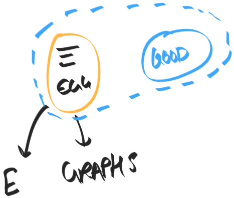
egg : Fast and extensible equality saturation

Siddharth Bhat

Monday, Jan 18 2021



How do compilers work?

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$P_0 \xrightarrow{T_1} P_1 \xrightarrow{T_2} P_2 \xrightarrow{T_3} P_3 \xrightarrow{T_4} P_4 \xrightarrow{\dots} \dots$

In what *order*?

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int foo() {
    int x = 1;
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        printf("foo")
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int bar() {
    int y = 42 == 42;
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int bar(int x) {
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int bar(int x) {
    // 4. Constant propagation
    int y = true;
    if (true)  // IS a constant now!
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int bar(int x) {
    int y = true;
    // 5. Control flow simplification
    printf("foo");
}
```

Solutions to the phase ordering problem

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Pass Arguments:  -tti -targetlibinfo -tbaa -scoped-noalias-aa
-instcombine -simplifycfg -basiccg -globals-aa -prune-eh -inline -openmpopt
-function-attrs -argpromotion -domtree -sroa -basic-aa -aa -memoryssa
-early-cse-memssa -aa -lazy-value-info -jump-threading -correlated-propagation
-simplifycfg -domtree -aggressive-instcombine -basic-aa -aa -loops
-lazy-branch-prob -lazy-block-freq -opt-remark-emitter -instcombine
-libcalls-shrinkwrap -loops -postdomtree -branch-prob -block-freq
-lazy-branch-prob -lazy-block-freq -opt-remark-emitter -pgo-memop-opt -basic-aa
-aa -loops -lazy-branch-prob -lazy-block-freq -opt-remark-emitter -tailcallelim
-simplifycfg -reassociate -domtree -loops -loop-simplify -lcssa-verification
-lcssa -basic-aa -aa -scalar-evolution -loop-rotate -memoryssa
-lazy-branch-prob -lazy-block-freq -licm -loop-unswitch -simplifycfg -domtree
-basic-aa -aa -loops -lazy-branch-prob -lazy-block-freq -opt-remark-emitter
-instcombine -loop-simplify -lcssa-verification -lcssa -scalar-evolution
-loop-idiom -indvars -loop-deletion -loop-unroll -sroa -aa -mldst-motion
-phi-values -aa -memdep -lazy-branch-prob -lazy-block-freq -opt-remark-emitter
-gvn -phi-values -basic-aa -aa -memdep -memcpyopt -sccp -demanded-bits -bdce
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-opt-remark-emitter -instcombine -barrier -elim-avail-extern -basiccg
-rpo-function-attrs -globalopt -globaldce -basiccg -globals-aa -domtree
-float2int -lower-constant-intrinsics -domtree -loops -loop-simplify
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-basic-aa -aa -loop-accesses -demanded-bits -lazy-branch-prob -lazy-block-freq
