

# Probabilistic Categorical Grammar

## From/to Intensional Categories to/from Probabilities

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The verb is the argument-taker par excellence.  
Its form (root, stem, finite) appears to make a difference IN REFERENCE in all languages.

- (1) a. Mary would \*to run/\*runs/run.  
b. Mary wants \*run/\*runs/to run.  
c. Mary \*run/\*to run/runs.

(a) is reference to construction of possible worlds, (b) to such constructed possible worlds, and (c) to actual or imaginarily realized worlds.

Category choice may reveal decision points for the INTENDED reference:

- (2) a. She played the piano for an hour/\*in an hour.
- b. She played the sonata \*for an hour/in an hour.
- c. She played the sonata for a year.
- d. She played the piano in a year.

They can be thought of arising from the following choices:

- (3) a. played ::  $(S \setminus NP) / NP: \lambda x \lambda y. play' xy$
- b. played ::  $(S \setminus NP) / Long / Piece: \lambda y \lambda x \lambda z. perform' (iter' x (play' yz)) z$
- c. played ::  
 $(S \setminus NP) / Period / Instrmnt: \lambda y \lambda x \lambda z. practice' (iter' x (play' yz)) z$
- d. played ::  
 $(S \setminus NP) / PP_{schdl} / NP_{score}: \lambda y \lambda x \lambda z. perform' (iter' x (play' yz)) z$
- e. played ::  
 $(S \setminus NP) / PP_{duration} / NP_{tool}: \lambda y \lambda x \lambda z. practice' (iter' x (play' yz)) z$

Compositionality in combinators may have something to say about these aspects.

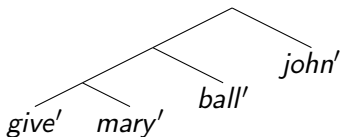
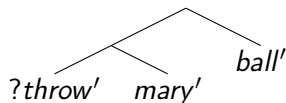
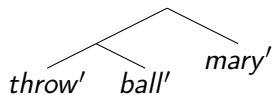
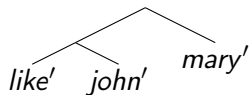
- Thematic structure
- Argument structure
- Word order
- Tense
- Aspect
- Mood

# A brief history of Linguistics

- Thematic structure from argument structure
- Argument structure from thematic structure
- syntax suffices
- semantics suffices
- Categorical grammarians: it is the correspondence, which seems necessary and sufficient
  - argument from necessity: syntactic identity of semantic differences (e.g. control verbs, ECM, raising)
  - argument from sufficiency: composition (surface constituency)
- How do we get syntactic forms, logical forms and phonological forms ON THE FLY?
- These “interfaces” must be quite trivial

- There seems to be a universal conceptual space (things, actions, events)
- That space seems to be asymmetric
- All human languages are probably homomorphic in that space
- Since languages differ in surface structure, surface structure cannot be homomorphic to that space
- That brings us to WORD ORDER
- How do we capture BOTH aspects if parsing is a reflex? (Garrett)

# Asymmetry and structure



$(((\text{give}' \text{mary}') \text{ball}') \text{john}') =$   
 $\text{give}' \text{mary}' \text{ball}' \text{john}'$

(4) a. The window broke.

$init'(broken'w')$

b. The stone broke the window.

$\lambda z.cause'(init'(broken'w'))st' z$

c. The man broke the window with a stone.

$cause'(init'(broken'w'))st'man'$

$init'state'$  : an inchoative event culminating in state  $state'$

What are these primes and LFs? Hypotheses about LOT.



