Intensity Transform and Spatial Filtering

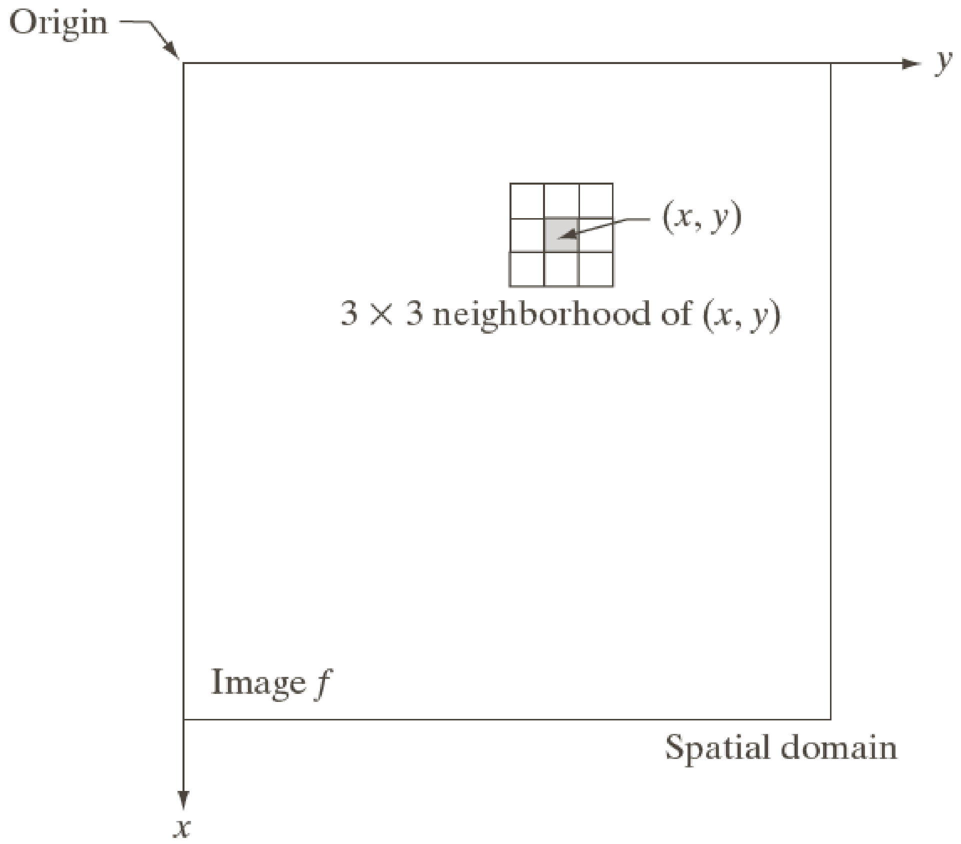
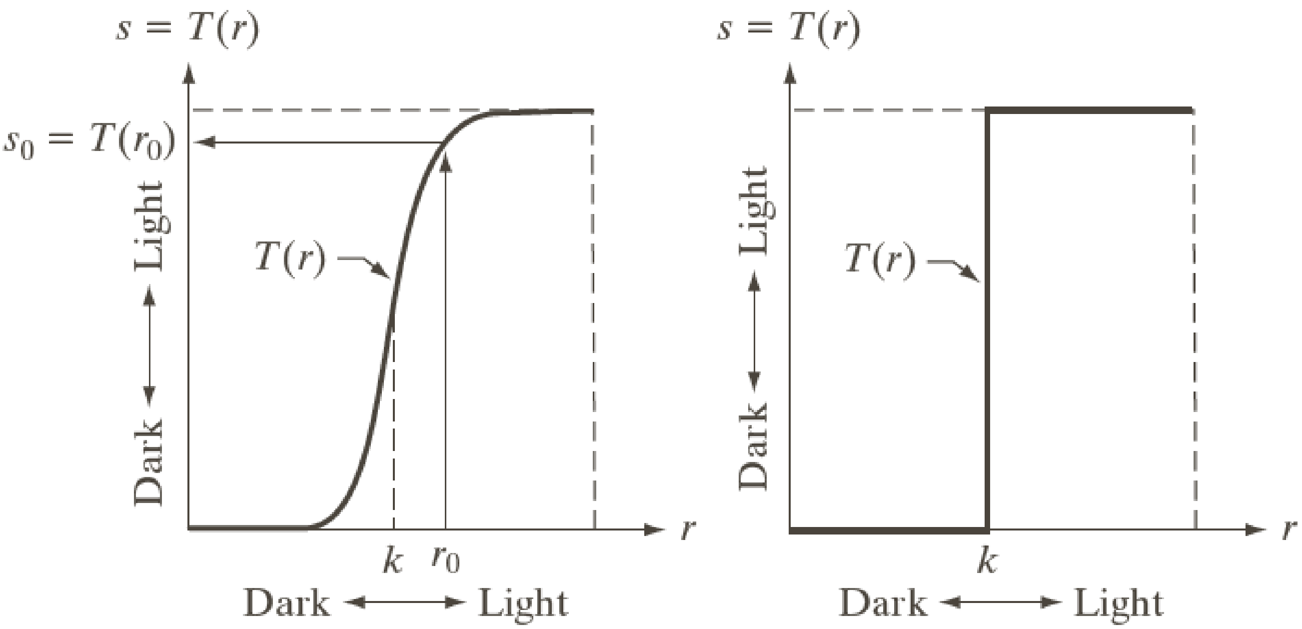


FIGURE 3.1 A 3×3 neighborhood about a point (x, y) in an image in the spatial domain. The neighborhood is moved from pixel to pixel in the image to generate an output image.

Contrast Stretching and Thresholding



a b

FIGURE 3.2 Intensity transformation functions. (a) Contrast-stretching function. (b) Thresholding function.

Some Intensity Transform Function

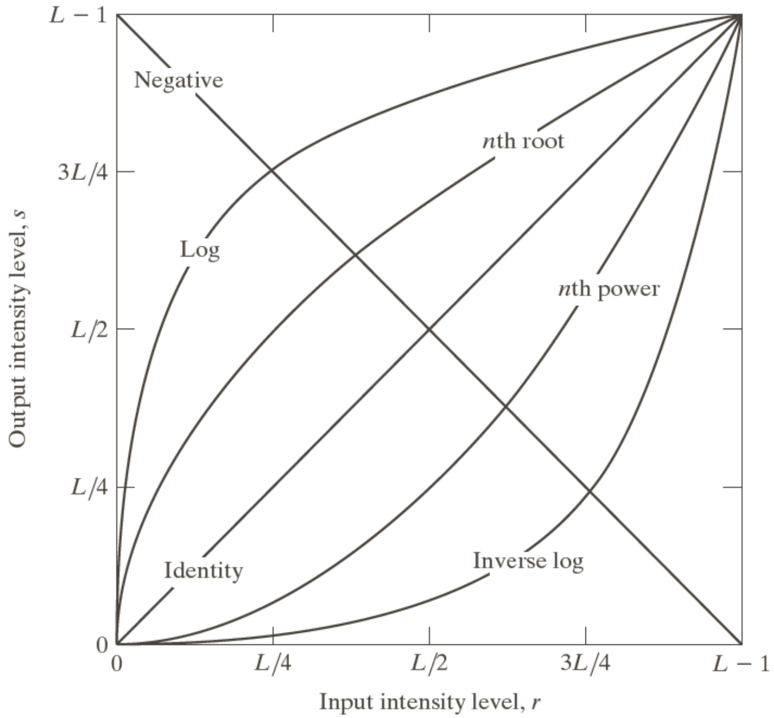
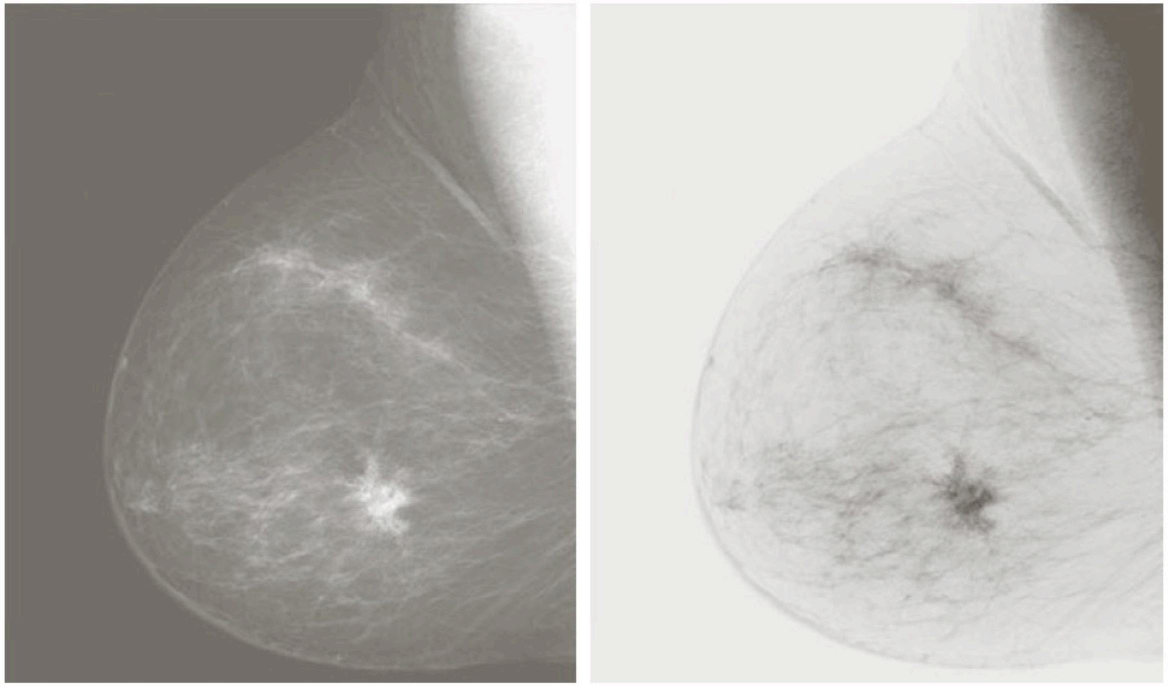


FIGURE 3.3 Some basic intensity transformation functions. All curves were scaled to fit in the range shown.

