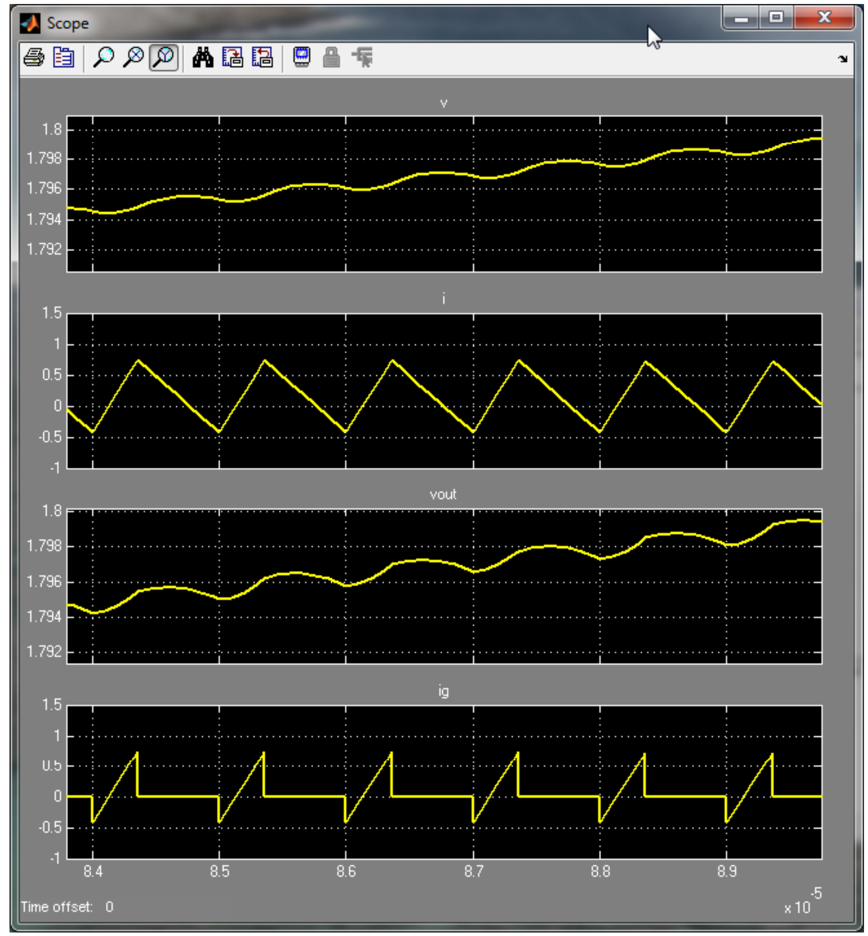


- “Masking” a Simulink subsystem allows parameterization
- Same subsystem model can be re-used
- Models and MATLAB scripts can be collected in a library

Switching simulation: open-loop start-up transient



20 μs/div



Zoom in, 1 μs/div

Averaged model

$$v_L = L\frac{di}{dt} = \begin{cases} v_g - (R_{on1} + R_L)i - v_{out} & (c=1) \\ -(R_{on2} + R_L)i - v_{out} & (c=0) \end{cases}$$

$$i_C = C\frac{dv}{dt} = i - i_{Load}$$

$$i_g = \begin{cases} i & (c=1) \\ 0 & (c=0) \end{cases}$$

$$v_{out} = v + R_{esr}(i - i_{Load})$$

Switching model

State-space averaging (review Textbook Sections 7.1-7.3)

$$\langle v_L \rangle_{T_s} = L\frac{d\langle i \rangle_{T_s}}{dt} = d\left(\langle v_g \rangle_{T_s} - (R_{on1} + R_L)\langle i \rangle_{T_s} - \langle v_{out} \rangle_{T_s}\right) + (1-d)\left(-(R_{on2} + R_L)\langle i \rangle_{T_s} - \langle v_{out} \rangle_{T_s}\right)$$

$$\langle i_C \rangle_{T_s} = C\frac{d\langle v \rangle_{T_s}}{dt} = \langle i \rangle_{T_s} - \langle i_{Load} \rangle_{T_s}$$

$$\langle i_g \rangle_{T_s} = d\langle i \rangle_{T_s}$$

$$\langle v_{out} \rangle_{T_s} = \langle v \rangle_{T_s} + R_{esr}\left(\langle i \rangle_{T_s} - \langle i_{Load} \rangle_{T_s}\right)$$

Large-signal averaged model

Converter averaged state equations: MATLAB

The MATLAB function stays exactly the same, except d (duty-cycle) replaces c (switch control)

Embedded MATLAB Editor - Block: syncbuck_OL/SyncBuck/CCM buck

File Edit Text Debug Tools Window Help

```
function y = CCMbuck(u,L,C,RL,Ron1,Ron2,Resr)
% State equations of a synchronous buck converter
% Conduction losses due to RL, Ron1, Ron2, Resr are included
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% Outputs: y = [iC/C vL/L vout ig]
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% variables
vg = u(1); % input voltage
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iLoad = u(3); % load current
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% state equations
vout = v + Resr*(i-iLoad); % output voltage
ig = d*i; % input current
iC = i - iLoad; % capacitor current
vL = d*(vg-(Ron1+RL)*i-vout)+(1-d)*(-(Ron2+RL)*i - vout); % inductor voltage
%
% output
y = [iC/C vL/L vout ig];
```
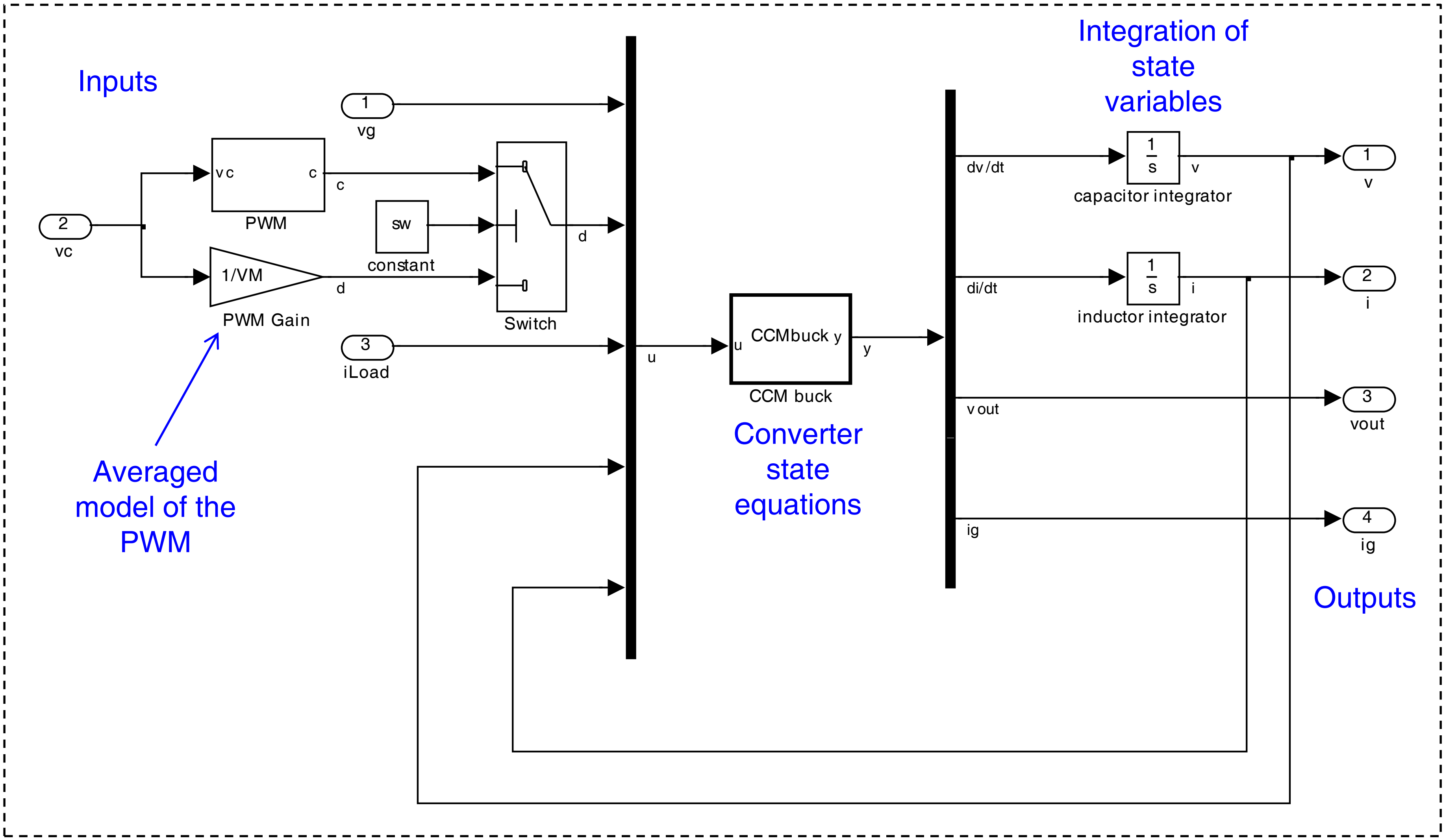
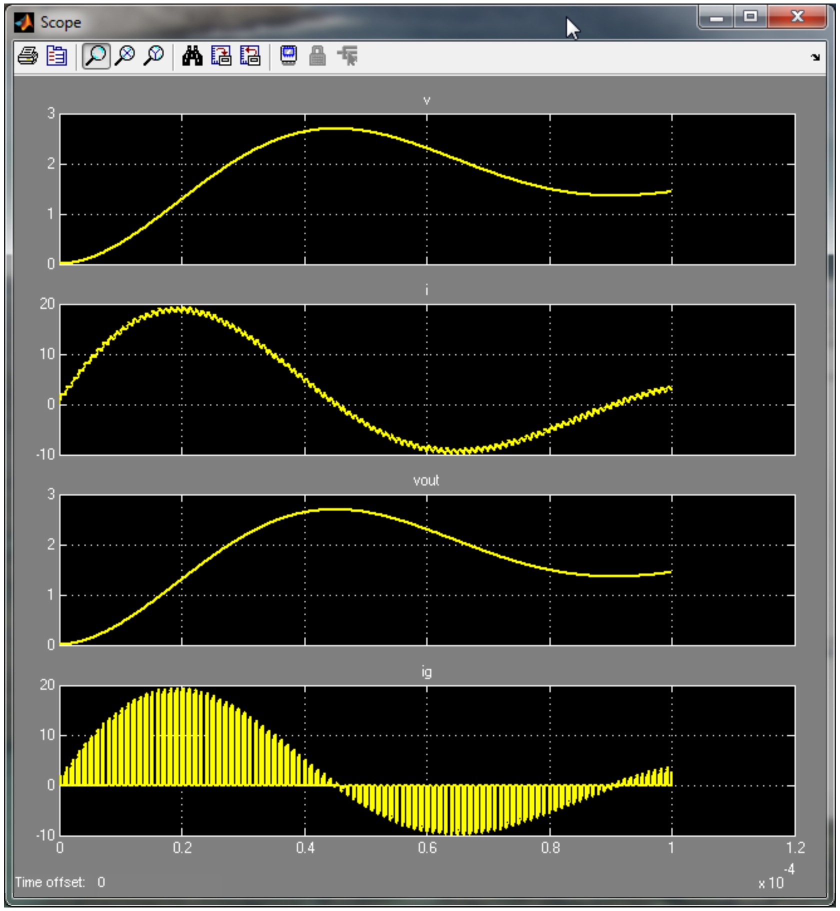