- (5) a. persuades ::  $(S \backslash NP_{3s}) / VP_{to-inf} / NP$  English  
b. promises ::  $(S \backslash NP_{3s}) / VP_{to-inf} / NP$   
c. expects ::  $(S \backslash NP_{3s}) / VP_{to-inf} / NP$   
d. broke ::  $(S \backslash NP) / PP / NP$   
e. broke ::  $(S \backslash NP) / NP$   
f. broke ::  $S \backslash NP$   
g. gwelodd (saw.3s) ::  $(S / NP) / NP_{3s}$  Welsh  
h. iniutusan (order) ::  $(S_{DV} / VP) / NP_{ANG,AGR} / NP_{NG}$  Tagalog  
i. savundum (defended.1s) ::  $S \backslash S'_{acc}$  Turkish  
j. viu (saw.3s) ::  $(S \backslash NP_{3s}) / NP$  Portuguese

(6) a. The window broke.

broke ::  $S \backslash NP : \lambda x. init'(broken' x)$

b. The stone broke the window.

broke ::  $(S \backslash NP) / NP : \lambda x \lambda y \lambda z. cause'(init'(broken' x)) y z$

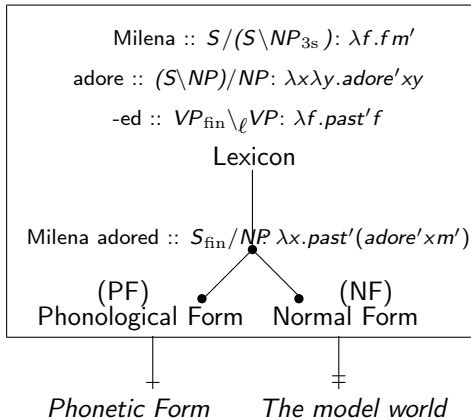
c. The man broke the window with a stone.

$(S \backslash NP) / PP / NP : \lambda x \lambda y \lambda z. cause'(init'(broken' x)) y z$

We get thematic structure as an inference over predicate-argument structure

If surface structure is not homomorphic to universal conceptual structure, how do children learn the word order of their language?

# Linguistic architecture



- projection to constituents of *string* :: *syn:sem*
- serialization of feature geometry from *string* and *syn*
- normalization from *syn* and *sem*
- + realization and intake
- ≠ inference and valuation

- Syntax is a HIDDEN variable for the child (NB. necessity of correspondence).
- She has to infer a CATEGORY from form-meaning pairs only (NB. sufficiency of correspondence).
- These pairs are subject to universal combinatorics. (ANALYSIS)
- These inferences set up a MENTAL GRAMMAR.
- Such grammars are necessarily PROBABILISTIC.
  
- Change the nature of categories, and we might end up with PLANS and SCRIPTS.

- (7) a. *persuades* ::  
 $(S \setminus NP_{3s}) / VP_{to-inf} / NP: \lambda x \lambda p \lambda y. persuade'(px)xy$
- b. *promises* ::  
 $(S \setminus NP_{3s}) / VP_{to-inf} / NP: \lambda x \lambda p \lambda y. promise'(py)xy$
- c. *expects* ::  $(S \setminus NP_{3s}) / VP_{to-inf} / NP: \lambda x \lambda p \lambda y. expect'(px)y$
- d. *broke* ::  $(S \setminus NP) / PP / NP: \lambda x \lambda y \lambda z. cause'(init' broken' x)yz$
- e. *broke* ::  $(S \setminus NP) / NP: \lambda x \lambda y \lambda z. cause'(init' broken' x)yz$
- f. *broke* ::  $S \setminus NP: \lambda x. init'(broken' x)$
- g. *gwelodd (saw.3s)* ::  $(S / NP) / NP_{3s}: \lambda x \lambda y. saw'yx$
- h. *iniutusan (order)* ::  
 $(S_{DV} / VP) / NP_{ANG,AGR} / NP_{NG}: \lambda x \lambda y \lambda p. order'(py)yx$
- i. *savundum (defended.1s)* ::  
 $S_{pd} \setminus S'_{acc}: \lambda p. defend' pi' \wedge topic' i'$
- j. *viu (saw.3s)* ::  $(S \setminus NP_{3s}) / NP_{rex}: \lambda x \lambda y. saw'xy$

