NOTES AND INSIGHTS

Improving Loops that Matter

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Abstract

The Loops that Matter (LTM) approach to understanding behavior has proven easy to use and broadly applicable, but it has a shortcoming in its original formulation. This is because the original formulation treats the impact of a flow on a stock relative to the net flow, so that all scores tend to get very large in magnitude as a stock approaches equilibrium, but how big depends strongly on how the flows are specified. By reformulating the link scores from a flow to a stock, this topological dependency is removed. The mathematics behind this approach makes clear the relationship of LTM to the Pathway Participation and Loop Impact analysis methods. The result of this, when applying the analysis to a variety of models, is that the determination of the structure responsible for behavior is clearer, and more clearly tied to work already documented using other techniques.

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Introduction

Understanding models, and therefore reality, from a structure-based feedback perspective is of tantamount importance to the system dynamics method. For over 50 years the field has worked to develop tools and methods to perform automated, objective, loop dominance analysis (Graham, 1977; Forrester, 1982; Eberlein, 1984; Davidsen, 1991; Mojtahedzadeh, 1996; Ford, 1999; Saleh, 2002; Mojtahedzadeh *et al.*, 2004; Güneralp, 2006; Gonçalves, 2009; Saleh *et al.*, 2010; Kampmann, 2012; Hayward and Boswell, 2014; Moxnes and Davidsen, 2016; Oliva, 2016; Sato, 2016; Hayward and Roach, 2017; Naumov and Oliva, 2018; Oliva, 2020 and Schoenberg *et al.*, 2020). A recent development in that long-standing stream of research is the invention of the Loops that Matter (LTM) method (Schoenberg *et al.*, 2020) and for the first time, the inclusion of an automated loop dominance analysis method (i.e. LTM) in commercially available software (Stella Architect & Professional 2.0) (Schoenberg and Eberlein, 2020). LTM advanced the state of the art, not only in its ability to analyze a wide range of models including those with discrete and discontinuous elements, but also by enabling powerful visualizations including animated stock and flow diagrams, as

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well as algorithmically generated, machine simplified, animated causal loop diagrams (Schoenberg, 2019; Schoenberg and Eberlein, 2020).

The LTM method uses link scores (a measure of a link's contribution to behavior) across not only individual links between auxiliaries, but also across links between flows and their stocks while still allowing those scores to be chained together via multiplication. To support this, LTM has two methods for measuring the link score, one for instantaneous connections (stock/auxiliary/flow to auxiliary/flow) and one for integration-based connections (flows to stocks). While these two forms for the link score were designed to measure the same concept, the specific mathematical steps taken to compute them are different, to account for the impact of the integration process.

During continued experimentation, we discovered a problem—the original LTM analysis is sensitive to the structure of the flows into a stock. This means that choices made about the aggregation of flow components (for example combing two flows into a net flow) can make a significant difference in the results of the analysis.

We address this shortcoming by updating the method used to measure the link score between a flow and a stock and demonstrate the efficacy of the improved formulation. We will show that the cause of the identified shortcoming in LTM is a product of the original formulation of the flow to stock link score being based on the value of the flows rather than the change in value. Furthermore, this paper will present an updated analysis of LTM's relationship to existing automated loop dominance analysis techniques such as the Pathway Participation Metric (PPM) (Mojtahedzadeh, 1997; Mojtahedzadeh *et al.*, 2004; Mojtahedzadeh, 2008; Mojtahedzadeh, 2011), and the Loop Impact method (Hayward and Boswell, 2014).

Problem demonstration

The link score is a measure which approximates the link gain and measures the "contribution of a value change in an independent variable to a value change in a dependent variable and also the associated polarity [of that relationship] " (Schoenberg *et al.*, 2020, p. 164). Link scores are measured for all links in the network of model equations, including those which exist between flows and stocks (and therefore represent the integration process). The original method for measuring the link score of a flow (i for inflow, o for outflow) to stock (S) relationship is reproduced below as Eq. (1):

$$Original-Inflow: LS(i \rightarrow S) = \left(\left| \frac{i}{i-o} \right| * 1 \right)$$

$$Original-Outflow: LS(o \rightarrow S) = \left(\left| \frac{o}{i-o} \right| * -1 \right) \tag{1}$$