Synchronous buck (SyncBuck) subsystem: switching or averaged model



Start-up transient simulations

Switching model



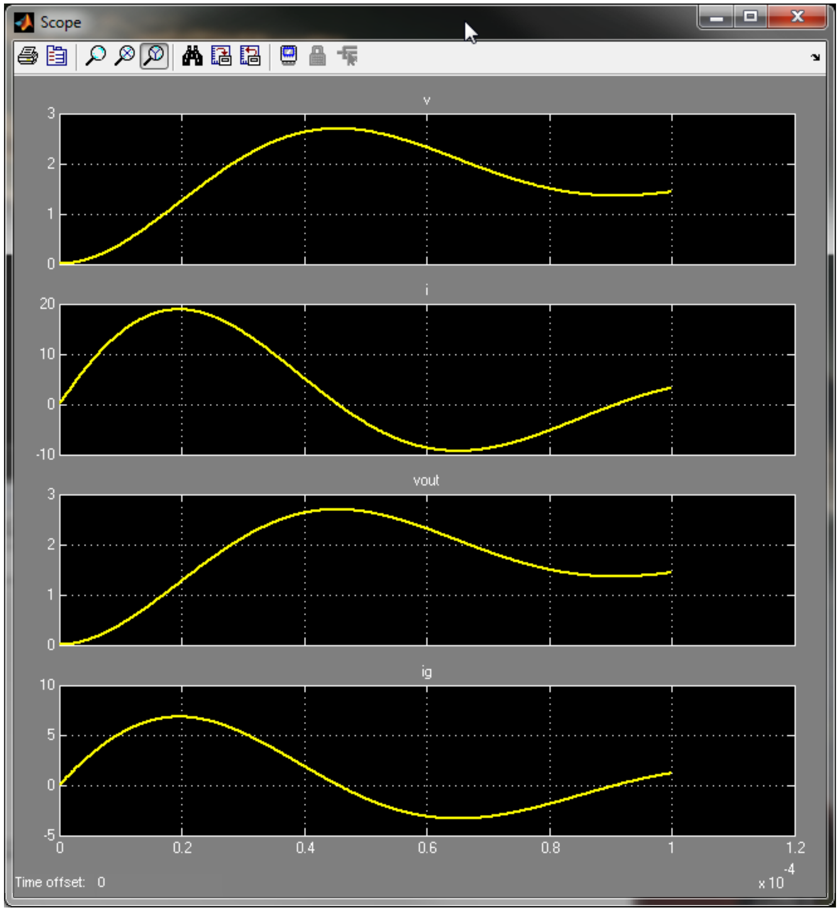
v

i

v_{out}

i_g

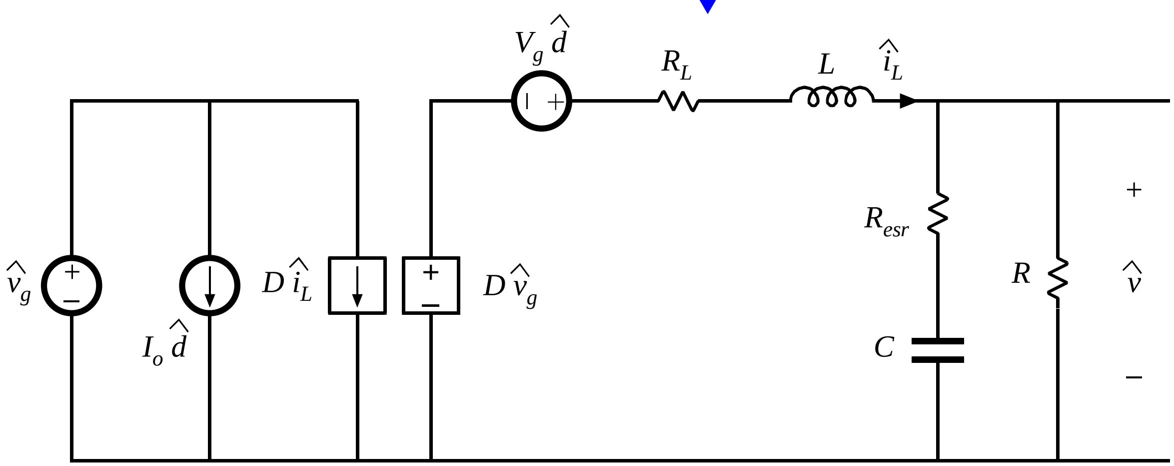
Averaged model



Linearization of the large-signal averaged model

Large-signal (nonlinear) averaged model

Linearization at an operating point



Small-signal averaged model

The small-signal model can be solved for all important converter transfer functions:

$$\boxed{G_{vd}(s) = \frac{\hat{v}}{\hat{d}}}$$

Control-to-output

$$\boxed{G_{vg}(s) = \frac{\hat{v}}{\hat{v}_g}}$$

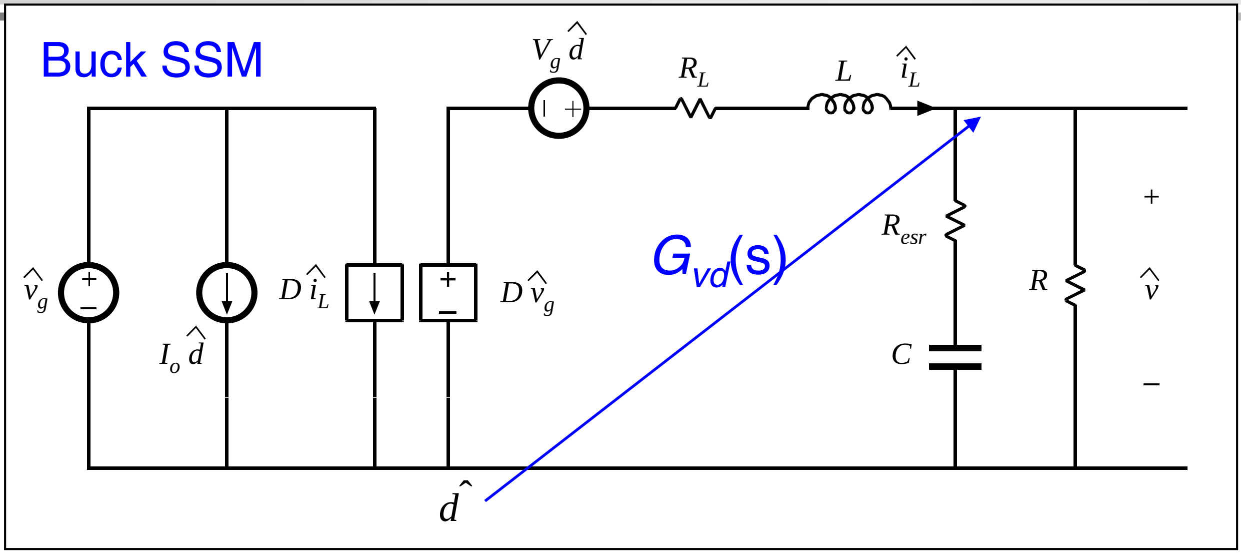
Line-to-output

$$\boxed{Z_{out}(s) = \frac{\hat{v}}{\hat{i}_{load}}}$$

Output impedance

Review textbook Chapter 8

Synchronous buck converter example



$$G_{vd}(s) = \frac{\hat{v}_o}{\hat{d}}$$

$$G_{vd}(s) = V_g \frac{1 + \frac{s}{\omega_{esr}}}{1 + \frac{1}{Q}\frac{s}{\omega_o} + \left(\frac{s}{\omega_o}\right)^2}$$

Pair of poles:

$$f_o = \frac{1}{2\pi\sqrt{CL}} = 11\,\text{kHz}$$

$$Q_{loss} = \frac{\sqrt{L/C}}{R_{esr} + R_L} = 2.3 \to 7.2\,\text{dB} \qquad Q_{load} = \frac{R}{\sqrt{L/C}} > 5$$

$$Q = Q_{loss} \parallel Q_{load} = \frac{Q_{loss}Q_{load}}{Q_{loss} + Q_{load}} < 2.3 \to 7.2\,\text{dB}$$

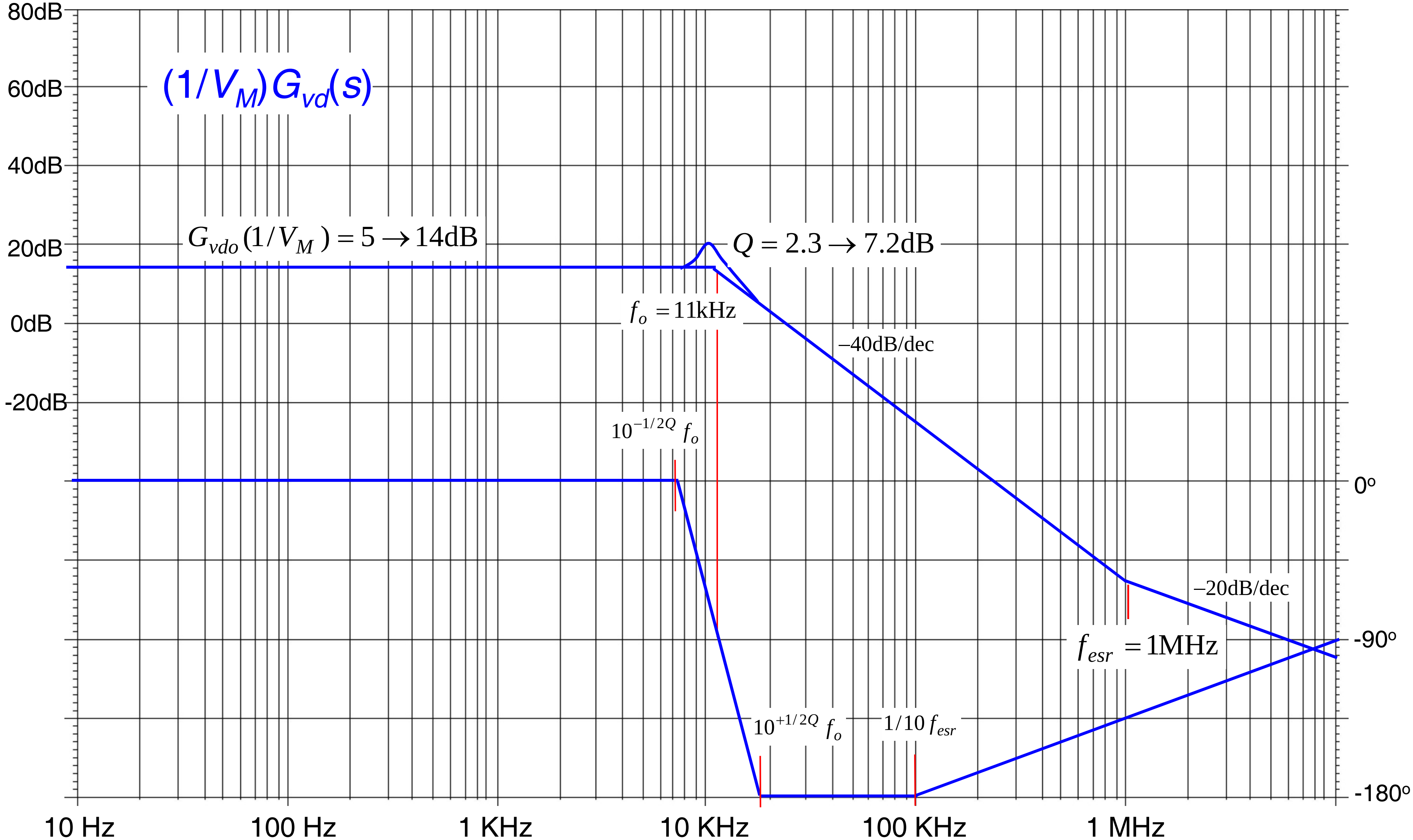
Low-frequency gain:

$$G_{vdo} = 5\text{V} \to 14\text{dBV}$$

ESR zero:

