

Probablistic Programming: Use cases and implementation

Siddharth Bhat

IIIT Hyderabad

November 16, 2019
(FunctionalConf '19)

Outline

- ▶ How is this different from `System.Random`?
- ▶ What is Probablistic programming, and when can I use it?
- ▶ A sketch of the algorithm.
- ▶ Hunt me down post-talk for a wealth of details.

Our primitives

```
data Rand a = ...
```

```
instance Functor Rand
```

```
instance Applicative Rand
```

```
instance Monad Rand
```

```
-- / Convert a pure value into a Rand value
```

```
return :: a -> Rand a
```

```
-- / Get a random number in [0, 1)
```

```
uniform01 :: Rand Float
```

```
-- / Take 'n' samples from a random variable
```

```
samples :: Int -> Rand a -> [a]
```

```
-- / take a Float, do *something*, and return no nothing
```

```
score :: Float -> Rand ()
```

First example – The same as `System.Random`

```
-- | dice
dice :: Rand Int
dice = do
  u <- uniform01
  return $ 1 + floor (6*u)

-- | sum of dice
tossDice :: Rand Int
tossDice = do
  d1 <- dice
  d2 <- dice
  return $ d1 + d2

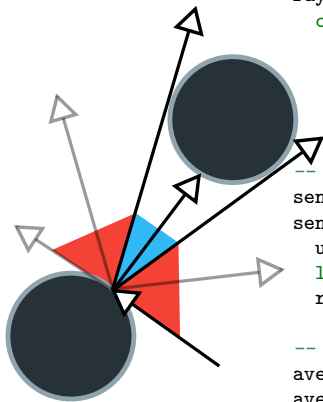
main :: IO ()
main = do
  print $ sample 10 tossDice
  drawHistogram $ sample 100 tossDice
```

Output:

[7,6,7,10,5,8,12,8,9,6]



Raytracing (Default)



```
-- / recursively raytrace
raytrace :: Ray -> Rand Color
raytrace r = do
  case getCollision r of
    Some (surface, loc) ->
      color' <- averageRays loc
      return $ mixColor surface color'
    None -> return backgroundColor

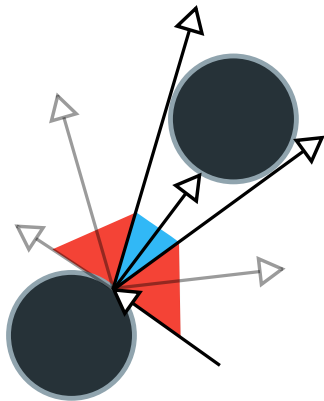
-- / Send a random ray
sendRandRay :: Position -> Rand Color
sendRandRay p =
  u <- uniform01
  let angle = 360 * u
  raytrace (makeRay p angle)

-- / Average rays sent from a location
averageRays :: Position -> Rand Color
averageRays p = do
  -- / computationally wasteful
  colors <- replicateM 100 (sendRandRay p)
  return $ averageColors colors

-- / Default background color.
backgroundColor = white
```

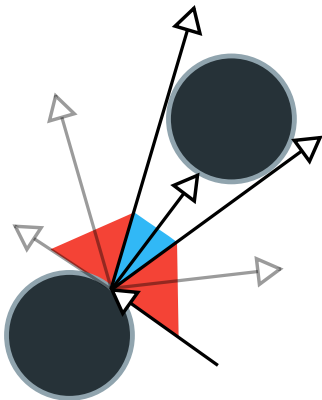
Raytracing (Scored)

```
raytrace :: Ray -> Rand Color
raytrace r = do
  case getCollision r of
    Some (surface, loc) ->
      color' <- averageRays loc
      return $ mixColor surface color'
    None -> return backgroundColor
```