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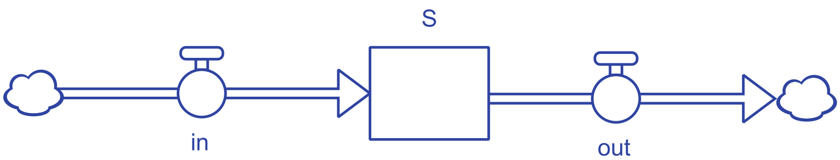

Fig. 1. Diagrammatic depiction of the system demonstrated in Table 1 [Color figure can be viewed at wileyonlinelibrary.com]

In Eq. (1) the contribution of a flow to the change in behavior of a stock is the portion of the net change in the stock resulting from the flow under analysis.

This is in contrast to Eq. (2), which defined the link score $x \rightarrow z$ where z is a flow or auxiliary defined by the equation $z = f(x, y)$.

$$LS(x \rightarrow z) = \begin{cases} \left(\left| \frac{\Delta_x z}{\Delta z} \right| \cdot sign\left(\frac{\Delta_x z}{\Delta x} \right) \right), \\ 0, \Delta z = 0 \text{ or } \Delta x = 0 \end{cases} \tag{2}$$

The first term in Eq. (2) measures the contribution of x to z by reporting the proportion of the change in z which originated from x, where the partial change in z, $\Delta_x z$, is the change in z due to x alone with y held constant. The second term measures the polarity of the link using Richardson's (1995) method. For an in-depth discussion see the section *Defining link scores for links without integration* in Schoenberg *et al*., 2020.

Let us now demonstrate the problem with the original flow to stock form of the link score shown in Eq. (1) using a simple model (pictured in Figure 1) with one stock (S), one inflow (*in*) and one outflow (*out*) with the values shown in Table 1:

Now let us change the network structure of the model but keep it mathematically identical by aggregating the flows into a net flow (*net*) and therefore turning the variables *in* and *out* into auxiliaries (shown in Figure 2). To get a value which is comparable to the link score between *out* and *s*, compute the link score from *out* to *net* which is shown in Table 2 (the link between *net* and S is 1 so it does not change the result when going to the stock).

Table 1. Link score between *out* and S with a disaggregated flow structure using Eq. (1) (Original Formulation). The link score magnitude results when compared to Table 2 demonstrate the flaw with the Original Link Score formulation

Variable	Time 1	Time 2	Link score magnitude Time 2
in	5	10	$\left\lvert \frac{in}{in-out} \right\rvert = \frac{10}{5}$
out	4	5	$\left\lvert \frac{in}{in-out} \right\rvert = \frac{5}{5}$
$\boldsymbol{S} = \int(\boldsymbol{in} - \boldsymbol{out})$	101	106	-
initial = 100			

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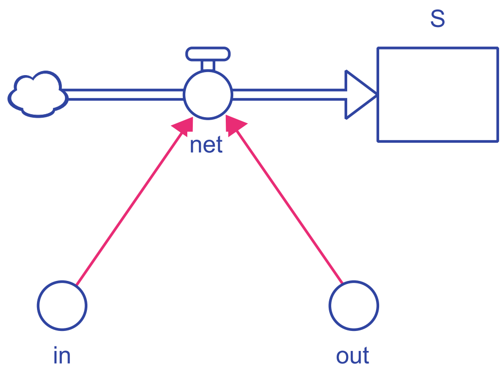

Fig. 2. Diagrammatic depiction of the system demonstrated in Table 2 [Color figure can be viewed at wileyonlinelibrary.com]

Comparing the analyses of the two mathematically equivalent models represented in Tables 1 and 2, we can clearly see that the link score magnitude from *out* or *in* to S is not equivalent (1 does not equal 0.25). The discrepancy is the result of the difference in the way that the link score is calculated for the flow to the stock. Focusing on just the outflow, the original flow to stock link score formulation (Eq. 1) overweighs the impact of the relatively small change in *out* on S. Focus for a moment on the outflow link score calculation demonstrated in Table 1. Using the original flow to stock link score formulation (Eq. 1), we are dividing *out* (5) by the change in S (5) to get a link score magnitude of 1. Now focusing on the instantaneous form of the outflow to net flow relationship demonstrated in Table 2, using the instantaneous link score equation (Eq. 2), we are dividing the change in *net* caused by *out* (−1) by the change in *net* (4) yielding 0.25. This demonstrates that for mathematically equivalent models different conclusions are reached, which is the flaw in having such different calculation methods for determining the link score.