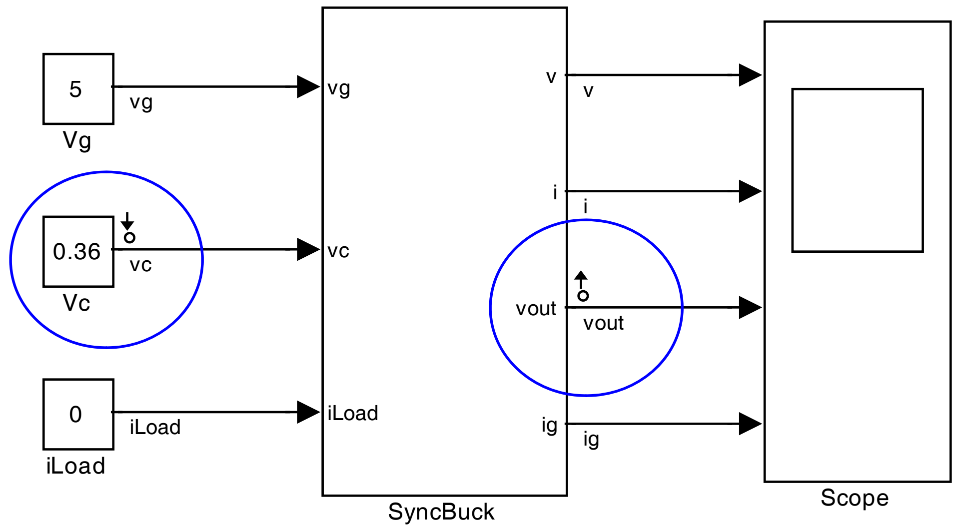
$$f_{esr} = \frac{1}{2\pi C R_{esr}} = 1\,\text{MHz}$$

Magnitude and phase Bode plots of G_{vd}



Linearization and frequency responses in MATLAB/Simulink

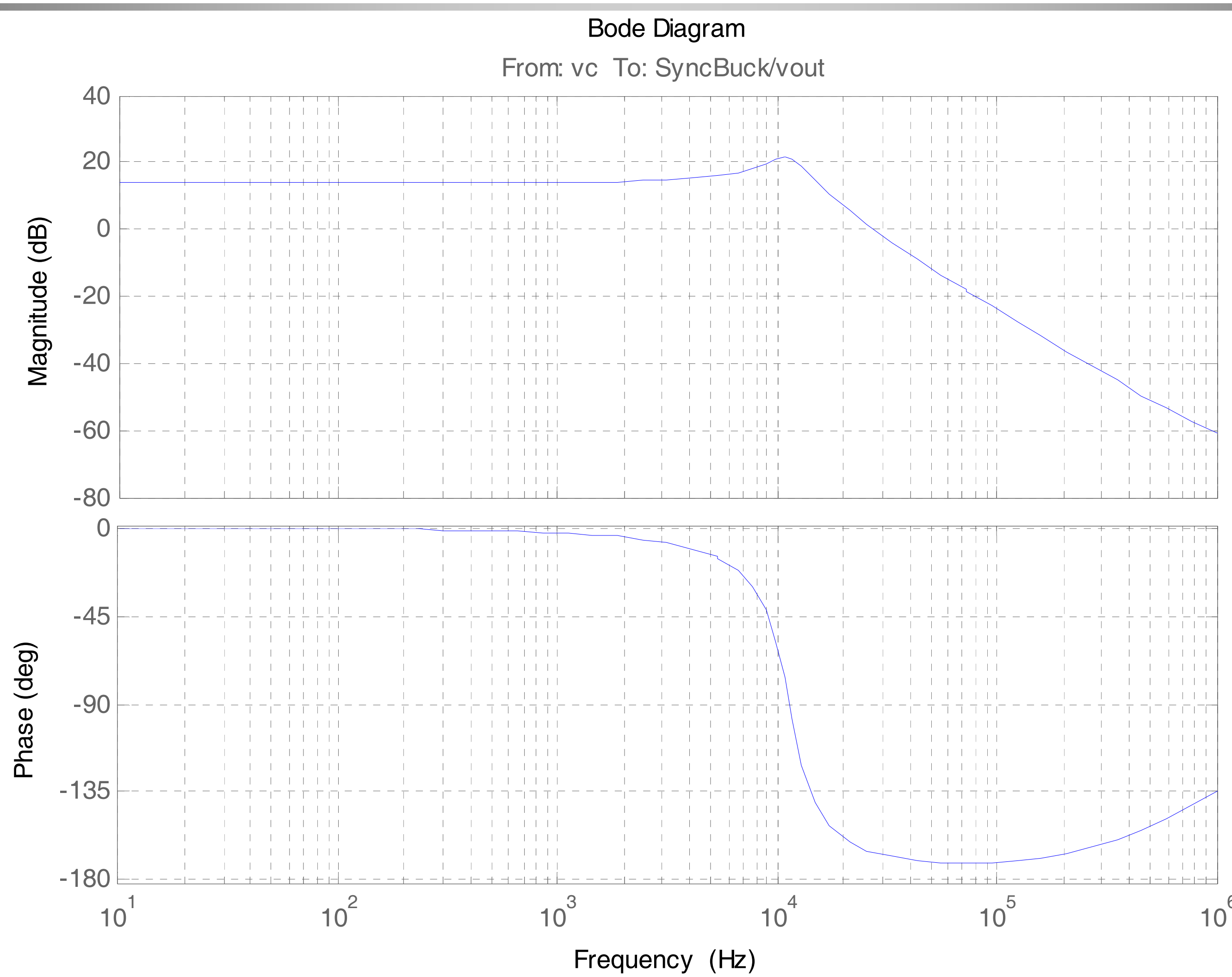
1. Set transfer function input and output points



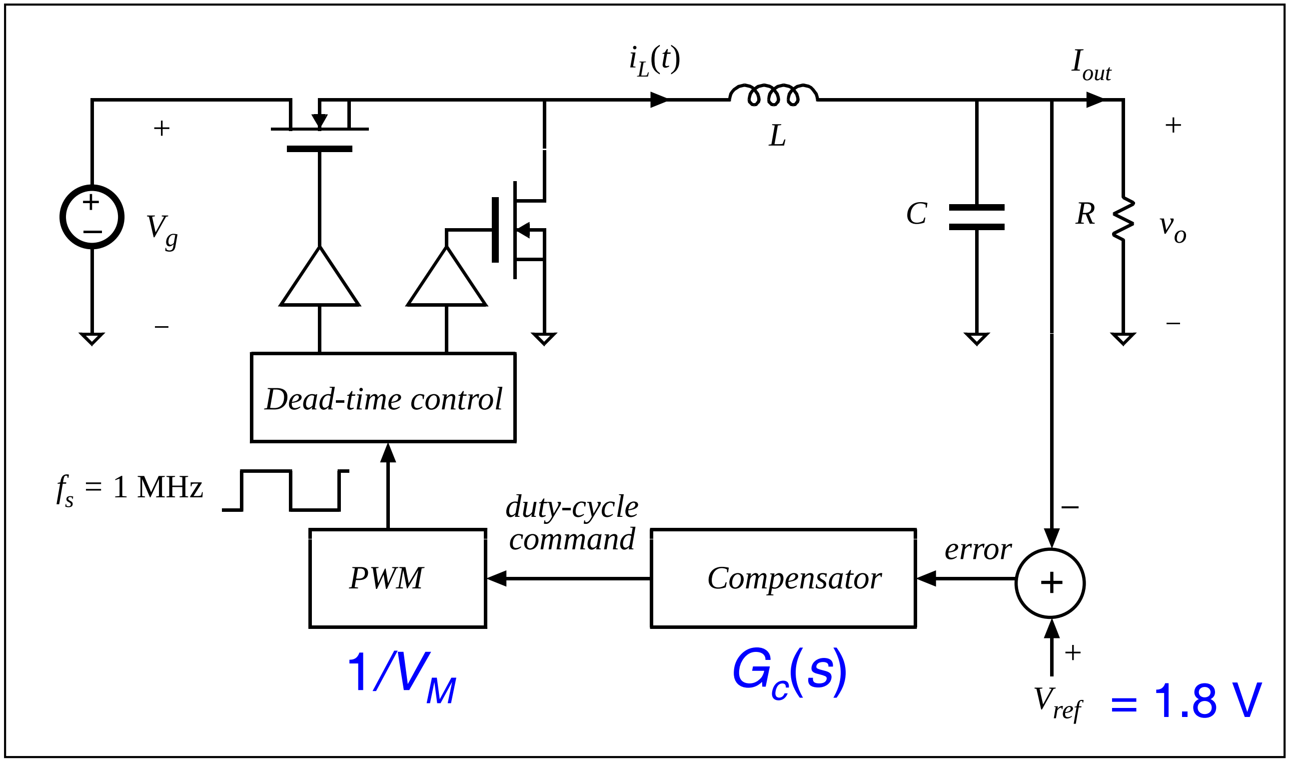
2. MATLAB script (BodePlotter_script.m) computes DC operating point, linearizes the model, computes and plots the transfer function magnitude and phase responses

```
%% Bode plotter using linearization tool
% requires Simulink Control Design toolbox
%
%
model = 'syncbuck_OL'; % set to file name of simulink model. Must have i/o points set within this model
io = getlinio(model)   % get i/o signals of model
op = operspec(model)
op = findop(model,op)   % calculate model operating point
ssm = linearize(model,op,io)   % compute state space model of linearized system
%
%
ltiview('bode',ssm)    % send linearized model to LTI Viewer tool
%
```

