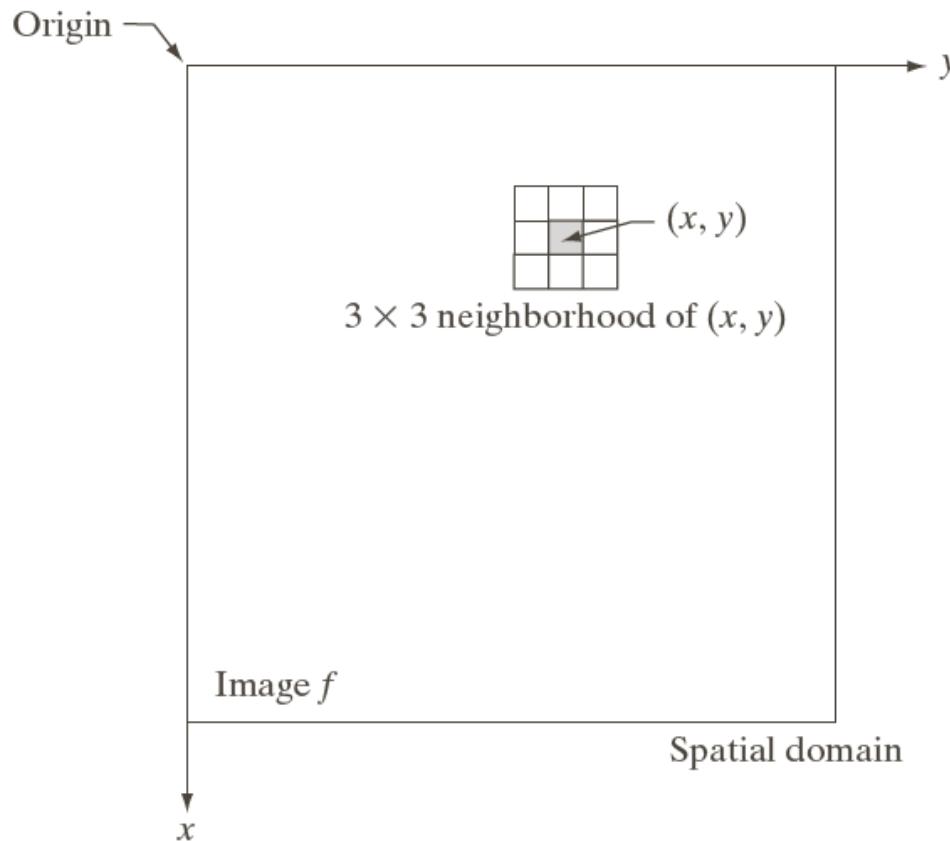


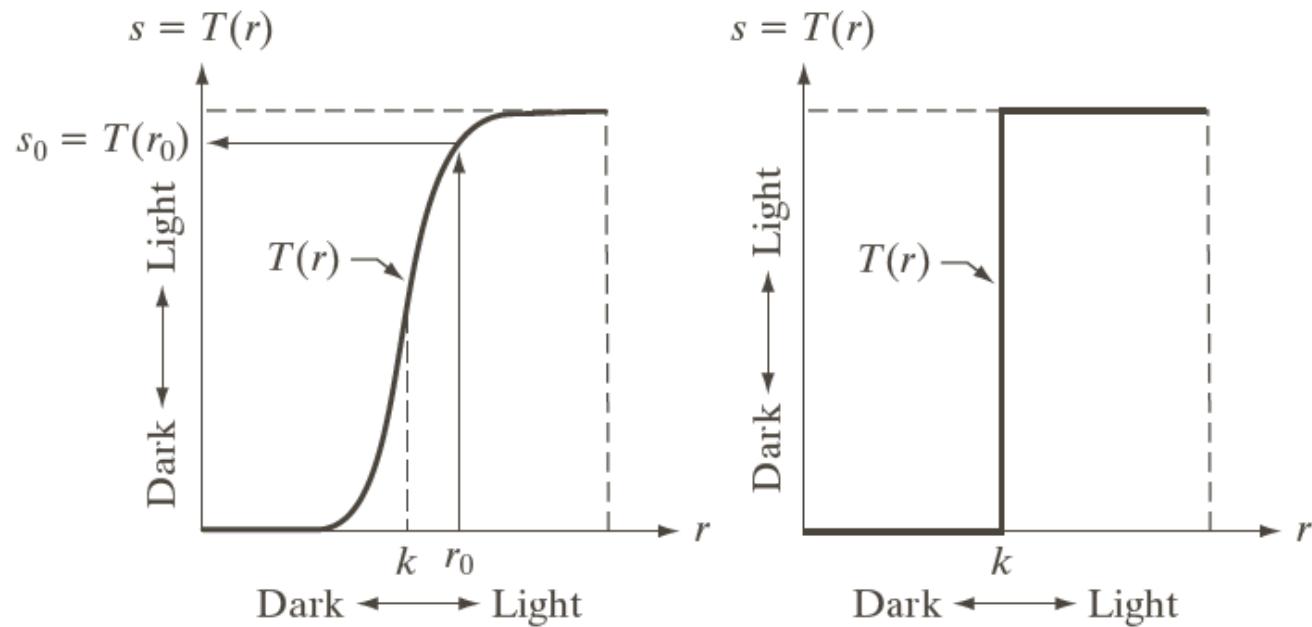
# Intensity Transform and Spatial Filtering

---



**FIGURE 3.1**  
A  $3 \times 3$  neighborhood about a point  $(x, y)$  in an image in the spatial domain. The neighborhood is moved from pixel to pixel in the image to generate an output image.