# Categorial Word order: not a metrical concept

- (8) a.  $(S \backslash NP) / NP: \lambda x \lambda y. verb' xy$  (SVO)  
b.  $(S / NP) \backslash NP: \lambda x \lambda y. verb' yx$  (SVO')  
c.  $(S / NP) \backslash NP: \lambda x \lambda y. verb' xy$  (OVS)  
d.  $(S \backslash NP) / NP: \lambda x \lambda y. verb' yx$  (OVS')  
e.  $(S \backslash NP) \backslash NP: \lambda x \lambda y. verb' xy$  (SOV)  
f.  $(S \backslash NP) \backslash NP: \lambda x \lambda y. verb' yx$  (OSV)  
g.  $(S / NP) / NP: \lambda x \lambda y. verb' xy$  (VOS)  
h.  $(S / NP) / NP: \lambda x \lambda y. verb' yx$  (VSO)

# Categorial Word order: not a metrical concept

(9) a.	$(S \backslash NP) / NP: \lambda x \lambda y. verb' xy$	(SVO)	English
b.	$(S / NP) \backslash NP: \lambda x \lambda y. verb' yx$	(SVO')	Huastec
c.	$(S / NP) \backslash NP: \lambda x \lambda y. verb' xy$	(OVS)	Hixkaryana
d.	$(S \backslash NP) / NP: \lambda x \lambda y. verb' yx$	(OVS')	Päri
e.	$(S \backslash NP) \backslash NP: \lambda x \lambda y. verb' xy$	(SOV)	Turkish
f.	$(S \backslash NP) \backslash NP: \lambda x \lambda y. verb' yx$	(OSV)	Dyirbal
g.	$(S / NP) / NP: \lambda x \lambda y. verb' xy$	(VOS)	Tagalog
h.	$(S / NP) / NP: \lambda x \lambda y. verb' yx$	(VSO)	Welsh



# A Micro Word-order Thought Experiment

- Syntax is a HIDDEN variable for the child.
- She has to infer a CATEGORY from form-meaning pairs only.
- These pairs are subject to universal combinatorics.  
(ANALYSIS)
- These inferences set up a MENTAL GRAMMAR.
- Such grammars are necessarily PROBABILISTIC.
- All word orders are available to the child IN THE BEGINNING.

`github.com/bozsahin/thebench`

L: Logical Form

S: Sentence

D: Derivation

- Hidden variable problem:  $\theta$  are weights of lexical assumptions

$$\arg \max_L P(L \mid S; \bar{\theta}) = \arg \max_L \sum_D P(L, D \mid S; \bar{\theta}) \quad (1)$$

- Ds are inferred, not observed.
- Relating probabilities and weights (hence categories):

$$P(L, D \mid S; \bar{\theta}) = \frac{e^{\bar{f}(L, D, S) \cdot \bar{\theta}}}{\sum_L \sum_D e^{\bar{f}(L, D, S) \cdot \bar{\theta}}} \quad (2)$$

- $f(L, D, S)$ : some local measure function (use counts etc.)

# Parameter estimation: weight update

- Assume a mental grammar.
- Assume some data to be true (training set)
- What is assumed is form-meaning correspondence
- Parse-to-learn to re-estimate the parameters

# Learning workflow

TRAINING SET

threw the dog mommy : throw (def dog) mommy;  
the dog ate the biscuit : eat (def biscuit) (def dog);  
the cat ate the biscuit : eat (def biscuit) (def cat);  
mommy ate the biscuit : eat (def biscuit) mommy;  
the biscuit ate mommy : eat (def biscuit) mommy; % not favor OVS  
mommy threw the ball : throw (def ball) mommy;  
mommy threw the dog : throw (def dog) mommy;