Magnitude and phase Bode plots of G_{vd}

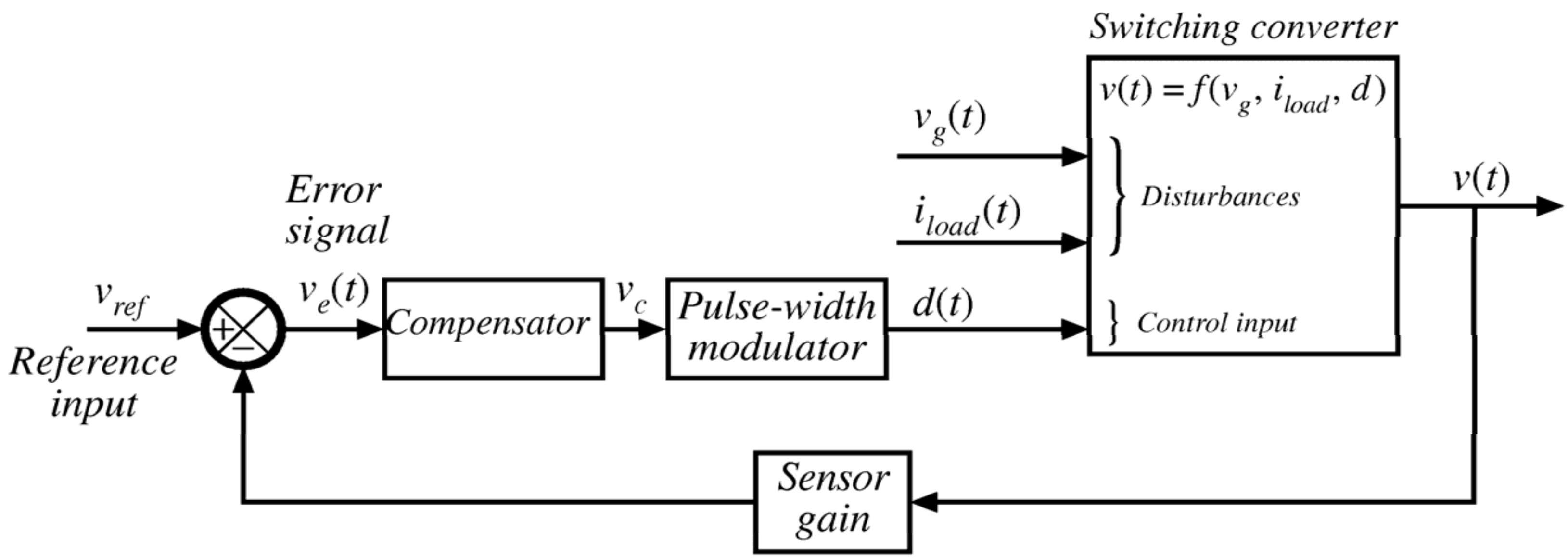


Closed-loop (voltage-mode) control



Point-of-Load (POL) Synchronous Buck Regulator

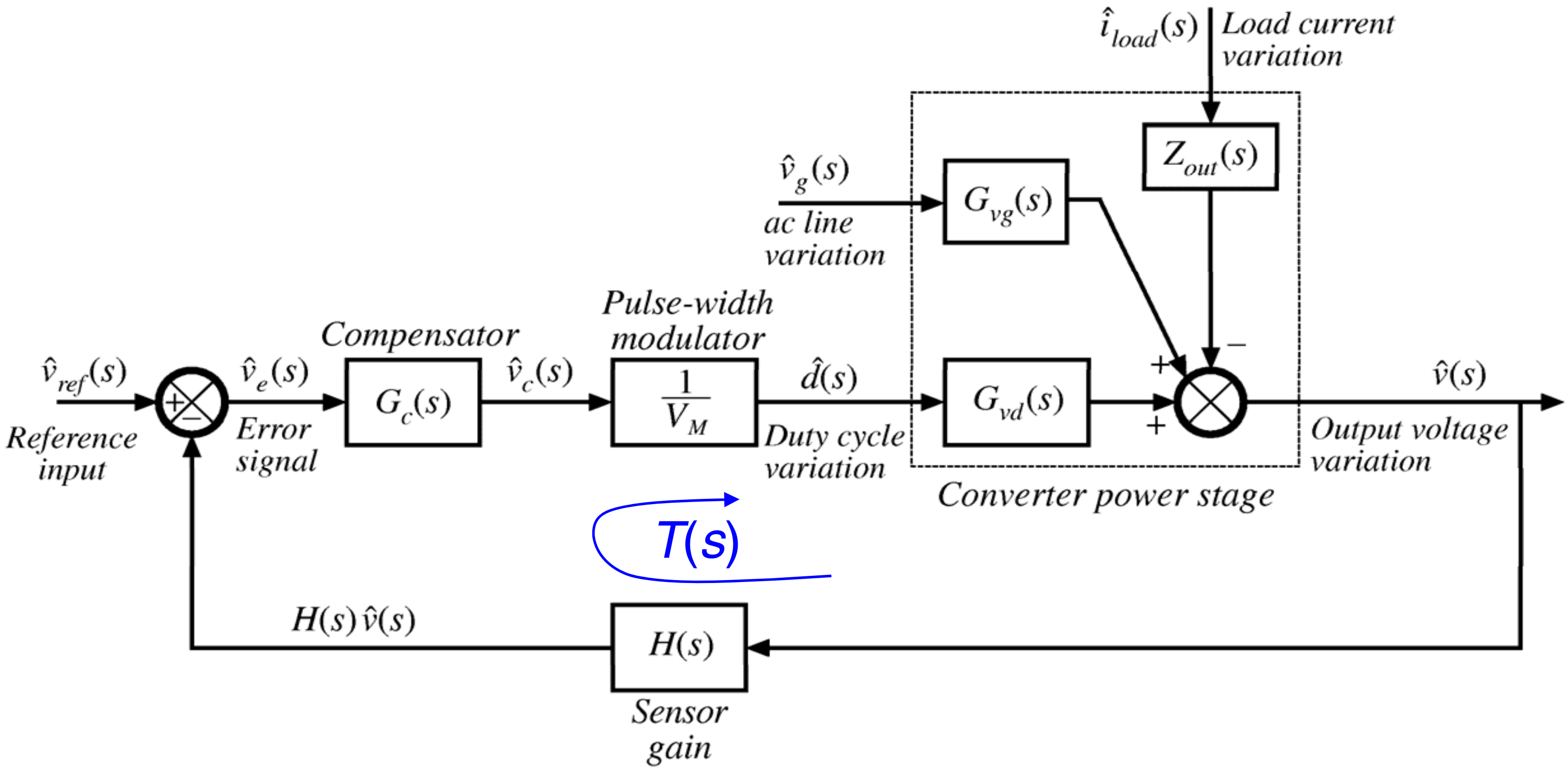
Closed-loop SMPS block diagram



Control objectives: tight output voltage regulation

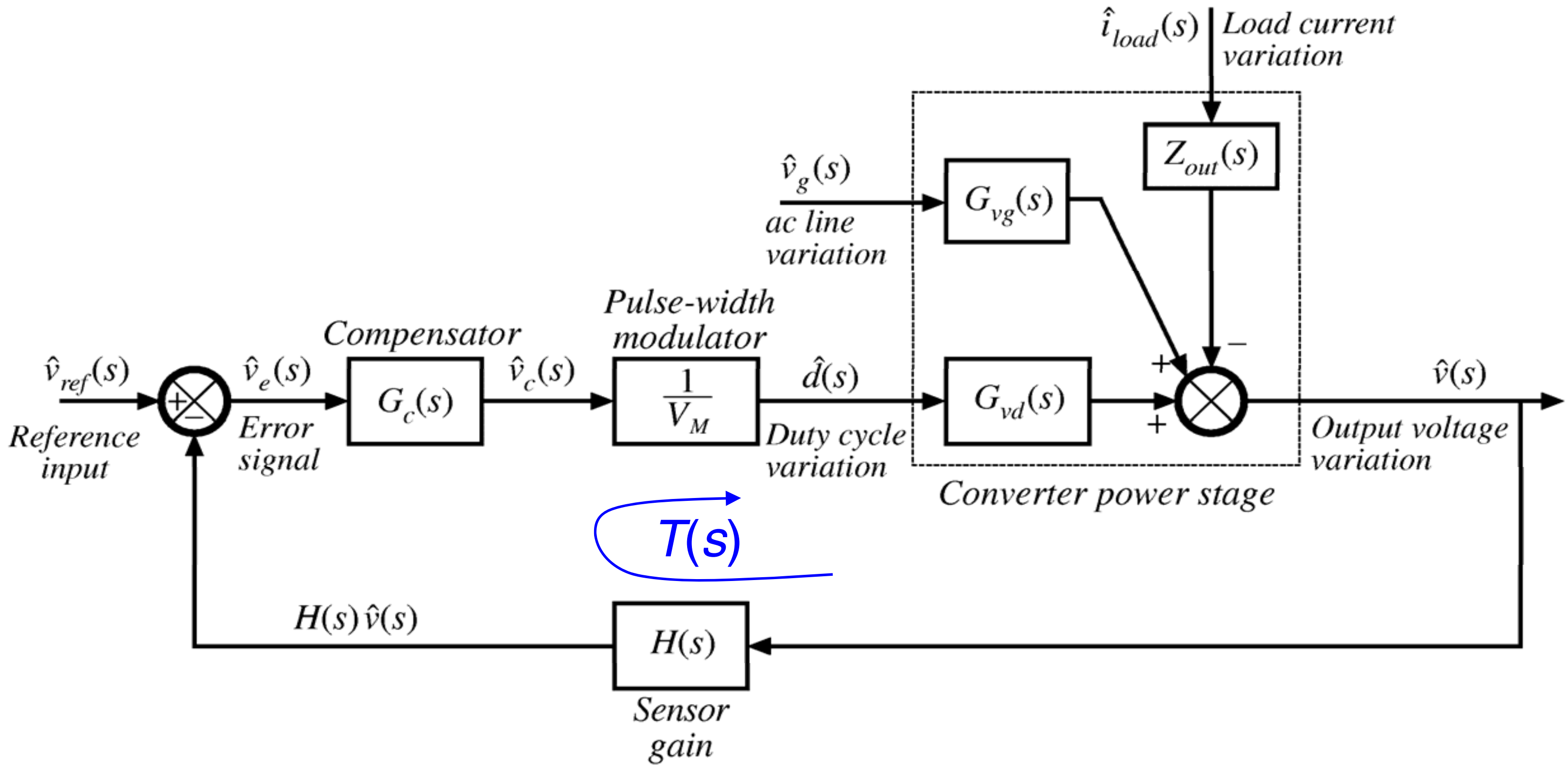
- Static or dynamic disturbances
  - – Input (line) voltage v_g
  - – Load current i_{load}
- Component tolerances