# Contrast Stretching and Thresholding

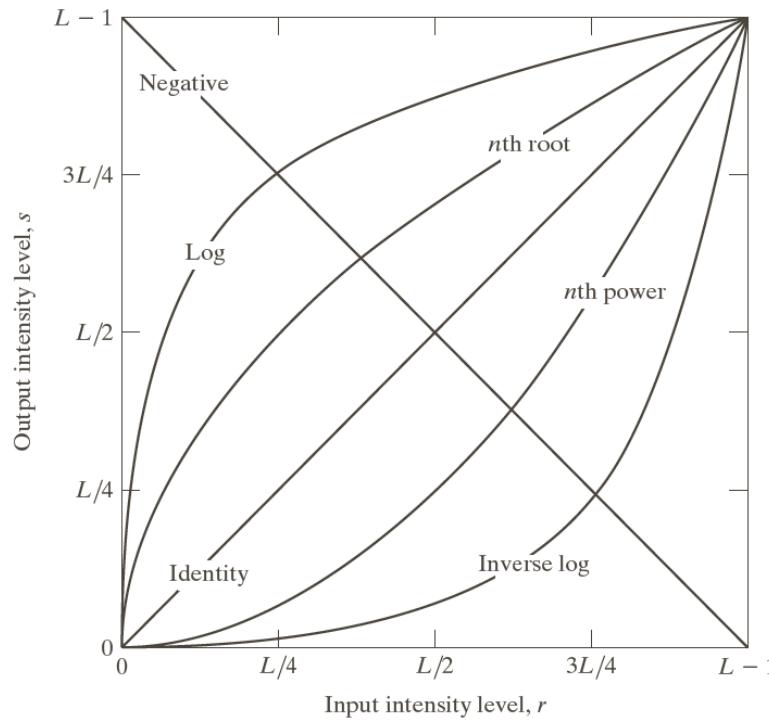


a | b

**FIGURE 3.2**  
Intensity transformation functions.  
(a) Contrast-stretching function.  
(b) Thresholding function.

# Some Intensity Transform Function

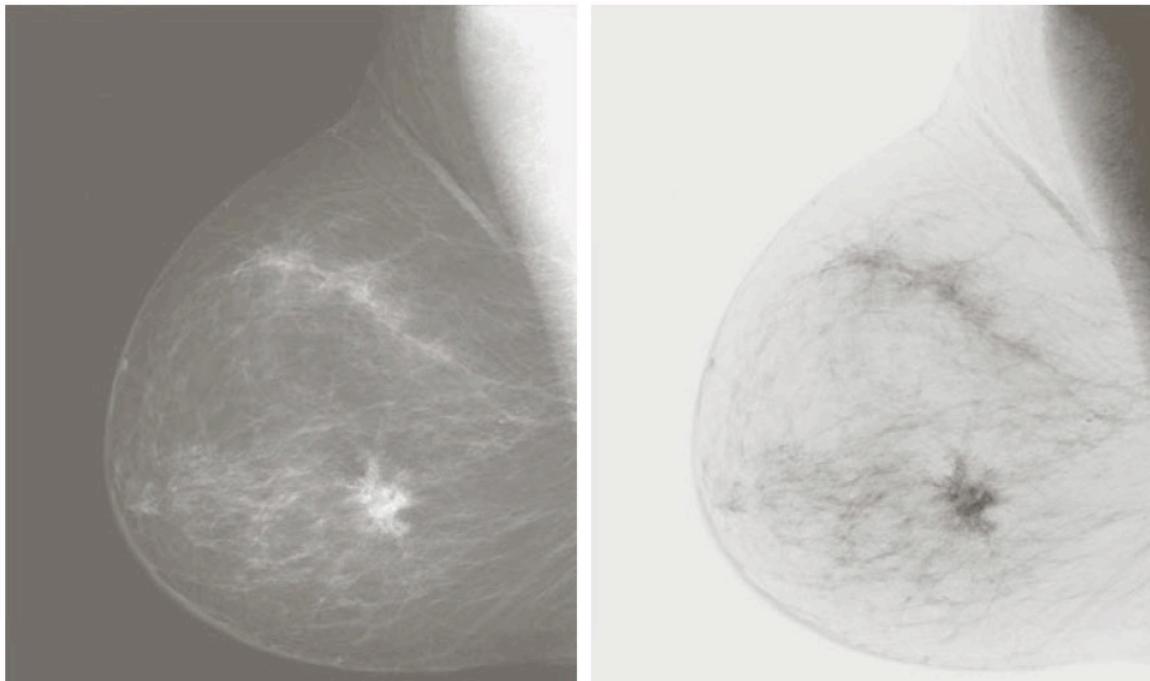
---



**FIGURE 3.3** Some basic intensity transformation functions. All curves were scaled to fit in the range shown.

# Negative

---



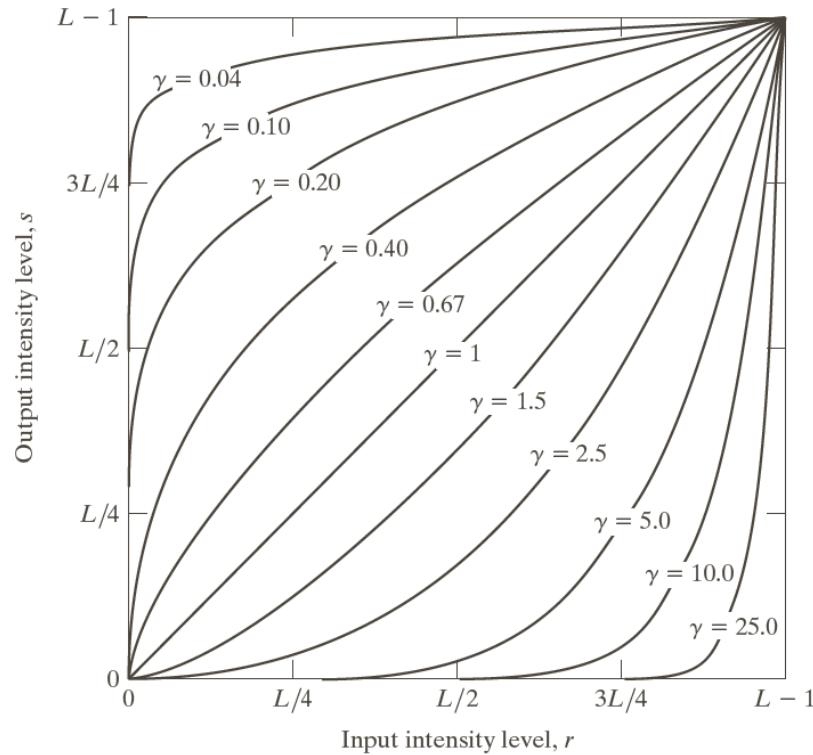
a | b

**FIGURE 3.4**  
(a) Original digital mammogram.  
(b) Negative image obtained using the negative transformation in Eq. (3.2-1).  
(Courtesy of G.E. Medical Systems.)

---

# Gamma correction

---



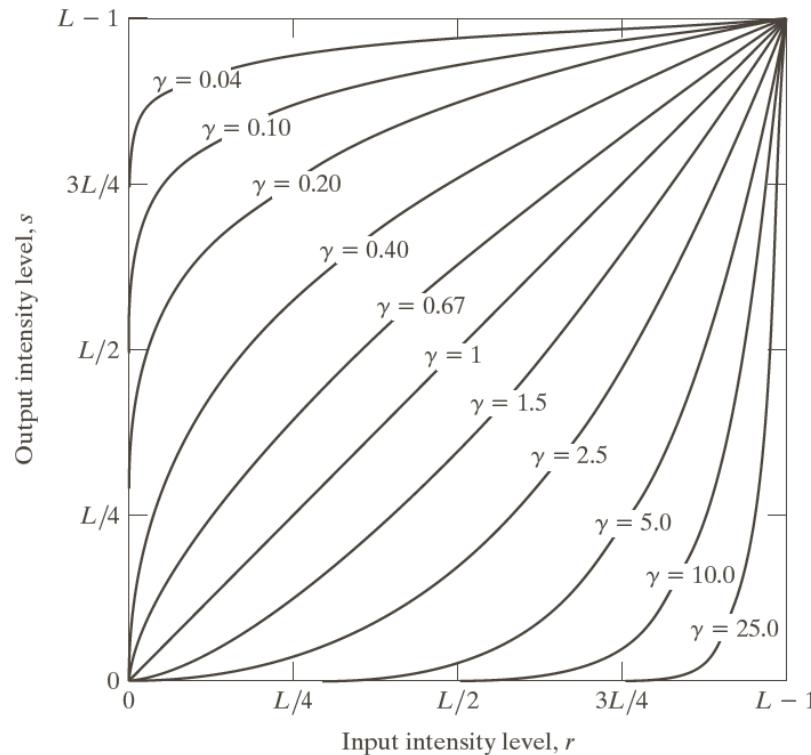
**FIGURE 3.6** Plots of the equation  $s = cr^\gamma$  for various values of  $\gamma$  ( $c = 1$  in all cases). All curves were scaled to fit in the range shown.

# Gamma correction

---



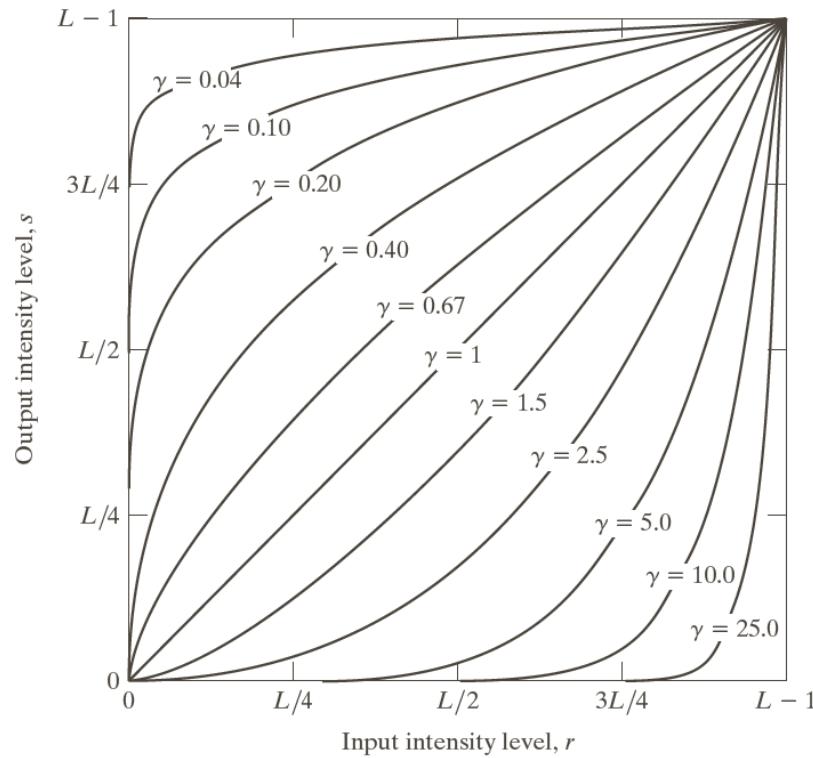
$$\gamma = ?$$



**FIGURE 3.6** Plots of the equation  $s = cr^\gamma$  for various values of  $\gamma$  ( $c = 1$  in all cases). All curves were scaled to fit in the range shown.

# Gamma Correction

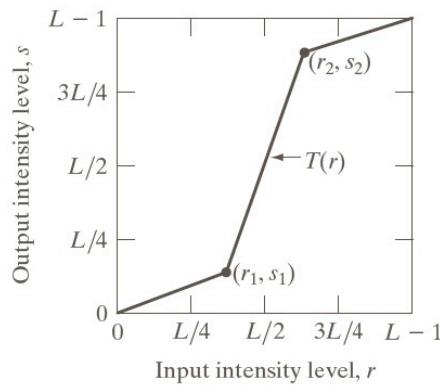
---



**FIGURE 3.6** Plots of the equation  $s = cr^\gamma$  for various values of  $\gamma$  ( $c = 1$  in all cases). All curves were scaled to fit in the range shown.