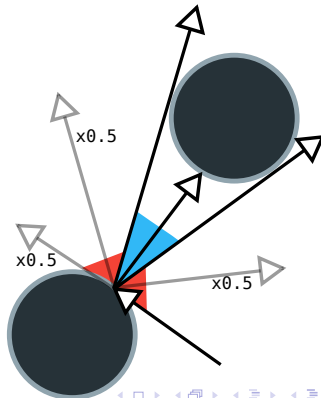


Raytracing (Scored)

```
raytrace :: Ray -> Rand Color
raytrace r = do
  case getCollision r of
    Some (surface, loc) ->
      color' <- averageRays loc
      return $ mixColor surface color'
    None -> return backgroundColor
```



```
raytrace' :: Ray -> Rand Color
raytrace' r = do
  case getCollision r of
    Some (surface, loc) ->
      color' <- averageRays loc
      return $ mixColor surface color'
    None -> do
      score 0.5 -- New!
      return backgroundColor
```



Exploring a complicated landscape

```
-- | Naive understanding / Little knowledge when we begin
prior :: Rand a
prior = ...

-- | Learn as you go!
learn :: Rand a
learn = do
  value <- prior
  score (usefulness value)
  return value

-- | Generate samples according to unknown distribution
-- (Rays from the raytracing)
landscape :: [a]
landscape = samples 1000 learn
```


Program optimisation

- ▶ A hypothetical stack-based machine (eg. JVM bytecode!)
- ▶ Instructions are Push, Add, Dup, Sub

$f(x) = x*2$

Program(f) = [Push 2; Mul]

[BOT x] -Push 2-> [BOT x; 2] -Mul-> [BOT x*2]

Program optimisation

- ▶ A hypothetical stack-based machine (eg. JVM bytecode!)
- ▶ Instructions are Push, Add, Dup, Sub

$f(x) = x*2$

Program(f) = [Push 2; Mul]

[BOT x] -Push 2-> [BOT x; 2] -Mul-> [BOT x*2]

- ▶ Want to automatically discover optimisations

$f(x) = x*2 = x+x$

Program(f) = [Dup; Add]

[BOT x] -Dup-> [BOT x; x] -Add-> [BOT x+x]

Faster (addition is faster than multiplication)

Program optimisation

- ▶ A hypothetical stack-based machine (eg. JVM bytecode!)
- ▶ Instructions are Push, Add, Dup, Sub

$$f(x) = x*2$$

Program(f) = [Push 2; Mul]

[BOT x] -Push 2-> [BOT x; 2] -Mul-> [BOT x*2]

- ▶ Want to automatically discover optimisations

$$f(x) = x*2 = x+x$$

Program(f) = [Dup; Add]

[BOT x] -Dup-> [BOT x; x] -Add-> [BOT x+x]

Faster (addition is faster than multiplication)

- ▶ Neural nets do not work - no gradient $\frac{\partial loss}{\partial program}$

Program optimisation Pseudocode (Random)

Key idea: Randomly change the initial program and pick the best.

```
-- | Randomly change programs and return their performance
equivRandomProgram :: Program -> Rand (Performance, Program)
equivRandomProgram p = do
  p' <- modifyProgram p
  if semanticsEqual p p'
  then return (performance p', p')
  else return (0, p') -- A program that does not work has 0 perf.

-- | Take the random samples and pick the good performing ones
optimise :: Program -> Program
optimise p =
  let ps' = sample 100 (equivRandomProgram p)
  in snd $ maximumBy (\a b -> compare (fst a) (fst b)) ps'
```

Program optimisation Pseudocode (Scored)

```
equivRandomProgram :: Program -> Rand (Performance, Program)
equivRandomProgram p = do
  p' <- modifyProgram p
  if semanticsEqual p p'
  then return (performance p', p')
  else return (0, p') -- A program that does not work has 0 perf.
```