Small-signal model: loop gain T



Loop gain: $T(s) = H(s)G_c(s)(1/V_M)G_{vd}(s)$

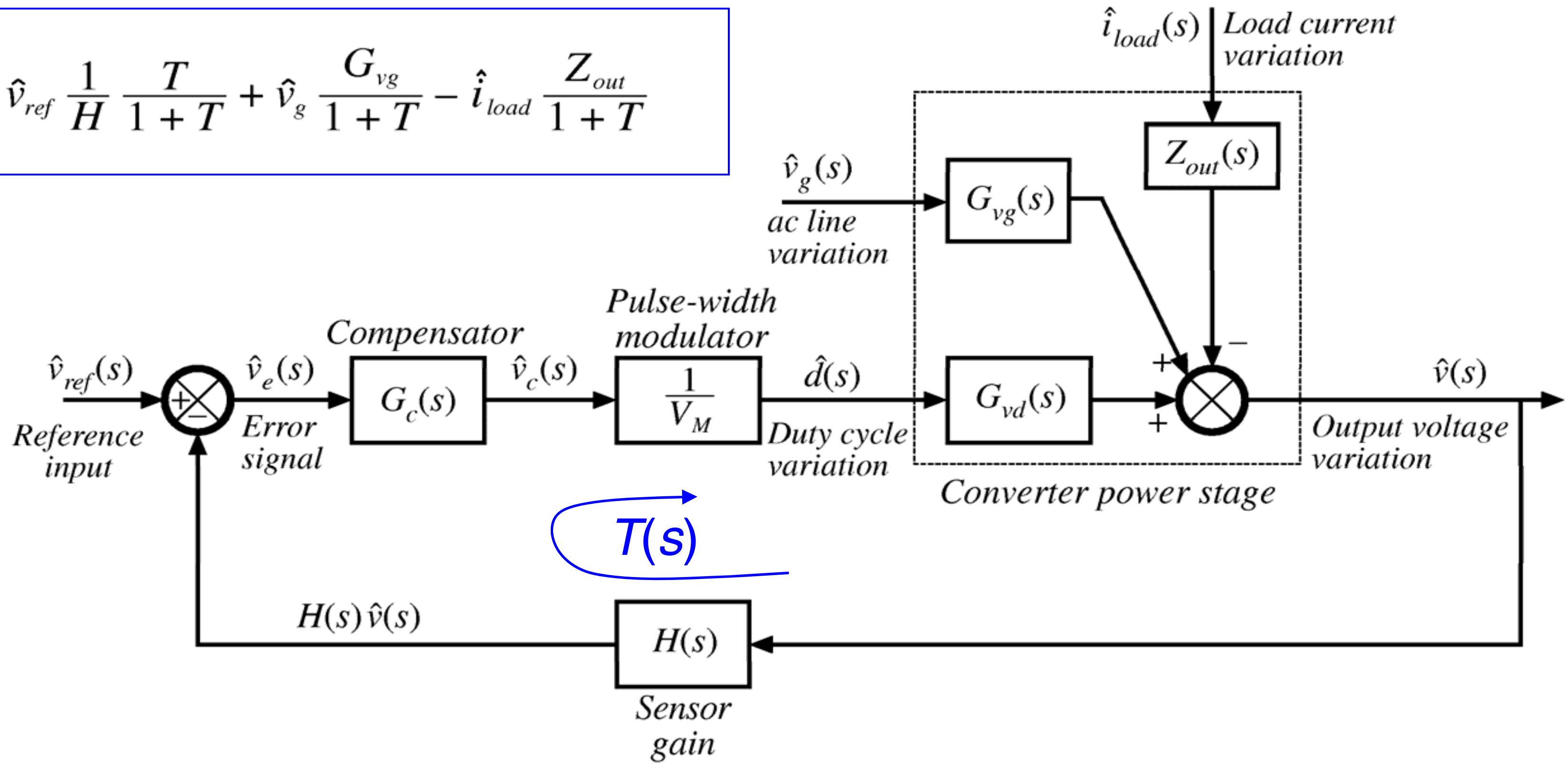
Small-signal model: closed-loop responses



$$\hat{v} = \hat{v}_{ref} \frac{1}{H} \frac{T}{1+T} + \hat{v}_g \frac{G_{vg}}{1+T} - \hat{i}_{load} \frac{Z_{out}}{1+T}$$

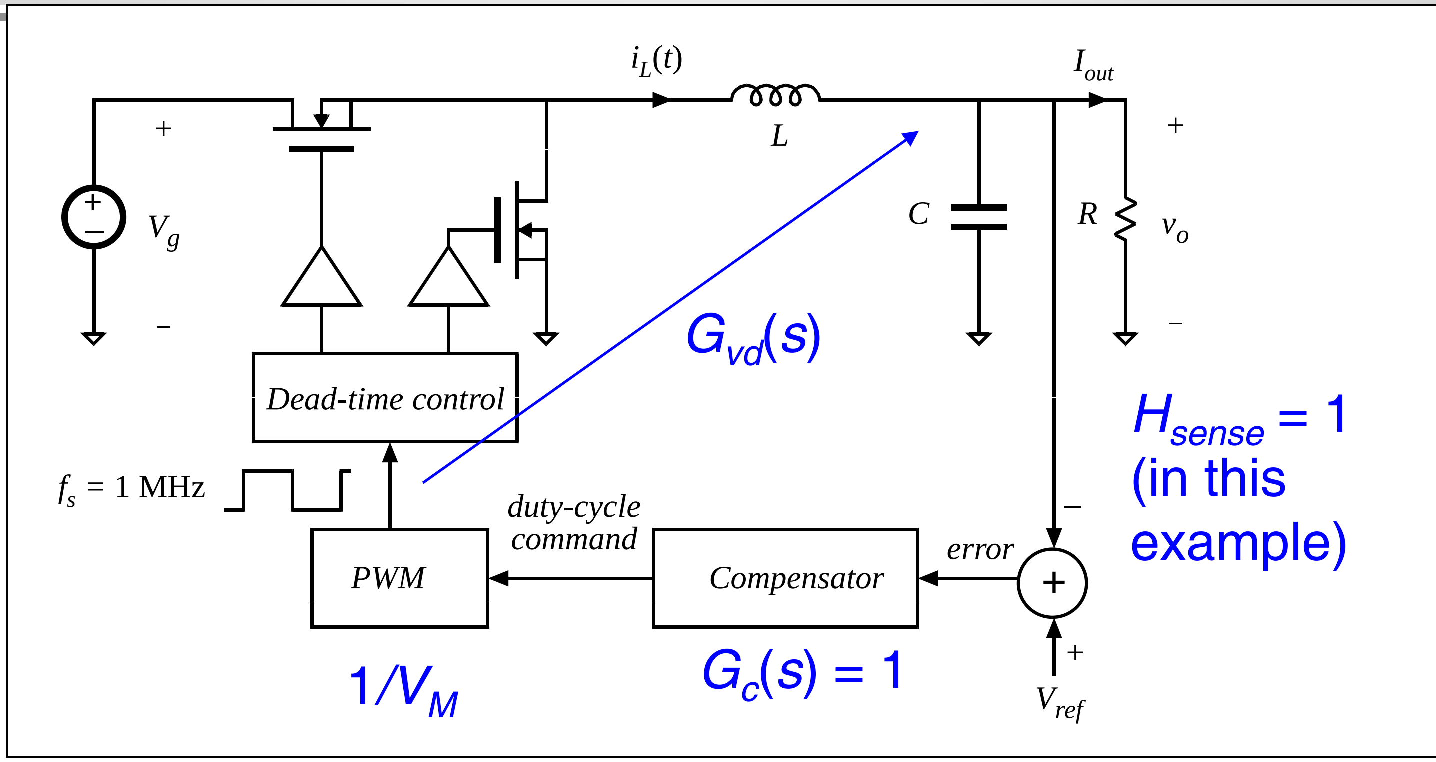
Feedback loop design objectives

$$\hat{v} = \hat{v}_{ref} \frac{1}{H} \frac{T}{1+T} + \hat{v}_g \frac{G_{vg}}{1+T} - \hat{i}_{load} \frac{Z_{out}}{1+T}$$



- To meet the control objectives, design *T* as large as possible in as wide frequency range as possible, i.e. with as high f_c as possible
- *Limitation*: stability and quality of closed-loop responses