# Contrast Stretching

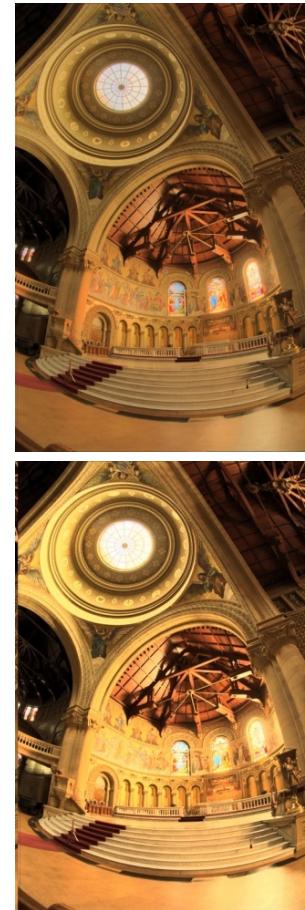
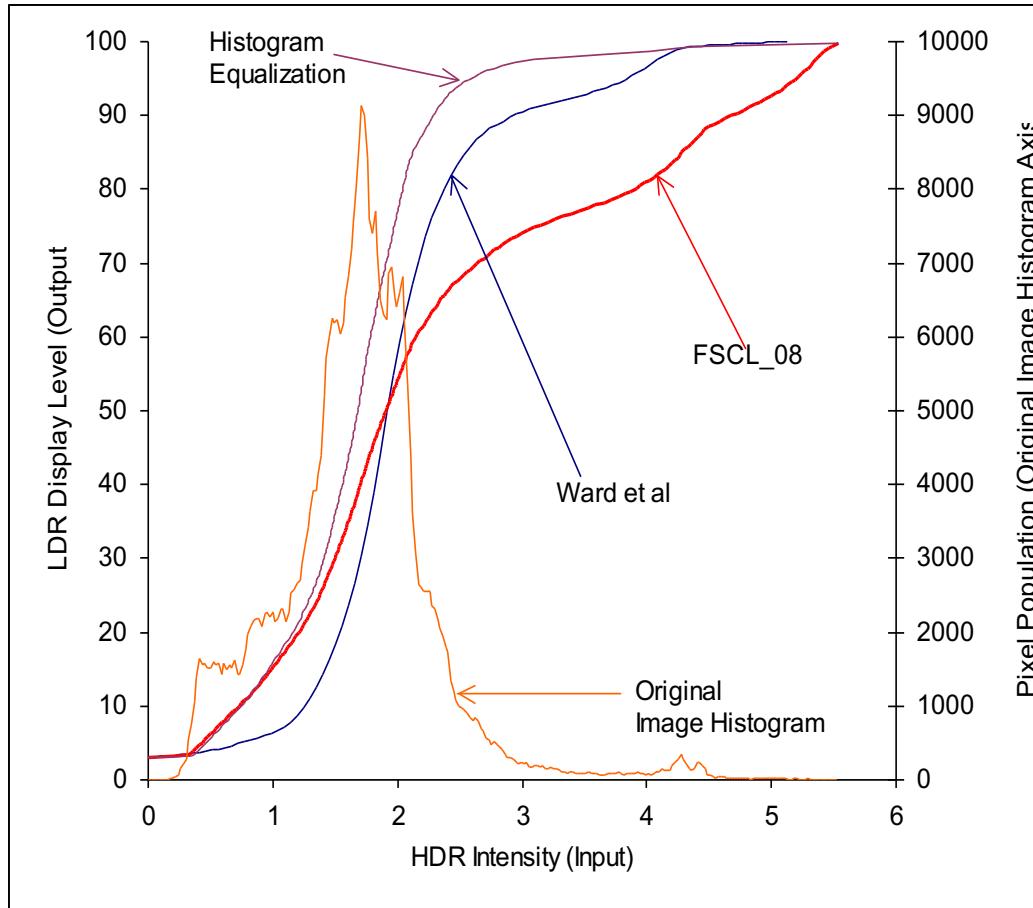
---



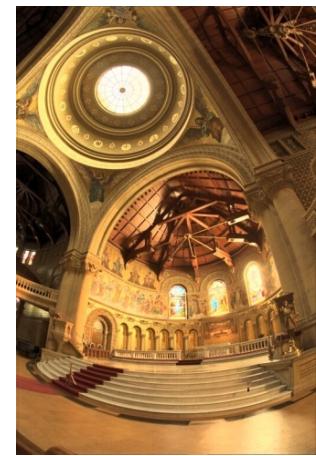
a  
b  
c  
d

**FIGURE 3.10**  
Contrast stretching.  
(a) Form of  
transformation  
function. (b) A  
low-contrast image.  
(c) Result of  
contrast stretching.  
(d) Result of  
thresholding.  
(Original image  
courtesy of Dr.  
Roger Heady,  
Research School of  
Biological Sciences,  
Australian National  
University,  
Canberra,  
Australia.)

# Dynamic Range Compression



The dynamic range of the image is 340,016:1



# Dynamic Range Compression

---

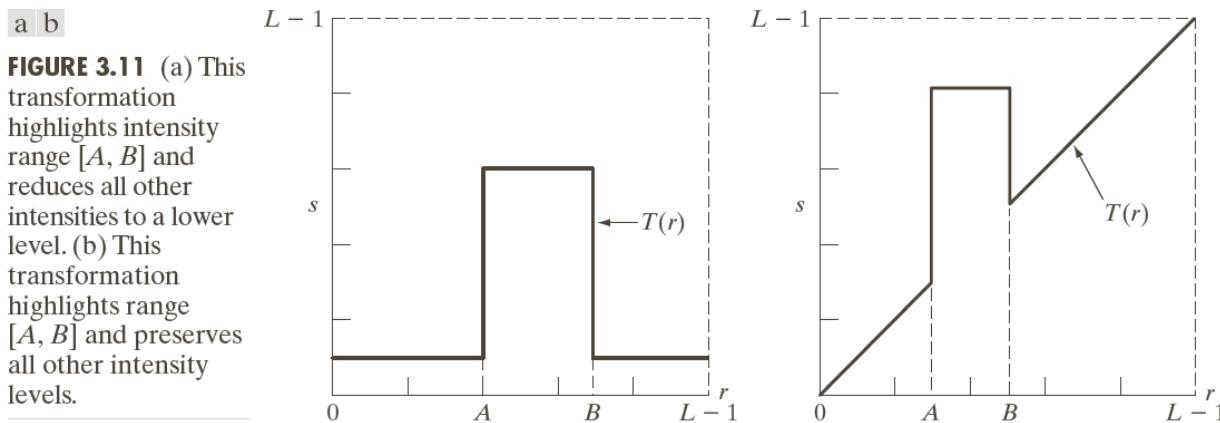
Simple dynamic range compression functions:

$$s = c * \log(1 + r)$$

# Grey-level Slicing

---

Highlighting a specific range of gray levels in an image



# Grey-level Slicing

---

Highlighting a specific range of gray levels in an image



a b c

**FIGURE 3.12** (a) Aortic angiogram. (b) Result of using a slicing transformation of the type illustrated in Fig. 3.11(a), with the range of intensities of interest selected in the upper end of the gray scale. (c) Result of using the transformation in Fig. 3.11(b), with the selected area set to black, so that grays in the area of the blood vessels and kidneys were preserved. (Original image courtesy of Dr. Thomas R. Gest, University of Michigan Medical School.)

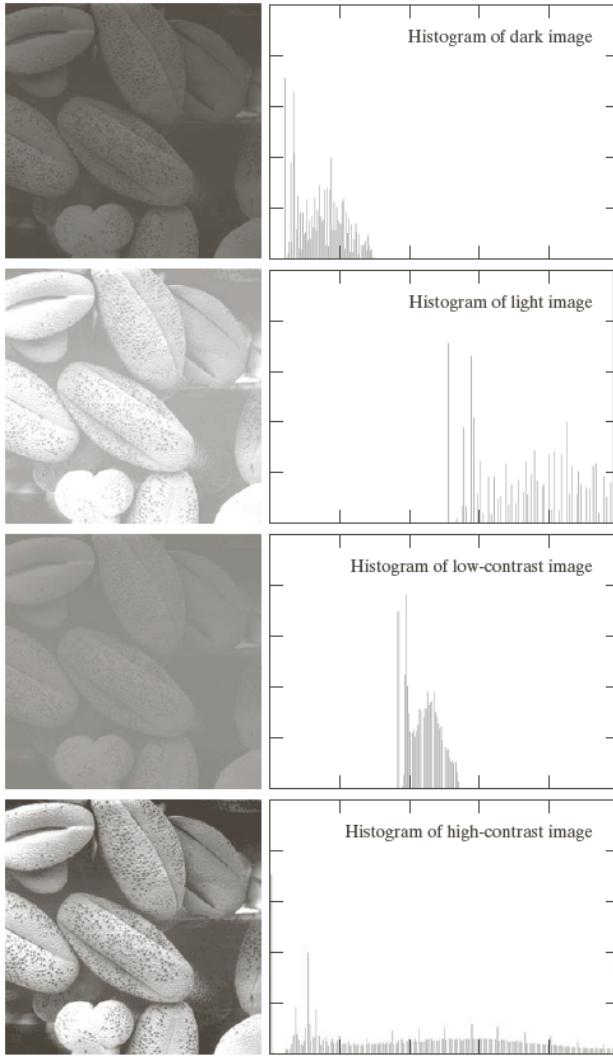
# Histogram Processing

---

The histogram of a digital image with gray levels in the range  $[0, L-1]$  is a discrete function  $p(r_k) = n_k/n$ , where  $r_k$  is the  $k_{th}$  gray level,  $n_k$  is the number of pixels in the image with that gray level,  $n$  is the total number of pixels in the image,  $k = 0, 1, \dots, L-1$

# Histogram Processing

---

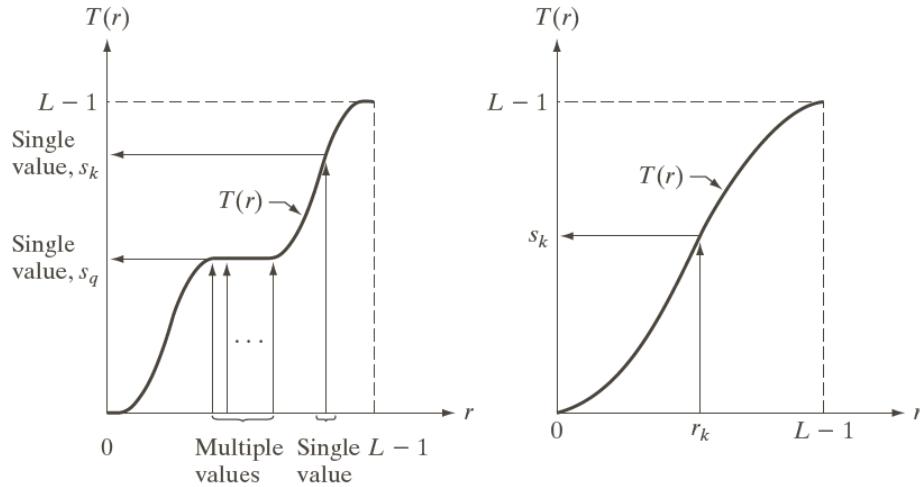


**FIGURE 3.16** Four basic image types: dark, light, low contrast, high contrast, and their corresponding histograms.

---

# Histogram Processing

## Gray-level transformation function



a b

**FIGURE 3.17**  
(a) Monotonically increasing function, showing how multiple values can map to a single value.  
(b) Strictly monotonically increasing function. This is a one-to-one mapping, both ways.

