Equivalence of categories

Siddharth Bhat

##harmless **Category Theory in Context**

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- Let $S \equiv \{c, d\} \in \mathrm{Set}_\partial$; $T \equiv \{3, 4\} \in \mathrm{Set}_\partial$; $g \in Hom_\partial(S, T)$; $g(c) \equiv 3$, $g(d) \not\equiv \_$
- $FS \equiv \{1, 2, \{1, 2\}_*\}$; $FT \equiv \{3, 4, \{3, 4\}_*\}$; $Fg \equiv c \mapsto 3, d \mapsto \{3, 4\}, \{1, 2\} \mapsto \{3, 4\}$.
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- $GFS \equiv \{1, 2\}$; $GFT \equiv \{3, 4\}$; $GFg \equiv c \mapsto 3, d \not\mapsto$.
- In general, may have needed a $\epsilon : FG \to Id_{\text{Set}_\partial}$

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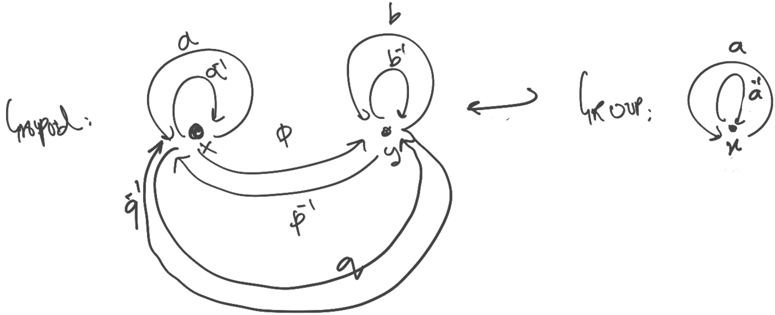

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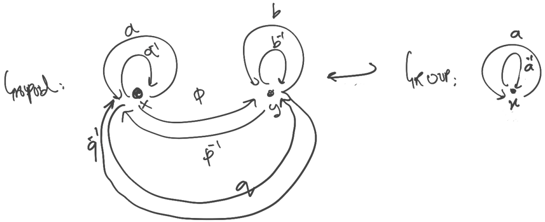

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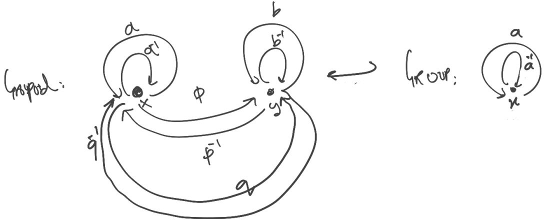

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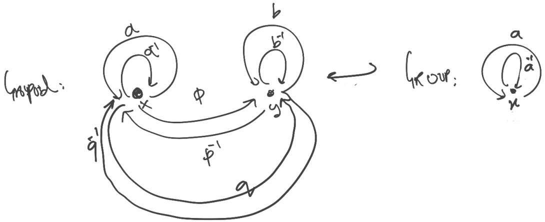

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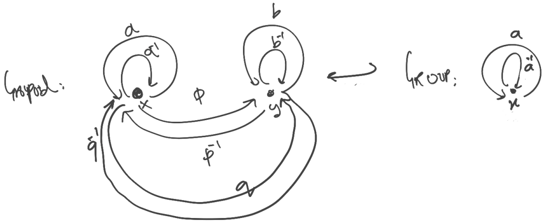

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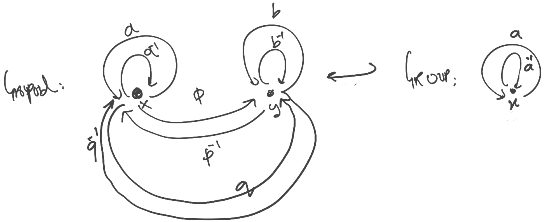