Problem solution: An improvement to calculating the link score from flows to stocks

Ultimately the solution to this problem is simple: Convert all disaggregated flows into a single aggregated net flow, then use a link score of 1 for all net

Table 2. Link score between *out* and *net*. The link score magnitude results when compared to Table 1 demonstrate the flaw with the Original Link Score formulation

Variable	Time 1	Time 2	Variable Change	Partial Change in *net*	Link score magnitude Time 2
in	5	10	$\Delta in = 5$	$\Delta_{in} net = (10-4) - 1 = 5$	$\left\|\frac{\Delta_{in} net}{\Delta \mathrm{net}}\right\| = \frac{5}{4}$
out	4	5	$\Delta out = 1$	$\Delta_{out} net = (5-5) - 1 = -1$	$\left\|\frac{\Delta_{out} net}{\Delta \mathrm{net}}\right\| = \frac{1}{4}$
$net = in$ - *out*	1	5	$\Delta net = 4$	–	–

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flow to stock links. At that point by definition the analysis is completely insensitive to the aggregation level of the flows because the flows are always aggregated before performing the analysis. While tidy, this solution deserves an explanation demonstrating that it is theoretically sound and not arbitrary.

To demonstrate that a special form for measuring the link score between flows and stocks is not necessary, we restate the instantaneous form of the link score (Eq. 2 into Eq. 3), the updated flow to stock link score, accounting for the integration process, but ultimately producing the same set of mathematical operations as if all flows were aggregated into a single net flow. Equation (3) uses i for an inflow, o for an outflow, (S) for the stock, and (t) represents time.

$$Updated-Inflow: LS(i \rightarrow S) = \left(\left|\frac{\Delta i}{\Delta S_t - \Delta S_{t-dt}}\right| * 1\right)$$
$$Updated-Outflow: LS(o \rightarrow S) = \left(\left|\frac{\Delta o}{\Delta S_t - \Delta S_{t-dt}}\right| * -1\right) \tag{3}$$

Comparing the updated flow to stock link score equation (Eq. 3) to the instantaneous link score equation (Eq. 2), Δi and Δo represent the first order partial change in the stock S with respect to the flow. The difference $\Delta S_t - \Delta S_{t-dt}$ is the change in the net flow which is the second order change in the stock S. The flow to stock link score magnitude in Eq. (3) now measures the first order partial change in the stock due to the flow relative to the second order change of the stock. This is a clear conceptual difference from the instantaneous form of the link score, which is necessary to account for the integration process, but from an arithmetic/operational perspective the instantaneous and updated flow to stock link score equations now produce the same set of calculations which is demonstrated in Table 3 using the same example values from above.

Table 3. Link score for *in* and *out* to S using Eq. (3) (Updated Formulation). The link score magnitude results match Table 2, demonstrating the Updated Formulation is not sensitive to the structure of the flows into the stock which was demonstrated in Table 1

Variable	Time 1	Time 2	Variable Change	Link score magnitude Time 2
in	5	10	$\Delta in = 5$	$\left\lvert\frac{\Delta in}{\Delta S_t - \Delta S_{t-dt}}\right\rvert = \frac{5}{4}$
out	4	5	$\Delta out = 1$	$\left\lvert\frac{\Delta out}{\Delta S_t - \Delta S_{t-dt}}\right\rvert = \frac{1}{4}$
$\boldsymbol{S} = \int(\boldsymbol{in} - \boldsymbol{out})$ *initial = 100*	101	106	$\Delta S_t - \Delta S_{t-dt} = 5 - 1$	–

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Observing Table 3 and Table 2, the result, and all the steps along the way, are now the same. Reinterpreting the instantaneous form of the link score, shown in Eq. (2), into the updated flow to stock link score equation, shown in Eq. (3), demonstrates that there is no need for a separate calculation method for measuring the link score between flows and stocks as long as all flows are aggregated during analysis. Ultimately it is now up to the implementor of the LTM method to determine whether or not automated flow aggregation is advantageous to their implementation, because the updated flow to stock link score equation (Eq. 3) demonstrates that the level of flow aggregation is now irrelevant to the analysis.