Program optimisation Pseudocode (Scored)

```
equivRandomProgram :: Program -> Rand (Performance, Program)
equivRandomProgram p = do
  p' <- modifyProgram p
  if semanticsEqual p p'
  then return (performance p', p')
  else return (0, p') -- A program that does not work has 0 perf.

equivRandomProgram' :: Program -> Rand (Performance, Program)
equivRandomProgram' p = do
  (perf, p) <- equivRandomProgram p
  let perf =
    if semanticsEqual p p'
    then 5 + performance p' -- Correct programs are given high score
    else performance p' -- Fast incorrect programs are also allowed
  score perf -- ^ Correct programs are more likely
  return (perf, p')
```

<http://stoke.stanford.edu/>

<https://github.com/bollu/blaze/blob/master/notebooks/tutorial.ipynb>

Transformations discovered by STOKe

- ▶ `f = 2 + 3`
(nparams: 0 | [IPush 2,IPush 3,IAdd])

Transformations discovered by STOKE

- ▶ `f = 2 + 3`
 `(nparams: 0 | [IPush 2,IPush 3,IAdd])`
 `[IPush 5] | score: 2.5`
 `// constant folding: 2 + 3 -> 5`

Transformations discovered by STOKE

- ▶ `f = 2 + 3`
`(nparams: 0 | [IPush 2,IPush 3,IAdd])`
`[IPush 5] | score: 2.5`
`// constant folding: 2 + 3 -> 5`
- ▶ `f(x) = 2 * x`
`(nparams: 1 | [IPush 2,IMul])`

Transformations discovered by STOKE

- ▶ `f = 2 + 3`
`(nparams: 0 | [IPush 2,IPush 3,IAdd])`
`[IPush 5] | score: 2.5`
`// constant folding: 2 + 3 -> 5`
- ▶ `f(x) = 2 * x`
`(nparams: 1 | [IPush 2,IMul])`
`[IDup,IAdd] | score: 2.25`
`// strength reduction: 2 * x -> x + x`

Transformations discovered by STOKE

- ▶ $f = 2 + 3$
(nparams: 0 | [IPush 2,IPush 3,IAdd])
[IPush 5] | score: 2.5
// constant folding: $2 + 3 \rightarrow 5$
- ▶ $f(x) = 2 * x$
(nparams: 1 | [IPush 2,IMul])
[IDup,IAdd] | score: 2.25
// strength reduction: $2 * x \rightarrow x + x$
- ▶ $f(x) = x \& x$
(nparams: 1 | progInsts = [IDup,IAnd])

Transformations discovered by STOKE

- ▶ $f = 2 + 3$
(nparams: 0 | [IPush 2,IPush 3,IAdd])
[IPush 5] | score: 2.5
// constant folding: $2 + 3 \rightarrow 5$
- ▶ $f(x) = 2 * x$
(nparams: 1 | [IPush 2,IMul])
[IDup,IAdd] | score: 2.25
// strength reduction: $2 * x \rightarrow x + x$
- ▶ $f(x) = x \& x$
(nparams: 1 | progInsts = [IDup,IAnd])
[] | score: 3.0
// algebraic rewrite: $x \& x == x$

STOKE: The intuition

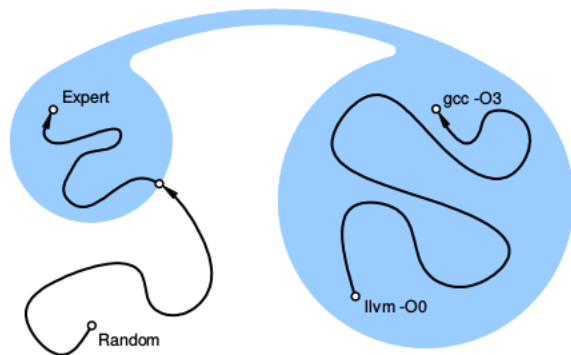


Figure 4. Abstract depiction of the search space for the Montgomery multiplication benchmark. O0 and O3 optimized codes occupy a densely connected part of the space which is easily traversed. Expert code occupies an entirely different region of the space which is reachable only by way of an extremely low probability path.

