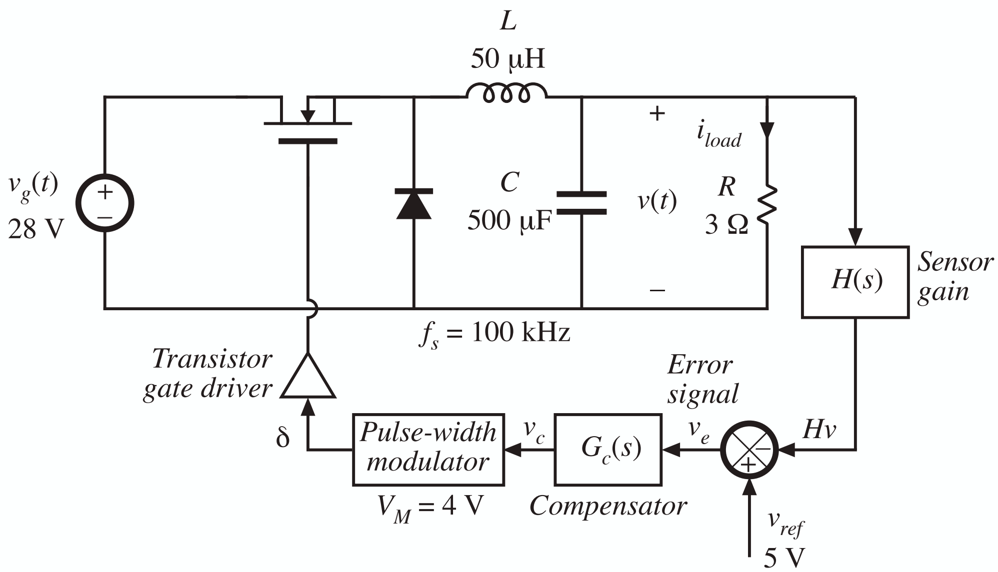
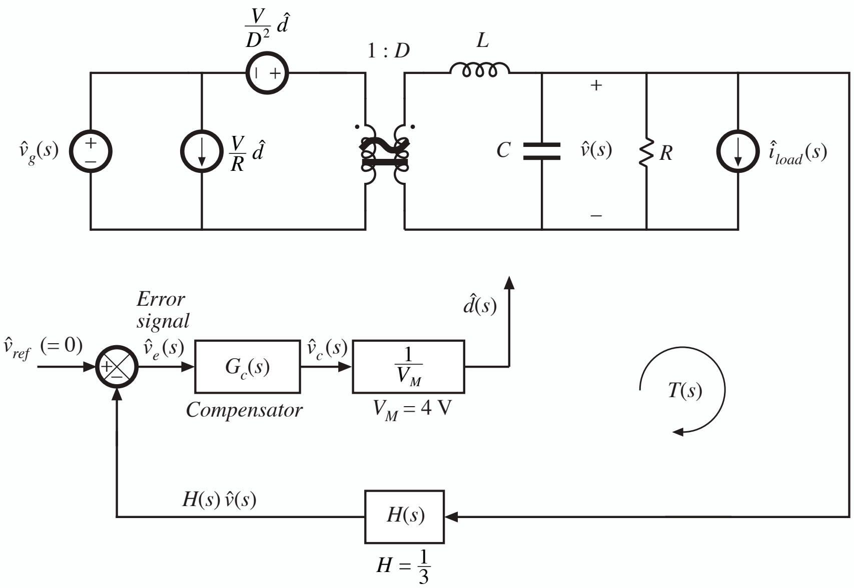
9.5.4. Design example



Quiescent operating point

Input voltage	$V_g = 28\text{V}$
Output	$V = 15\text{V}$, $I_{load} = 5\text{A}$, $R = 3\Omega$
Quiescent duty cycle	$D = 15/28 = 0.536$
Reference voltage	$V_{ref} = 5\text{V}$
Quiescent value of control voltage	$V_c = DV_M = 2.14\text{V}$
Gain $H(s)$	$H = V_{ref}/V = 5/15 = 1/3$