Negative



a b

FIGURE 3.4
(a) Original digital mammogram.
(b) Negative image obtained using the negative transformation in Eq. (3.2-1).
(Courtesy of G.E. Medical Systems.)

Gamma correction

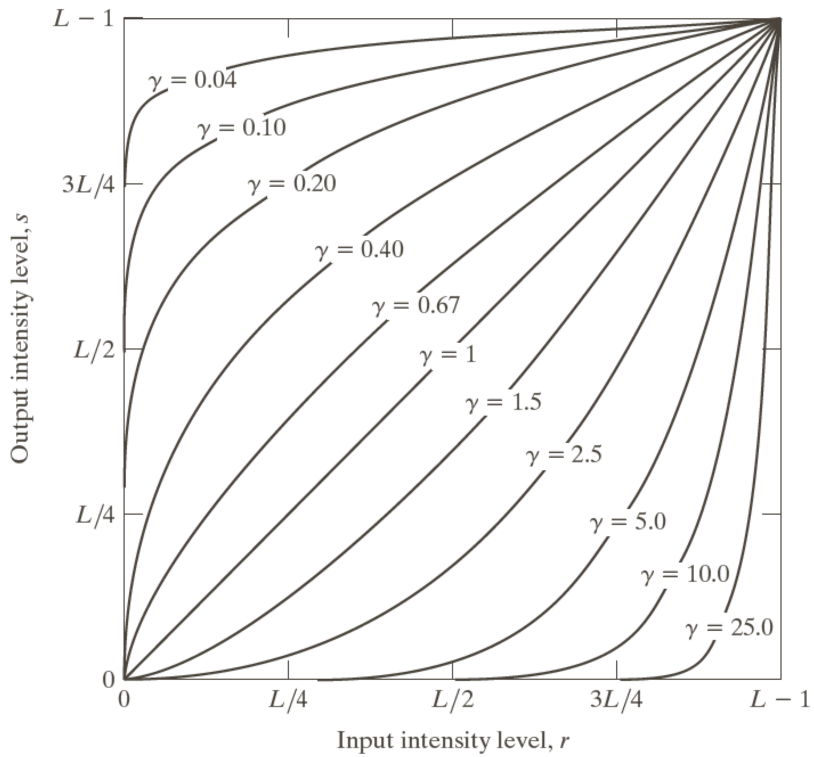


FIGURE 3.6 Plots of the equation $s = cr^{\gamma}$ for various values of γ ($c = 1$ in all cases). All curves were scaled to fit in the range shown.

Gamma correction

γ = ?

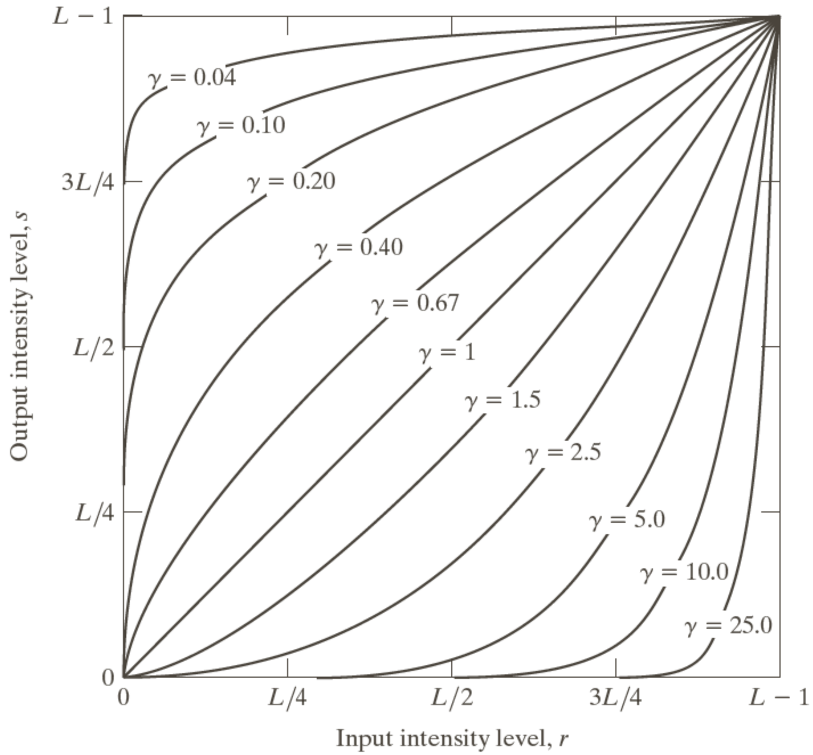
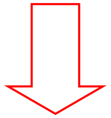


FIGURE 3.6 Plots of the equation $s = cr^{\gamma}$ for various values of γ ($c = 1$ in all cases). All curves were scaled to fit in the range shown.

Gamma Correction



$\gamma = ?$

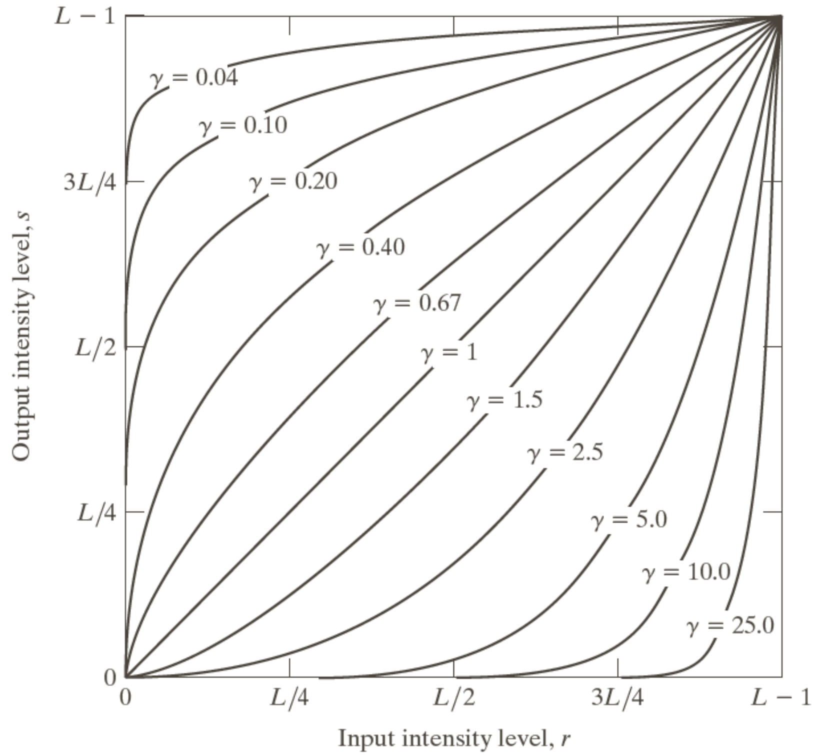
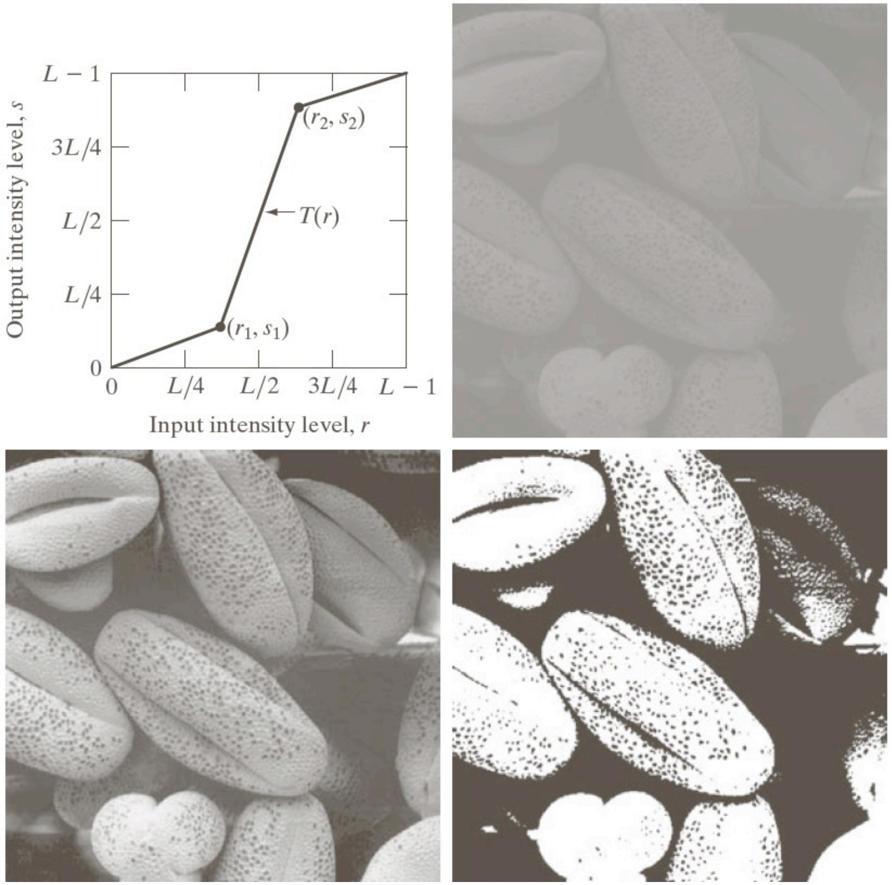


FIGURE 3.6 Plots of the equation $s = cr^{\gamma}$ for various values of γ ($c = 1$ in all cases). All curves were scaled to fit in the range shown.