STOKE: The intuition

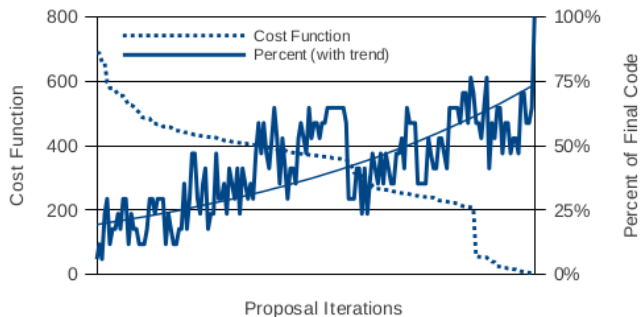


Figure 8. Cost function versus percentage of instructions which appear in the final zero-cost rewrite. Random search is an effective method for performing synthesis insofar as it is able to discover partially correct rewrites incrementally.

STOKE: A real-world example

```
1 # gcc -O3                                1 # STOKE
2                                           2
3 .L0:                                       3 .L0:
4     movq rsi, r9                          4     shlq 32, rcx
5     movl ecx, ecx                        5     movl edx, edx
6     shrq 32, rsi                         6     xorq rdx, rcx
7     andl 0xffffffff, r9d                 7     movq rcx, rax
8     movq rcx, rax                        8     mulq rsi
9     movl edx, edx                        9     addq r8, rdi
10    imulq r9, rax                       10    adcq 0, rdx
11    imulq rdx, r9                       11    addq rdi, rax
12    imulq rsi, rdx                       12    adcq 0, rdx
13    imulq rsi, rcx                       13    movq rdx, r8
14    addq rdx, rax                        14    movq rax, rdi
15    jae .L2
16    movabsq 0x100000000, rdx
17    addq rdx, rcx
18 .L2:
19    movq rax, rsi
20    movq rax, rdx
21    shrq 32, rsi
22    salq 32, rdx
23    addq rsi, rcx
24    addq r9, rdx
25    adcq 0, rcx
26    addq r8, rdx
27    adcq 0, rcx
28    addq rdi, rdx
29    adcq 0, rcx
30    movq rcx, r8
31    movq rdx, rdi
```

Gradient-free optimisation on a complicated landscape

```
-- / Naive understanding / Little knowledge when we begin
prior :: Rand a
prior = ...

-- / Learn as you go!
learn :: Rand (Score, a)
learn = do
  value <- prior
  let s = score (usefulness value)
  return (s, value)

-- / Sample and pick best value (random programs)
-- / Works because sampler will "move" towards
-- scored regions!
best :: (Score, a)
best = maximumBy (\a b -> compare (fst a) (fst b))
  (samples 1000 learn)
```



```
data Rand x where
    Ret :: x -> Rand x
    SampleUniform01 :: (Double -> Rand x) -> Rand x
    Score :: Float -> Rand x -> Rand x
```

```
instance Functor Rand
instance Applicative Rand
instance Monad Rand
```

(Rand is a free monad)

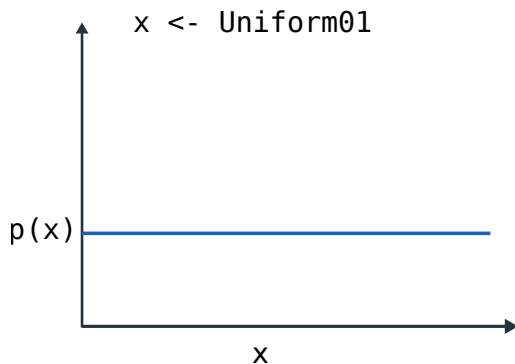
```
return :: x -> Rand x
return = Ret
```

```
uniform01 :: Rand Float
uniform01 = SampleUniform01 Ret
```