Placing LTM into the literature

With the removal of the special case of the link score between flows and stocks, the relationship of the link score to the PPM (Mojtahedzadeh, 1997; Mojtahedzadeh *et al.*, 2004; Mojtahedzadeh, 2008; Mojtahedzadeh, 2011) and Loop Impact (Hayward and Boswell, 2014) becomes clearer and easier to understand. The link score is closely related to the PPM with a single key difference in the interpretation of the sign of a link or path score. In the PPM the sign measures the effect that the causal pathway has on the behavior of the stock; a positive value means the behavior of the stock is exponential (increasing or decreasing), whereas a negative value means the stock's behavior is logarithmic (increasing or decreasing). In LTM, the sign measures the structural polarity of the causal pathway.

To demonstrate this relationship lets again look at the link $\mathrm{x} \rightarrow \mathrm{z}$ where z is characterized by the equation $z = f(x,y)$. Assume z is not a stock. The link score for this link is shown in Eq. (2), which can be restated as

$$LS(x \rightarrow z) = \begin{cases} \left(\frac{\Delta_x z}{\Delta \mathrm{x}} \cdot \left| \frac{\Delta \mathrm{x}}{\Delta \mathrm{z}} \right| \right), \\ 0, \Delta z = 0 \text{ or } \Delta x = 0 \end{cases} . \tag{4}$$

Or if we let all our deltas approach 0 (fundamentally dt approaches 0), the result becomes

$$LS(x \rightarrow z) = \begin{cases} \left(\frac{\partial z}{\partial \mathrm{x}} \cdot \left| \frac{\dot{x}}{\dot{z}} \right|, \right), \\ 0, z \doteq 0 \text{ or } x \doteq 0 \end{cases} . \tag{5}$$

This expression of the link score (Eq. 5) contains the gain between adjacent auxiliary variables $\frac{\partial z}{\partial x}$ (Kampmann, 2012, p. 373; Richardson, 1995, p. 75). These link gains are used in the PPM (Mojtahedzadeh *et al.*, 2004, eq. 3) and

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the definition of impact (Hayward and Boswell, 2014, appendix 2). The link gains obey the chain rule of partial differentiation so that $\frac{\partial z}{\partial x}$ remains the gain regardless of the number of auxiliary variables between x and z. Although loop score weights these gains by the value of time derivatives of the variables, $|\frac{\dot{x}}{\dot{z}}|$, these weights cancel each other when applying the chain rule so the path score is always the gain multiplied by the relative time derivative of the two variables.

For the link between a flow and stock, the denominator of Eq. (3) is the change in ΔS over time, or said another way, the second order change in the stock. Letting dt approach zero allows us to restate Eq. (3) as

$$Inflow: LS(i \rightarrow S) = \left| \frac{\frac{di}{dt}}{\frac{d^2 S}{dt^2}} \right|, \tag{6}$$

where we have assumed i is an inflow (there would be a corresponding formula for outflows).

We next consider the link score between adjacent stocks in a causal chain in order to compare the LTM metric with the PPM and Loop Impact.

Let S_1 be a source stock and S_2 be the target stock with flow f. Stock S_1 is connected to S_2 through f. The link score between S_1 and S_2 is the link score between S_1 and f using Eq. (5) multiplied by the link score between f and S_2 using Eq. (6), our revised formula. Ignoring the cases where the derivatives are zero,

$$LS(S_1 \rightarrow S_2) = LS(S_1 \rightarrow f) \times LS(f \rightarrow S_2) = \frac{\partial f}{\partial S_1} \left| \frac{\dot{S}_1}{\dot{f}} \right| \left| \frac{\dot{f}}{\ddot{S}_2} \right| = \frac{\partial f}{\partial S_1} \left| \frac{\dot{S}_1}{\ddot{S}_2} \right|. \tag{7}$$

We note that the PPM and Loop Impact methods are related. Whereas *impact* measures the *absolute* value of the curvature in stock behavior, due to a source stock, the PPM, from which Loop Impact is derived, measures the *relative* change in curvature compared with other influences on the stock. As the link score in Eq. (7) is an absolute measure, we first compare it with the impact between the two stocks. The relationship between stocks S_2 and S_1 can be written as

$$\frac{dS_2}{dt} = f(S_1, \ldots) + \ldots \tag{8}$$

where the ellipsis indicates the possible presence of other variables. The impact between the stocks is obtained by differentiating Eq. (8) (Hayward and Roach, 2017, appendix C; c.f. Mojtahedzadeh *et al.*, 2004, appendix 1):

