MILP Approach for Custom Datapath Design

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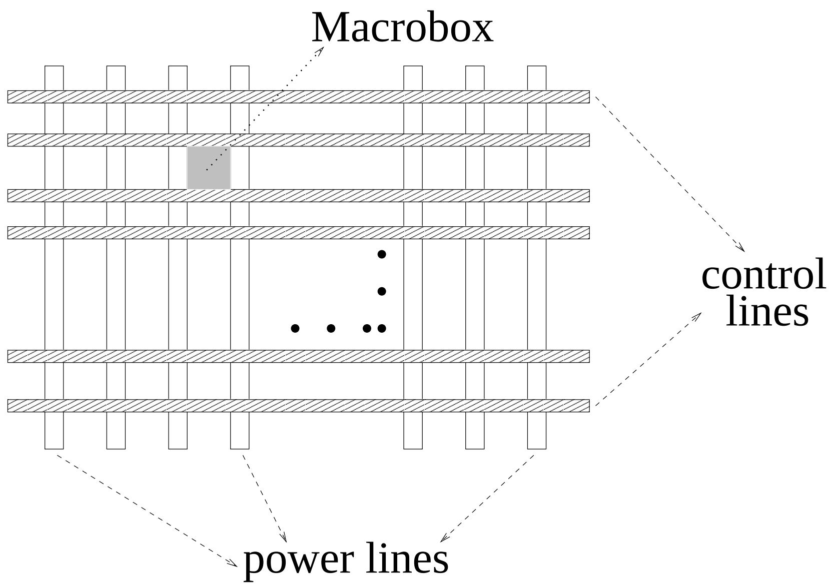
Datapath Synthesis Problem

- Designing layout for high-performance datapaths.
- **Approach:** Mixed Integer Linear Programming.
- Difficult problem
  - Datapath constraints
  - Constraints of custom design
  - Problem complexity

Structure of datapaths

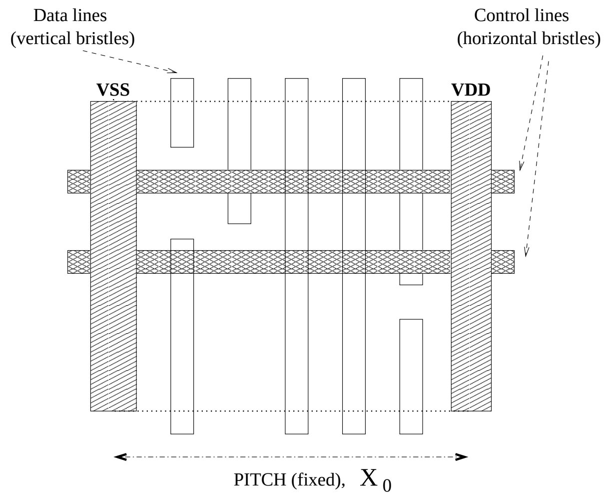
- Highly regular layout structures.
- Floorplan:
  - Array of bit slices
  - Words of identical bit cells (macrobox).

Floorplan of a Datapath



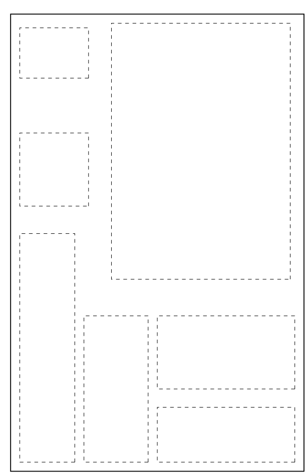
Inside The Macrobox

- Power supply rails
- Bristles
  - vertical (data lines)
  - horizontal (control lines)
- Pitch (fixed)



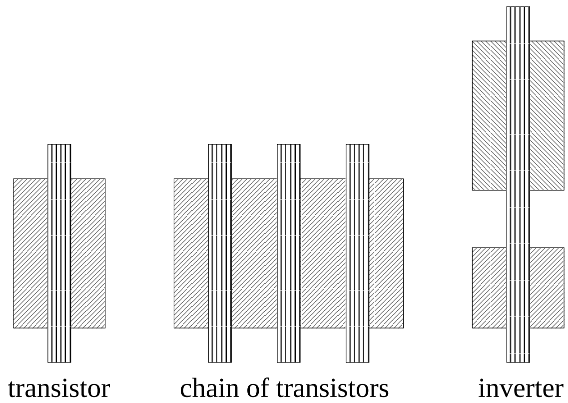
Problem Definition

- Generate macrobox layout
- Components:
  Rectangular objects



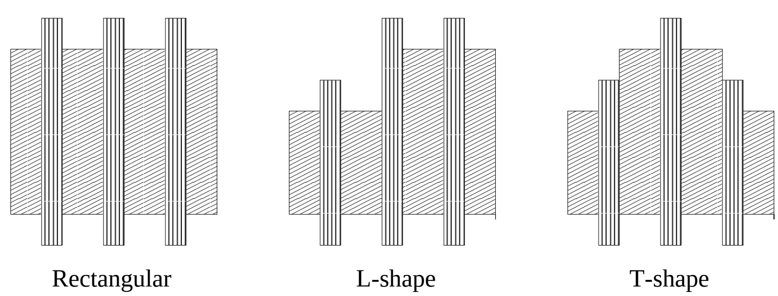
Components - 1

- Single transistor, chain of transistors
- Inverters, logic gates



Components - 2

- Rectangular
- L-shape
- T-shape.