# Histogram Equalization

---

## Principle

- Let  $r$  represent the gray levels in the image to be processed (enhanced).
- Assuming  $r$  is continuous and has been normalized to lie in the interval  $[0, 1]$ ,  $r = 0$ = black,  $r = 1$ =white
- The following transformation  
 $S = T(r)$  satisfies
  1.  $T(r)$  is single-valued and monotonically increasing in the interval  $0 \leq r \leq 1$ , and
  2.  $0 \leq T(r) \leq 1$  for  $0 \leq r \leq 1$
- Condition 1 preserve the order from black to white in the grayscale whilst condition 2 guarantees a mapping that is consistent with the allowed range of the pixel values

# Histogram Equalization

---

## Principle

- The inverse transformation from  $s$  back to  $r$  is denoted as

$$r = T^{-1}(s) \quad 0 \leq s \leq 1$$

- It is assumed that  $T^{-1}$  also satisfies above two conditions

# Histogram Equalization

---

## Principle

- The gray level of an image may be viewed as random quantities in the interval  $[0, 1]$
- If they are continuous variables, the original and transformed gray levels can be characterized by their probability density functions  $p_r(r)$  and  $p_s(s)$ , respectively
- If  $p_r(r)$  and  $T(r)$  are known and  $T^{-1}(r)$  satisfies the condition above, then we have

# Histogram Equalization

---

## Principle

$$p_s(s) = \left[ p_r(r) \frac{dr}{ds} \right]_{r=T^{-1}(s)}$$

# Histogram Equalization

---

## Principle

- Consider the transformation function

$$s = T(r) = \int_0^r p_r(w) dw \quad 0 \leq r \leq 1$$

Cumulated  
distribute  
function (CDF)

# Histogram Equalization

---

## Principle

- Consider the transformation function

$$s = T(r) = \int_0^r p_r(w) dw \quad 0 \leq r \leq 1$$

$$\frac{ds}{dr} = p_r(r)$$

# Histogram Equalization

---

## Principle

- Consider the transformation function

$$\frac{ds}{dr} = p_r(r)$$

$$p_s(s) = \left[ p_r(r) \frac{dr}{ds} \right]_{r=T^{-1}(s)} = \left[ p_r(r) \frac{1}{\frac{ds}{dr}} \right]_{r=T^{-1}(s)} = \left[ p_r(r) \frac{1}{p_r(r)} \right]_{r=T^{-1}(s)} = 1$$

$$0 \leq s \leq 1$$

# Histogram Equalization

---

## Principle

- Consider the transformation function

$$\frac{ds}{dr} = p_r(r)$$

Uniform  
distribution

$$p_s(s) = \left[ p_r(r) \frac{dr}{ds} \right]_{r=T^{-1}(s)} = \left[ p_r(r) \frac{1}{\frac{ds}{dr}} \right]_{r=T^{-1}(s)} = \left[ p_r(r) \frac{1}{p_r(r)} \right]_{r=T^{-1}(s)} = 1$$

$$0 \leq s \leq 1$$

# Histogram Equalization

---

## Principle

- Use the cumulative distribution of  $r$  produces as a transform function
- Produces an image whose gray levels have a uniform density function

$$s = T(r) = \int_0^r p_r(w) dw \quad 0 \leq r \leq 1$$

- An image whose gray levels have a uniform density function appears to have high contrast

# Histogram Equalization

---

## Principle

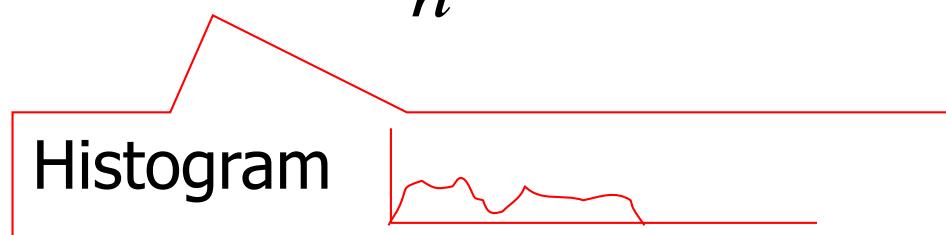
# Histogram Equalization

---

## Practice

- Digital images
- Assume  $n_k$  is the number of pixels have vale  $r_k$ , there are  $n$  pixels in the image, the probabilities (frequencies of occurrence) of gray levels  $r_k$

$$p_r(r_k) = \frac{n_k}{n} \quad 0 \leq r_k \leq 1 \quad k = 0, 1, \dots, L-1$$



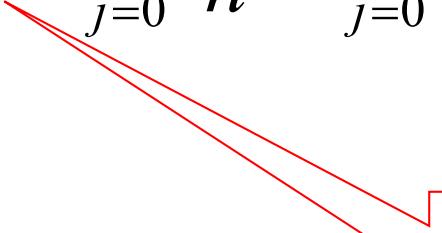
# Histogram Equalization

---

## Practice

- Cumulative histogram

$$s_k = \sum_{j=0}^k \frac{n_j}{n} = \sum_{j=0}^k p_r(r_j) \quad 0 \leq r_k \leq 1 \quad k = 0, 1, \dots, L-1$$



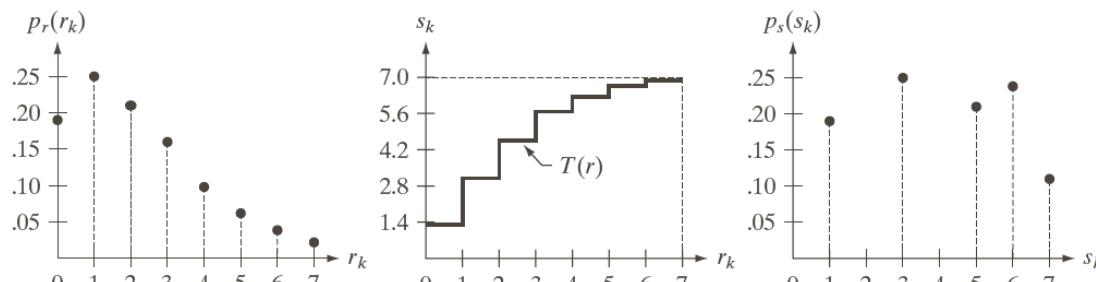
Histogram  
equalization

# Histogram Equalization

## Example

$r_k$	$n_k$	$p_r(r_k) = n_k/MN$
$r_0 = 0$	790	0.19
$r_1 = 1$	1023	0.25
$r_2 = 2$	850	0.21
$r_3 = 3$	656	0.16
$r_4 = 4$	329	0.08
$r_5 = 5$	245	0.06
$r_6 = 6$	122	0.03
$r_7 = 7$	81	0.02

**TABLE 3.1**  
Intensity distribution and histogram values for a 3-bit,  $64 \times 64$  digital image.

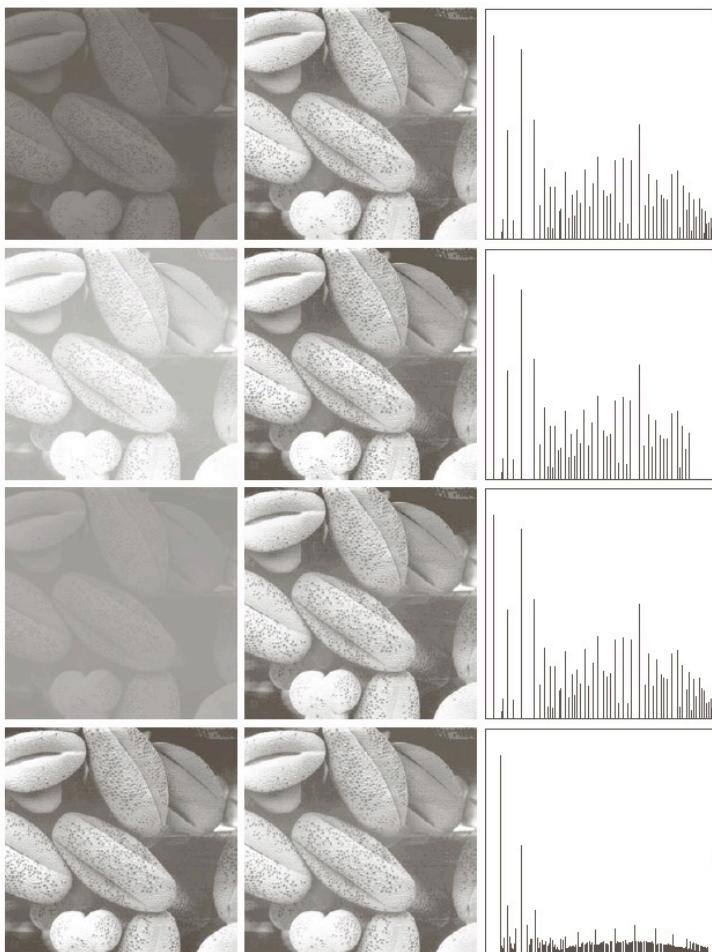


a | b | c

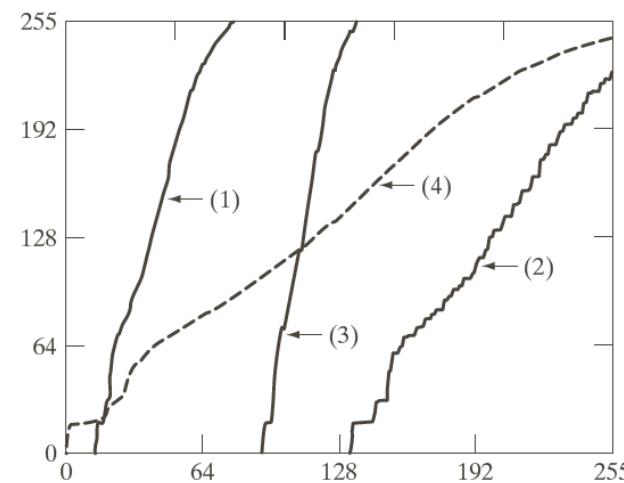
**FIGURE 3.19** Illustration of histogram equalization of a 3-bit (8 intensity levels) image. (a) Original histogram. (b) Transformation function. (c) Equalized histogram.