Small-signal model



Open-loop control-to-output transfer function $G_{vd}(s)$

$$G_{vd}(s) = \frac{V}{D}\,\frac{1}{1 + s\frac{L}{R} + s^2 LC}$$

standard form:

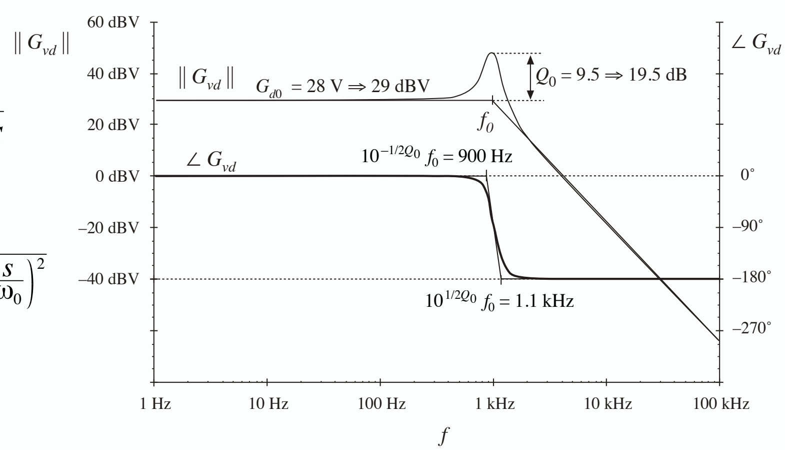
$$G_{vd}(s) = G_{d0}\,\frac{1}{1 + \frac{s}{Q_0\omega_0} + \left(\frac{s}{\omega_0}\right)^2}$$

salient features:

$$G_{d0} = \frac{V}{D} = 28\text{V}$$

$$f_0 = \frac{\omega_0}{2\pi} = \frac{1}{2\pi\sqrt{LC}} = 1\text{kHz}$$

$$Q_0 = R\sqrt{\frac{C}{L}} = 9.5 \Rightarrow 19.5\text{dB}$$



Open-loop line-to-output transfer function and output impedance

$$G_{vg}(s) = D \frac{1}{1 + s\frac{L}{R} + s^2 LC}$$

—same poles as control-to-output transfer function standard form:

$$G_{vg}(s) = G_{g0} \frac{1}{1 + \frac{s}{Q_0\omega_0} + \left(\frac{s}{\omega_0}\right)^2}$$

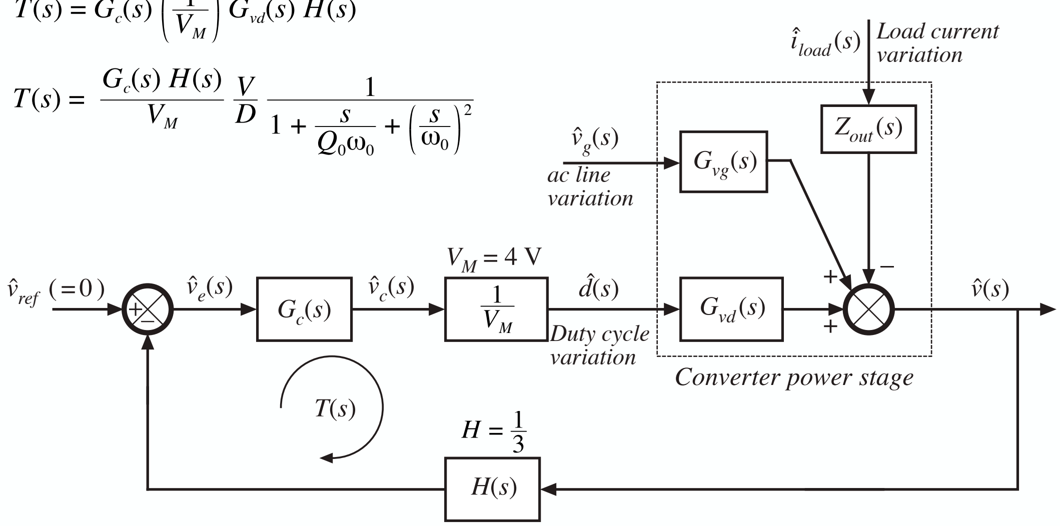
Output impedance:

$$Z_{out}(s) = R \parallel \frac{1}{sC} \parallel sL = \frac{sL}{1 + s\frac{L}{R} + s^2 LC}$$