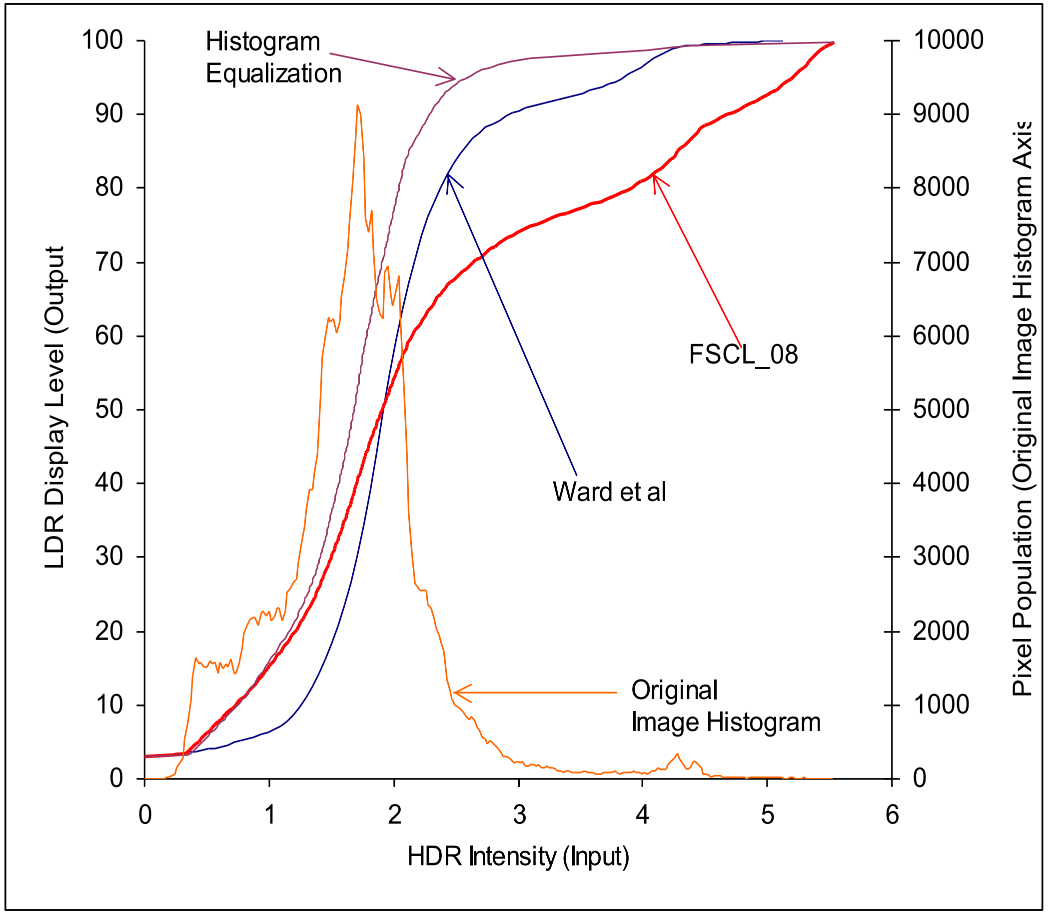
Contrast Stretching



a b
c d

FIGURE 3.10 Contrast stretching. (a) Form of transformation function. (b) A low-contrast image. (c) Result of contrast stretching. (d) Result of thresholding. (Original image courtesy of Dr. Roger Heady, Research School of Biological Sciences, Australian National University, Canberra, Australia.)

Dynamic Range Compression



The dynamic range of the image is 340,016:1

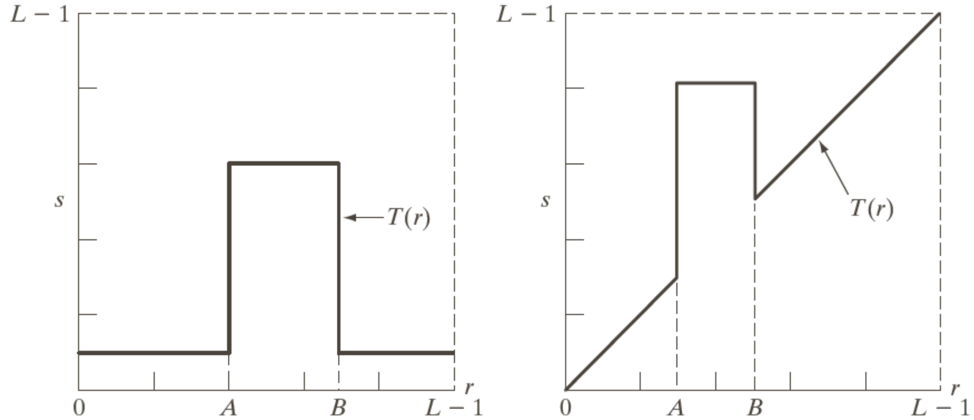
Dynamic Range Compression

Simple dynamic range compression functions:

$s = c*\log(1 + r)$

Grey-level Slicing

Highlighting a specific range of gray levels in an image

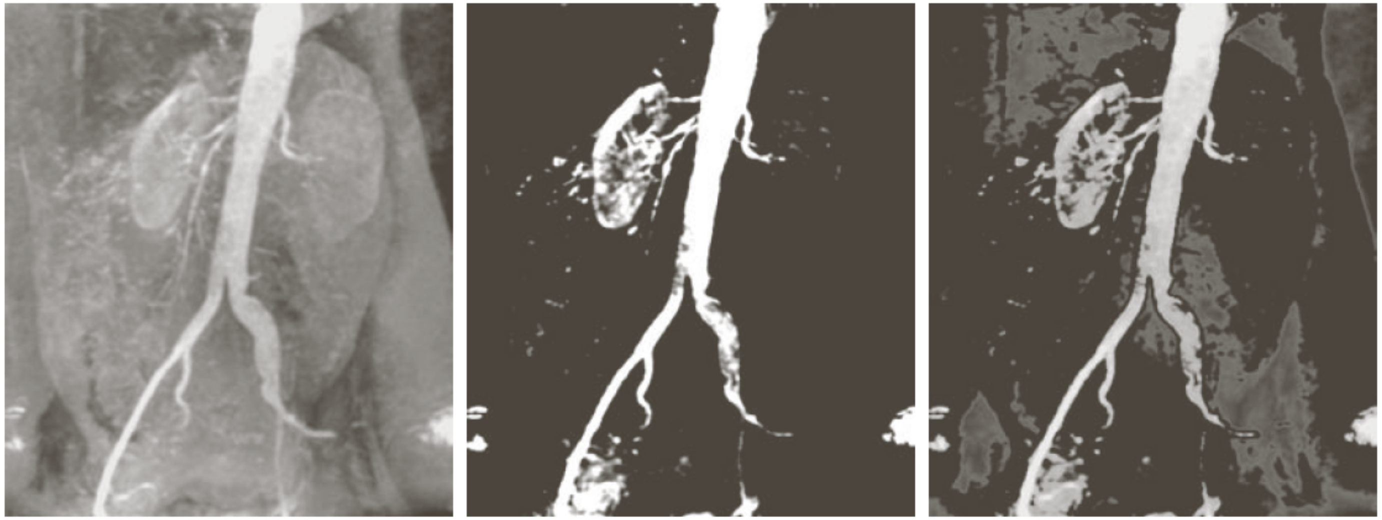


a b

FIGURE 3.11 (a) This transformation highlights intensity range [A, B] and reduces all other intensities to a lower level. (b) This transformation highlights range [A, B] and preserves all other intensity levels.

Grey-level Slicing

Highlighting a specific range of gray levels in an image



a b c

FIGURE 3.12 (a) Aortic angiogram. (b) Result of using a slicing transformation of the type illustrated in Fig. 3.11(a), with the range of intensities of interest selected in the upper end of the gray scale. (c) Result of using the transformation in Fig. 3.11(b), with the selected area set to black, so that grays in the area of the blood vessels and kidneys were preserved. (Original image courtesy of Dr. Thomas R. Gest, University of Michigan Medical School.)

Histogram Processing

The histogram of a digital image with gray levels in the range [0, L-1] is a discrete function $p(r_k) = n_k/n$, where r_k is the k_{th} gray level, n_k is the number of pixels in the image with that gray level, n is the total number of pixels in the image, k = 0, 1, …, L-1

Histogram Processing

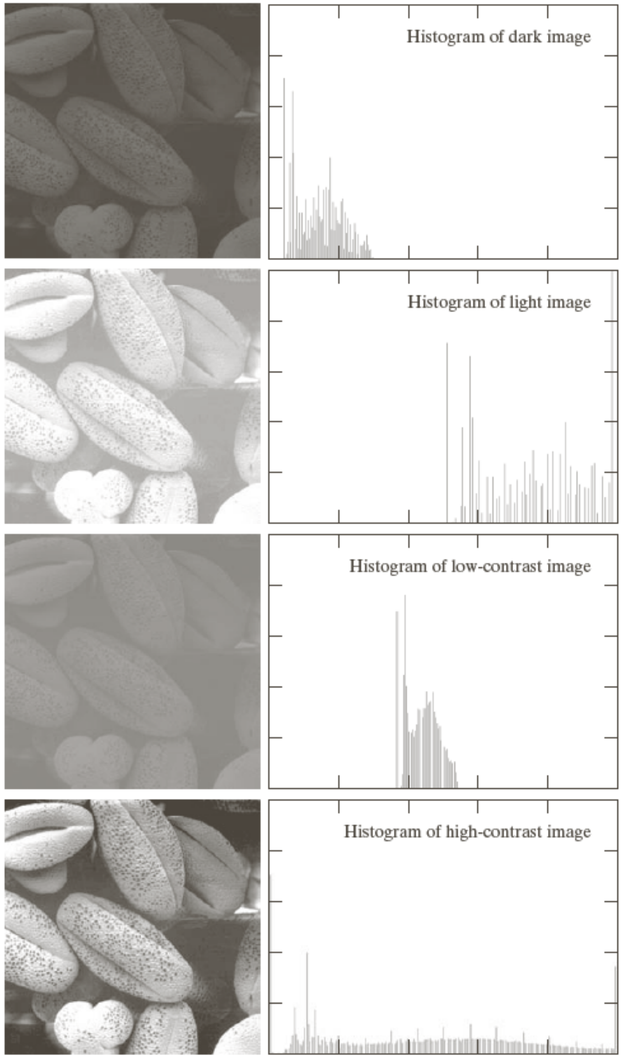
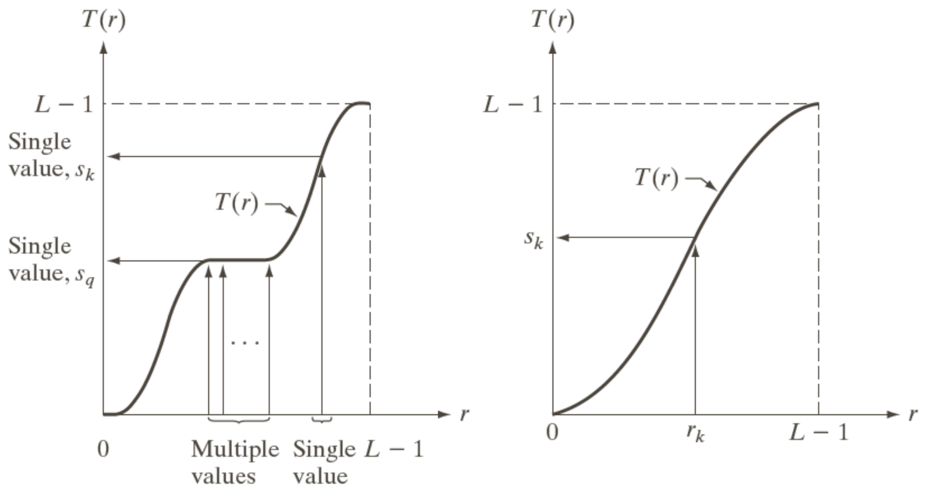


FIGURE 3.16 Four basic image types: dark, light, low contrast, high contrast, and their corresponding histograms.