# Histogram Equalization

## Example



**FIGURE 3.20** Left column: images from Fig. 3.16. Center column: corresponding histogram-equalized images. Right column: histograms of the images in the center column.



**FIGURE 3.21**  
Transformation functions for histogram equalization. Transformations (1) through (4) were obtained from the histograms of the images (from top to bottom) in the left column of Fig. 3.20 using Eq. (3.3-8).

# Histogram Specification

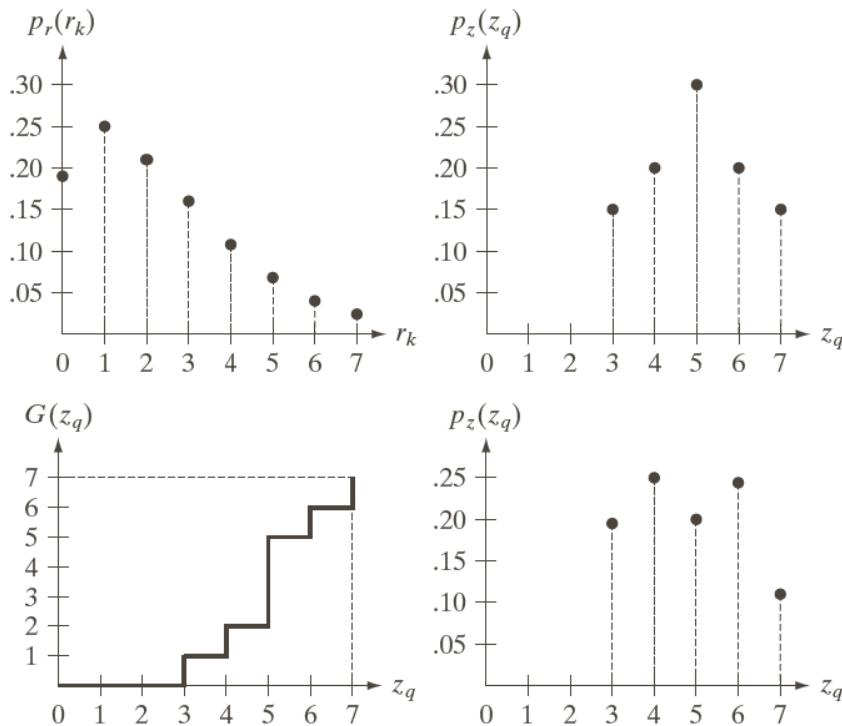
---

## Principle and practice

histogram specification (matching) is the transformation of an image so that its histogram matches a specified histogram. The well-known histogram equalization method is a special case in which the specified histogram is uniformly distributed.

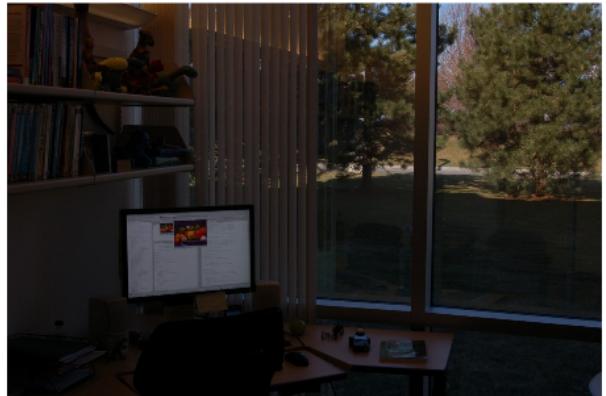
# Histogram Specification

## Example



a b  
c d

**FIGURE 3.22**  
(a) Histogram of a 3-bit image. (b) Specified histogram.  
(c) Transformation function obtained from the specified histogram.  
(d) Result of performing histogram specification. Compare (b) and (d).



input



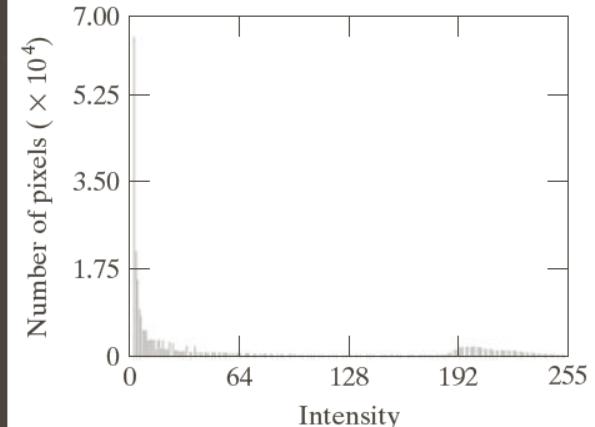
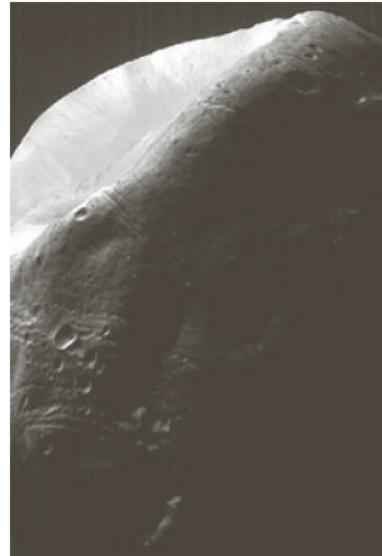
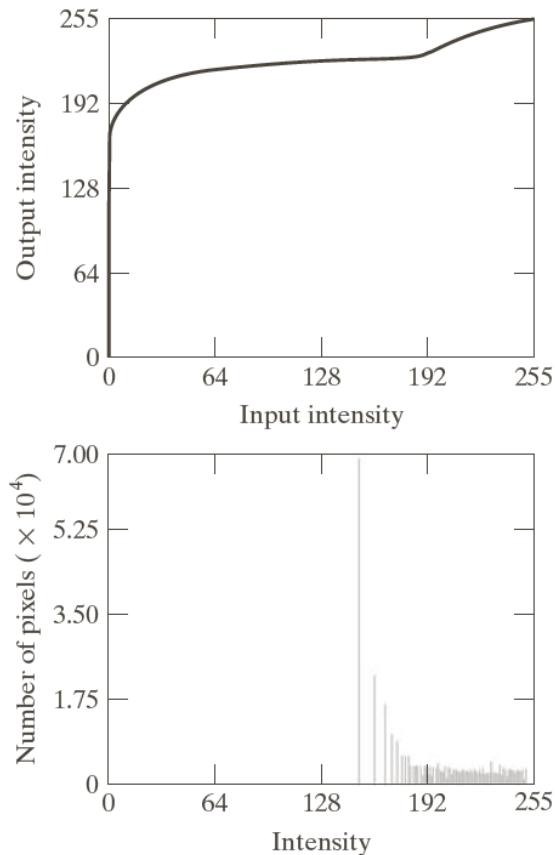
reference



result

# Histogram Processing

## Drawbacks/Artefacts



a  
b  
c

**FIGURE 3.24**  
(a) Transformation function for histogram equalization.  
(b) Histogram-equalized image (note the washed-out appearance).  
(c) Histogram of (b).

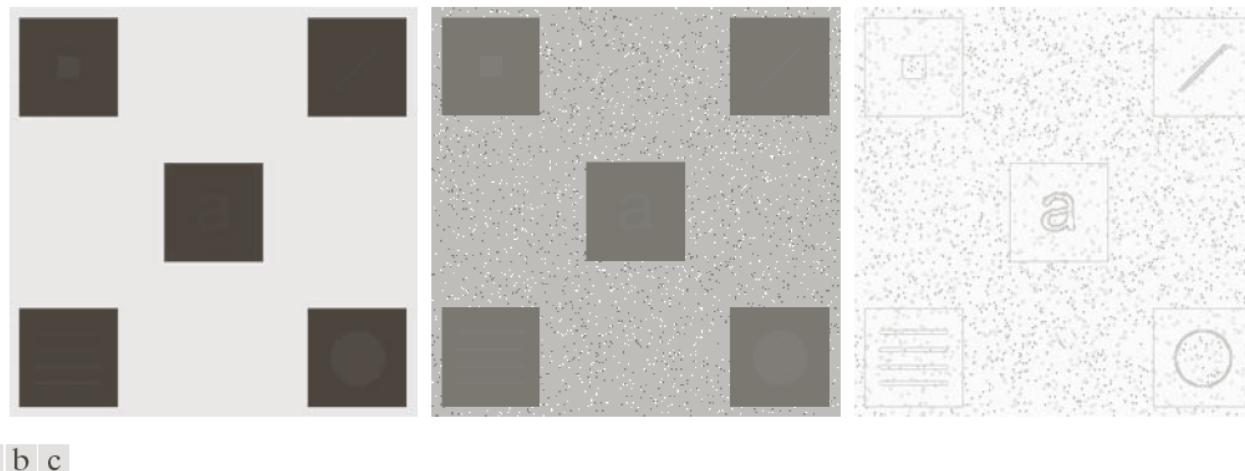
a  
b

**FIGURE 3.23**  
(a) Image of the Mars moon Phobos taken by NASA's *Mars Global Surveyor*.  
(b) Histogram. (Original image courtesy of NASA.)