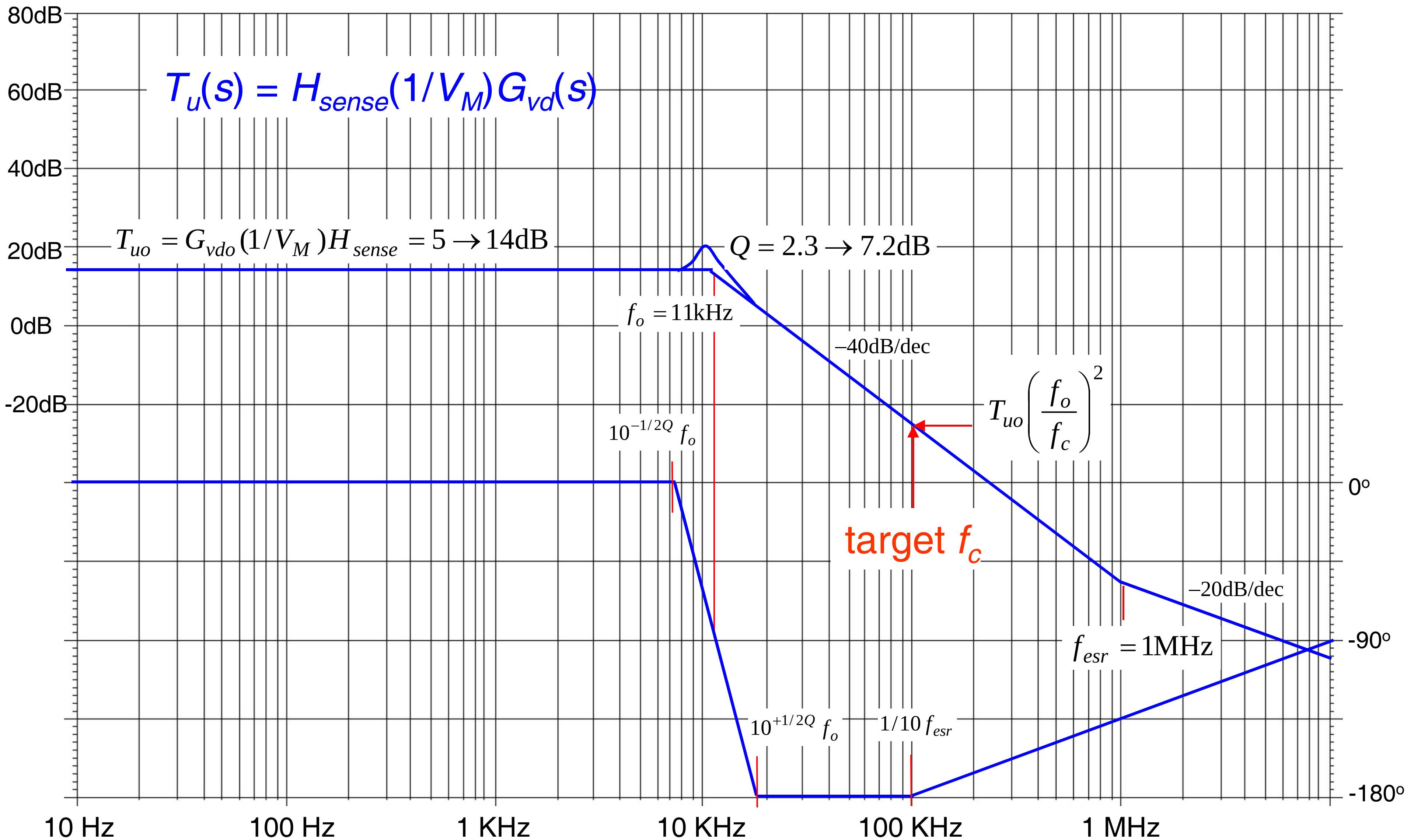
Uncompensated loop gain T_u



$$T_u(s) = H_{sense}(1/V_M)G_{vd}(s)$$

Plot magnitude and phase responses of $T_u(s)$ to plan how to design $G_c(s)$

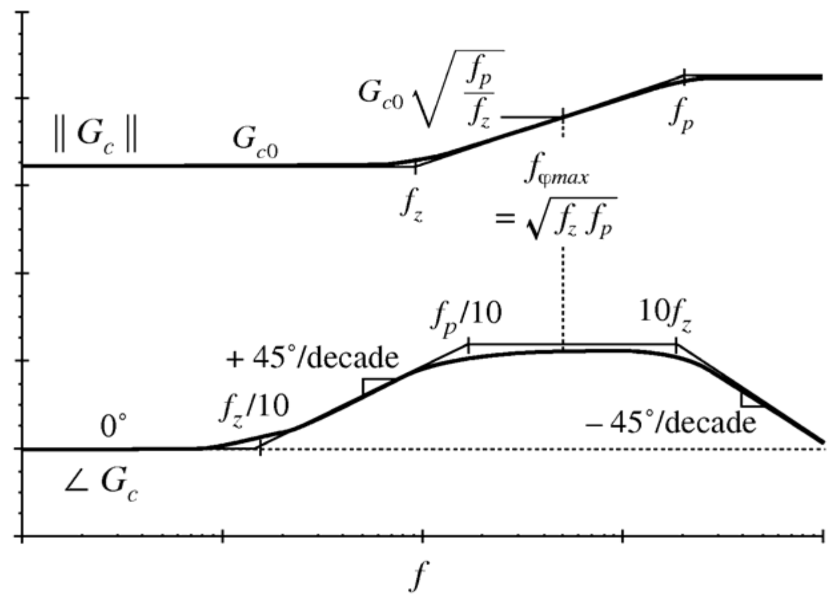
Magnitude and phase Bode plots of T_u



Lead (PD) compensator design

1. Choose: $f_c = 100\ \text{kHz}$

$\theta = \varphi_m = 53^o$



2. Compute:

$$f_z = f_c \sqrt{\frac{1-\sin(\theta)}{1+\sin(\theta)}} = 33\ \text{kHz}$$

$$f_p = f_c \sqrt{\frac{1+\sin(\theta)}{1-\sin(\theta)}} = 300\ \text{kHz}$$

3. Find G_{co} to position the crossover frequency:

$$\underbrace{T_{uo}\left(\frac{f_o}{f_c}\right)^2}_{\text{Magnitude of } T_u \text{ at } f_c} \underbrace{G_{co}\sqrt{\frac{f_p}{f_z}}}_{\text{Magnitude of } G_c \text{ at } f_c} = 1 \longrightarrow G_{co} = \frac{1}{T_{uo}}\left(\frac{f_c}{f_o}\right)^2 \sqrt{\frac{f_z}{f_p}} = 5.45 \rightarrow 15\ \text{dB}$$

Lead (PD) compensator summary

$$G_c(s) = \underbrace{G_{co}\frac{\left(1+\frac{s}{\omega_z}\right)}{\left(1+\frac{s}{\omega_{p1}}\right)}}_{\text{Lead compensator}}\underbrace{\frac{1}{\left(1+\frac{s}{\omega_{p2}}\right)}}_{\text{HF pole}}$$

$$G_{co} = 5.45 \rightarrow 15\,\text{dB}$$

$$f_z = 33\,\text{kHz}$$

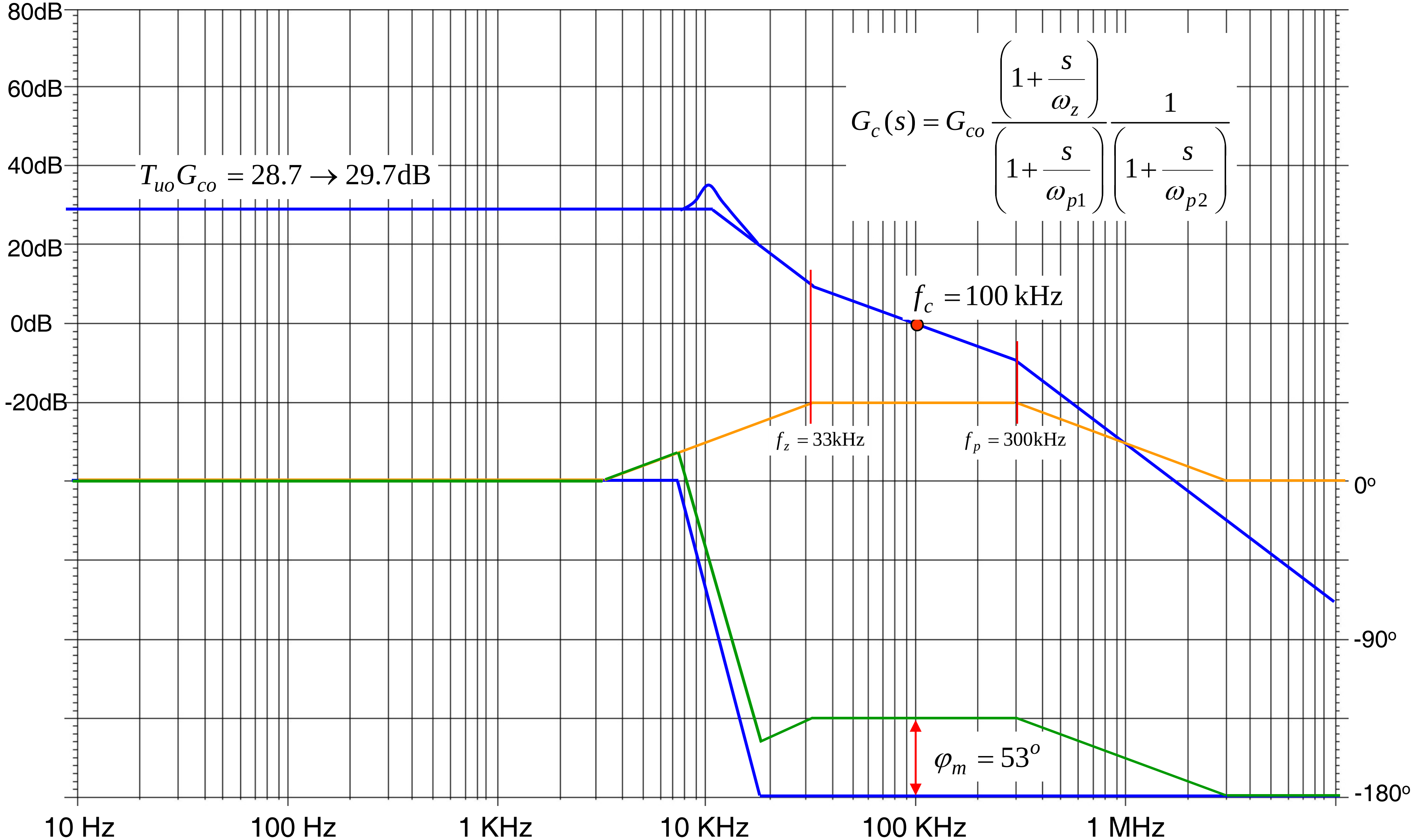
$$f_{p1} = 300\,\text{kHz}$$

$f_c = 100\,\text{kHz}$ (=1/10 of f_s)

High-frequency gain of the lead compensator: $G_{co}\, f_{p1}/f_z = 49$ (34 dB)

Added high-frequency pole: $f_{p2} = 1\,\text{MHz}$ (= f_{esr} = f_s in this example)