Histogram Processing

Gray-level transformation function



a b

FIGURE 3.17
(a) Monotonically increasing function, showing how multiple values can map to a single value.
(b) Strictly monotonically increasing function. This is a one-to-one mapping, both ways.

Histogram Equalization

Principle

- Let r represent the gray levels in the image to be processed (enhanced).

- Assuming r is continuous and has been normalized to lie in the interval [0, 1], r =0= black, r =1=white

- The following transformation
  S = T(r) satisfies
  1. T(r) is single-valued and monotonically increasing in the interval 0<= r <=1, and
  2. 0<= T(r) <=1 for 0<= r <=1

- Condition 1 preserve the order from black to white in the grayscale whilst condition 2 guarantees a mapping that is consistent with the allowed range of the pixel values

Histogram Equalization

Principle

- The inverse transformation from s back to r is denoted as

  $r = T^{-1}(s)$ 0 <= s <= 1

- It is assume that T^{-1} also satisfied above two conditions

Histogram Equalization

Principle

- The gray level of an image may be viewed as random quantities in the interval [0, 1]
- If they are continuous variables, the original and transformed gray levels can be characterized by their probability density functions $p_r(r)$ and $p_s(s)$, respectively
- If $p_r(r)$ and $T(r)$ are known and $T^{-1}(r)$ satisfies the condition above, then we have

Histogram Equalization

Principle

$$p_s(s) = \left[p_r(r) \frac{dr}{ds} \right]_{r=T^{-1}(s)}$$

Histogram Equalization

Principle

- Consider the transformation function

$$s = T(r) = \int_0^r p_r(w)\,dw \qquad 0 \le r \le 1$$

Cumulated distribute function (CDF)

Histogram Equalization

Principle

- Consider the transformation function

$$s = T(r) = \int_0^r p_r(w)\,dw \qquad 0 \le r \le 1$$

$$\frac{ds}{dr} = p_r(r)$$

Histogram Equalization

Principle

- Consider the transformation function

$$\frac{ds}{dr} = p_r(r)$$

$$p_s(s) = \left[p_r(r)\frac{dr}{ds} \right]_{r=T^{-1}(s)} = \left[p_r(r)\frac{1}{\frac{ds}{dr}} \right]_{r=T^{-1}(s)} = \left[p_r(r)\frac{1}{p_r(r)} \right]_{r=T^{-1}(s)} = 1$$

$$0 \le s \le 1$$

Histogram Equalization

Principle

- Consider the transformation function

$$\frac{ds}{dr} = p_r(r)$$

Uniform distribution

$$p_s(s) = \left[p_r(r)\frac{dr}{ds} \right]_{r=T^{-1}(s)} = \left[p_r(r)\frac{1}{\frac{ds}{dr}} \right]_{r=T^{-1}(s)} = \left[p_r(r)\frac{1}{p_r(r)} \right]_{r=T^{-1}(s)} = 1$$

$$0 \le s \le 1$$

Histogram Equalization

Principle

- Use the cumulate distribution of r produces as a transform function

- Produces an image whose gray levels have a uniform density function

$$s = T(r) = \int_0^r p_r(w)\,dw \qquad 0 \le r \le 1$$

- An image whose gray levels have a uniform density function appears to have high contrast

Histogram Equalization

Principle

Histogram Equalization

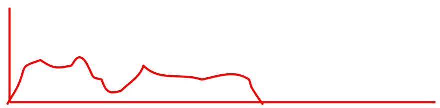
Practice

- Digital images

- Assume n_k is the number of pixels have vale r_k, there are n pixels in the image, the probabilities (frequencies of occurrence) of gray levels r_k

$$p_r(r_k) = \frac{n_k}{n} \qquad 0 \le r_k \le 1 \qquad k = 0,1,\cdots L-1$$

Histogram



Histogram Equalization

Practice

- Cumulative histogram

$$s_k = \sum_{j=0}^{k} \frac{n_j}{n} = \sum_{j=0}^{k} p_r\left(r_j\right) \qquad 0 \le r_k \le 1 \qquad k = 0,1,\cdots L-1$$

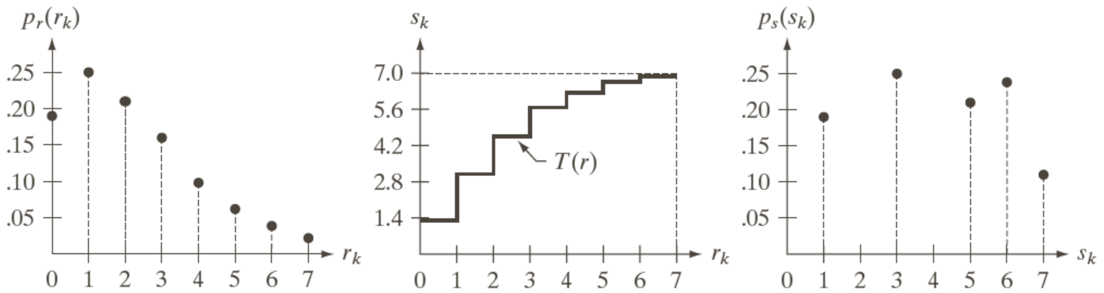
Histogram equalization

Histogram Equalization

Example

r_k	n_k	$p_r(r_k) = n_k/MN$
$r_0 = 0$	790	0.19
$r_1 = 1$	1023	0.25
$r_2 = 2$	850	0.21
$r_3 = 3$	656	0.16
$r_4 = 4$	329	0.08
$r_5 = 5$	245	0.06
$r_6 = 6$	122	0.03
$r_7 = 7$	81	0.02

TABLE 3.1 Intensity distribution and histogram values for a 3-bit, 64×64 digital image.



a b c

FIGURE 3.19 Illustration of histogram equalization of a 3-bit (8 intensity levels) image. (a) Original histogram. (b) Transformation function. (c) Equalized histogram.

Histogram Equalization

Example

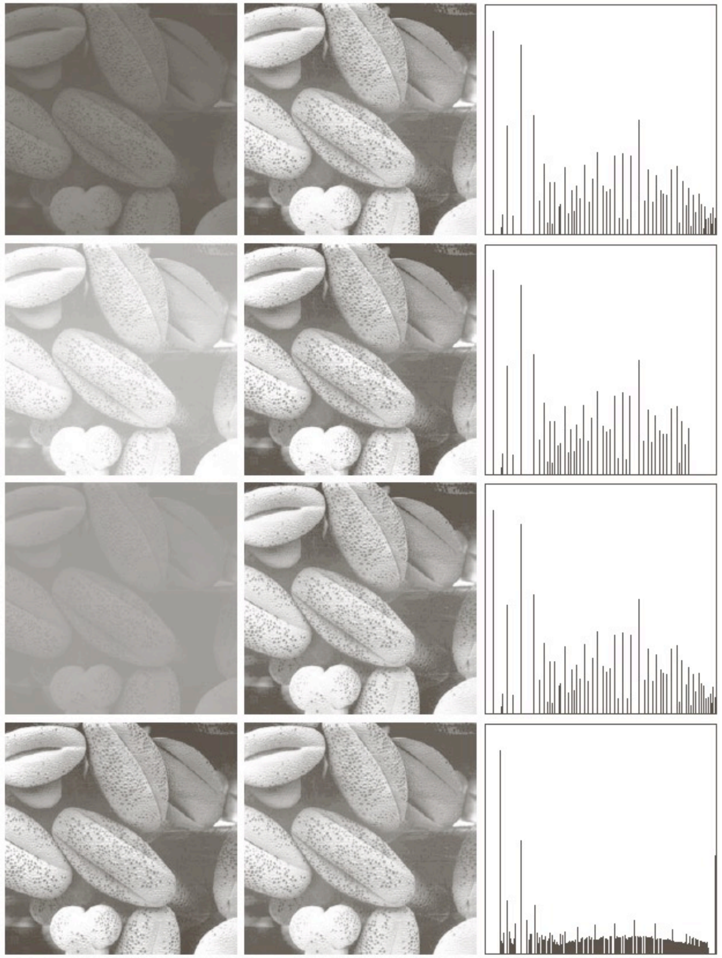


FIGURE 3.20 Left column: images from Fig. 3.16. Center column: corresponding histogram-equalized images. Right column: histograms of the images in the center column.

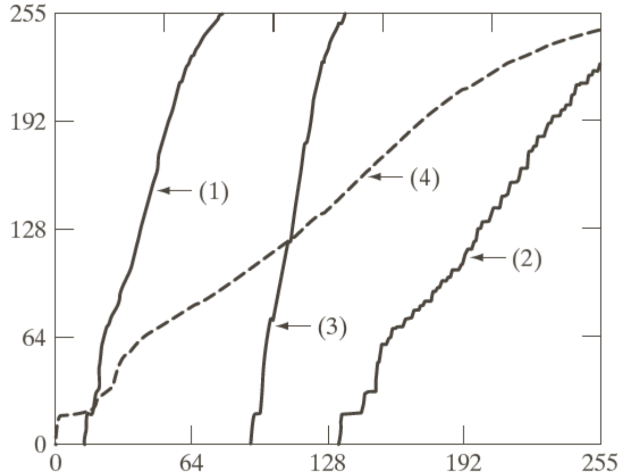


FIGURE 3.21 Transformation functions for histogram equalization. Transformations (1) through (4) were obtained from the histograms of the images (from top to bottom) in the left column of Fig. 3.20 using Eq. (3.3-8).