# Histogram Processing

---

## Global vs Local Processing



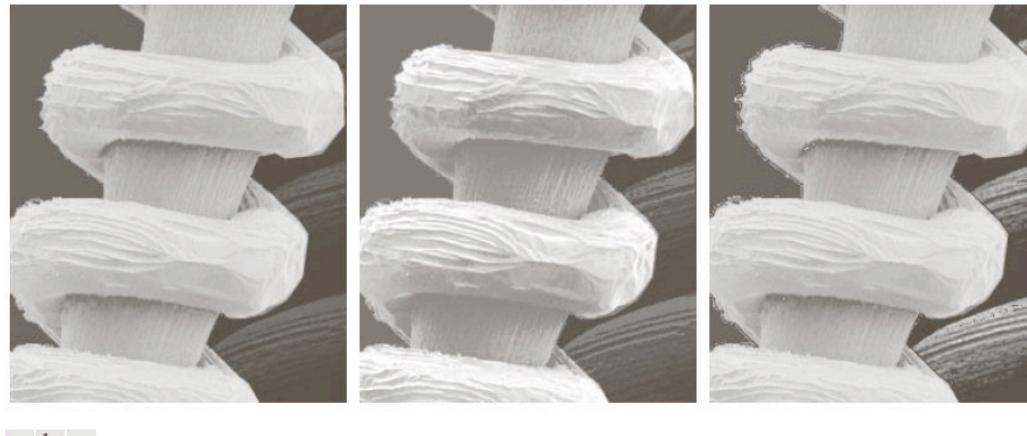
a b c

**FIGURE 3.26** (a) Original image. (b) Result of global histogram equalization. (c) Result of local histogram equalization applied to (a), using a neighborhood of size  $3 \times 3$ .

# Histogram Processing

---

## Global vs Local Processing

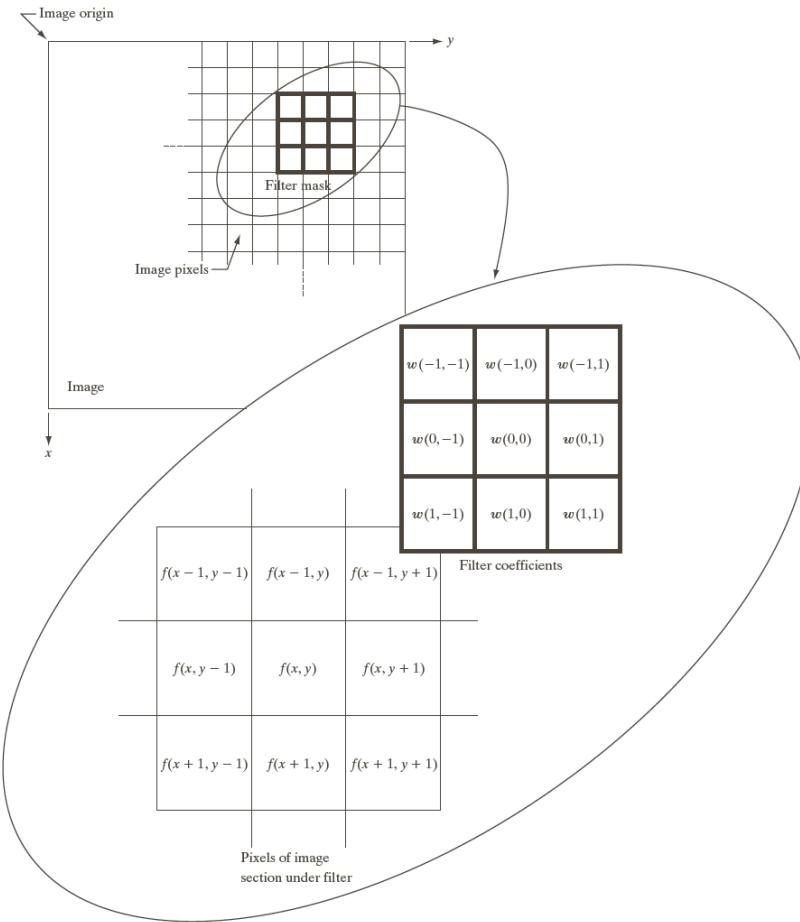


a b c

**FIGURE 3.27** (a) SEM image of a tungsten filament magnified approximately 130×. (b) Result of global histogram equalization. (c) Image enhanced using local histogram statistics. (Original image courtesy of Mr. Michael Shaffer, Department of Geological Sciences, University of Oregon, Eugene.)

# Spatial Filtering

---



**FIGURE 3.28** The mechanics of linear spatial filtering using a  $3 \times 3$  filter mask. The form chosen to denote the coordinates of the filter mask coefficients simplifies writing expressions for linear filtering.

# Spatial Filtering

---

$w_1$	$w_2$	$w_3$
$w_4$	$w_5$	$w_6$
$w_7$	$w_8$	$w_9$

**FIGURE 3.31**  
Another representation of a general  $3 \times 3$  filter mask.

# Spatial Filtering

---

## Low-pass (Smoothing) Filtering

$$\frac{1}{9} \times \begin{array}{|c|c|c|} \hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline \end{array} \quad \frac{1}{16} \times \begin{array}{|c|c|c|} \hline 1 & 2 & 1 \\ \hline 2 & 4 & 2 \\ \hline 1 & 2 & 1 \\ \hline \end{array}$$

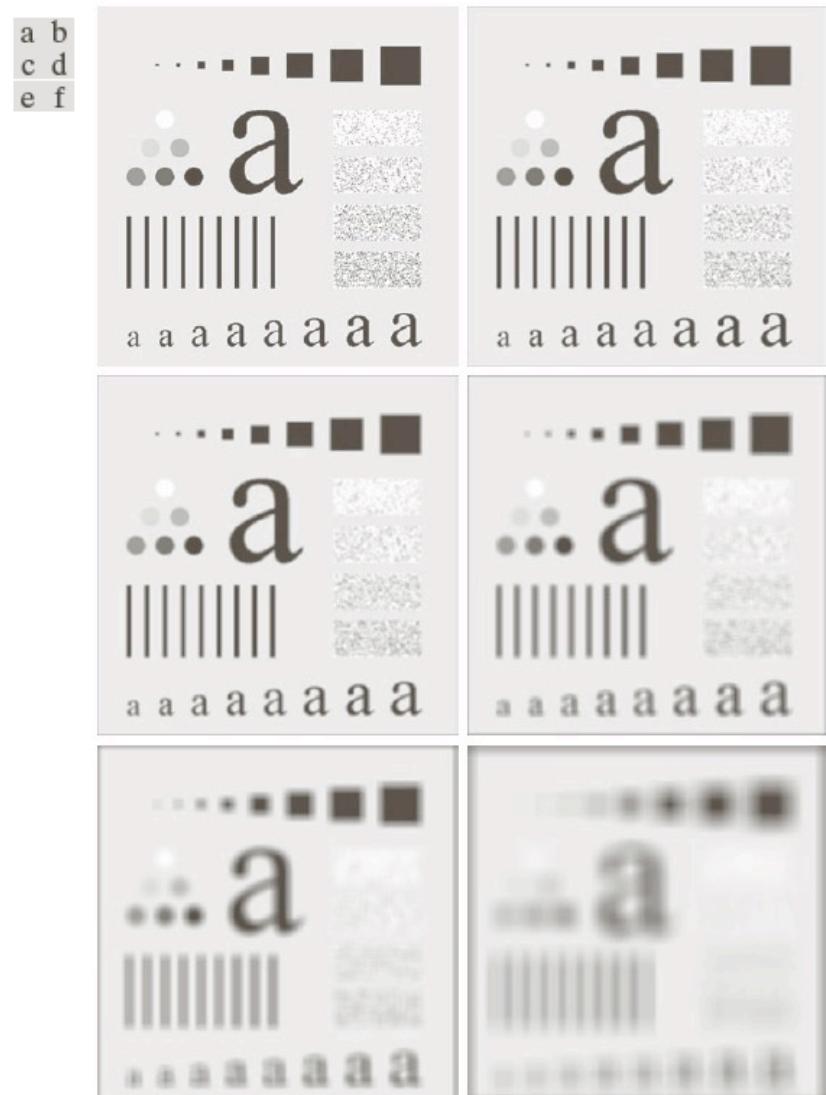
a b

**FIGURE 3.32** Two  $3 \times 3$  smoothing (averaging) filter masks. The constant multiplier in front of each mask is equal to 1 divided by the sum of the values of its coefficients, as is required to compute an average.

# Spatial Filtering

## Low-pass (Smoothing) Filtering

**FIGURE 3.33** (a) Original image, of size  $500 \times 500$  pixels. (b)–(f) Results of smoothing with square averaging filter masks of sizes  $m = 3, 5, 9, 15, 25, 35$ , and  $45$ , respectively. The black squares at the top are of sizes  $3, 5, 9, 15, 25, 35, 45$ , and  $55$  pixels; their borders are 25 pixels apart. The letters at the bottom range in size from 10 to 24 points, in increments of 2 points; the large letter at the top is 60 points. The vertical bars are 5 pixels wide and 100 pixels high; their separation is 20 pixels. The diameter of the circles is 25 pixels, and their borders are 15 pixels apart; their intensity levels range from 0% to 100% black in increments of 20%. The background of the image is 10% black. The noisy rectangles are of size  $50 \times 120$  pixels.