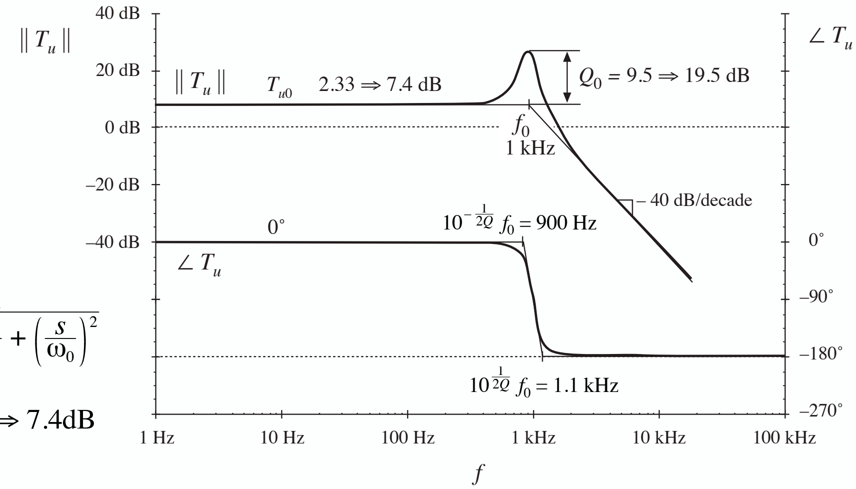
System block diagram

$$T(s) = G_c(s) \left(\frac{1}{V_M}\right) G_{vd}(s)\, H(s)$$

$$T(s) = \frac{G_c(s)\, H(s)}{V_M} \frac{V}{D} \frac{1}{1 + \frac{s}{Q_0\omega_0} + \left(\frac{s}{\omega_0}\right)^2}$$



Uncompensated loop gain (with $G_c = 1$)



With $G_c = 1$, the loop gain is

$$T_u(s) = T_{u0} \frac{1}{1 + \frac{s}{Q_0\omega_0} + \left(\frac{s}{\omega_0}\right)^2}$$