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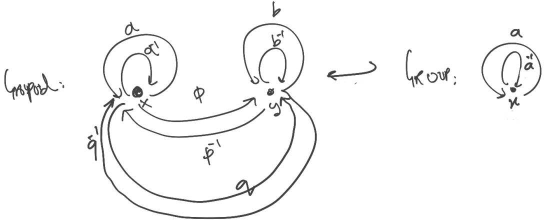

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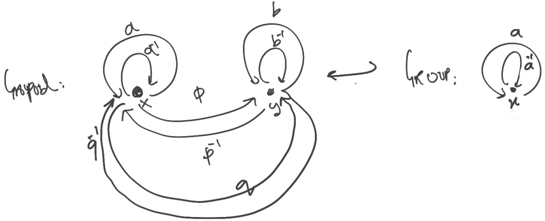

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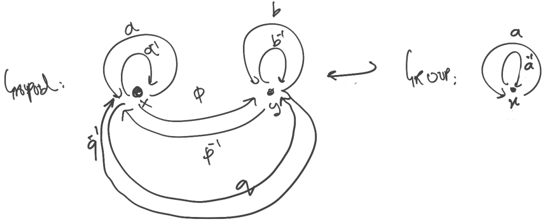

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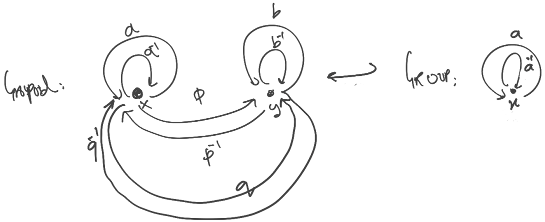

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- Philosophically, equivalence of categories does not need to preserve size.

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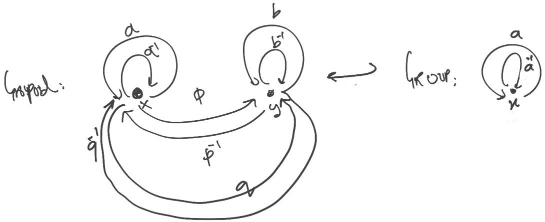

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- The embedding functor $F : G \to O$ is full and faithful since its image contains a single object (x) where it preserves all arrows.
- Also see that it is essentially surjective. For any other $y \in O$, we have a path $x \xrightarrow{p} y$ (as O is connected). since it is a groupoid, all morphisms are isos, and thus $y \simeq x$.
- Soo, this is an equivalence of categories?!
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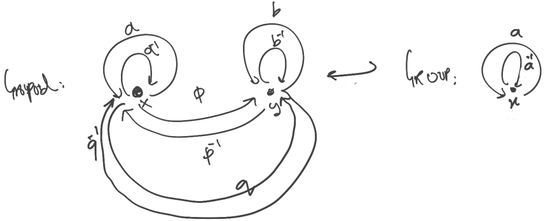
Equivalence of categories is Unintuitive (to me)



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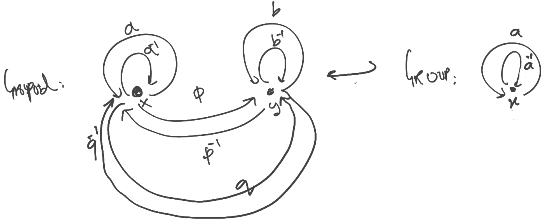

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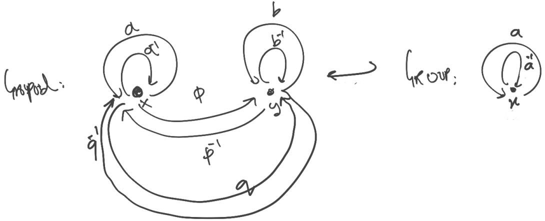

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Skeleta

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- *Mat*, category of numbers and materices is the skeleton of FinVectBasis, category of finite vector spaces with bases, and morphisms as matrices encoding linear operators relative to the basis.
- Can build $sk(C)$ (Skeleton of C). Crush each isomorphism class into a single object.
- The inclusion $sk(C) \hookrightarrow C$ defines an equivalence of categories.