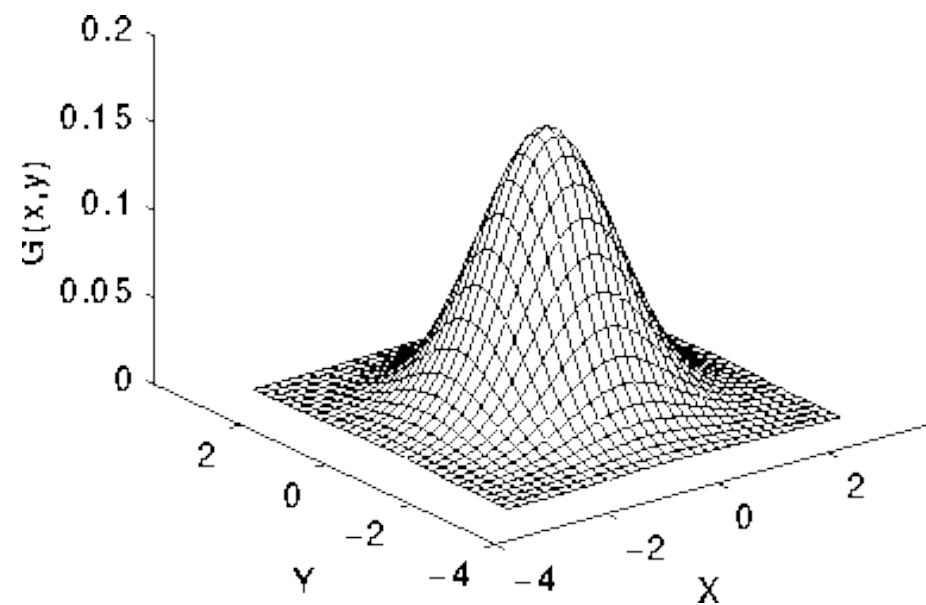


# Spatial Filtering

---

## Low-pass (Smoothing) Filtering

- Gaussian Smoothing



# Spatial Filtering

---

Drawbacks of low-pass filtering

# Spatial Filtering

---

Median (Smoothing) Filtering

Median vs Low-pass

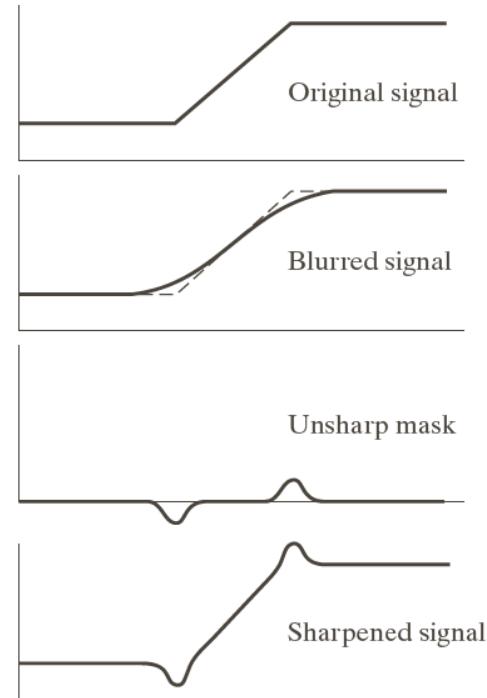
# Spatial Filtering

---

High-pass (sharpening) filtering

# Spatial Filtering

## Unsharp masking



a  
b  
c  
d

**FIGURE 3.39** 1-D illustration of the mechanics of unsharp masking. (a) Original signal. (b) Blurred signal with original shown dashed for reference. (c) Unsharp mask. (d) Sharpened signal, obtained by adding (c) to (a).

# Spatial Filtering

---

## Unsharp masking



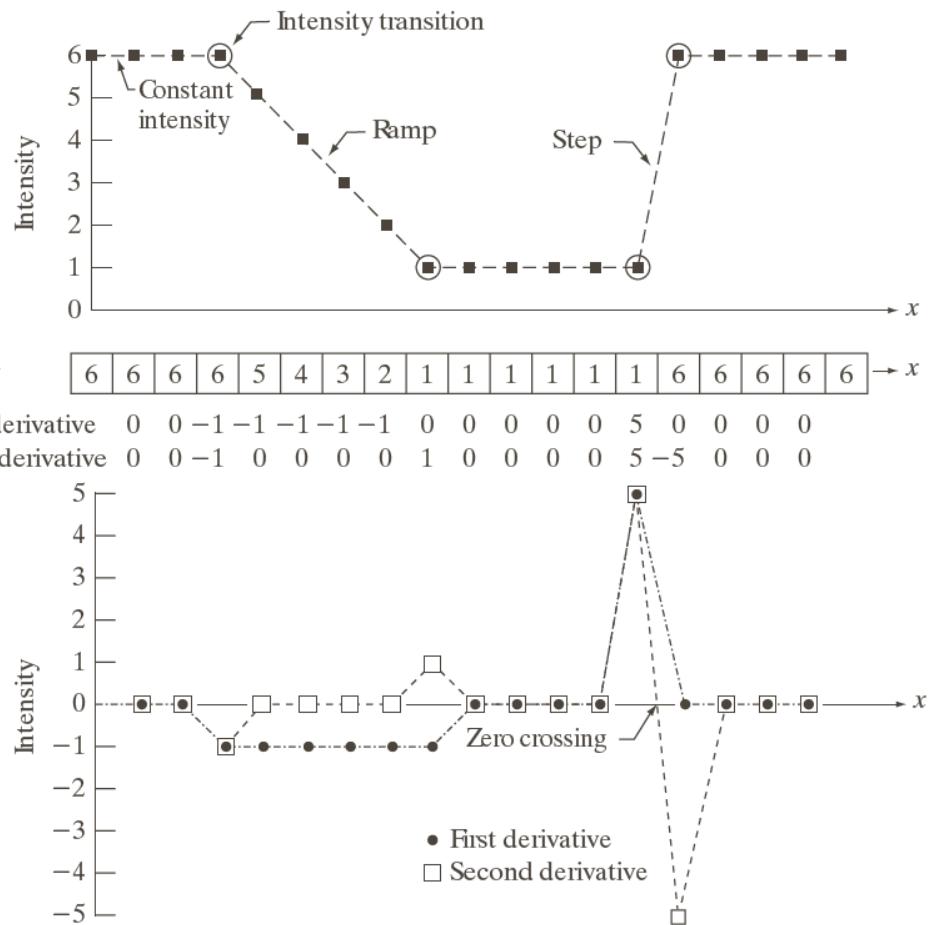
a  
b  
c  
d  
e

**FIGURE 3.40**  
(a) Original image.  
(b) Result of blurring with a Gaussian filter.  
(c) Unsharp mask.  
(d) Result of using unsharp masking.  
(e) Result of using highboost filtering.

---

# Spatial Filtering

## Derivative Filters



a  
b  
c

**FIGURE 3.36**  
Illustration of the first and second derivatives of a 1-D digital function representing a section of a horizontal intensity profile from an image. In (a) and (c) data points are joined by dashed lines as a visualization aid.

# Spatial Filtering

---

## Derivative Filters

0	1	0	1	1	1
1	-4	1	1	-8	1
0	1	0	1	1	1
0	-1	0	-1	-1	-1
-1	4	-1	-1	8	-1
0	-1	0	-1	-1	-1

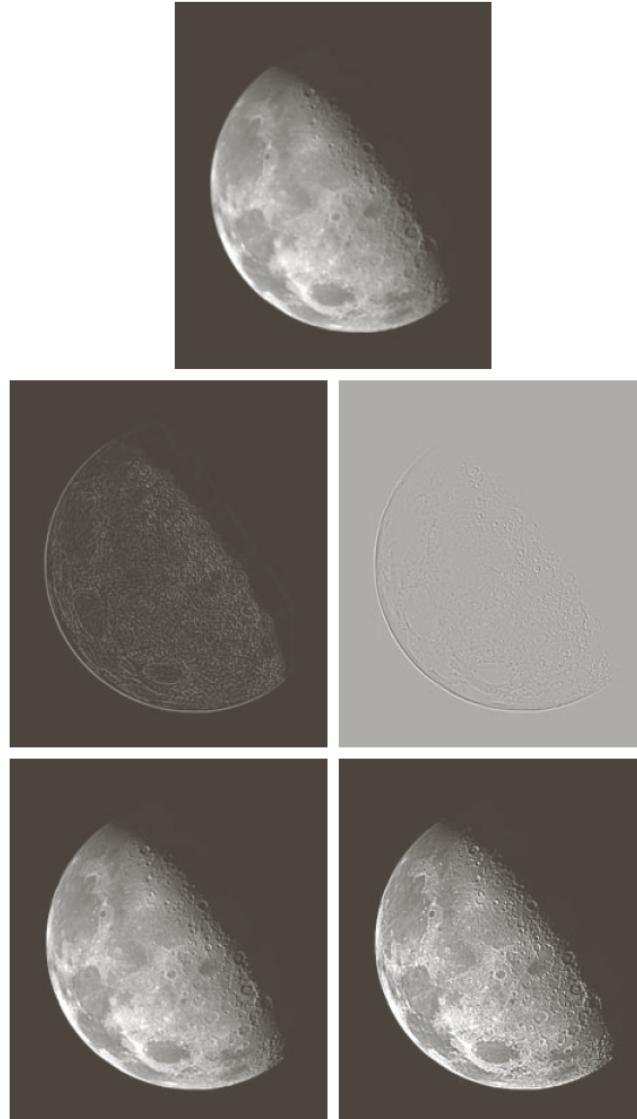
a b  
c d

**FIGURE 3.37**  
(a) Filter mask used to implement Eq. (3.6-6).  
(b) Mask used to implement an extension of this equation that includes the diagonal terms.  
(c) and (d) Two other implementations of the Laplacian found frequently in practice.

# Spatial Filtering

---

## Derivative Filters



**FIGURE 3.38**  
(a) Blurred image  
of the North Pole  
of the moon.  
(b) Laplacian  
without scaling.  
(c) Laplacian with  
scaling. (d) Image  
sharpened using  
the mask in Fig.  
3.37(a). (e) Result  
of using the mask  
in Fig. 3.37(b).  
(Original image  
courtesy of  
NASA.)

# Spatial Filtering

---

## More Derivative Filters

$z_1$	$z_2$	$z_3$
$z_4$	$z_5$	$z_6$
$z_7$	$z_8$	$z_9$

-1	0		0	-1
0	1		1	0

-1	-2	-1	-1	0	1
0	0	0	-2	0	2
1	2	1	-1	0	1

a  
b c  
d e

**FIGURE 3.41**

A  $3 \times 3$  region of an image (the  $z$ s are intensity values).