$$T_{u0} = \frac{H}{D} \frac{V}{V_M} = 2.33 \Rightarrow 7.4\text{dB}$$

$f_c = 1.8$ kHz, $\varphi_m = 5^\circ$

Lead compensator design

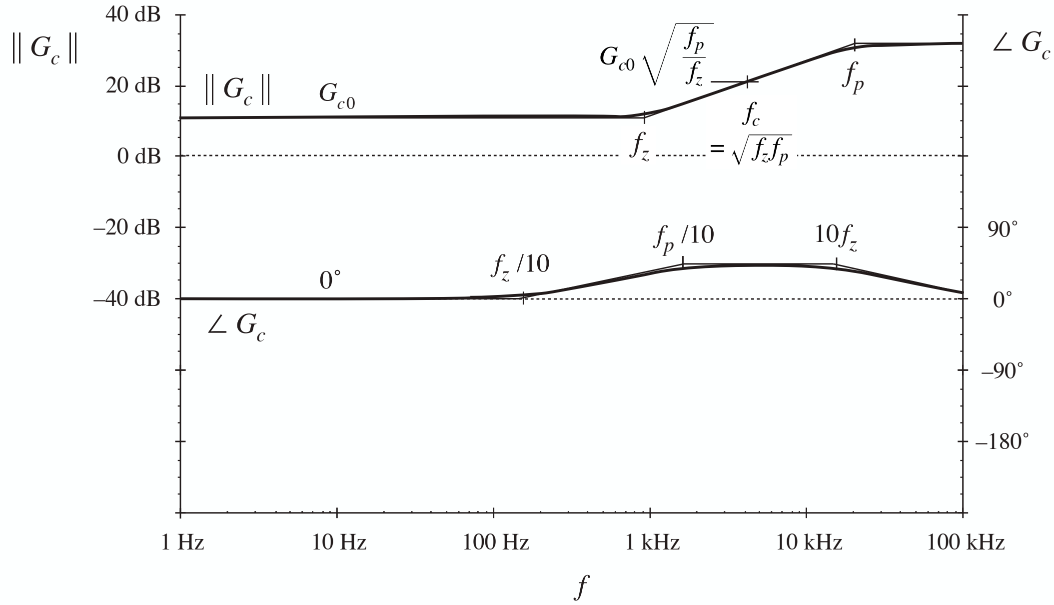
- Obtain a crossover frequency of 5 kHz, with phase margin of 52˚
- T_u has phase of approximately – 180˚ at 5 kHz, hence lead (PD) compensator is needed to increase phase margin.
- Lead compensator should have phase of + 52˚ at 5 kHz
- T_u has magnitude of – 20.6 dB at 5 kHz
- Lead compensator gain should have magnitude of + 20.6 dB at 5 kHz
- Lead compensator pole and zero frequencies should be

$$f_z = (5\text{kHz}) \sqrt{\frac{1 - \sin(52°)}{1 + \sin(52°)}} = 1.7\text{kHz}$$

$$f_p = (5\text{kHz}) \sqrt{\frac{1 + \sin(52°)}{1 - \sin(52°)}} = 14.5\text{kHz}$$

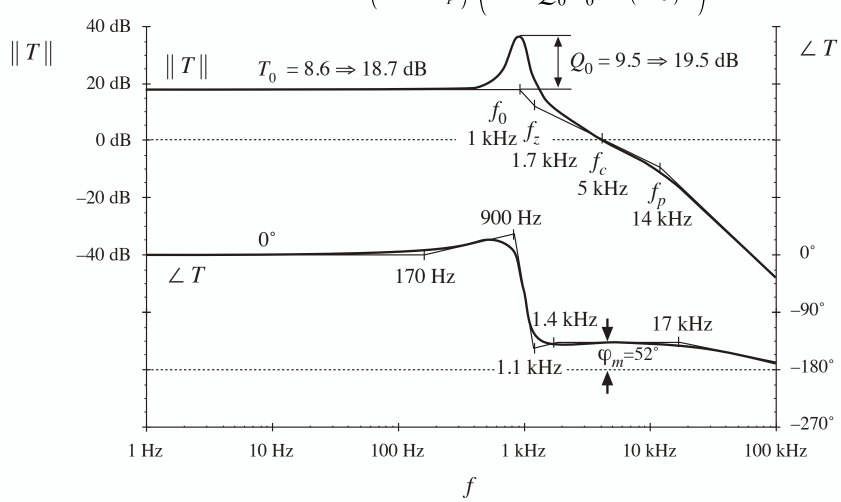
- Compensator dc gain should be $G_{c0} = \left(\frac{f_c}{f_0}\right)^2 \frac{1}{T_{u0}} \sqrt{\frac{f_z}{f_p}} = 3.7 \Rightarrow 11.3\text{dB}$