Method for Macrobox Layout

1. Initial Relative Placement
   - connectivity
   - no geometrical information
2. Geometric Placement
   - fixed connectivity
   - geometrical information
3. Post-processing
   - compaction
   - final orientation

Initial Relative Placement

GOAL:

- To obtain a relative initial placement to simplify geometric placement.

METHOD:

- Force directed technique

Force Directed Placement - 1

- Based on analogy to classical mechanics problem of a system of bodies.
- The components sharing the same net excersize forces on each other.

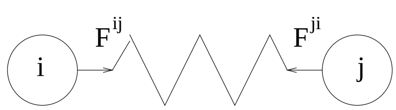
Hooke's Law

$$F_{ij} = k_{ij} * \Delta d_{ij}$$

$$\text{where } \Delta d_{ij} = \sqrt{{\Delta x_{ij}}^2 + {\Delta y_{ij}}^2}$$

- The final placement is when all components are at equilibrium.

Force Directed Placement - 2



$\vec{F^{ij}}$, attractive force applied on i by j

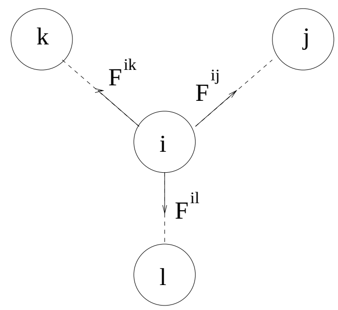
$$F_x^{ij} = k_{ij} * \Delta x_{ij}$$

$$F_y^{ij} = k_{ij} * \Delta y_{ij}$$

where k_{ij} is the proportionality constant (connectivity), $\Delta x_{ij} = x_j - x_i$ and $\Delta y_{ij} = y_j - y_i$

Force Directed Placement - 3

At Equilibrium: The net force acting on each component $\Rightarrow 0$



$$\forall i, \qquad \vec{F^i} = \vec{F_x^i} + \vec{F_y^i} = 0$$
$$F_x^i = \sum_{j=1}^{n}[k_{ij} * \Delta x_{ij}] = 0$$
$$F_y^i = \sum_{j=1}^{n}[k_{ij} * \Delta y_{ij}] = 0$$

Force Directed Placement - 4

$$F_x^i = \sum_{j=1}^{n}[k_{ij} * \Delta x_{ij}] = 0$$
$$k_{i1} * (x_1 - x_i) + k_{i2} * (x_2 - x_i) + ... + k_{in} * (x_n - x_i) = 0$$
$$\sum_{j=1}^{n}[k_{ij} * x_j] = \sum_{j=1}^{n} k_{ij} * x_i$$
$$x_i = \frac{\displaystyle\sum_{j=1}^{n}[k_{ij} * x_j]}{\displaystyle\sum_{j=1}^{n} k_{ij}}$$

Same for y direction

$$y_i = \frac{\displaystyle\sum_{j=1}^{n}[k_{ij} * y_j]}{\displaystyle\sum_{j=1}^{n} k_{ij}}$$

Force Directed Placement - 5

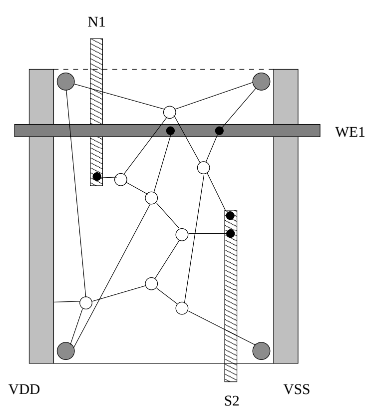
- The avoid trivial solution

$$x_i = x_j = ... = x_n$$

  the location of some components must be fixed.
- Fixed components are known as **anchors**.
- Anchor assignment is done based on connectivity information. (heuristic)

Result of Force Directed Placement

- Initial relative placement of components



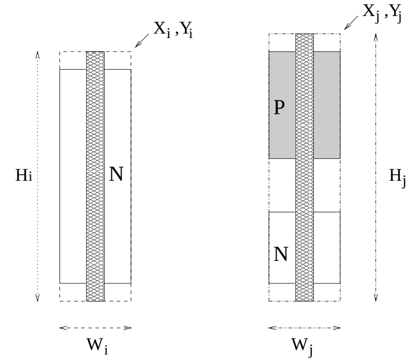
Geometric Placement

Objective: To minimize the height of the placement, Y_0

- It concentrates on the non-overlapping placement
- It takes into consideration of geometrical information and relative positions of components
- Formulation includes
  - Component modeling
  - Component rotation
  - Non-overlapping constraints
  - Boundary constraints (fixed pitch)
- MILP is used to solve this problem