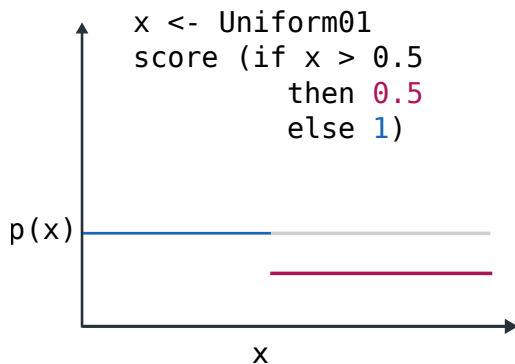
```
score :: Float -> Rand ()
score s = Score s (return ())
```

```
-- | Run the computation _unweighted_,  
-- | ignores scores.  
sampleUnweighted :: RandomGen g => g -> Rand a -> (a, g)  
sampleUnweighted g (Ret a) = (a, g)  
sampleUnweighted g (SampleUniform01 f2my) =  
    let (f, g') = random g in sample g' (f2my f)  
sampleUnweighted g (Score f mx) =  
    error "unable to handle Score"
```

Semantics of Score: Uniform probability



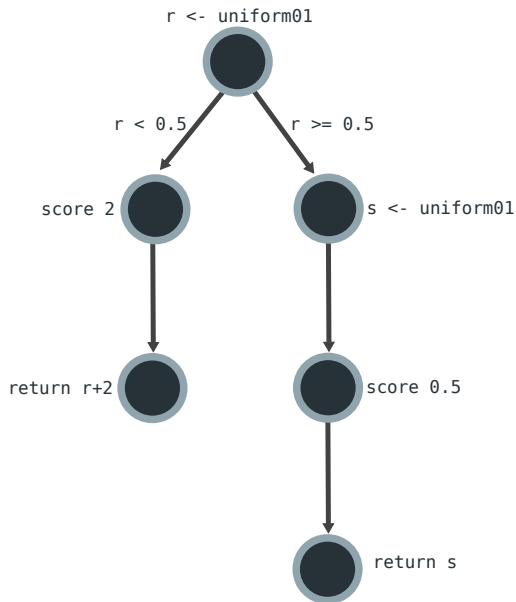
Semantics of Score: Custom ratios of probabilities!



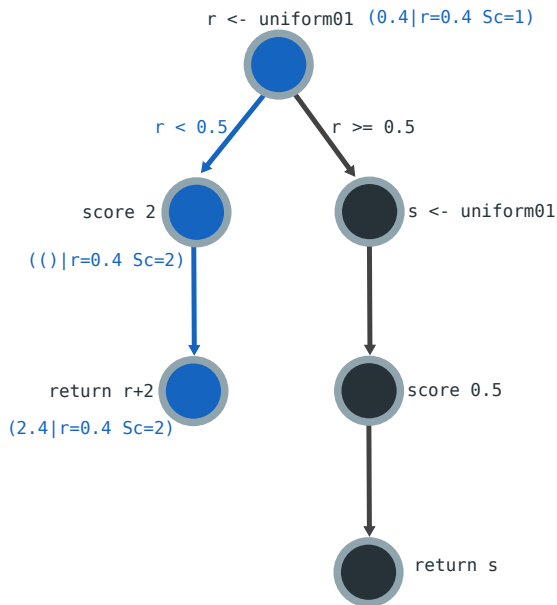
How do we handle Score?

- ▶ Game plan: convert `Rand a` to `Rand (Trace a)`.
- ▶ `Rand a` may contain `Score`, `Rand (Trace a)` will not.
- ▶ `Rand (Trace a)` will have extra information to make `Score` work.
- ▶ Can use `sampleUnweighted` on `Rand (Trace a)`. It will respect `Score`.

