Haskell Road; Getting Started Part 2

Getting started part 2

Monday at the library part 2

Breaktime

The von der Surwitzes pop over to the student center cafe for a break. They grab a large mineral water, a brand they knew in Germany, and Ute has packed some *Vollkornbrot* sandwiches of hummus and cucumber. They sit at a table and pour the water and pass around out the sandwiches.

: All right, so I emailed the professor about a couple of questions from that first chapter of *The Haskell Road*, and she replied saying, first, she's happy we're tackling the material early. And she mentioned some resources — a few texts she has on reserve at the library.
: Sort of like, I'm not going to give you the answers. I'm going to point you in the right direction. What books are they?
[murmurs of acknowledgement]
: Math. Upper level college texts. Abstract algebra and number theory.
: I've heard computer science has all these higher math concepts, but then you don't go as far as a math major does.
[silence, eating and drinking]
: [continuing] I guess you're just supposed to learn as much as you can. But like she said at the open house, a computer scientist is really an *applied* mathematician.
[murmurs of agreement]
: And the math is the hardest part for incoming CS students, those first four semesters, ergo, that's what we're emphasizing it in this course.
[nods of agreement then silence as they eat and drink]
: [continuing] So no hand-waving. And she doesn't have a set amount she wants us to get through. The course is open-ended. I just find that amazing.
[murmurs of agreement]
: But I'm sure we'll need to keep moving and not be laggards about it.
[murmurs of agreement]
: A whole year, the whole school year. Her sabbatical ends next summer, but I'm pretty sure I'll want to continue. I don't know if I want to be a computer scientist or software engineer, but learning this can't hurt.

: I guess you could say Novalis is sort of an open *Gynmasium.*
[soft laughter]
: And what happens afterward? They definitely want you to just keep going at the U. Which I wouldn't mind at all.
[looks about the table]
: Yes, and lots of people just drift into a half-and-half situation where there taking courses over at the U.
: Well, Father has tenure now. But I don't know if Mutti can go on working from here. [shrugging and sighing] Anyway, I guess you two will cross that bridge before I will.
: [laughing] Hardly! You're right there with us in everything we're doing this coming year.

Divided by

Back at the library study room they've checked out the reserved books and are looking through sections of those that deal with the basic theory of division.

: [reading from the Divisibility section of *Proofs, A Long-Form Mathematical Textbook*[1]] All right, so Professor Chandra wants us to understand divisibility before we go to *greatest common divisor*, and before we talk about primes. She said, You have to know all of the implications of "divided by" before you can advance. And like it's saying in here, [reading] you could just say, *a divides b if $\frac{a}{b}$ is an integer.*
[Ursula and Uwe read the section from a second copy]
: [continuing] But we don't want that definition, we want *this* definition [getting up and writing on the board]

$$\exists\, k \in \mathbb{Z},\ a \neq 0,\ \ a \mid b \ \text{ if } \ b = a \cdot k \tag{1}$$

: [continuing] The symbol $a \mid b$ means a divides b for some k where $b = a \cdot k$ and a is not equal to zero. [pausing] Right, all that makes sense. So basically, this turns the whole question of divisibility into finding a proper integer value for k to *multiply* with . Now we have a math-formalist way of seeing divisibility.
[murmurs of approval]
: I like how he says good definitions don't just fall out of the sky.
[murmurs of agreement]
: Then the examples, like $2 \mid 14$ is true because $14 = 2 \cdot 7$, in other words we've found a whole number integer, $k = 7$ and we're happy.
: Again, we've turned division into an issue of true-false logic and multiplication. [writing on the board] So $7 \mid 23$ doesn't work because we have no solution for $7k = 23$.
: And look at that last one where it's $a \mid 0$. That's true, for a non-zero a since we can say $0 = a \cdot 0$ is always true for any a as long as $k = 0$.
[murmurs of agreement]

[1] *Proofs; A Long-Form Mathematics Textbook* by Jay Cummings

: And like he says we're not supposed to look at $2 \mid 14$ and just say it *equals* 7 . It's not supposed to be seen as a calculation, it's a logic *expression* that is true or false — for some value k .
: Right. We're in the world of logic now, not grade school arithmetic. So everything has to be reexplained and reworked.
[murmurs of agreement]
: Good, now he's talking about the *transitive* property of divisibility. It is a *proposition*, which is a type of theorem, and that means it comes with a proof. [writing on the board] Here it is in the compact math logic form

$$a, b, c \in \mathbb{Z}, \ \ a \mid b \ \wedge \ b \mid c \implies a \mid c$$

: [continuing] And then he goes on to prove it by saying assume the *if* part, the $a \mid b \ \wedge \ b \mid c$ part is true, that means the *then* part, the $a \mid c$ part is true. So [writing]

$$b = a \cdot s$$

$$c = b \cdot t$$

: [continuing] for some integers s and t . And now [writing]

$$\begin{aligned} c &= b \cdot t \\ &= (a \cdot s) \cdot t \\ &= a \cdot (s \cdot t) \qquad \ldots \text{ associativity} \end{aligned}$$

