Learning Program Synthesis for Integer Sequences

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MSR
AI4Code

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PROG

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(Axiom) [factorial(n) for n in 0..10]
(Magma) a:= func< n | Factorial(n) >; [ a(n) : n in [0..10]];
(Haskell)
a000142 :: (Enum a, Num a, Integral t) => t -> a
a000142 n = product [1 .. fromIntegral n]
a000142_list = 1 : zipWith (*) [1..] a000142_list
-- Reinhard Zumkeller, Mar 02 2014, Nov 02 2011, Apr 21 2011
(Python)
for i in range(1, 1000):
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import math
for i in range(1, 1000):
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(PARI) a(n)=prod(i=1, n, i) \\ Felix Fröhlich, Aug 17 2014
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(Sage) [factorial(n) for n in (1..22)] # Giuseppe Coppoletta, Dec 05 2014
(GAP) List([0..22], Factorial); # Muniru A Asiru, Dec 05 2018
(Scala) (1: BigInt).to(24: BigInt).scanLeft(1: BigInt)(_ * _) // Alonso del Arte,
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- What language is p written in?
- What is our training data set of (s, p) pairs?
- What is our architecture for generating a program p_t for a novel sequence t during test time?

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- Randomly explore space of possible programs.

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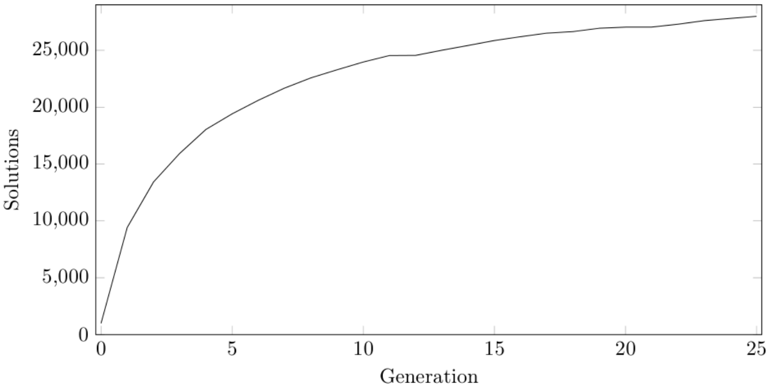

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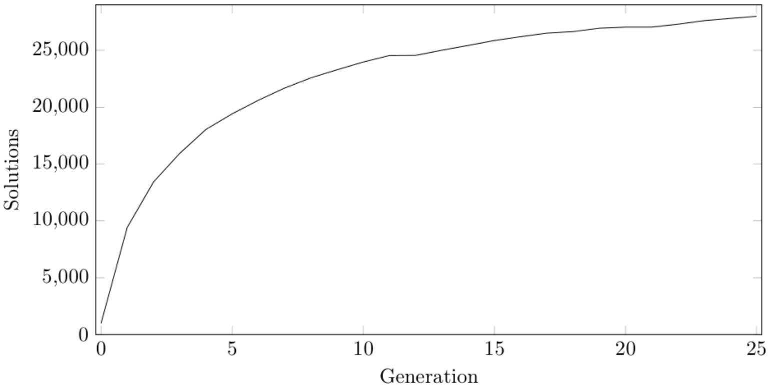

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  Thus, we can say that selecting the smallest solutions instead of the random ones for training helps finding new solutions in later generations.

10 Conclusion

Our system has created from scratch programs that gener
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shorter programs is more likely to generate better explanat
have also shown that the solutions discovered are correct fo
have observed that preferring shorter programs during the
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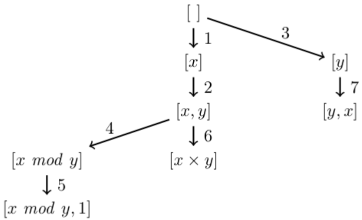

Figure 3: 7 iterations of the search loop gradually extending the search tree. The iteration number leading to the creation of a given node/stack is indicated on the arrow/action leading to it. The set of the synthesized programs after the 7th iteration is $\{1, x, y, x \times y, x \ mod \ y\}$.

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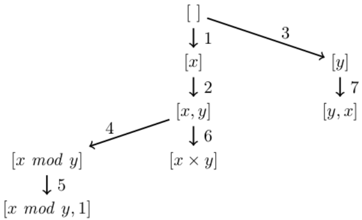

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compr

```
compr(f, a) :=
| failure if a < 0
| min{ m | m >= 0 and f(m, 0) <= 0 } if a = 0
| min{ m | m > compr(f, a-1) and f(m, 0) <= 0} otherwise
```