-opt-remark-emitter -inject-tli-mappings -loop-vectorize -loop-simplify
-scalar-evolution -aa -loop-accesses -lazy-branch-prob -lazy-block-freq
-loop-load-elim -basic-aa -aa -lazy-branch-prob -lazy-block-freq
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- (a*2)/2 $\xrightarrow{B}$ a*(2/2) $\rightarrow$ a*1

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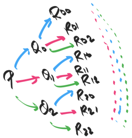

Figure: all possible transforms

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- *saturate* the original program with new programs *equivalent* to the original program.

Figure: classical

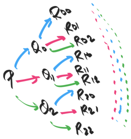


Figure: all possible transforms

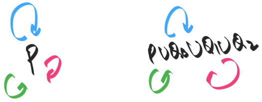
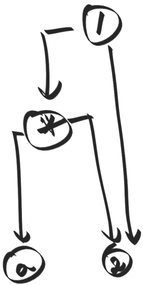


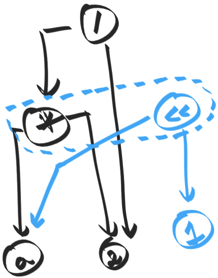
Figure: equality saturation



$(a*2)/2$:

(/ (* a 2) 2

egg : Fast and extensible equality saturation (2)



$(a*2)/2$:

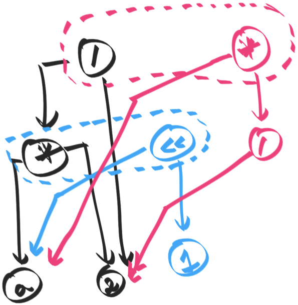
$(/\ (*\ a\ 2)\ 2)$

$p*2 \rightarrow p \ll 1$

$(/\ (*\ a\ 2)\ 2)$

$(\ll a\ 1)$

egg : Fast and extensible equality saturation (3)



$(a*2)/2$:

(/ (* a 2) 2)

$p*2 \rightarrow p<<1$

(/ (* a 2) 2)

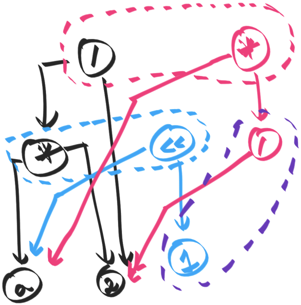
(<< a 1)

$(p*q)/r \rightarrow p*(q/r)$

(/ (* a 2) 2)

(* a (/ 2 2)

egg : Fast and extensible equality saturation (4)



x/x → 1

(* a (/ 2 2))
1

(a*2)/2 :

(/ (* a 2) 2)

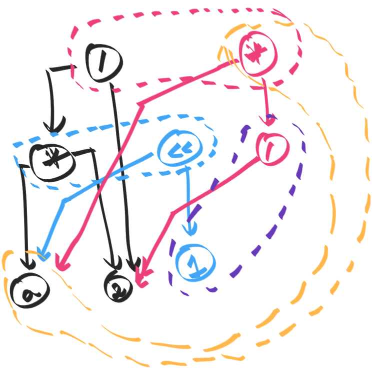
p*2 → p<<1

(/ (* a 2) 2)
(<< a 1)

(p*q)/r → p*(q/r)

(/ (* a 2) 2)
(* a (/ 2 2))

egg : Fast and extensible equality saturation (5)



$x/x \to 1$

(* a (/ 2 2))

~ 1

(a*2)/2 :

(/ (* a 2) 2)

$p*2 \to p \ll 1$

$x*1 \to x$

(* a (/ 2 2))

~ 1

~ a

(/ (* a 2) 2)

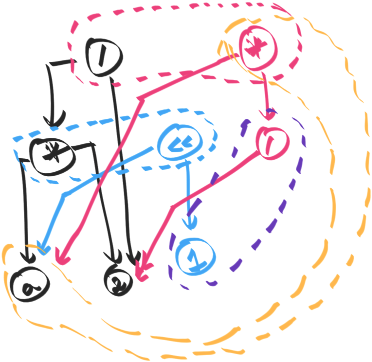
~ (<< a 1)

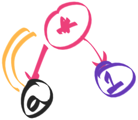
$(p*q)/r \to p*(q/r)$

(/ (* a 2) 2)

~ (* a (/ 2 2))

egg : Fast and extensible equality saturation (6)





$x/x \to 1$

(* a (/ 2 2))
~ 1

$x*1 \to x$

(* a (/ 2 2))
~ 1
~ a

$(a*2)/2$:

(/ (* a 2) 2)

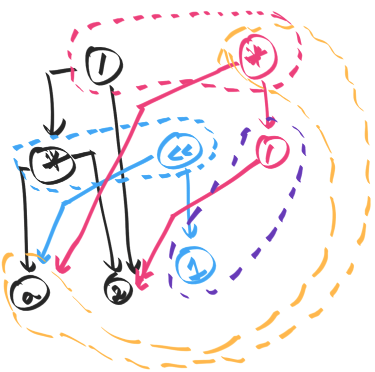
$p*2 \to p \ll 1$

(/ (* a 2) 2)
~ (<< a 1)

$(p*q)/r \to p*(q/r)$

(/ (* a 2) 2)
~ (* a (/ 2 2))

egg : Fast and extensible equality saturation (7)



$x/x \to 1$

(* a (/ 2 2))

~ 1

$x*1 \to x$

(* a (/ 2 2))

~ 1

~ a

(a*2)/2:

(/ (* a 2) 2)

$p*2 \to p<<1$

(/ (* a 2) 2)

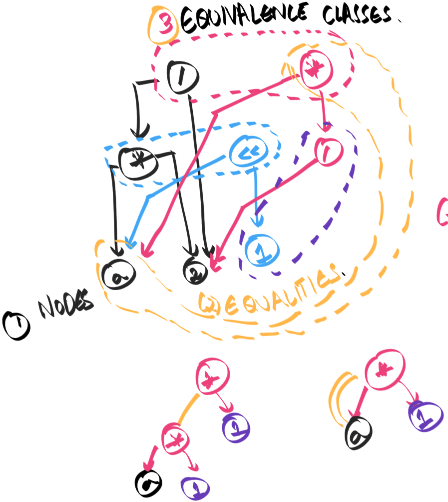
(<< a 1)

$(p*q)/r \to p*(q/r)$

(/ (* a 2) 2)

~ (* a (/ 2 2))

egg : Fast and extensible equality saturation (8)



x/x → 1

(* a (/ 2 2))

1

x*1 → x

(* a (/ 2 2))

1

a

(a*2)/2:

(/ (* a 2) 2)

p*2 → p<<1

(/ (* a 2) 2)

(<< a 1)

(p*a)/r → p*(a/r)

(/ (* a 2) 2)

(* a (/ 2 2))

Evaluation: Herbie