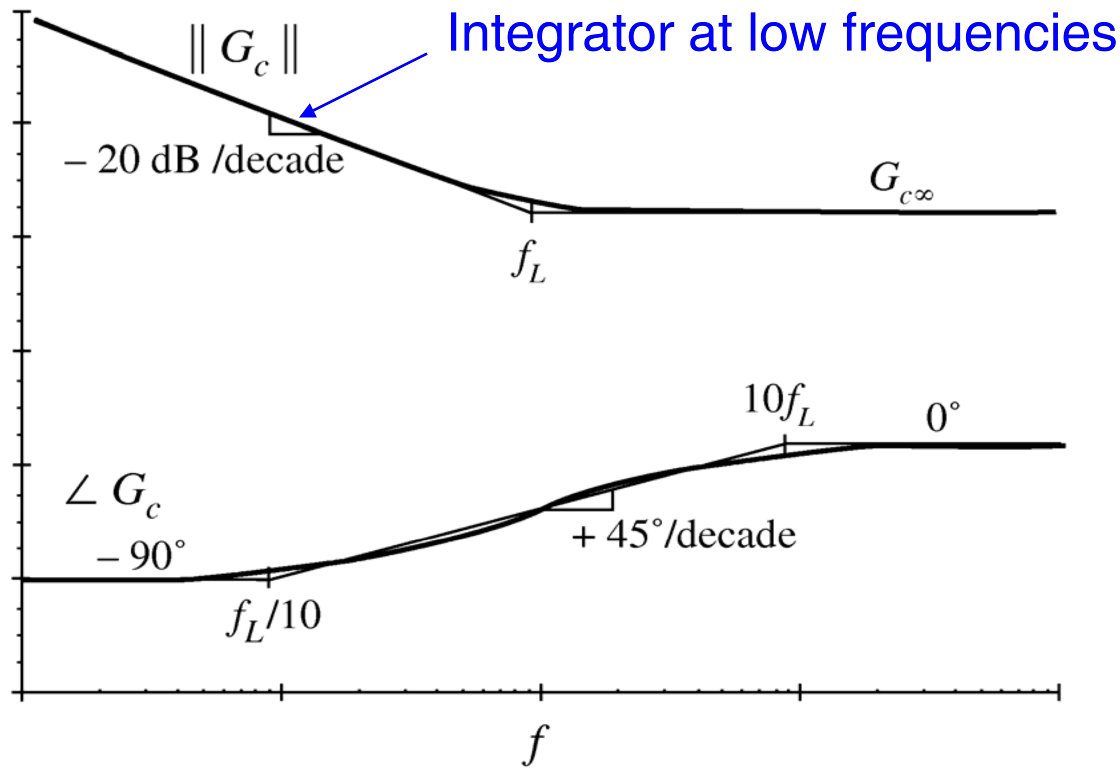
Loop gain with lead (PD) compensator



Add lag (PI) compensator

$$G_c(s) = G_{c\infty}\left(1 + \frac{\omega_L}{s}\right)$$

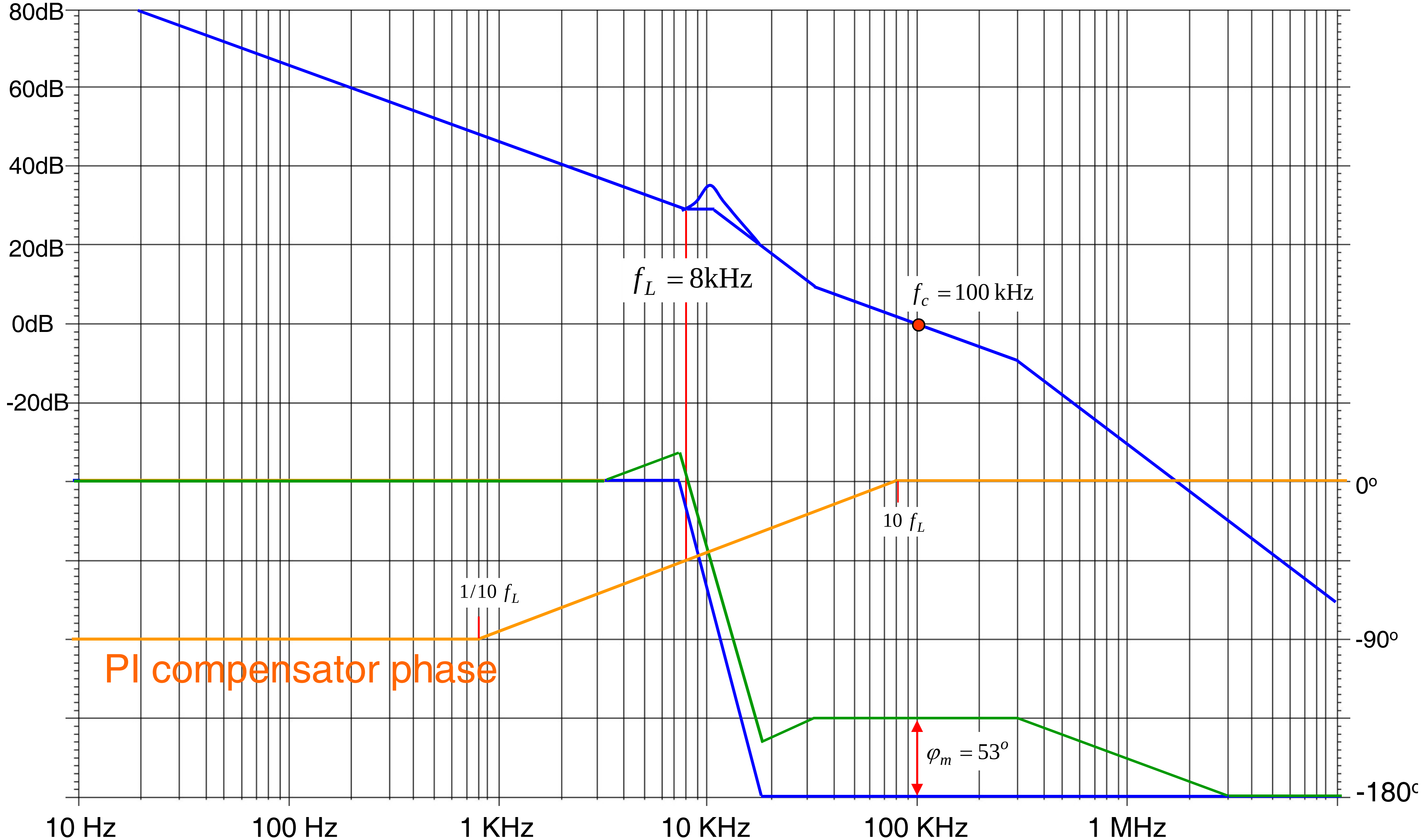
Improves low-frequency loop gain and regulation



Choose $10f_L < f_c$ so that phase margin stays approximately the same: f_L = 8 kHz

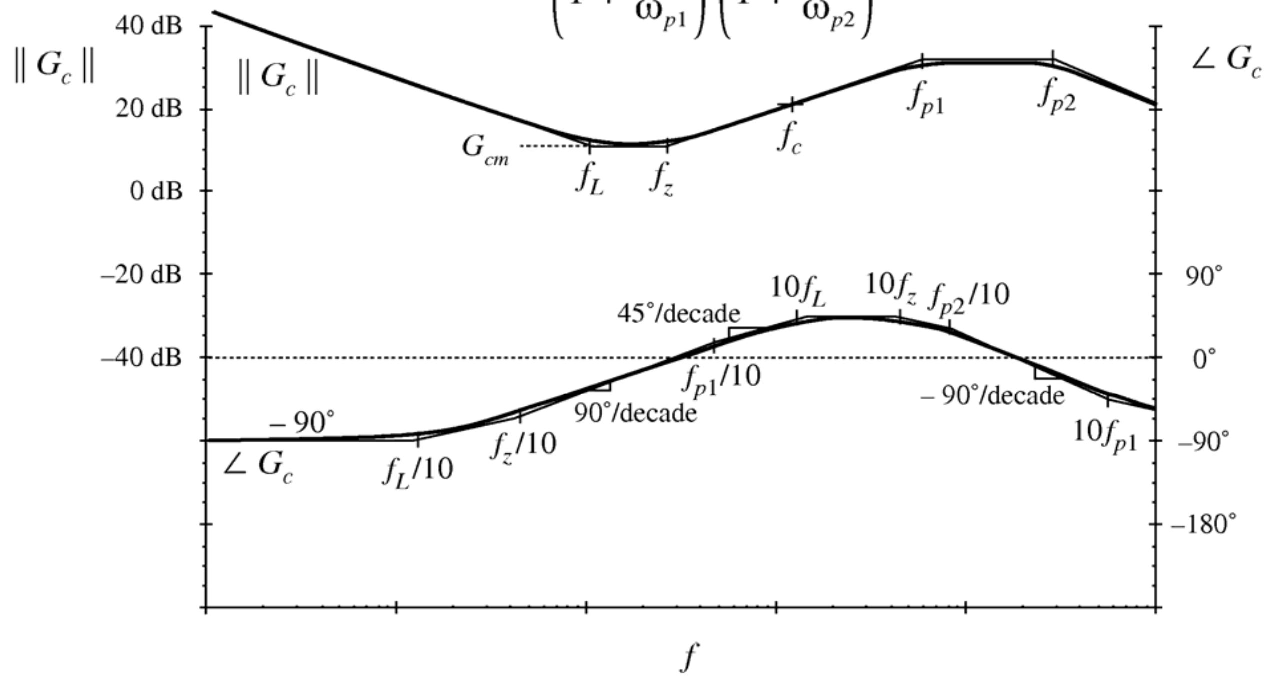
Keep the same cross-over frequency: $G_{c\infty} = G_{co} = G_{cm} = 5.45 \rightarrow 15\,\mathrm{dB}$

Adding PI Compensator



Complete PID compensator: summary

$$G_c(s) = G_{cm} \frac{\left(1 + \frac{\omega_L}{s}\right)\left(1 + \frac{s}{\omega_z}\right)}{\left(1 + \frac{s}{\omega_{p1}}\right)\left(1 + \frac{s}{\omega_{p2}}\right)}$$