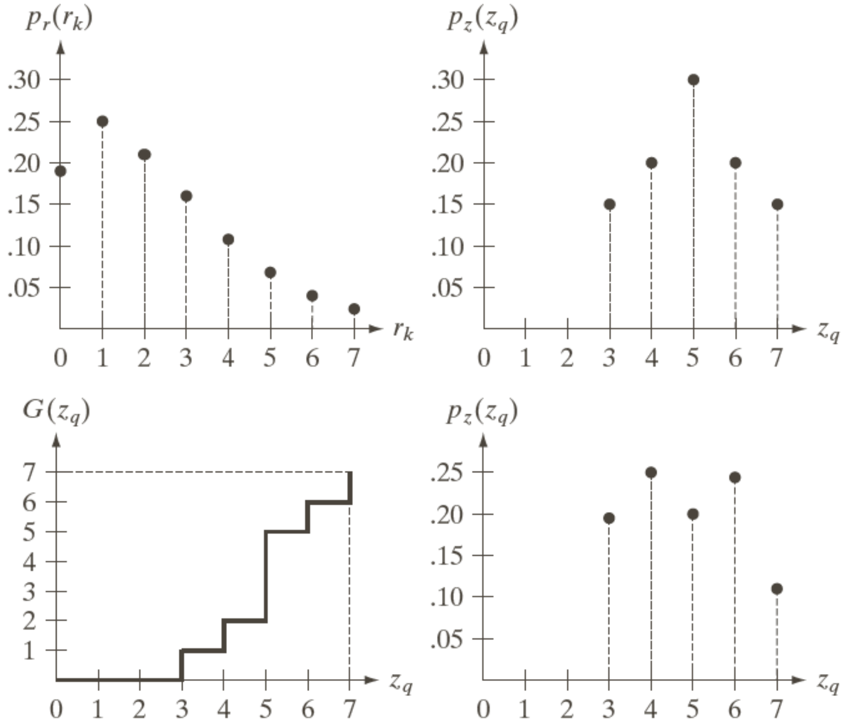
Histogram Specification

Principle and practice

histogram specification (matching) is the transformation of an image so that its histogram matches a specified histogram. The well-known histogram equalization method is a special case in which the specified histogram is uniformly distributed.

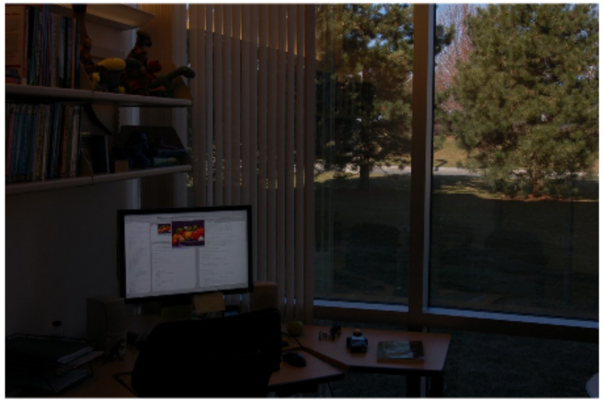
Histogram Specification

Example

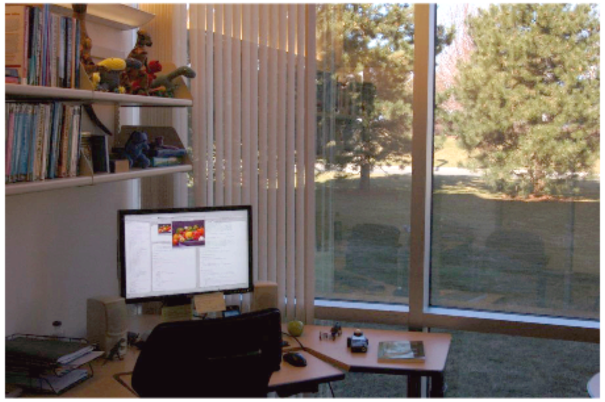


a b
c d

FIGURE 3.22 (a) Histogram of a 3-bit image. (b) Specified histogram. (c) Transformation function obtained from the specified histogram. (d) Result of performing histogram specification. Compare (b) and (d).



input

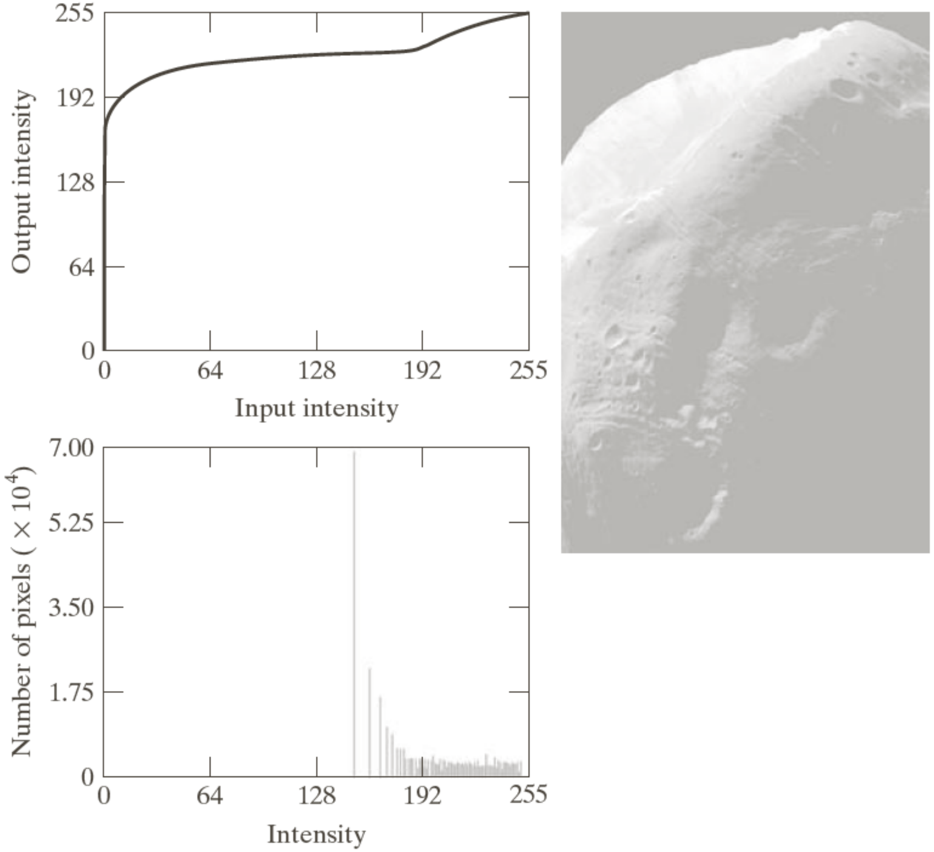


reference

result

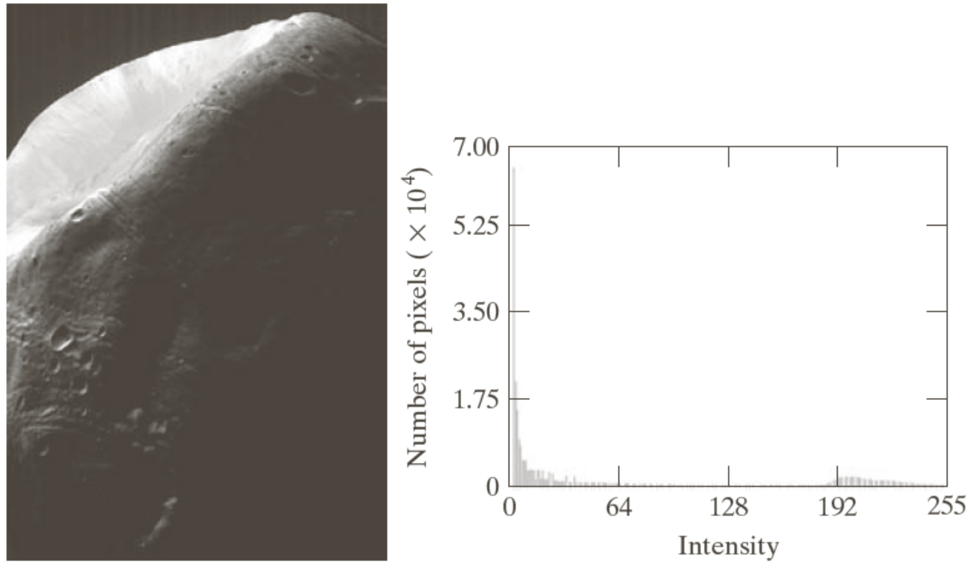
Histogram Processing

Drawbacks/Artefacts



a b
c

FIGURE 3.24
(a) Transformation function for histogram equalization.
(b) Histogram-equalized image (note the washed-out appearance).
(c) Histogram of (b).

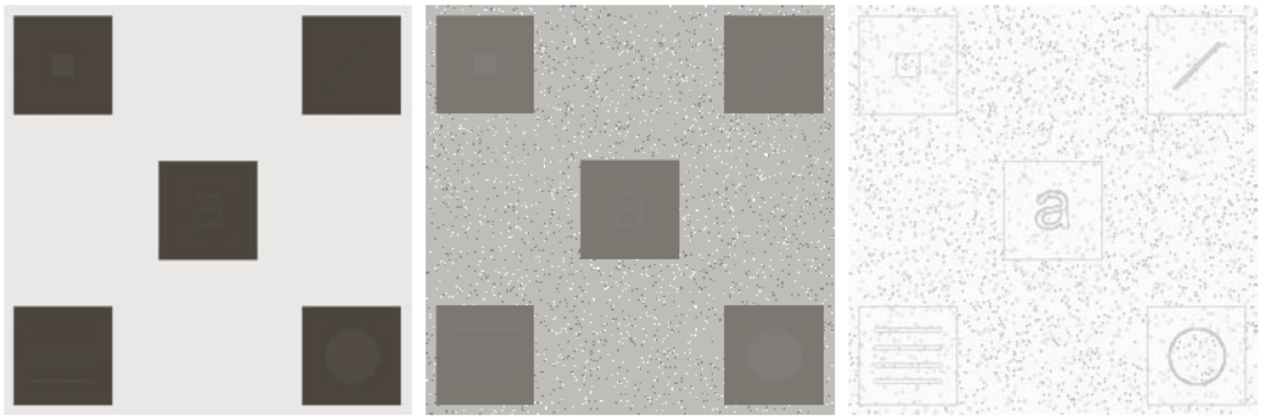


a b

FIGURE 3.23
(a) Image of the Mars moon Phobos taken by NASA's *Mars Global Surveyor.*
(b) Histogram.
(Original image courtesy of NASA.)