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$$\frac{d^2S_2}{dt^2}=\frac{\partial f}{\partial S_1}\frac{dS_1}{dt}+\ldots=\left(\frac{\partial f}{\partial S}\frac{\dot{S}_1}{\dot{S}_2}\right)\times\frac{dS_2}{dt}+\ldots \tag{9}$$

Impact measures the contribution of the stock S_1 to the acceleration of S_2 relative to its rate change $\frac{dS_2}{dt}$. Thus, the impact of S_1 on S_2 is given by the bracketed expression in Eq. (9):

$$\text{Impact}(S_1\rightarrow S_2)=\frac{\partial f}{\partial S_1}\frac{\dot{S}_1}{\dot{S}_2} \tag{10}$$

Comparing Eqs. (7) and (10) gives:

$$LS(S_1\rightarrow S_2)=\text{Impact}(S_1\rightarrow S_2)\times\left|\frac{\dot{S}_2}{\ddot{S}_2}\right|\text{Sign}(\dot{S}_1)\text{Sign}(\dot{S}_2). \tag{11}$$

Link score and impact differ in two aspects, the weighting by acceleration of the target stock, $|\frac{\dot{S}_2}{\ddot{S}_2}|$, and the polarity of the link, noted by the presence of the Sign functions in Eq. (11). Link score measures the impact between the stocks relative to the acceleration of the stock due to all influences. If the influence from stock S_1 were the only influence, link score would be unity (Schoenberg *et al*., 2020). Additionally, the polarity of the link score reflects the *structure* of the model, whereas impact (and thus PPM) measures the polarity (curvature) of the link's *behavior*. Ultimately this difference is due to a difference in goals and design. LTM is designed to report the polarity of the causal pathways it measures. By contrast, PPM and Loop Impact are designed to measure if a causal pathway is contributing to or working against a chosen stock's behavior.

For single stock models, the source and target stocks are the same, $S_2=S_1$, and form a first-order loop. Thus, from Eq. (10), impact is the loop gain G_1 as defined by Kampmann, 2012:

$$\text{Impact}(S_1\rightarrow S_1)=\frac{\partial f}{\partial S_1}\triangleq G_1. \tag{12}$$

From Eq. (11), link score is a weighted loop gain:

$$LS(S_1\rightarrow S_1)=\frac{G_1}{\left|\frac{\ddot{S}_1}{\dot{S}_1}\right|}, \tag{13}$$

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referred to as the *loop score*. In models with many loops, both the PPM and LTM present normalized measures of loop influence. The PPM is the percentage loop impact, Eq. (12), compared with all loops on a given stock (Mojtahedzadeh *et al*., 2004; Hayward and Boswell, 2014). In LTM, the relative loop score is the percentage form of the loop score, Eq. (13), (Schoenberg *et al*., 2020). Because the loop scores of all loops in a single stock will be weighted by $|\frac{\ddot{S}_1}{\dot{S}_1}|$, then the relative loop score will be identical to the PPM in single stock models. Thus, we expect LTM to produce the same analysis as the PPM and Loop Impact method for first-order systems.

For models with more than one stock, LTM differs from the PPM and Loop Impact by providing a single measure for the whole loop. By contrast, the other two methods have a measure for each stock in the loop. For example, consider a two-stock loop where S_2 in Equations (10, 11) is connected back to S_1. Using the loop impact theorem (Hayward and Boswell, 2014, appendix 3), the product of loop impacts in the loop equals the loop gain G_2, thus $\text{Impact}(S_1 \to S_2) \times \text{Impact}(S_2 \to S_1) = G_2$. The loop score of the loop is the product of the link scores, which becomes:

$$\begin{aligned} LS(S_1 \to S_1) &= LS(S_1 \to S_2) \times LS(S_2 \to S_1) \\ &= \frac{\text{Impact}(S_1 \to S_2) \times \text{Impact}(S_2 \to S_1)}{\left|\frac{\ddot{S}_1}{\dot{S}_1}\right|\left|\frac{\ddot{S}_2}{\dot{S}_2}\right|} \times \left(\text{Sign}(\dot{S}_1)\text{Sign}(\dot{S}_2)\right)^2 \\ &= \frac{G_2}{\left|\frac{\ddot{S}_1}{\dot{S}_1}\right|\left|\frac{\ddot{S}_2}{\dot{S}_2}\right|}, \end{aligned} \tag{14}$$

Using Eq. (11) and the loop impact theorem. Again, the loop score is directly related to the loop gain. In models with more than one stock, a loop dominance analysis using LTM will give different results to that of the PPM and Loop Impact. However, if the link scores on a single stock were compared, the results would be the same as that of the other two methods, except for the link polarities following the model structure rather than behavior.