$$G_{cm} = 5.45 \rightarrow 15\,\mathrm{dB}$$

$$f_L = 8\,\mathrm{kHz}$$

$$f_z = 33\,\mathrm{kHz}$$

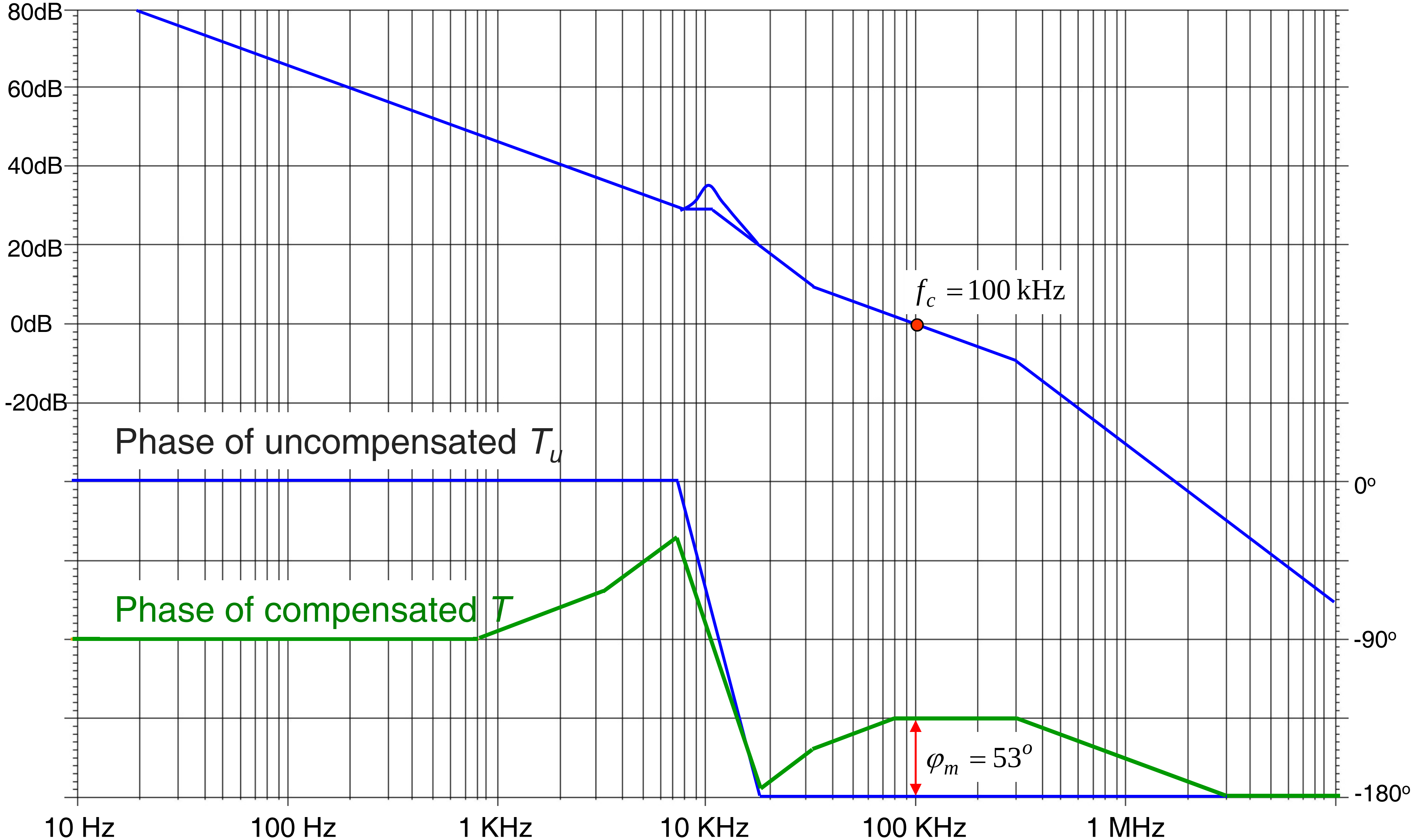
$$f_{p1} = 300\,\mathrm{kHz}$$

$$f_{p2} = 1\,\mathrm{MHz}$$

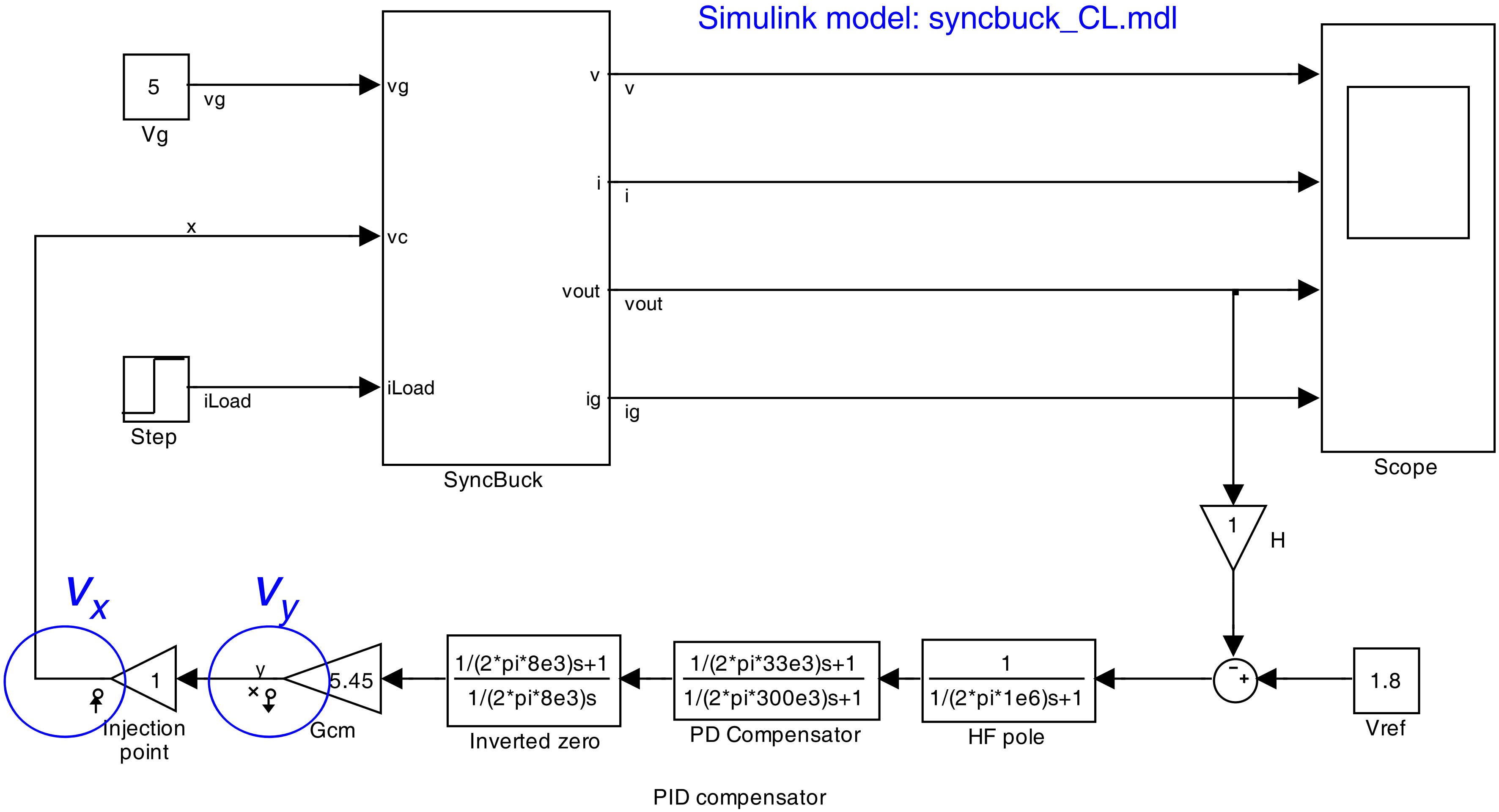
Crossover frequency: $f_c = 100\,\mathrm{kHz}$ (=1/10 of f_s)

Phase margin: $\varphi_m = 53^o$

Magnitude and phase Bode plots of T



Closed-loop voltage regulator in Simulink



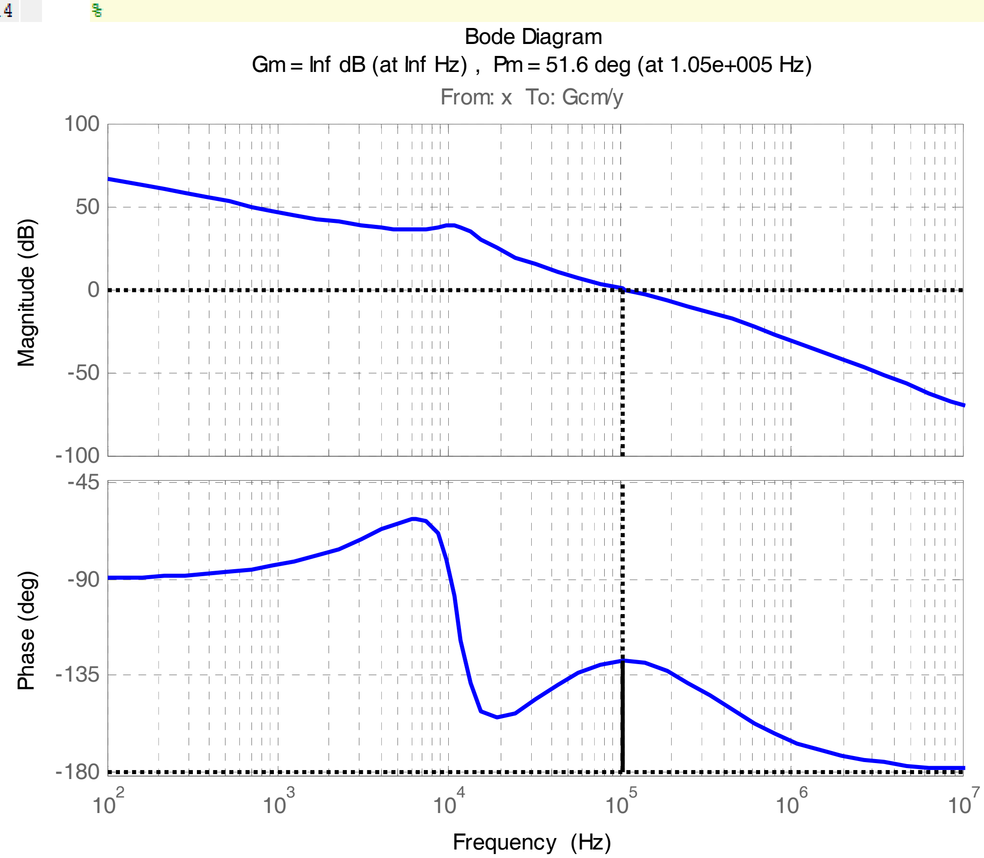
Input and output linearization points for finding the loop-gain, $T = -v_y/v_x$

The output point (y) should be "Open Loop" , as shown by an x symbol next to the output arrow

Loop gain and stability margins

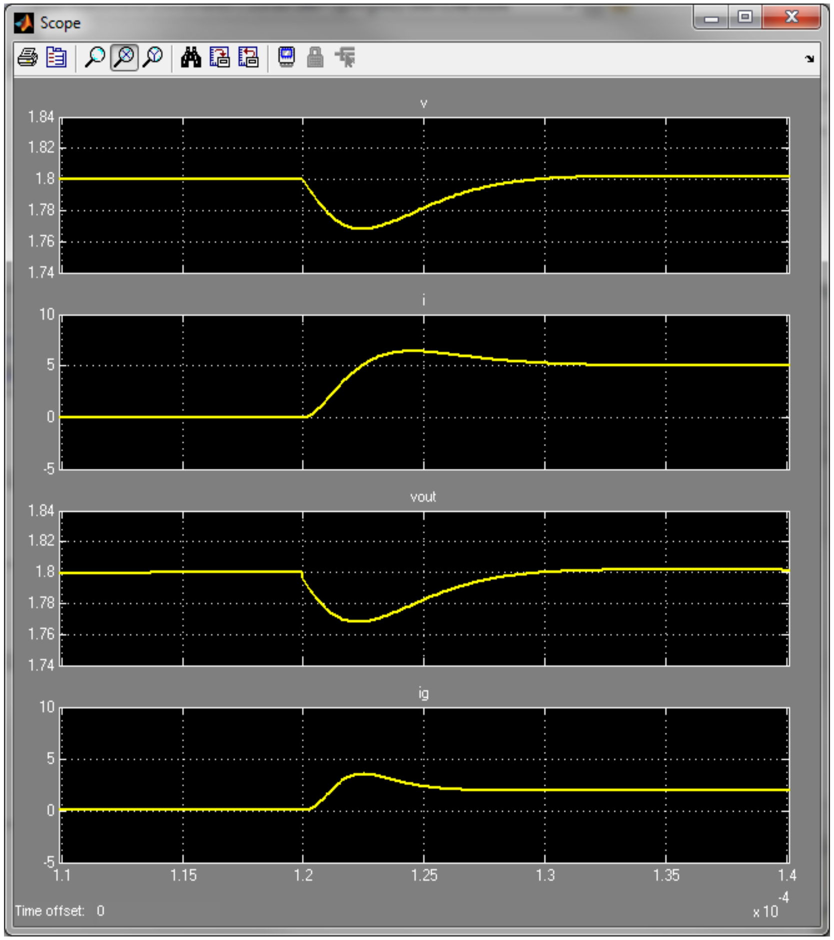
MATLAB script BodePlotter_scriptT.m (computes dc op, linearizes, calculates and plots frequency response and stability margins)

```
%% Loop gain bode plotter using linearization tool
% requires Simulink Control Design toolbox
%
%
model = 'syncbuck_CL'; % set to file name of simulink model. Must have i/o points set within this model
io = getlinio(model)  % get i/o signals of model
op = operspec(model)
op = findop(model,op)  % calculate model operating point
ssm = linearize(model,op,io)  % compute state space model of linearized system
%
%
%ltiview('bode',-ssm)   % send linearized model to LTI Viewer tool
margin(-ssm) % show loop-gain magnitude and phase responses and calculate fc, PM and GM
%
```



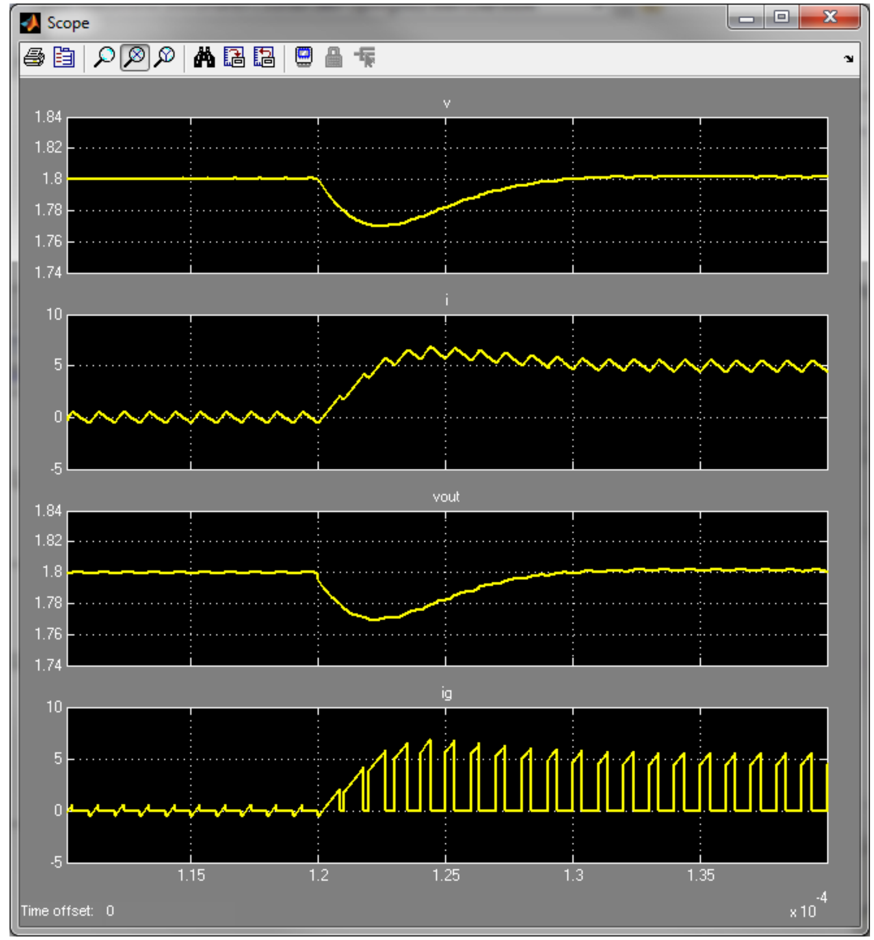
Closed-loop 0-5 A step-load transient responses

Averaged model



5 μs/div

Switching model



5 μs/div

v

i

v_{out}

i_g

See MATLAB/Simulink page on the course website ("Materials" page) for complete step-by-step details, and to download the example files