Histogram Processing

Global vs Local Processing



a b c

FIGURE 3.26 (a) Original image. (b) Result of global histogram equalization. (c) Result of local histogram equalization applied to (a), using a neighborhood of size 3 × 3.

Histogram Processing

Global vs Local Processing

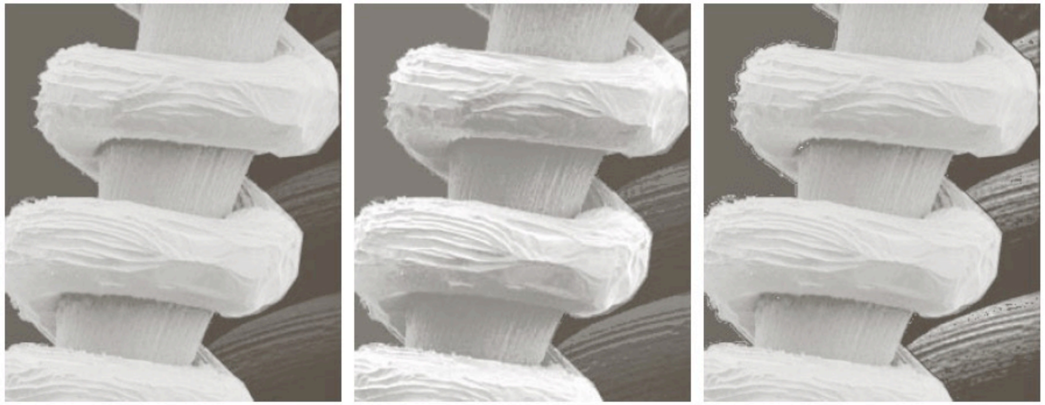


FIGURE 3.27 (a) SEM image of a tungsten filament magnified approximately 130×. (b) Result of global histogram equalization. (c) Image enhanced using local histogram statistics. (Original image courtesy of Mr. Michael Shaffer, Department of Geological Sciences, University of Oregon, Eugene.)

Spatial Filtering

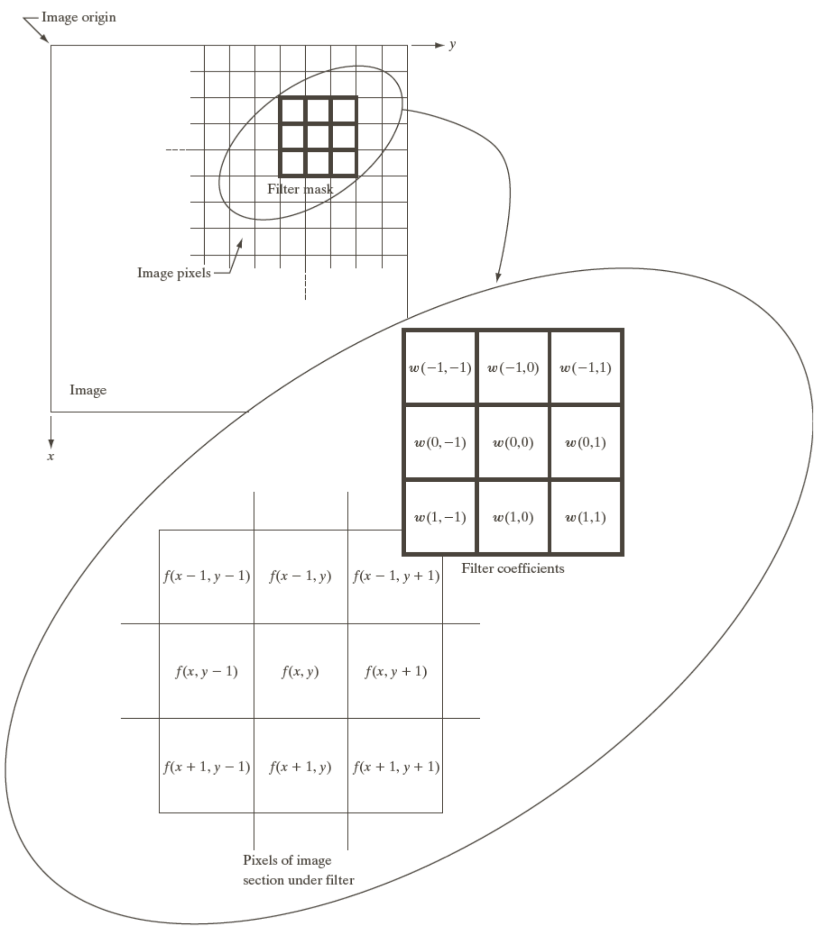


FIGURE 3.28 The mechanics of linear spatial filtering using a 3×3 filter mask. The form chosen to denote the coordinates of the filter mask coefficients simplifies writing expressions for linear filtering.

Spatial Filtering

w_1	w_2	w_3
w_4	w_5	w_6
w_7	w_8	w_9

FIGURE 3.31 Another representation of a general 3×3 filter mask.

Spatial Filtering

Low-pass (Smoothing) Filtering

$$\frac{1}{9} \times \begin{array}{|c|c|c|}\hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline\end{array} \qquad \frac{1}{16} \times \begin{array}{|c|c|c|}\hline 1 & 2 & 1 \\ \hline 2 & 4 & 2 \\ \hline 1 & 2 & 1 \\ \hline\end{array}$$

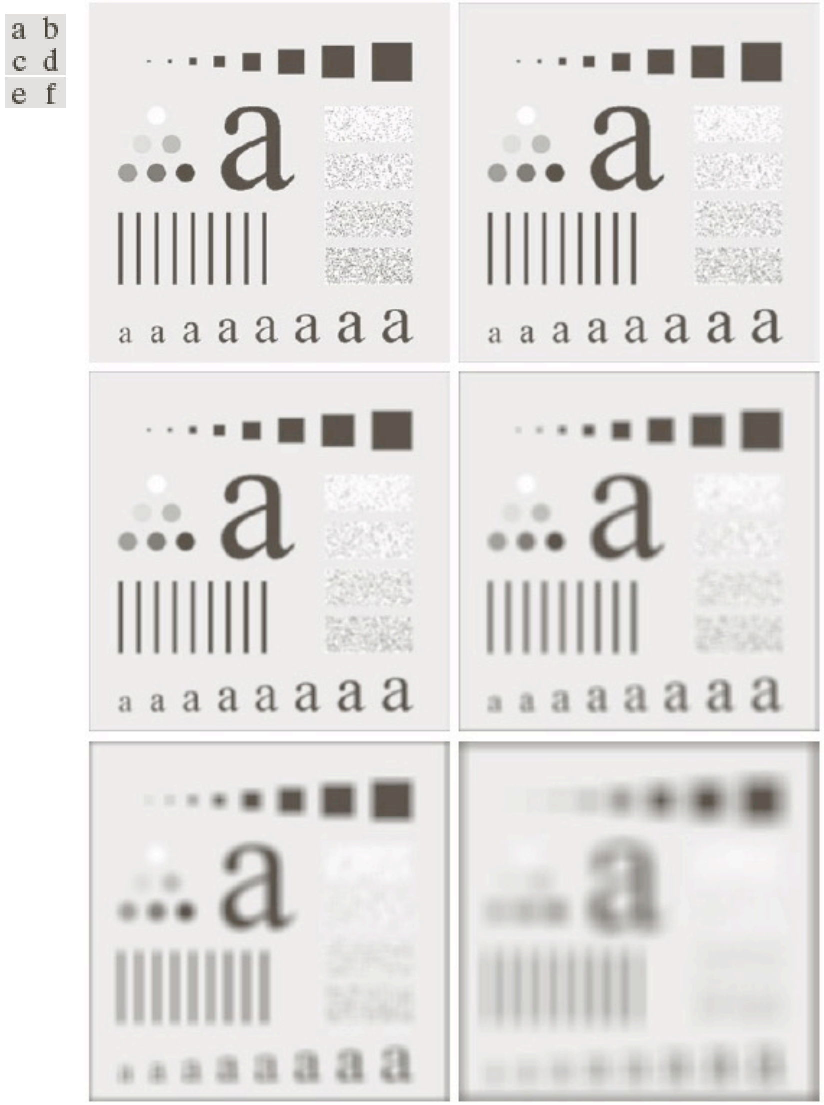
a b

FIGURE 3.32 Two 3×3 smoothing (averaging) filter masks. The constant multiplier in front of each mask is equal to 1 divided by the sum of the values of its coefficients, as is required to compute an average.

