Rectangular Duals and Box-representation in the Cylinder

(Brief abstract)

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1 Introduction

In this paper we consider two representations of a planar graph, called *rectangular dual* and *box-representation* in the cylinder.

A *rectangular dual system* of a rectangle R is a partition of R into a set $\Gamma = \{R_1, \ldots, R_n\}$ of non-overlapping rectangles such that no four rectangles in Γ meet at the same point.

A *rectangular dual* of a planar graph G is a rectangular subdivision system Γ and an one-to-one correspondence $R : V \longrightarrow \Gamma$ such that two vertices u and v are adjacent in G if and only if their corresponding rectangles $R(u)$ and $R(v)$ share a common boundary. This is applied in the design of floor planning of electronic chips and in architectural design [2, 6, 4, 5].

This problem was firstly studied by Bhasker & Sahni [1] and Koźmiński & Kinnen [3] who gave a linear time algorithm to construct rectangular duals on the Euclidean plane.

We also study the concept of *box-representation*, more general than a rectangular dual. A box-representation for a planar graph G is a set of non-overlapping rectangles such that no four rectangles meet at the same point, each one associated with a vertex of G, such that topological incidences in G correspond to geometric adjacencies between rectangles. Thomassen [7] characterizes the graphs that admit a box-representation in the plane.

The concepts of rectangular dual and box-representation can be naturally extended to other surfaces where two families of parallel lines are defined. The motivation for that extension is its numerous applications in several areas such as the layout of regular VLSI circuits.

We characterize graphs that admit a rectangular dual and a box-representation and we give linear time algorithms that test the existence and construct a rectangular dual or a box-representation for a planar graph in the cylinder.

2 Rectangular Dual

A *section* of a cylinder C, is the portion of C generated by the rotation of an interval of C around the axis. A *rectangular dual system* of a section S of a cylinder is a partition of S into a set $\Gamma = \{R_1, R_2, \ldots, R_n\}$ of non-overlapping rectangles such that no four rectangles in Γ meet at the same point. A *rectangular dual* in the cylinder of a planar graph G is a rectangular subdivision system Γ and an one-to-one correspondence $R : V \longrightarrow \Gamma$ such that two vertices u and v are adjacent in G if and only if their corresponding rectangles $S(u)$ and $S(v)$ share a common boundary.

Let $G = (V, E)$ be a planar graph. Consider a fixed cylindrical embedding of G. Unlike planar embeddings, cylindrical embeddings can have two unbounded faces, which are referred to as the *upper face* and *lower face* of the embedding. We assume throughout the paper that cylindrical embeddings have two unbounded faces. A cycle of G divides the

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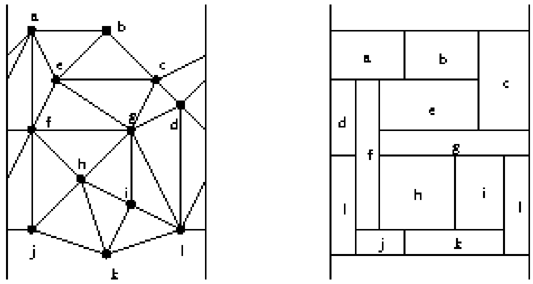

Figure 1: Rectangular dual in the cylinder.

cylinder into two regions (at least one of them is unbounded). A *cut subgraph* H of G is a subgraph whose removal disconnects G. If H is a cycle consisting of tree edges, then it is called a *separating triangle.* If T is a separating triangle and it divides the cylinder into two unbounded regions we say that T *wraps around* the cylinder.

Consider a cylindrical embedding of a planar graph $H = (V, E)$. We seek a rectangular dual in the cylinder of H. In order to simplify the problem, we modify H as follows: Add two new vertices N in the upper face and S in the lower face, and connect N (S, respectively) to every vertex on the upper face (lower face, respectively). Let G be the resulting graph. It is easy to see that H has a rectangular dual in the cylinder if and only if G has a rectangular dual in the cylinder with just one vertex on the unbounded faces.

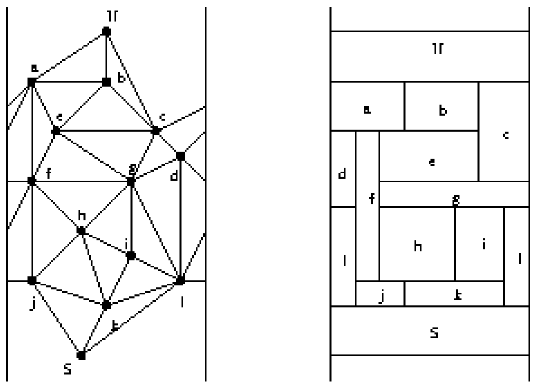

Figure 2: The new graph G.

Theorem 1 *A planar graph $G = (V, E)$ has a rectangular dual in the cylinder with just one vertex on the unbounded faces if and only if every face of G is a triangle and every separating triangle of G wraps around the cylinder. There is also a linear time algorithm for testing the existence of and constructing a rectangular dual for G in the cylinder.*

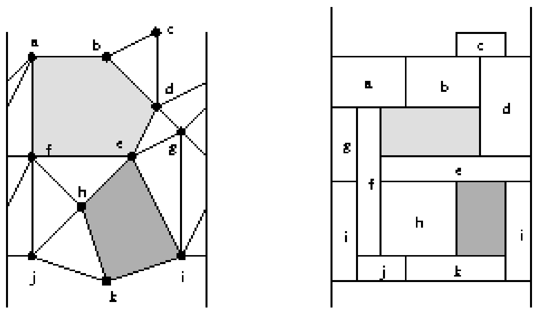

Figure 3: Box-representation in the cylinder.

3 Box-representation

A *box-representation* of a planar graph G is a set $\Gamma = \{R_1, \ldots, R_n\}$ of non-overlapping rectangles such that no four rectangles in Γ meet at the same point and an one-to-one correspondence $R : V \longrightarrow \Gamma$ such that two vertices u and v are adjacent in G if and only if their corresponding rectangles $R(u)$ and $R(v)$ share a common boundary.

Theorem 2 *A planar graph $G = (V, E)$ has a box-representation in the cylinder if and only if every separating triangle of G wraps around the cylinder. There is also a linear time algorithm for testing the existence of and constructing a box-representation for G in the cylinder.*

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