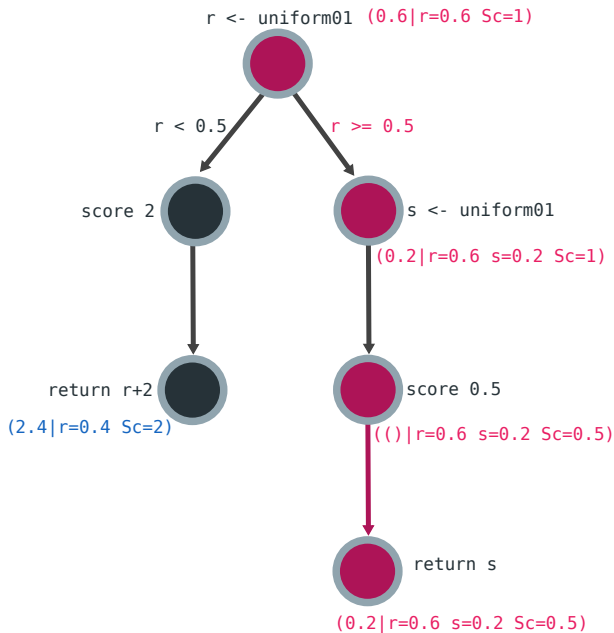
Traced Computations - Program



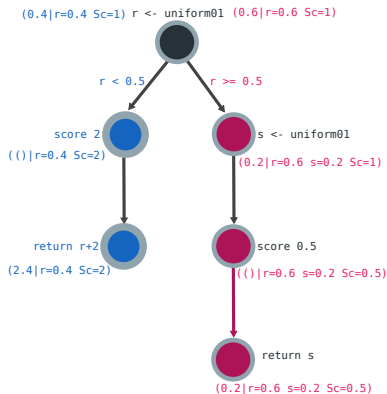
Traced Computations - Trace 1



Traced Computations - Trace 2

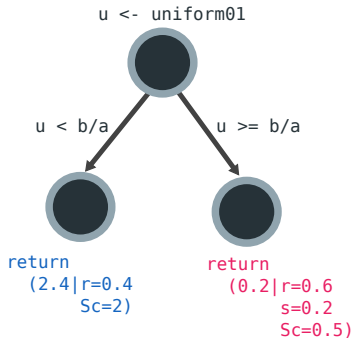


Traced Computations - Trace 3



$a = \text{Accept}(2.4|r=0.4 \text{ Sc}=2)$
 $= 0.4 \times \text{len}([0.4])$
 $= 0.4 \times 1 = 0.4$

$b = \text{Accept}(0.2|r=0.6 \text{ s}=0.2 \text{ Sc}=0.5)$
 $= 0.5 \times \text{len}([0.6 \text{ } 0.2])$
 $= 0.5 \times 2$
 $= 1$



Tracing: the data structure

```
-- / Trace all random choices made
data Trace a =
  Trace { tval :: a, -- ^ The value itself
          tscore :: Double, -- ^ The total score
          trs :: [Double] -- ^ The random numbers used
        }
```

Introspection of a Free monad

```
-- | Trace a random computation (remove Score)
traceR :: Rand x -- ^ Original computation
  -> Rand (Trace x) -- ^ Traced computation
traceR (Ret x) = Ret (Trace x 1.0 [])
traceR (SampleUniform01 mx) = do
  r <- sample01
  trx <- traceR $ mx r
  return $ trx { trs=trs ++ [r]}
traceR (Score s mx) = do -- ^ No score in the RHS
  trx <- traceR $ mx
  return $ trx { tscore = tscore*s}
```

Introspection of a Free monad

```
-- | Trace a random computation (remove Score)
traceR :: Rand x -- ^ Original computation
  -> Rand (Trace x) -- ^ Traced computation
traceR (Ret x) = Ret (Trace x 1.0 [])
traceR (SampleUniform01 mx) = do
  r <- sample01
  trx <- traceR $ mx r
  return $ trx { trs=trs ++ [r]}
traceR (Score s mx) = do -- ^ No score in the RHS
  trx <- traceR $ mx
  return $ trx { tscore = tscore*s}

-- | Send a particular sequence of randomness
feedRandomness :: [Float] -- ^ Randomness to use
  -> Rand a -- ^ Original computation
  -> Rand a -- ^ Fixed computation
feedRandomness _ (Ret x) = Ret x
feedRandomness
  (r:rs) (SampleUniform01 f) =
    feedRandomness rs (f r)
feedRandomness rs (Score s plx) =
  Score s $ injectRandomness rs plx
```