0.1 ate v := (s\np)\np: \x\y.eat x y; % SVO  
0.1 ate v := (s\np)\np: \x\y.eat y x; % SVO'  
0.1 ate v := (s\np)\np: \x\y.eat y x; % OVS  
0.1 ate v := (s\np)\np: \x\y.eat x y; % OVS'  
0.1 ate v := (s\np)\np: \x\y.eat x y; % SOV  
0.1 ate v := (s\np)\np: \x\y.eat y x; % OSV  
0.1 ate v := (s\np)\np: \x\y.eat x y; % VOS  
0.1 ate v := (s\np)\np: \x\y.eat y x; % VSO

GRAMMAR

PARSER

2.46 ate v := (s\np)\np: \x\y.eat x y; % SVO  
2.71 ate v := (s\np)\np: \x\y.eat y x; % SVO'  
-0.07 ate v := (s\np)\np: \x\y.eat y x; % OVS  
-0.23 ate v := (s\np)\np: \x\y.eat x y; % OVS'  
-1.04 ate v := (s\np)\np: \x\y.eat x y; % SOV  
-1.04 ate v := (s\np)\np: \x\y.eat y x; % OSV  
-1.04 ate v := (s\np)\np: \x\y.eat x y; % VOS  
-1.04 ate v := (s\np)\np: \x\y.eat y x; % VSO

updated GRAMMAR

Log-linear model of structure prediction

$$O(\bar{\theta}) = \sum_{i=1}^n \log P(L_i \mid S_i; \bar{\theta}) = \sum_{i=1}^n \log \left( \sum_D P(L_i, D \mid S_i; \bar{\theta}) \right) \quad (3)$$

## Derivative: How well each parameter serves training

$$\frac{\partial O}{\partial \theta_j} = E_{f_j(L_i, D, S_i)P(D|S_i, L_i; \bar{\theta})} - E_{f_j(L, D, S_i)P(L, D|S_i; \bar{\theta})} \quad (4)$$

Which is

$$\frac{\partial O}{\partial \theta_j} = \sum_{i=1}^n \sum_D f_j(L_i, D, S_i)P(D | S_i, L_i; \bar{\theta}) - \sum_{i=1}^n \sum_L \sum_D f_j(L, D, S_i)P(L, D | S_i; \bar{\theta}) \quad (5)$$

Roughly, the expected value of  
item  $j$ 's contribution to correct parses - contribution to all parses

NB. Both are expected values, not counts.

Too many items: we apply Stochastic Gradient to re-estimate

Initialize  $\bar{\theta}$  to some value. (6)

for  $k = 0 \dots N - 1$

  for  $i = 1 \dots n$

$$\bar{\theta} = \bar{\theta} + \frac{\alpha_0}{1 + c(i + kn)} \frac{\partial \log P(L_i | S_i; \bar{\theta})}{\partial \bar{\theta}}$$

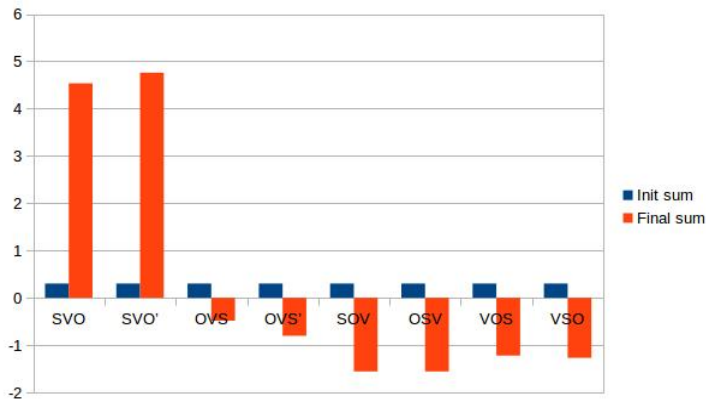


- All possibilities for all verbs are entered in initial mental grammar.
- Give a set of utterances whose meanings are assumed to be known.
- Train the categories on probabilities coming from the parses.
- Some will increase, some decrease, some stay the same.
- How big is the difference from initial grammar?

