# A new locally linear embedding scheme in light of Hessian eigenmap

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**Abstract**

We provide a new interpretation of Hessian locally linear embedding (HLLE), revealing that it is essentially a variant way to implement the same idea of locally linear embedding (LLE). Based on the new interpretation, a substantial simplification can be made, in which the idea of "Hessian" is replaced by rather arbitrary weights. Moreover, we show by numerical examples that HLLE may produce projection-like results when the dimension of the target space is larger than that of the data manifold, and hence one further modification concerning the manifold dimension is suggested. Combining all the observations, we finally achieve a new LLE-type method, which is called tangential LLE (TLLE). It is simpler and more robust than HLLE.

## 1 Introduction

Let $\mathcal{X} = \{x_i\}_{i=1}^N$ be a collection of data points in some $\mathbb{R}^D$. The goal of nonlinear dimensionality reduction (or manifold learning) is to find for $\mathcal{X}$ a representation $\mathcal{Y} = \{y_i\}_{i=1}^N$ in some lower dimensional $\mathbb{R}^d$, under the assumption that $\mathcal{X}$ lies on some unknown submanifold $\mathcal{M}$ in $\mathbb{R}^D$.

Among the several existing manifold learning methods, Hessian eigenmap [2], also called Hessian locally linear embedding (HLLE), is one that exhibits prominent performance on the popular synthetic data "Swiss roll with a hole". It can be regarded as a generalization of Laplacian eigenmap [1] in some respect or LLE [4] in another. However, its procedure concerning the construction and minimization of "Hessian" is much more sophisticated.

In this paper, we will provide a new interpretation of the mechanism behind HLLE, revealing that what it really does follows the same idea as LLE: Asking $\mathcal{Y}$ to satisfy the local linear relations for $\mathcal{X}$ as best as possible. The main differences lie in their ways of describing the local linear relations. Roughly speaking, HLLE only fits local

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