Metropolis Hastings

```
mhStep :: Rand (Trace x) -- ^ proposal
      -> Trace x -- ^ current position
      -> Rand (Trace x) -- ^ new
```

```
mhStep r trace = do
  -- / Return the original randomness, perturbed
  rands' <- perturbRandomness (trs trace)
  -- / Run the original computation with the perturbation
  trace' <- feedRandomness rands' r
  let ratio = trAccept trace' / trAccept trace
  r <- sample01
  return $ if r < ratio then trace' else trace
```

```
trAccept :: Trace x -> Double
trAccept tx =
  tscore tx *
  fromIntegral (length (trs tx))
```

```
perturbRandomness :: [Double]
  -> Rand [Double]
perturbRandomness rands = do
  -- / Random index
  ix <- choose [0..(length rands-1)]
  r <- sample01 -- ^ random val
  -- / Replace random index
  -- with random val.
  return $ replaceListAt ix r rands
```

Odds and Ends

```
-- | run the computation after taking weights into account
samples :: Int -> Rand x -> Rand [x]
samples 0 _ = return []
samples n r =
  let tracedR = traceR r
      -- go :: Int -> Rand (Trace x) -> Rand (Trace [x])
      go 0 _ = return []
      go n tx = do
        tx' <- repeatM 10 (mhStep tracedR) $ tx -- !
        txs <- go (n-1) tx'
        return (tx:txs)
  in do
    seed <- findNonZeroTrace $ tracedR
    tracedRs <- go n seed
    return $ map tval tracedRs

-- | Find a starting position that does not have probability 0
findNonZeroTrace :: Rand (Trace x) -> Rand (Trace x)
findNonZeroTrace tracedR = do
  trace <- tracedR
  if tscore trace /= 0
  then return $ trace
  else findNonZeroTrace tracedR
```

Takeaways

- ▶ Probabilistic Programming is a powerful model of computation.
- ▶ Free monads is essential: allows for introspection.
- ▶ Implementation is deceptively simple (hopefully!)

Thank you!

Questions?



tweag.io

(A huge thank you to everyone at tweag.io who read the literature with me!)

References I

Use case: Bayesian updates

```
predictCoinBias :: [Int] -> Rand Double
predictCoinBias flips = do
  b <- sample01
  forM_ flips $ \f -> do
    -- / Maximum a posterior
    score $ if f == 1 then b else (1 - b)
  return $ b

predictCoinBiasNoData :: Rand Double
predictCoinBiasNoData = predictCoinBias []

████████████████████████████████████████████████████████████████████████████████
████████████████████████████████████████████████████████████████████████████████

predictCoinBias0 :: Rand Double
predictCoinBias0 = predictCoinBias [0]

████████████████████████████████████████████████████████████████████████████████
████████████████████████████████████████████████████████████████████████████████

predictCoinBias01 :: Rand Double
predictCoinBias01 = predictCoinBias [0, 1]

████████████████████████████████████████████████████████████████████████████████
████████████████████████████████████████████████████████████████████████████████
```

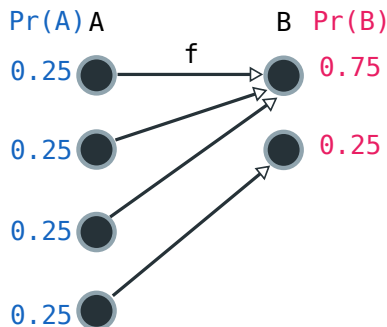
Use case: Sample from arbitrary distributions

```
sampleSinSq :: Rand Double
sampleSinSq = do
  x <- (6 *) <$> sample01
  score $ (sin x) * (sin x)
  return $ x
```



Deep dive into MCMC methods

Probability Distributions: the tension



- ▶ $f : A \rightarrow B$ is known.
- ▶ $f^{-1} : B \rightarrow A$ is required for the density.
- ▶ This is intractable.