Lead compensator Bode plot

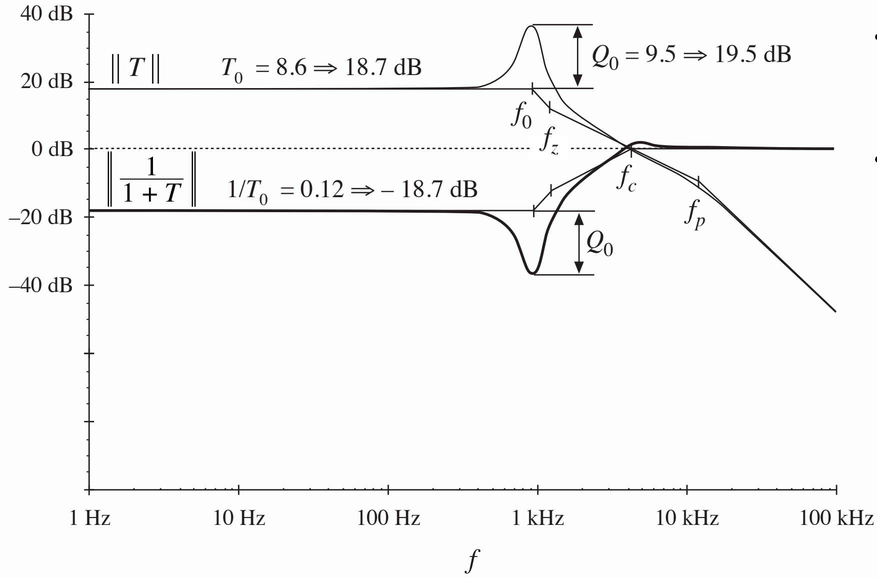


Loop gain, with lead compensator

$$T(s) = T_{u0}\, G_{c0} \frac{\left(1 + \frac{s}{\omega_z}\right)}{\left(1 + \frac{s}{\omega_p}\right)\left(1 + \frac{s}{Q_0\omega_0} + \left(\frac{s}{\omega_0}\right)^2\right)}$$



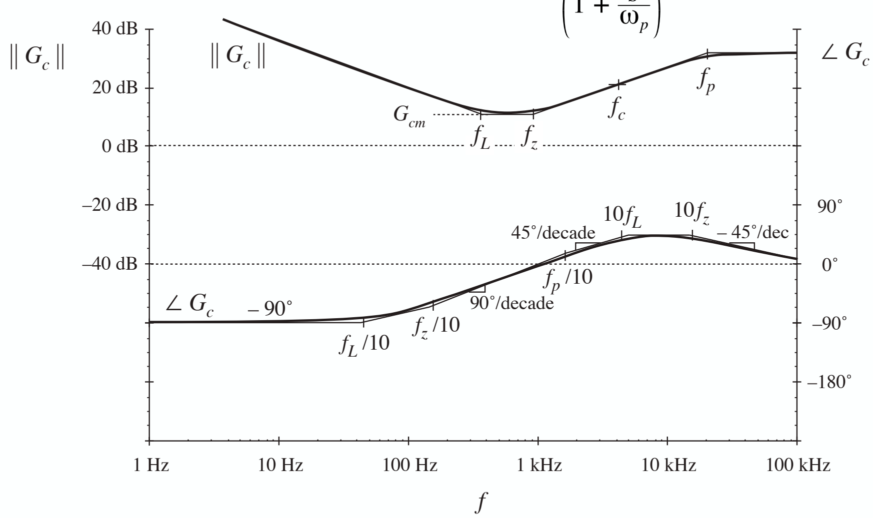
1/(1+T), with lead compensator



- need more low-frequency loop gain
- hence, add inverted zero (PID controller)