: [continuing] So since we have the form $c = a \cdot (s \cdot t)$ where we assumed s and t are integers, and that's the basic form of divisibility, so yes, $a \mid c$ since we've shown $c = a \cdot k$ where $k = (s \cdot t)$.
: Good. Let's switch over to this other book [she picks up a Springer Verlag book[2] and pages through it] Ah, in this book there's a section called *Divisors and the Greatest Common Divisor*. [paging to section, reading] Oh, here's one, *Determine whether true or false* [writing on the board]

$$2 \mid (6n + 4)$$

: Interesting. So writing it in the divisibility way [gets up and writes on the board]

$$(6n + 4) = 2k$$

[2] *The Whole Truth About Whole Numbers* by Sylvia Forman and Agnes M. Rash;

: So before we freak out and get lost, let's just notice that [writing]

$$2(3n + 4) = 2k \tag{2}$$

$$3n + 4 = k \tag{3}$$

: [continuing] I'd say we don't need to go any further with this. $2 \mid (6n + 4)$ is true — which means it's got solutions — because 2 will go into $(6n + 4)$ for whatever n wants to be.
: And this whole formal divisibility thing helps because if you just saw this one day [writing on the board]

$$\frac{(6n + 4)}{2} = 3n + 2 \tag{4}$$

: [continuing] you've now got a second way to see the idea that the equation is true for any n, that it's dependent on n .
: [looking ironically] Thanks, Uwe, Ute, for keeping it real.
[laughter]
: [reading text] All right, we have this example [writing on board]

$$0 \mid 11$$

: [continuing] which is false because there can't be any k where $k \cdot 0$ equals 11 . Agreed?
[nods of agreement]
: [continuing] All right, how about this?

Prove that if $a \mid b$ then $-a \mid b$

: Let's just logic it out [getting up and writing on the board]

$$b = a \cdot k$$

$$b = (-a) \cdot (-k)$$

$$b = -(a) \cdot (k)$$

$$b = -a \cdot k$$

then

$$-a \mid b \quad \text{for some } k \in \mathbb{Z}$$

: [continuing] So k by virtue of being an integer, which can be either positive or negative, we've derived $-a \mid b$ from $a \mid b$.

[silence while the others study the board]
: Hold it. I'm not sure we've got the spirit of this, quite.
: How so?
: [going to the board] We need to make sure we understand this as [writing] $(-a) \mid b$ and not as $-(a \mid b)$, right?
[murmurs of agreement]
: So that means we've got [writing] $b = (-a)(-k)$ as the only possible solution to keep that b positive. And I don't think you meant to factor out -1 like you did. So k must be negative to go with the $-a$, which then gives positive b . That's what is meant, I think. Yes, k being an integer allows this. But again, we're dealing with a multiplicative relationship, we're not doing division. And I'm sure we'll find out why this is so important in a while.
: Oh, I think that was in here. [pulling a large-format book from her messenger bag[3] and pages to tabbed page]. Right, and he shows that $0 \mid 0$, that zero divides zero, is true — because [writing on board] $0 = 0 \cdot k$, meaning k can be anything and the expression remains true. [reading further] And he's calling k the *accessory number*. [reading further] So his wording is the integers x that satisfy $7 \mid x$ are $x = 7 \cdot k$ — and that will be the arithmetic progression of the multiples of 7 . They're evenly spaced. Good. And there's this [going to the board and writing]

Plot the integers x which satisfy $5 \mid (x - 2)$

: [going to the board and writing] So if that's to be true then we've got $x - 2 = 5k$, and that means for the multiples of 5 , the *set* of integers x must keep $x - 2$ multiples of 5 also. So for example

$$-3 - 2 = 5 \cdot -1$$

$$2 - 2 = 5 \cdot 0$$

$$7 - 2 = 5 \cdot 1$$

$$12 - 2 = 5 \cdot 2$$

$$\dots$$

: [continuing] And the so-called *geometric* view of this set of xs would be a number line with points [writing on the board]

[width=.9]images/testsine1

[width=.9]tikztest3

[width=.9]tikztest2

[3] *An Illustrated Theory of Numbers* by Martin H. Weissman.

[width=.9]tikztest4

[width=.9]tikztest5a

[width=.9]tikztest5b

[width=.9]tikztest5c

: Good. gold standard for figuring out lowest common denominator.
: I'd say so, but now we need to see how Haskell does it internally, and how *The Haskell Road...* does it and stop being amateurs.
[laughter]

: I feel like you and the professor are like very strong bakers kneading and kneading and kneading my brain [demonstrates with imaginary brain-dough]
[laughter]
: No, this had really worked out, you, Ursula, racing ahead with the Haskell. And I going ahead with the set theory, and you, Ute, going on ahead with the math logic. I mean, we're definitely making progress. It's just that we have so much to learn!
[affirmations]