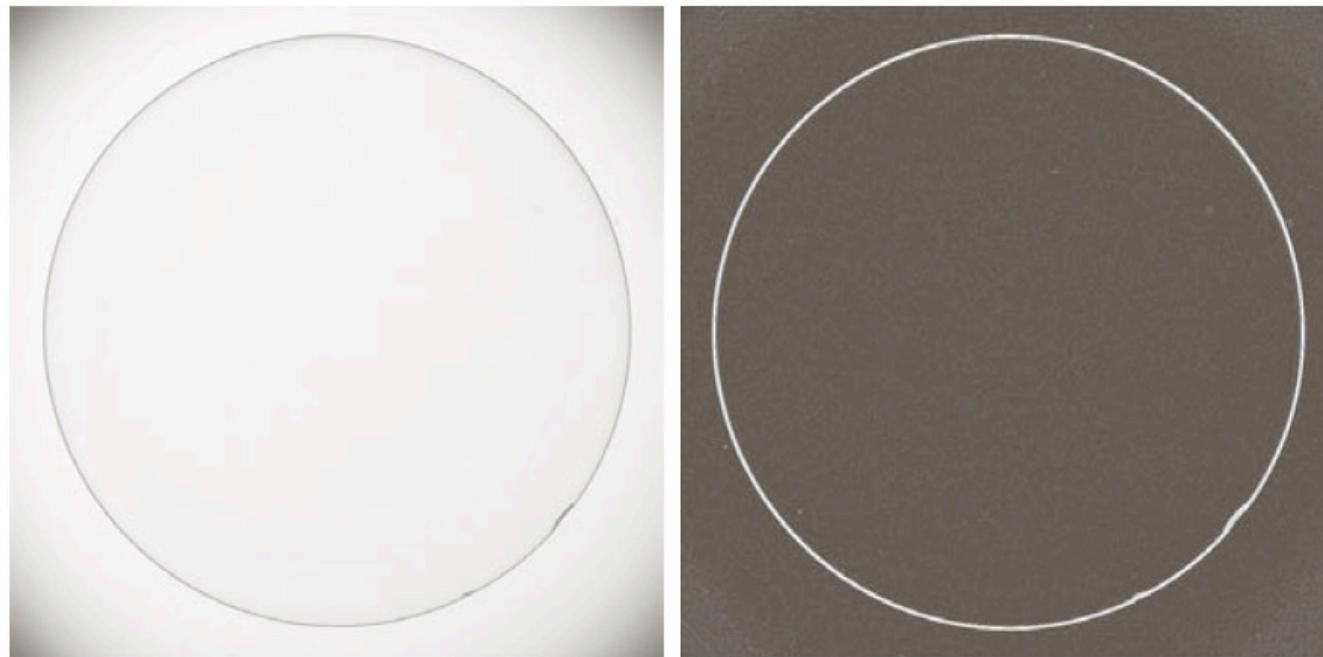
(b)–(c) Roberts cross gradient operators.

(d)–(e) Sobel operators. All the mask coefficients sum to zero, as expected of a derivative operator.

# Spatial Filtering

---

## Derivative Filters



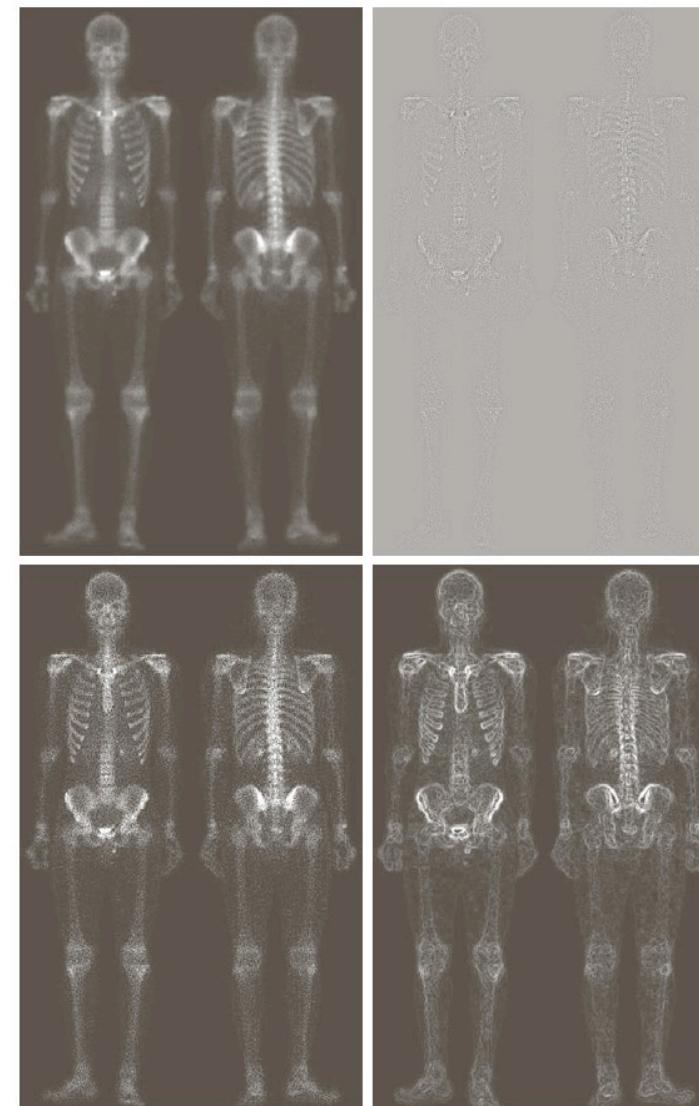
a b

**FIGURE 3.42**  
(a) Optical image of contact lens (note defects on the boundary at 4 and 5 o'clock).  
(b) Sobel gradient.  
(Original image courtesy of Pete Sites, Perceptics Corporation.)

# Spatial Filtering

---

## Derivative Filters



a b  
c d

**FIGURE 3.43**  
(a) Image of whole body bone scan.  
(b) Laplacian of (a). (c) Sharpened image obtained by adding (a) and (b).  
(d) Sobel gradient of (a).

# Spatial Filtering

---

## Derivative Filtering vs Edge Detection

- More later in the course

# Spatial Filtering

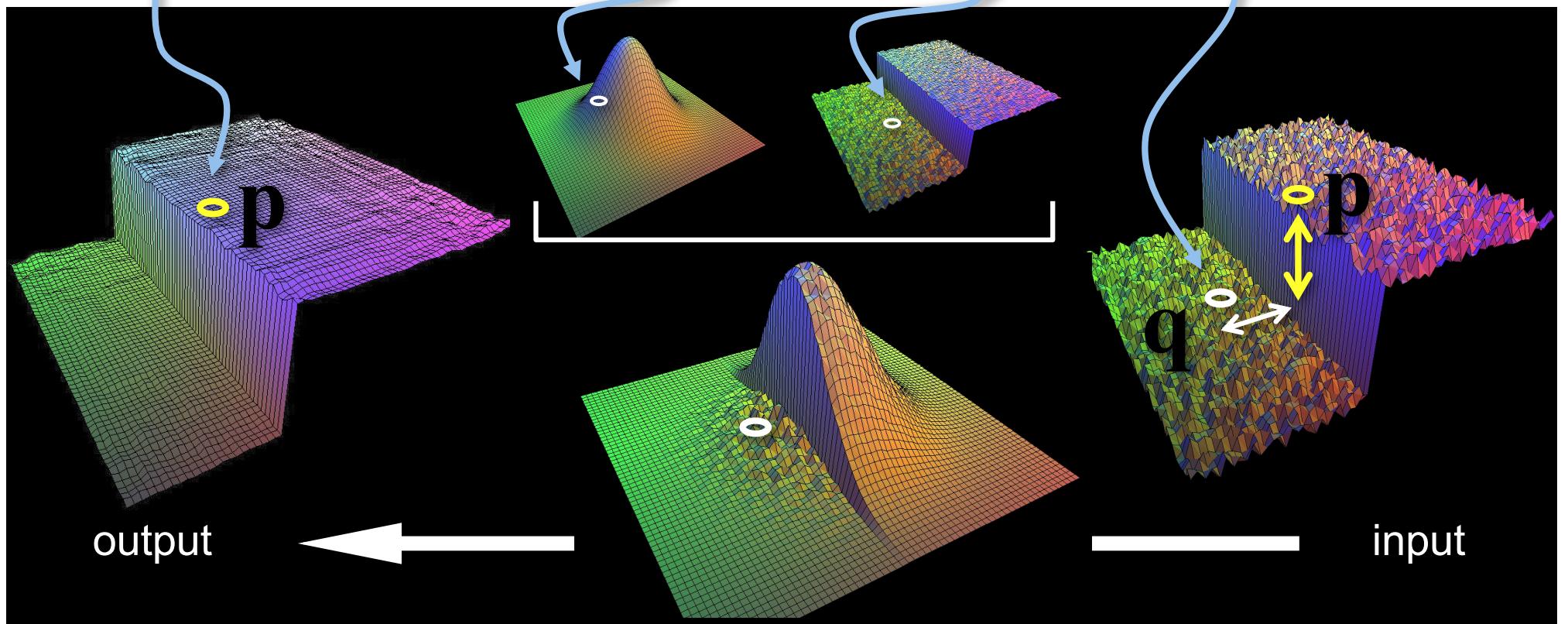
---

Latest development in image filtering

- Bilateral filtering

# Bilateral Filtering

$$BF[I]_p = \frac{1}{W_p} \sum_{q \in S} G_{\sigma_s}(\| p - q \|) G_{\sigma_r}(|I_p - I_q|) I_q$$



Source: [http://people.csail.mit.edu/sparis/bf\\_course/](http://people.csail.mit.edu/sparis/bf_course/)