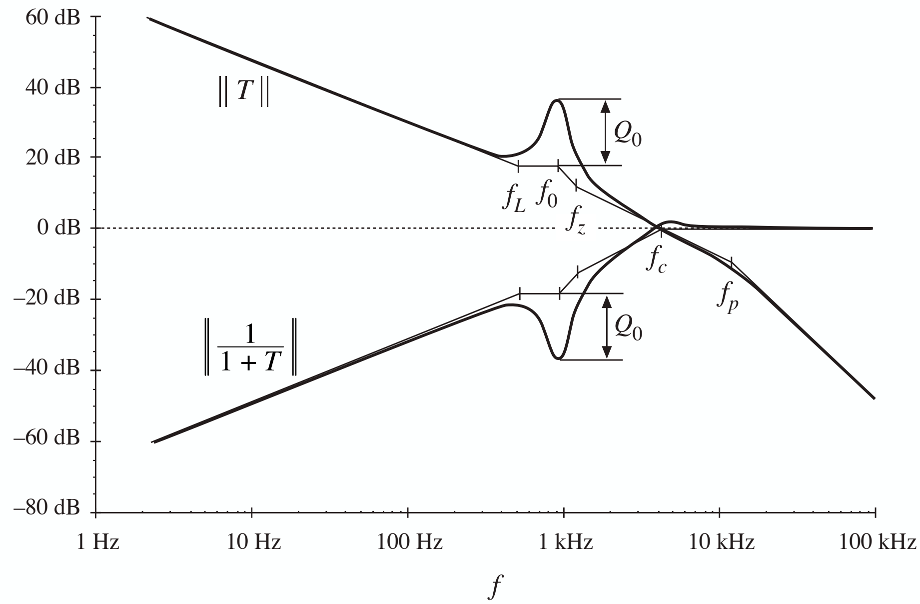
Improved compensator (PID)

$$G_c(s) = G_{cm} \frac{\left(1 + \frac{s}{\omega_z}\right)\left(1 + \frac{\omega_L}{s}\right)}{\left(1 + \frac{s}{\omega_p}\right)}$$

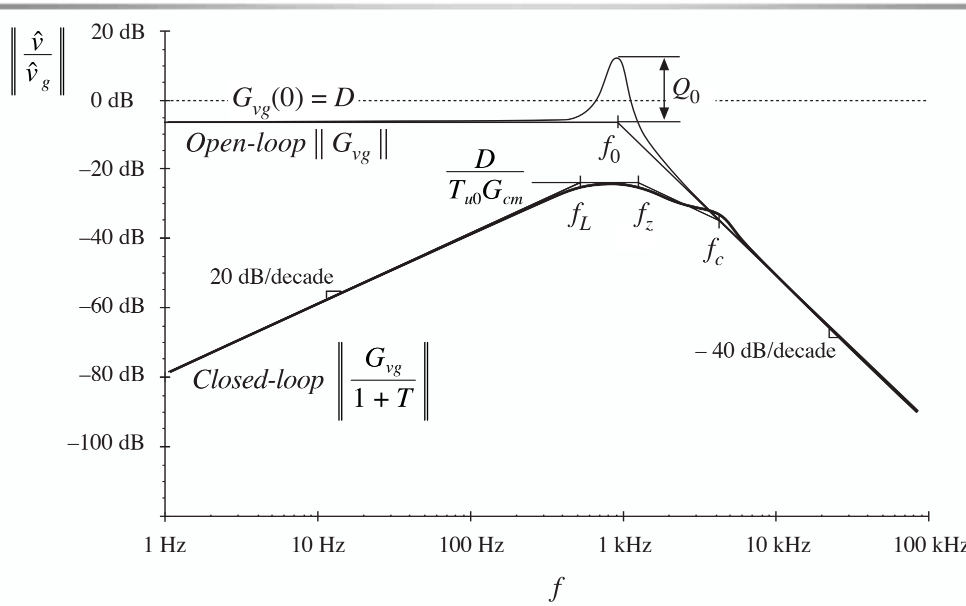


- add inverted zero to PD compensator, without changing dc gain or corner frequencies
- choose f_L to be $f_c/10$, so that phase margin is unchanged