```
sqrt(x+1) - sqrt(x)  →  1/(sqrt(x+1) + sqrt(x))
```

Herbie detects inaccurate expressions and finds more accurate replacements. The red expression is inaccurate when $x > 1$; Herbie's replacement, in blue, is accurate for all x.

Evaluation: Herbie

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sqrt(x+1) - sqrt(x)  →  1/(sqrt(x+1) + sqrt(x))
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Herbie detects inaccurate expressions and finds more accurate replacements. The red expression is inaccurate when $x > 1$; Herbie's replacement, in blue, is accurate for all x.

$$\sqrt{x+1}-\sqrt{x}=\frac{(\sqrt{x+1}-\sqrt{x})(\sqrt{x+1}+\sqrt{x})}{\sqrt{x+1}+\sqrt{x}}=\frac{(x+1)-x}{\sqrt{x+1}+\sqrt{x}}=\frac{1}{(\sqrt{x+1}+\sqrt{x})}$$

Evaluation: Herbie

sqrt(x+1) - sqrt(x) → **1/(sqrt(x+1) + sqrt(x))**

Herbie detects inaccurate expressions and finds more accurate replacements. The red expression is inaccurate when *x* > *1*; Herbie's replacement, in blue, is accurate for all *x*.

$$\sqrt{x+1}-\sqrt{x}=\frac{(\sqrt{x+1}-\sqrt{x})(\sqrt{x+1}+\sqrt{x})}{\sqrt{x+1}+\sqrt{x}}=\frac{(x+1)-x}{\sqrt{x+1}+\sqrt{x}}=\frac{1}{(\sqrt{x+1}+\sqrt{x})}$$

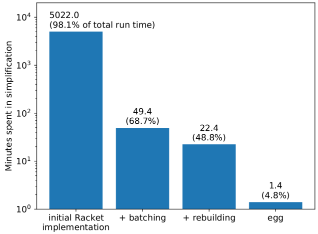
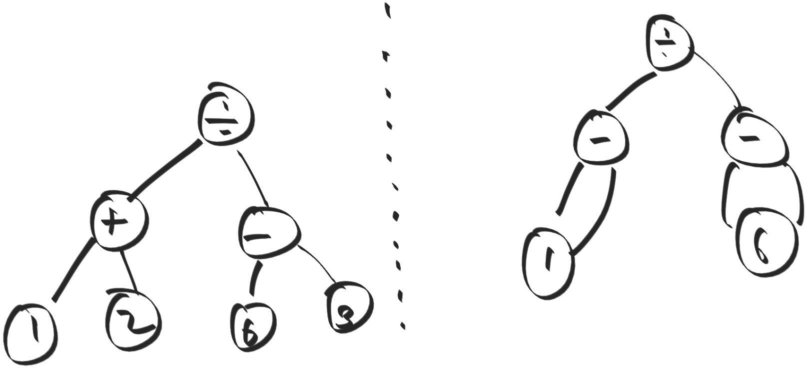


Fig. 12. Herbie sped up its expression simplification phase by adopting egg-inspired features like batched simplification and rebuilding into its Racket-based e-graph implementation. Herbie also supports using egg itself for additional speedup. Note that the y-axis is log-scale.

Herbie: Using egg — Analysis and rewrite

$$\frac{x}{x} = \begin{cases} 1 & : x \neq 0 \\ \mathrm{NaN} & x = 0 \end{cases}$$



Herbie: Using egg — Analysis and rewrite

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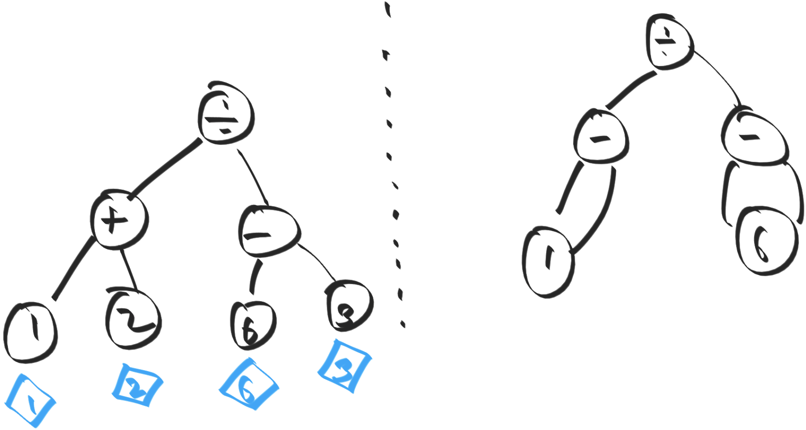

Herbie: Using egg — Analysis and rewrite

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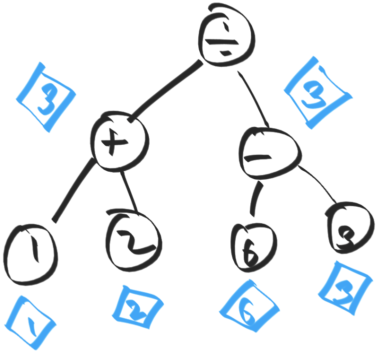