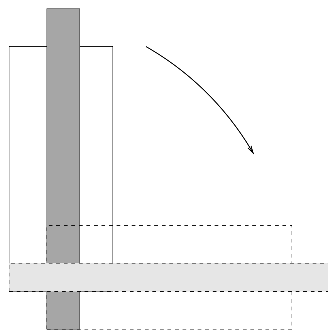
Modeling of Components - 1

- Each rectangular component is represented by the coordinates of their uppermost corner, (X_i, Y_i)
- T-shape and L-shape components can be represented as connected rectangles.



Modeling of Components - 2

- Rotation of components can also be formulated using this model.
- Rotation of the L-shape and T-shape components needs further analysis

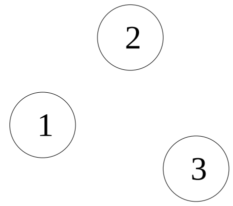


Non-overlapping Constraints

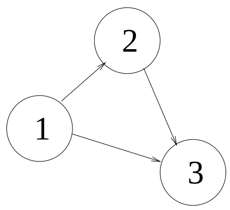
- The relative position of components are obtained from the initial relative placement.
- Horizontal (G_x) and Vertical (G_y) Adjacency Graphs are used to determine the geometric locations of the components.

$$G_x(V, E_x) : V = \{components\}, E_x = \{\{i, j\} : i \rightarrow j \qquad if x_i \leq x_j\}$$
$$G_y(V, E_y) : V = \{components\}, E_y = \{\{i, j\} : i \rightarrow j \qquad if y_i \leq y_j\}$$

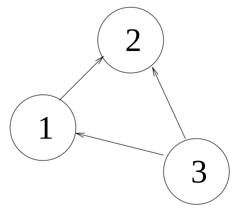
Example 1 - 1



Initial Relative Placement



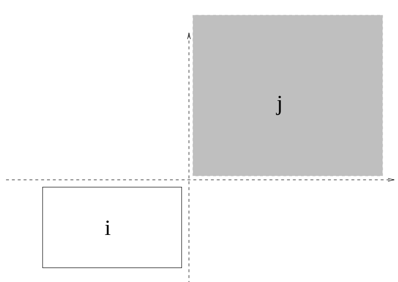
Horizontal Adjacency Graph



Vertical Adjacency Graph

Example 1 - 2

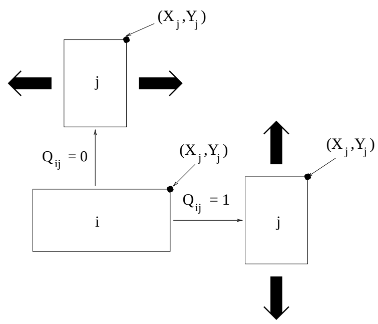
- If components are forced to satisfy both adjacency relations, the constraint set is convex.
- LP can be used



- Solution will be over-constrained.

Example 1 - 3

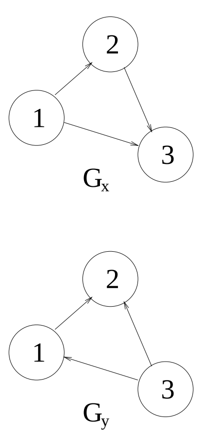
- One relation needs to be satisfied.
- The solution of resides in the non-convex area



- ILP is can not be applied. Integer variables, Q_{ij} are introduced.

Non-overlapping Constraints - 2

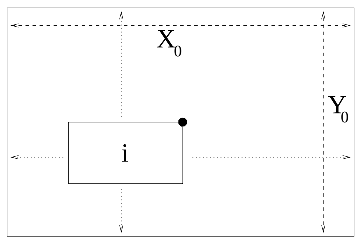
- MILP non-overlapping constraints can be written as follows



$$x_2 - x_1 \geq W_2 - L * (1 - Q_{12})$$
$$y_2 - y_1 \geq H_2 - L * Q_{12}$$
$$x_3 - x_1 \geq W_1 - L * (1 - Q_{13})$$
$$\vdots$$

Boundary Constraints

- Each component must be placed within the boundaries of the macrobox.



$$x_i \geq W_i$$
$$x_i \leq X_0$$
$$y_i \geq H_i$$
$$y_i \leq Y_0$$

MILP Formulation

- Minimize Y_0
  s.t.
  - Boundary constraints
  - Non-overlapping constraints

Post-processing

- Finding final component orientation
  - To simplify routability
  - To minimize P/N islands
- Iterative improvement
- Compaction