Thus, we have shown that LTM is derived from PPM/Loop Impact apart from its treatment of link polarity. But LTM is distinct in its application, measuring dominance across the entire model (or connected components for models which are not fully connected via feedback). It is this very difference between the link score and the PPM, the sign of the link score measuring structural polarity, which yields the ability to chain through multiple stocks (e.g. Eq. 14) making for the largest difference between the other PPM-based methods and LTM. This is what allows for dominance to be measured model-wide as described by Schoenberg *et al*., 2020:

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> [In LTM] We define loop dominance as a concept which relates to the entirety of a model, as opposed to loop dominance being something that affects a single stock. For loop dominance to apply to the entire model, we require that all stocks are connected to each other by the network of feedback loops in the model. For models where there are stocks that do not share feedback loops, we consider each subcomponent of interrelated feedback loops individually, and we refer to each model substructure as having a separate loop dominance profile. [In LTM,] our measurement of loop dominance is specific to the particular time period selected for analysis. We say that a loop (or set of loops) is dominant if the loop(s) describe at least 50% of the observed change in behavior across all stocks in the model over the selected time period. (p. 159)

The implications of Eq. (14), and therefore the meaning of the loop score, are threefold. First, loop scores always measure the structural polarity of loops because of the absolute values of the loop impacts in the denominator. Second, if a stock is not changing (reaching a maximum, minimum, or equilibrium value), e.g. as $\dot{S}_1$ or $\dot{S}_2$ approaches 0, then the loop score approaches 0. As a corollary, when loop gains are 0, the loop score is 0, which means inactive loops are never explanatory. Third, as the acceleration in a stock ceases, for instance at inflection points, when stocks are changing the most, e.g. as $\ddot{S}_1$ or $\ddot{S}_2$ approaches 0, then the loop score approaches infinity. All of this aligns well with the goal of the loop score, to measure the change in the behavior of the stocks in the model. This leads to the understanding that LTM favors loops with large gains, which pass through stocks changing the most. It also demonstrates the necessity of the relative loop score (where the loop scores are normalized across all feedback loops which interact) in order to make the infinities which happen at inflection points more easily interpreted. Ultimately, dominance is a measure of relative importance, and therefore the values of the loop scores are only meaningful in relation to each other as relative numbers. The only exception to that rule may be that the sum of the absolute values of the loops scores may have some use to express the magnitude of the change in the model at an instant in time.

Conclusions

The updates to the LTM method have resolved the identified shortcoming that the aggregation level of flows changes the analysis. This change makes analysis more straightforward and has helped to better situate the LTM method within the preexisting literature, bringing it closer to other PPM based methods in its theoretical underpinnings. This change is included in Stella Architect version 2.1 and all subsequent versions (isee Systems, 2021).

Conflict of interest

No conflict of interest.

Biographies

William (Billy) Schoenberg is a lead software engineer at isee Systems inc. He holds a PhD in system dynamics from the University of Bergen. He is a key member of the team that designs and develops Stella Architect and the isee Exchange. He has always been fascinated by the relationship between structure and behavior and is especially interested in communicating the nature of that relationship.

John Hayward is a visiting fellow in mathematics based at the University of South Wales. Before retirement, he was a lecturer at the university, teaching specialist courses in system dynamics, mathematical modeling, and agent-based methods. He was awarded his BSc in astrophysics and PhD in applied mathematics from the University of London. His research is in the growth of religious and political movements, historical modeling, and integrating mathematical approaches into system dynamics practice.

Robert (Bob) Eberlein is Co-President of isee Systems inc., where he oversees technology development. He holds a PhD from MIT and has worked in the field of system dynamics for over 30 years.