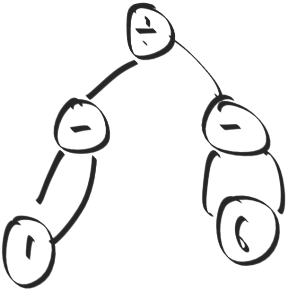
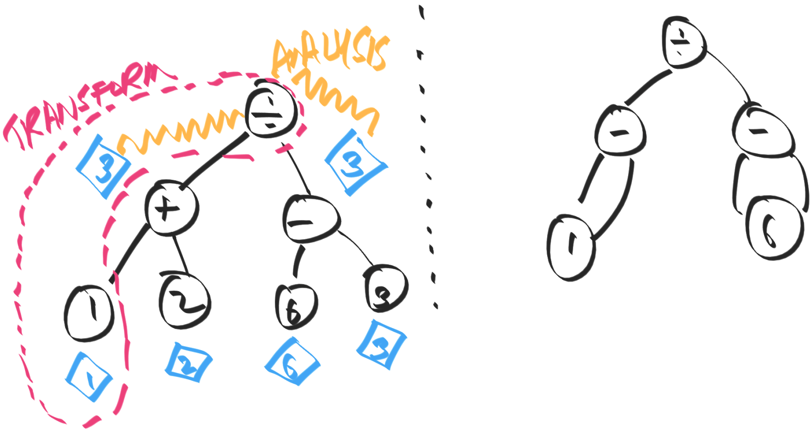




Herbie: Using egg — Analysis and rewrite

$$\frac{x}{x} = \begin{cases} 1 & : x \neq 0 \\ \mathrm{NaN} & x = 0 \end{cases}$$



Herbie: Using egg — Analysis and rewrite

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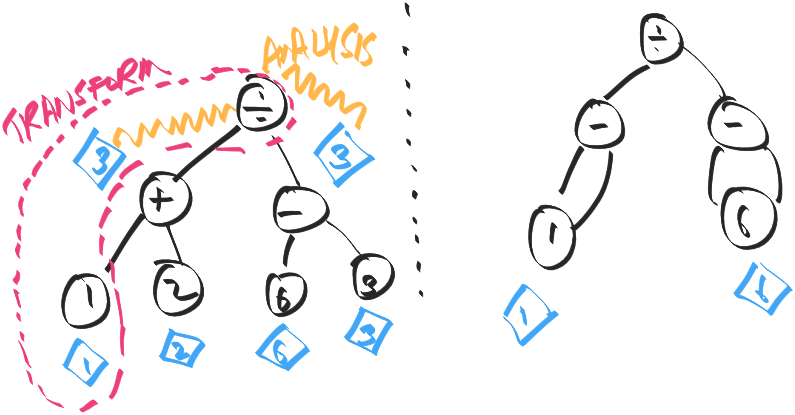
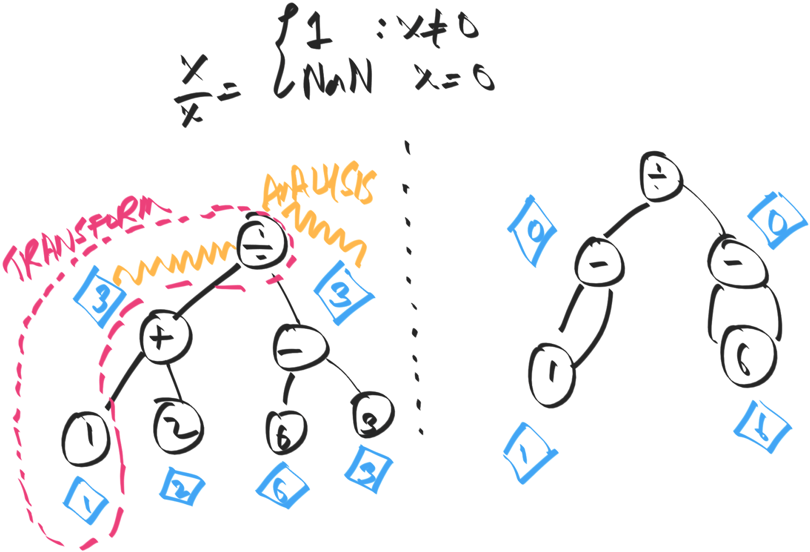
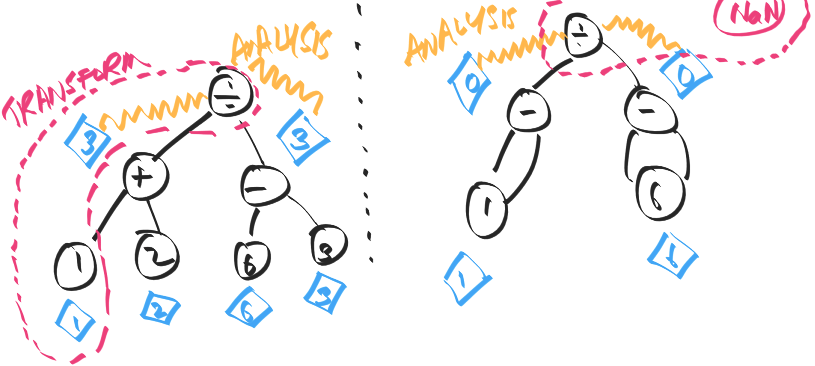


Herbie: Using egg — Analysis and rewrite



Herbie: Using egg — Analysis and rewrite

$$\frac{x}{x} = \begin{cases} 1 & : x \neq 0 \\ \text{NaN} & x = 0 \end{cases}$$



Speedup over prior art: Merging

acac F_1

adad F_2

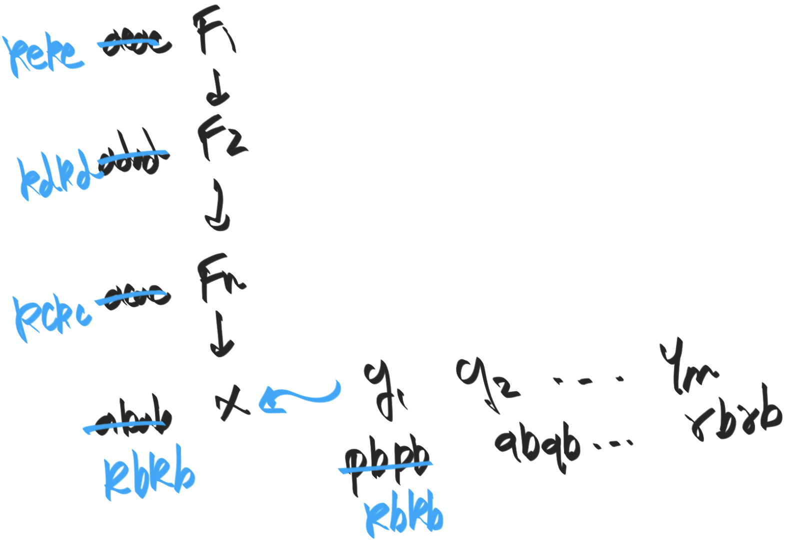
acac F_n

abab x

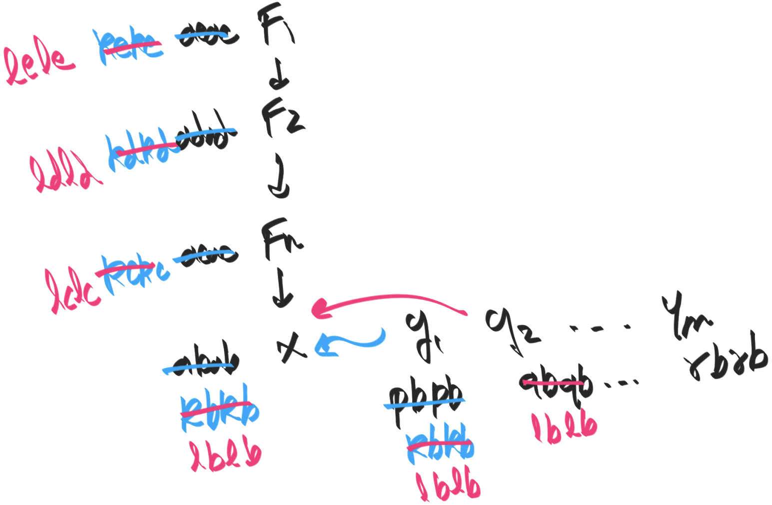
y_1 y_2 ... y_m

pbpb qbqb ... rbrb

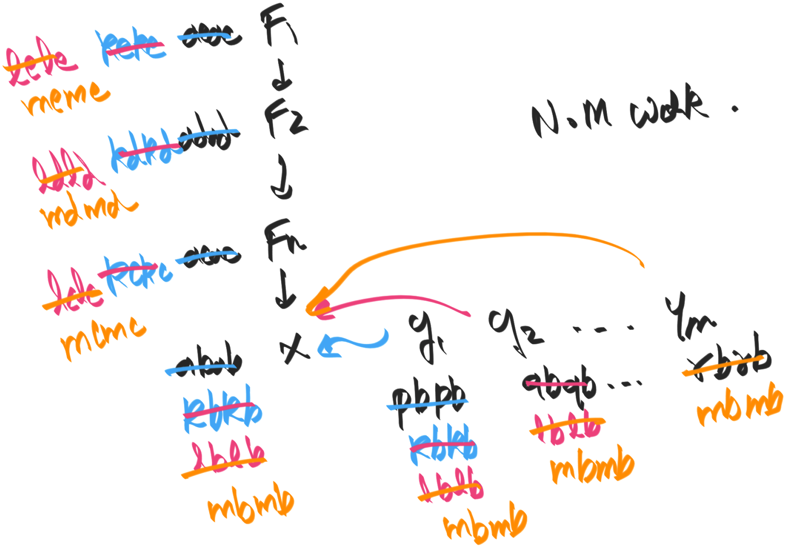
Speedup over prior art: Merging



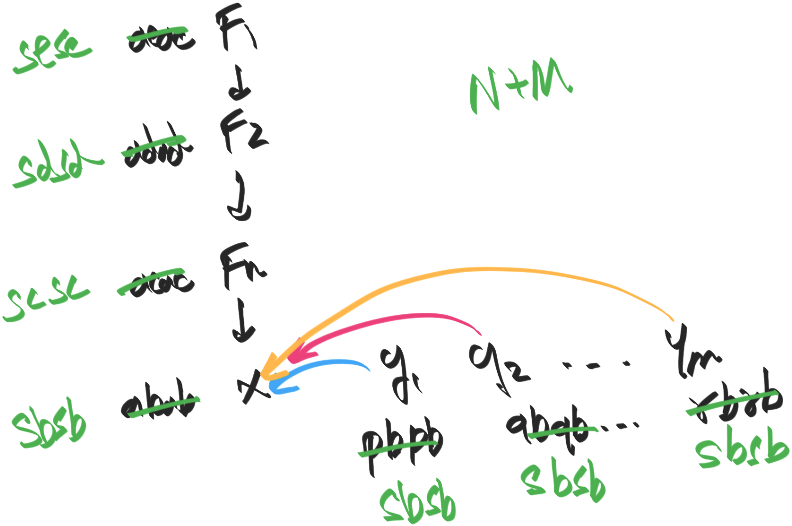
Speedup over prior art: Merging



Speedup over prior art: Merging



Speedup over prior art: Merging



Speedup over prior art: Merging

3.2.1 Examples of Rebuilding. Deferred rebuilding speeds up congruence maintenance by amortizing the work of maintaining the hashcons invariant. Consider the following terms in an e-graph: $f_1(x), \ldots, f_n(x),\ y_1, \ldots, y_n$. Let the workload be `merge`$(x, y_1), \ldots,$ `merge`(x, y_n). Each merge may change the canonical representation of the $f_i(x)$s, so the traditional invariant maintenance strategy could require $O(n^2)$ hashcons updates. With deferred rebuilding the `merges` happen before the hashcons invariant is restored, requiring no more than $O(n)$ hashcons updates.

Takeaways

- Don't stick to one order; try everything at once!
- Solution to phase ordering!
- Scales reasonably well if you engineer it well
- Well-designed API that enables analysis and rewrites

Lambda calculus in egg : Dynamic rewrites redux