Spatial Filtering

Low-pass (Smoothing) Filtering

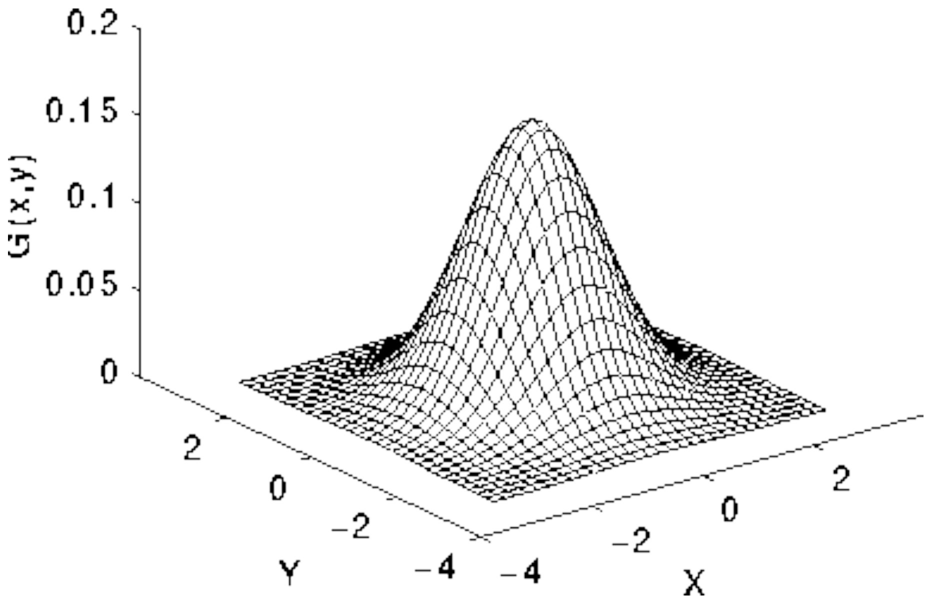
FIGURE 3.33 (a) Original image, of size 500 × 500 pixels. (b)–(f) Results of smoothing with square averaging filter masks of sizes $m = 3, 5, 9, 15$, and 35, respectively. The black squares at the top are of sizes 3, 5, 9, 15, 25, 35, 45, and 55 pixels, respectively; their borders are 25 pixels apart. The letters at the bottom range in size from 10 to 24 points, in increments of 2 points; the large letter at the top is 60 points. The vertical bars are 5 pixels wide and 100 pixels high; their separation is 20 pixels. The diameter of the circles is 25 pixels, and their borders are 15 pixels apart; their intensity levels range from 0% to 100% black in increments of 20%. The background of the image is 10% black. The noisy rectangles are of size 50 × 120 pixels.



Spatial Filtering

Low-pass (Smoothing) Filtering

- Gaussian Smoothing



Spatial Filtering

Drawbacks of low-pass filtering

Spatial Filtering

Median (Smoothing) Filtering

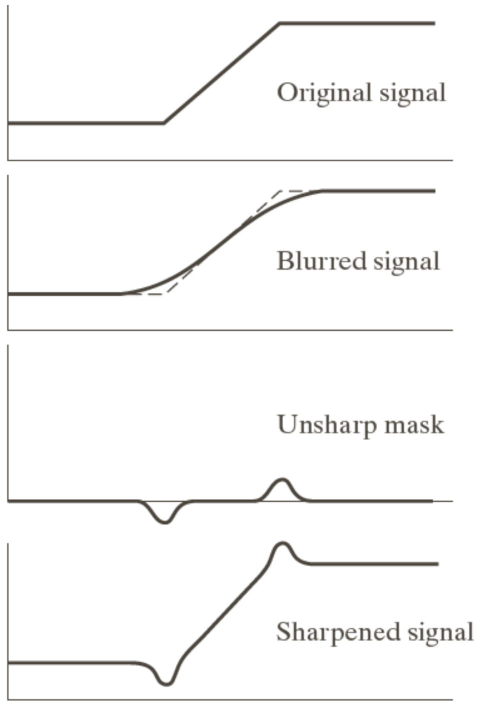
Median vs Low-pass

Spatial Filtering

High-pass (sharpening) filtering

Spatial Filtering

Unsharp masking



a
b
c
d

FIGURE 3.39 1-D illustration of the mechanics of unsharp masking. (a) Original signal. (b) Blurred signal with original shown dashed for reference. (c) Unsharp mask. (d) Sharpened signal, obtained by adding (c) to (a).

Spatial Filtering

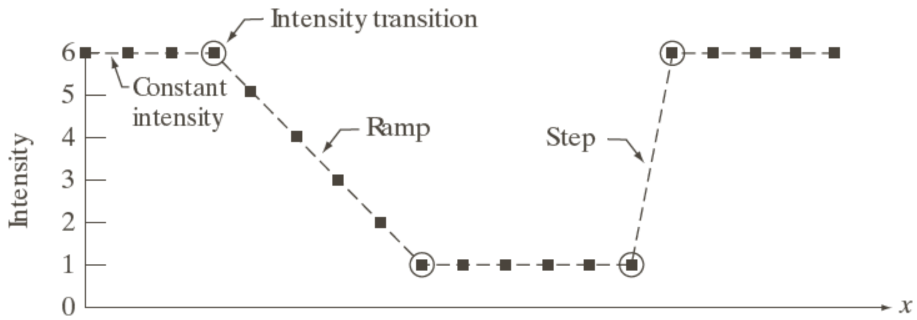
Unsharp masking

a
b
c
d
e

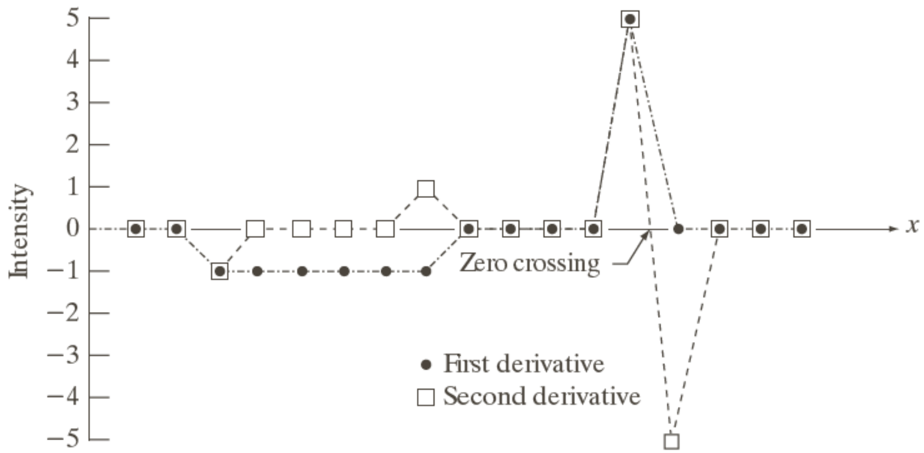
FIGURE 3.40 (a) Original image. (b) Result of blurring with a Gaussian filter. (c) Unsharp mask. (d) Result of using unsharp masking. (e) Result of using highboost filtering.

Spatial Filtering

Derivative Filters



Scan line	6	6	6	6	5	4	3	2	1	1	1	1	1	1	6	6	6	6	6
1st derivative		0	0	−1	−1	−1	−1	−1	0	0	0	0	0	5	0	0	0	0	
2nd derivative		0	0	−1	0	0	0	0	1	0	0	0	0	5	−5	0	0	0	



a
b
c

FIGURE 3.36 Illustration of the first and second derivatives of a 1-D digital function representing a section of a horizontal intensity profile from an image. In (a) and (c) data points are joined by dashed lines as a visualization aid.

Spatial Filtering

Derivative Filters

(a)

0	1	0
1	−4	1
0	1	0

(b)

1	1	1
1	−8	1
1	1	1

(c)

0	−1	0
−1	4	−1
0	−1	0

(d)

−1	−1	−1
−1	8	−1
−1	−1	−1

a b
c d

FIGURE 3.37 (a) Filter mask used to implement Eq. (3.6-6). (b) Mask used to implement an extension of this equation that includes the diagonal terms. (c) and (d) Two other implementations of the Laplacian found frequently in practice.

Spatial Filtering

Derivative Filters

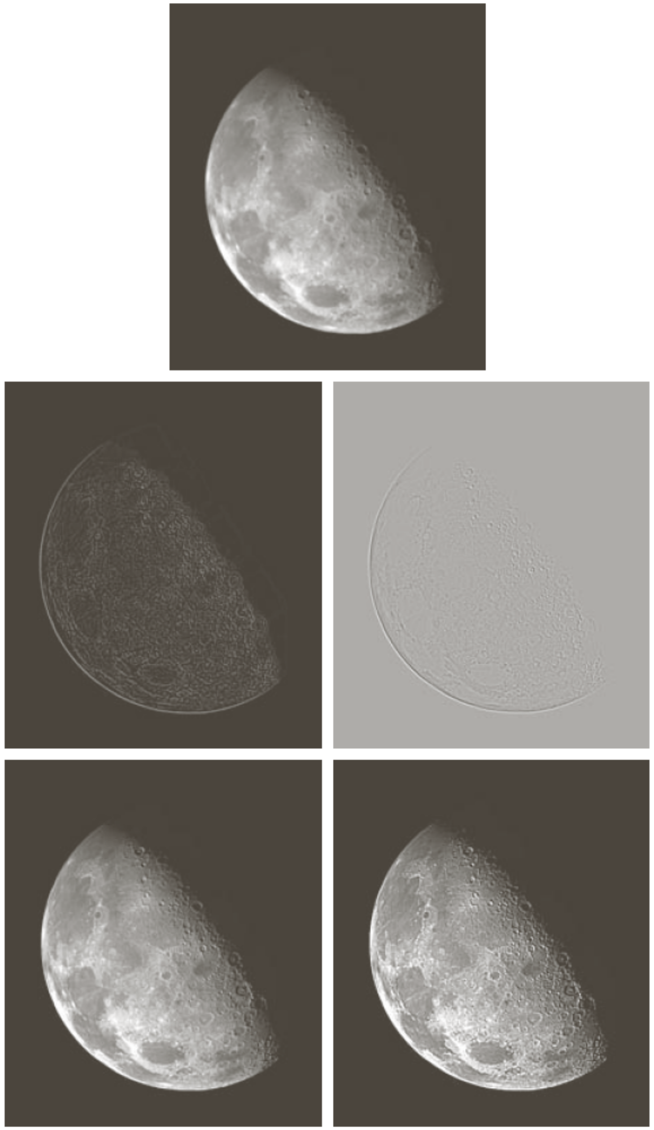


FIGURE 3.38
(a) Blurred image of the North Pole of the moon. (b) Laplacian without scaling. (c) Laplacian with scaling. (d) Image sharpened using the mask in Fig. 3.37(a). (e) Result of using the mask in Fig. 3.37(b). (Original image courtesy of NASA.)