References

Davidsen P. 1991. *The Structure-Behavior Graph. The System Dynamics Group*. MIT: Cambridge, MA.

Eberlein RL. 1984. *Simplifying Dynamic Models by Retaining Selected Behavior Modes* (Doctoral dissertation). Massachusetts Institute of Technology: Cambridge, MA.

Ford D. 1999. A behavioral approach to feedback loop dominance analysis. *System Dynamics Review* **15**(1): 3–36.

Forrester N. 1982. *A Dynamic Synthesis of Basic Macroeconomic Theory: Implications for Stabilization Policy Analysis*. PhD Thesis. Massachusetts Institute of Technology: Cambridge, MA.

Gonçalves P. 2009. Behavior modes, pathways and overall trajectories: Eigenvector and eigenvalue analysis of dynamic systems. *System Dynamics Review* **25**(1): 35–62.

Graham AK. 1977. *Principles of the Relationship between Structure and Behavior of Dynamic Systems*. PhD Thesis. Massachusetts Institute of Technology: Cambridge, MA.

DOI: 10.1002/sdr

Güneralp B. 2006. Towards coherent loop dominance analysis: Progress in eigenvalue elasticity analysis. *System Dynamics Review* **22**(3): 263–289.

Hayward J, Boswell GP. 2014. Model behaviour and the concept of loop impact: A practical method. *System Dynamics Review* **30**(1): 29–57.

Hayward J, Roach PA. 2017. Newton's laws as an interpretive framework in system dynamics. *System Dynamics Review* **33**(3–4): 183–218.

isee Systems (2021). "Stella Architect" version 2.1. Available at: https://iseesystems.com/.

Kampmann CE. 2012. Feedback loop gains and system behaviour (1996). *System Dynamics Review* **28**(4): 370–395.

Mojtahedzadeh MT. 1996. *Structural Analysis of the URBAN1 Model*. University at Albany, SUNY: Albany, NY.

Mojtahedzadeh M. 1997. *A Path Taken: Computer Assisted Heuristics for Understanding Dynamic Systems*. PhD dissertation. University at Albany, SUNY: Albany, NY.

Mojtahedzadeh M. 2008. Do the parallel lines meet? How can pathway participation metrics and eigenvalue analysis produce similar results? *System Dynamics Review* **24**(4): 451–478.

Mojtahedzadeh M. 2011. Consistency in explaining model behavior based on its feedback structure. *System Dynamics Review* **27**(4): 358–373.

Mojtahedzadeh MT, Andersen D, Richardson GP. 2004. Using digest to implement the pathway participation method for detecting influential system structure. *System Dynamics Review* **20**(1): 1–20.

Moxnes E, Davidsen PI. 2016. Intuitive understanding of steady-state and transient behaviors. *System Dynamics Review* **32**(2): 128–153.

Naumov S, Oliva R. 2018. Refinements on eigenvalue elasticity analysis: Interpretation of parameter elasticities. *System Dynamics Review* **34**(3): 426–437.

Oliva R. 2016. Structural dominance analysis of large and stochastic models. *System Dynamics Review* **32**(1): 26–51.

Oliva R. 2020. On structural dominance analysis. *System Dynamics Review* **36**(1): 8–28.

Richardson GP. 1995. Loop polarity, loop dominance, and the concept of dominant polarity. *System Dynamics Review* **11**(1): 67–88.

Saleh MM. 2002. *The characterization of model behavior and its Causal Foundation*. PhD Thesis. University of Bergen: Bergen.

Saleh MM, Oliva R, Kampmann CE, Davidsen PI. 2010. A comprehensive analytical approach for policy analysis of system dynamics models. *European Journal of Operational Research* **203**(3): 673–683.

Sato JB. 2016. *State space analysis of dominant structures in dynamic social systems*. PhD Thesis. University of Washington: St. Louis, MO.

Schoenberg, W. (2019). LoopX: Visualizing and understanding the origins of dynamic model behavior. *arXiv preprint arXiv:1909.01138*.

Schoenberg W, Davidsen P, Eberlein R. 2020. Understanding model behavior using the loops that matter method. *System Dynamics Review* **36**(2): 158–190.

Schoenberg, W., & Eberlein, R. (2020). Seamlessly integrating loops that matter into model development and analysis. *arXiv preprint arXiv:2005.14545*.

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