```
-- v2=x is not free in (\x. x + 1)
x_bound = let v1 = (\x. x*2) in (\x.   x+1)
x_bound = let v1 = e         in (\v2.  body)
```

Lambda calculus in egg : Dynamic rewrites redux

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-- v2=x is not free in (\x. x + 1)
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x_bound = let v1 = e          in (\v2.  body)
```

How to push `let` into `lambda`?

```
x_bound' = \x.  let v1 = (\x. x*2) in x + 1
x_bound' = \v2. let v1 = e          in body
```

Lambda calculus in egg : Dynamic rewrites redux

```
-- v2=x is not free in (\x. x + 1)
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How to push `let` into `lambda`?

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x_bound' = \x.  let v1 = (\x. x*2) in x + 1
x_bound' = \v2. let v1 = e         in body
```

```
-- v2=x is free in e=x*2
x :: Int; x = 42
x_free = let v1 = x*2 in (\x.   x+1)
x_free = let v1 = e   in (\v2.  body)
```

Lambda calculus in egg : Dynamic rewrites redux

```
-- v2=x is not free in (\x. x + 1)
x_bound = let v1 = (\x. x*2) in (\x.   x+1)
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-- v2=x is free in e=x*2
x :: Int; x = 42
x_free = let v1 = x*2 in (\x.   x+1)
x_free = let v1 = e   in (\v2.  body)
```

```
-- v2=x is free in e=x*2
x :: Int; x = 42
x_free'_wrong = \x. let v1 = x*2 in x + 1
```