Spatial Filtering

More Derivative Filters

(a)

z_1	z_2	z_3
z_4	z_5	z_6
z_7	z_8	z_9

(b)

−1	0
0	1

(c)

0	−1
1	0

(d)

−1	−2	−1
0	0	0
1	2	1

(e)

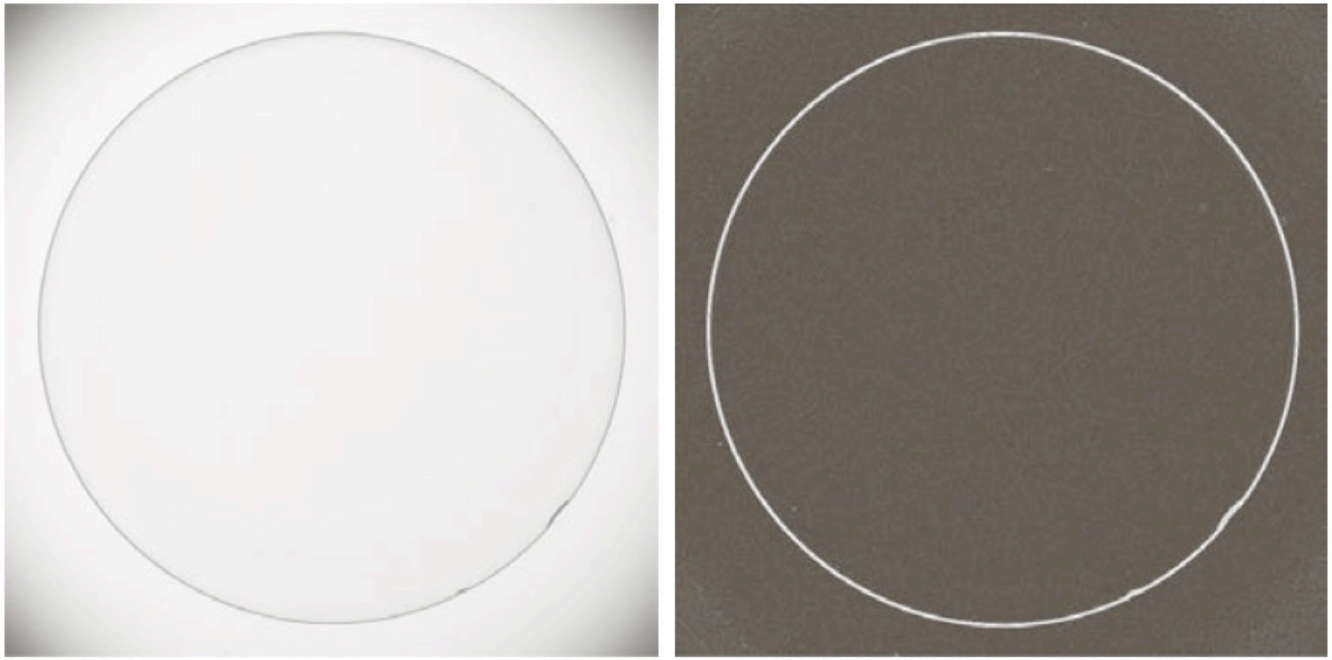
−1	0	1
−2	0	2
−1	0	1

a
b c
d e

FIGURE 3.41 A 3 × 3 region of an image (the zs are intensity values). (b)–(c) Roberts cross gradient operators. (d)–(e) Sobel operators. All the mask coefficients sum to zero, as expected of a derivative operator.

Spatial Filtering

Derivative Filters

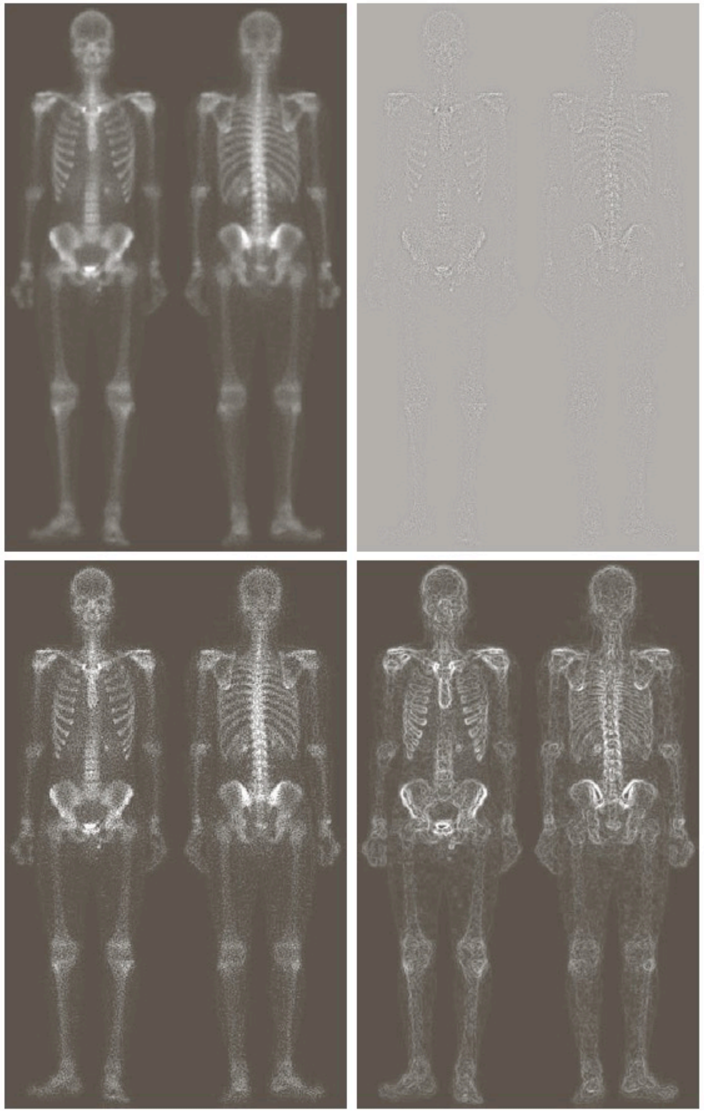


a b

FIGURE 3.42 (a) Optical image of contact lens (note defects on the boundary at 4 and 5 o'clock). (b) Sobel gradient. (Original image courtesy of Pete Sites, Perceptics Corporation.)

Spatial Filtering

Derivative Filters



a b
c d

FIGURE 3.43
(a) Image of whole body bone scan.
(b) Laplacian of (a). (c) Sharpened image obtained by adding (a) and (b).
(d) Sobel gradient of (a).

Spatial Filtering

Derivative Filtering vs Edge Detection

- More later in the course

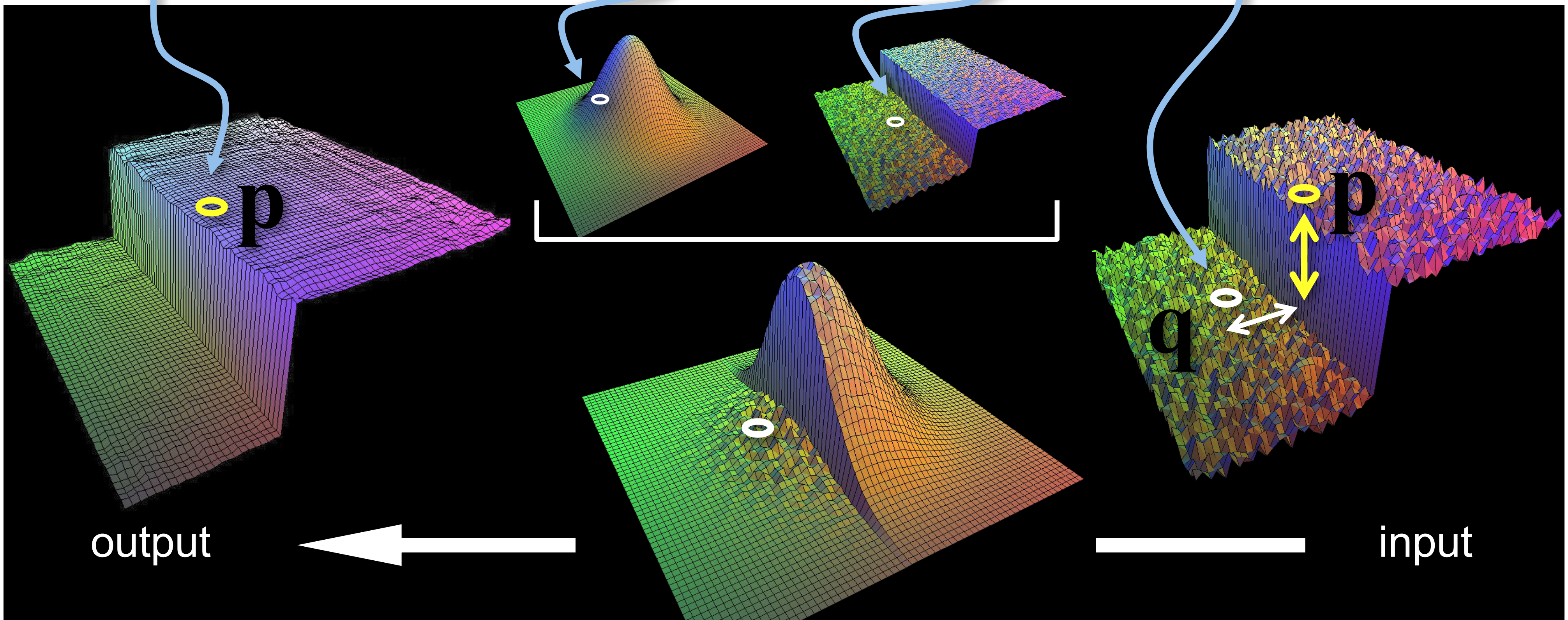
Spatial Filtering

Latest development in image filtering

- Bilateral filtering

Bilateral Filtering

$$\underbrace{BF[I]_{\mathbf{p}}}\ =\ \frac{1}{W_{\mathbf{p}}}\sum_{\mathbf{q}\in S}\underbrace{G_{\sigma_s}(\|\mathbf{p}-\mathbf{q}\|)}\ \underbrace{G_{\sigma_r}(|I_{\mathbf{p}}-I_{\mathbf{q}}|)}\ I_{\mathbf{q}}$$



Source: http://people.csail.mit.edu/sparis/bf_course/