$T(s)$ and $1/(1+T(s))$, with PID compensator

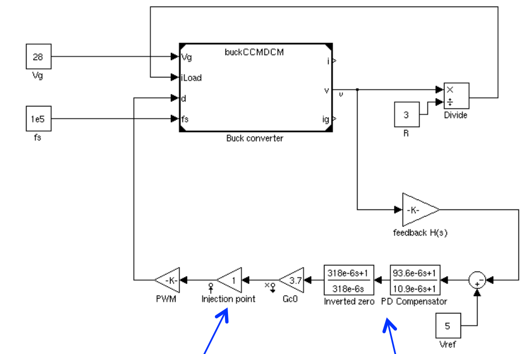


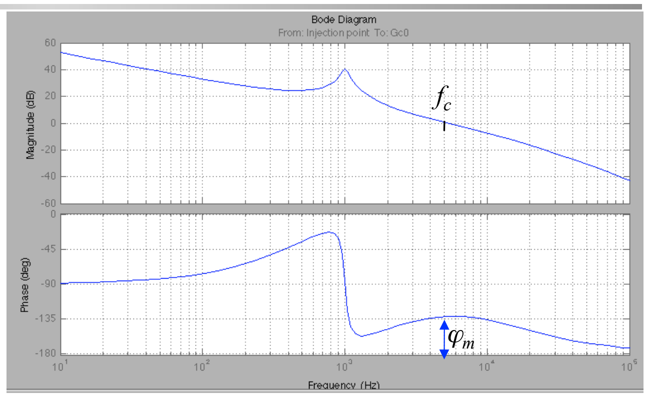
Line-to-output transfer function



Closed-loop buck converter

Simulink frequency domain simulation, averaged model





Loop gain: Bode plot

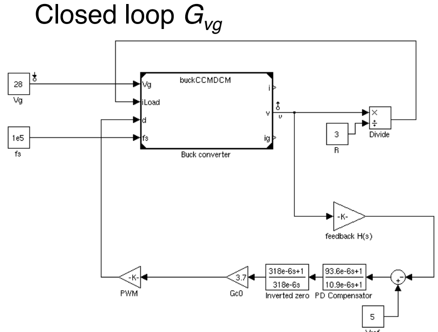
Injection point for measurement of loop gain $T(s)$

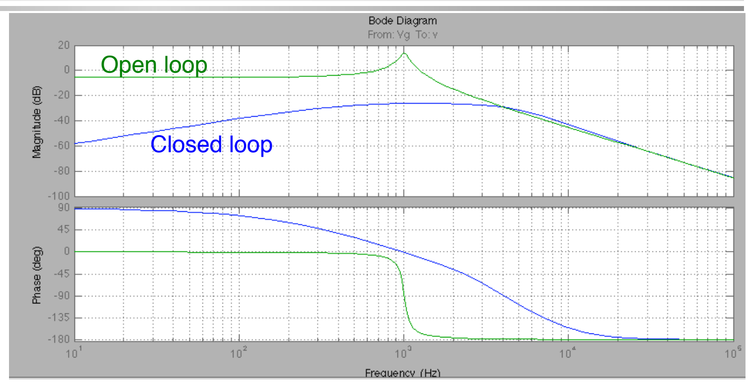
Transfer function blocks: Implementing the PID compensator

Closed-loop line-to-output transfer function

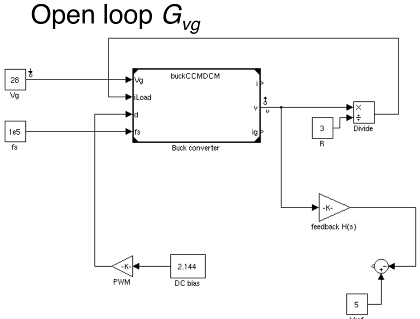
Simulink frequency domain simulation

Closed loop G_{vg}





Open loop G_{vg}



Script that generates both plots

```
%% Bode plotter using linearization tool
% requires simulink control design toolbox
mdl = 'BuckFeedbackExampleCh9PIDGvg'; % set
io = getlinio(mdl)  % get i/o signals of md
op = operspec(mdl)
op = findop(mdl,op)  % calculate model opera
lin = linearize(mdl,op,io)  % compute state
mdl = 'BuckFeedbackExampleCh9GvgOpenLoop'; 
io = getlinio(mdl)  % get i/o signals of md
op = operspec(mdl)
op = findop(mdl,op)  % calculate model opera
lino = linearize(mdl,op,io)  % compute stat
ltiview(lin,lino)  % send linearized model
```