ERR!

Lambda calculus in egg : Dynamic rewrites redux

```
-- v2=x is not free in (\x. x + 1)
x_bound = let v1 = (\x. x*2) in (\x.   x+1)
x_bound = let v1 = e           in (\v2.  body)
```

How to push let into lambda?

```
x_bound' = \x.  let v1 = (\x. x*2) in x + 1
x_bound' = \v2. let v1 = e          in body
```

```
-- v2=x is free in e=x*2
x :: Int; x = 42
x_free = let v1 = x*2 in (\x.   x+1)
x_free = let v1 = e   in (\v2.  body)
```

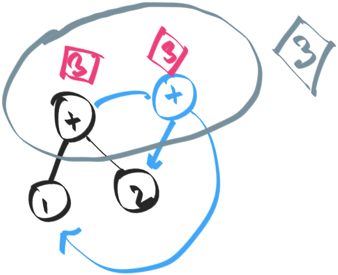
```
-- v2=x is free in e=x*2
x :: Int; x = 42
x_free'_wrong = \x. let v1 = x*2 in x + 1
```

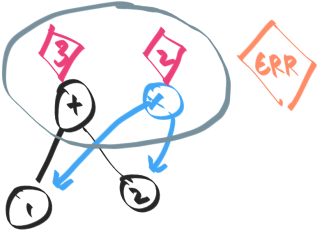
ERR!

How to push let into lambda?

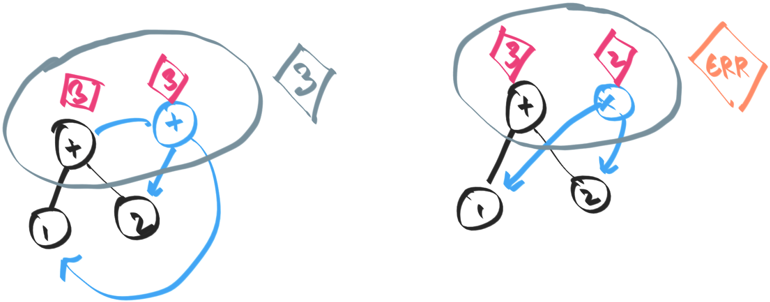
```
x :: Int; x = 42
x_free' = \fresh. let v1 = e   in (let v2 = fresh in body  )
x_free' = \fresh. let v1 = x*2 in (let x  = fresh in (x + 1))
```

Herbie: Using egg — Lattices





Herbie: Using egg — Lattices



- Abstract interpretation of equivalence classes.
- For each node, provide function $\alpha : \texttt{node} \to L$ (Abstraction function)
- $(L, \cap)$ is a join-semilattice.
- egg provides for each equivalence class $\texttt{class} \mapsto \bigcap_{\texttt{node} \in \texttt{class}} \alpha(\texttt{node}) \in L$