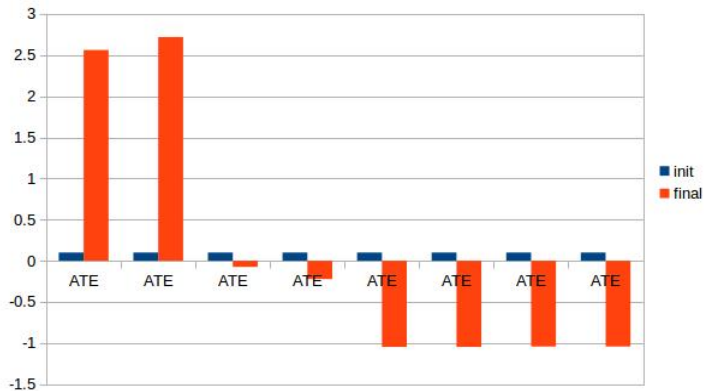
# The training set

```
the dog ate the biscuit : eat (def biscuit) (def dog);
the cat ate the biscuit : eat (def biscuit) (def cat);
mommy ate the biscuit : eat (def biscuit) mommy;
the biscuit ate mommy : eat (def biscuit) mommy;
mommy threw the ball : throw (def ball) mommy;
mommy threw the dog : throw (def dog) mommy;
threw the dog mommy : throw (def dog) mommy;
the dog saw mommy : see mommy (def dog) ;
mommy saw the dog : see (def dog) mommy;
the dog walked : walk (def dog);
the cat slept : sleep (def cat);
the cat walked : walk (def cat);
the dog : def dog;
the cat : def cat;
mommy : mommy;
```

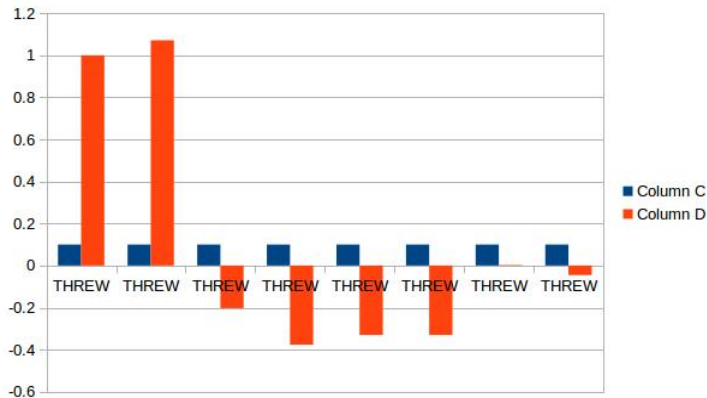
# Verb categories update: 3 verbs total $\times$ 8 categories



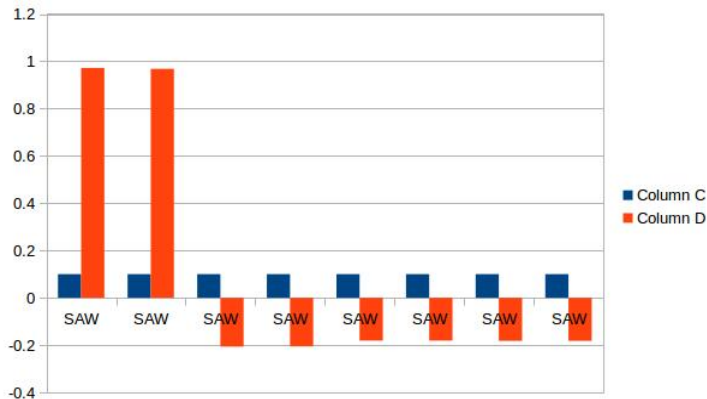
# Ate parameters



# Threw parameters



# Saw parameters



- These conditions are quite realistic for the child.
- A computationally efficient grammatical theory can attack the hidden-variable problem in syntax
  - to fast convergence without brain switches,
  - to meet the timeframe of language acquisition and the size of grammars children learn.
- Having a correspondence is the prerequisite (necessity).
- Having a compositional correspondence may be sufficient.