

# decompr: GVC decomposition in R

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## Abstract

Global Value Chains have become a central unit of analysis in research on international trade. However, the complex matrix transformations at the basis of most Value Chain indicators still constitute a significant entry barrier to the field. The R package **decompr** solves this problem by implementing the algorithms for the analysis of Global Value Chains as R procedures, thereby simplifying the decomposition process.. Two methods for gross export flow decomposition using Inter-Country Input-Output tables are provided. The first method is the **Wang-Wei-Zhu** algorithm which splits bilateral gross exports into 16 value added components. These components can broadly be divided into domestic and foreign value added in exports. The second method concerns a decomposition based on the classical Leontief (1936) insight. It derives the value added origins of an industry's exports by source country and source industry, using easily available gross trade data. This article summarises the methodology of the algorithms, describes the format of the input and output data, and exemplifies the usefulness of the two methods on the basis of a simple example data set.

# 1 Introduction

Hello....is is me you're looking for?

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Lionel Richie

Global Value Chains (GVCs) have become a central topic in research on trade and development policy. Both policy makers and academia increasingly value the growth opportunities GVCs offer to global trade and, especially, to developing countries. However, analysing this phenomenon empirically requires complex matrix manipulations, since the relevant data is only available in the form of gross flows. This package enables researchers with little background in matrix algebra and linear programming to easily derive standard GVC indicators for statistical analysis.

The **decompr** package uses Inter-Country Input-Output tables (ICIOs), such as those published by the OECD and WTO (TiVA), the World Input Output Database (Timmer et al. 2012), or national statistics bureaus, as input. These tables state supply and demand relationships in gross terms between industries within and across countries.

For instance, let us look at the example of the leather used in German manufactured car seats. The ICIOs quantify the value of inputs that the Turkish leather and textiles industry supplies to the German transport equipment industry.

The problem of these gross trade flows, is that they do not reveal how much of the value was added in the supplying industry, and how much of the value was added in previous stages of production, performed by other industries or even countries.

The **Leontief** decomposition of gross trade flows solves this problem by reallocating the value of intermediate goods used by industries to the original producers. In our example, the use of Argentinian agricultural produce (raw hides) is subtracted from the Turkish leather industry and added to the Argentinian agricultural industry.

The **Wang-Wei-Zhu** (henceforth WWZ) decomposition goes a step further by not only revealing the source of the value added, but also breaking down exports into different categories, according to final usage and destination. It implements the theoretical work of Wang, Wei, and Zhu (2014). The high-level divisions in this framework are listed below.

1. domestic value added in exports
2. foreign value added in exports
3. pure double counted terms

## 1.1 Package Details

The **decompr** package implements the algorithms for these decompositions as R procedures and provides example data sets. The R procedures are implemented as functions, the included functions are listed below.

- **load\_tables\_vectors**; transforms the input objects to an object used for the decompositions (class: `decompr`)
- **leontief**; takes a `decompr` object and applies the Leontief decomposition
- **wwz**; takes a `decompr` object and applies the Wang-Wei-Zhu decomposition

- **vertical\_specialisation**; takes a `decompr` object and returns a measure of vertical specialisation (also works with `vertical_specialization`)
- **leontief\_output**; takes a `decompr` object and applies the Leontief decomposition using final output
- **decompr**; a wrapper function which integrates the use of **load\_tables\_vectors** with the various decompositions, using a argument **method** to specify the desired decomposition (default **leontief**)

For legacy purposes, several deprecated functions / methods are available under their original names. In addition to this, two example data sets are included.

- **World Input-Output Database** (Timmer et al. 2012)
- **Leather**; a fictional three-country, three-sector data set

Section 2 introduces the data as it is used by the package, as well as an example data set provided by the WIOD. After which sections 3 and 4 summarise the theoretical derivation for the two decompositions, and show how these can be performed in R using **decompr**. We conclude with a discussion of potential uses and limitations of this approach.

## 2 Data

Two data sets are included in the package, one real world data set, and one minimal data set (fictional), for training purposes. The former is the WIOD regional Inter-Country Input Output tables for the year 2011 (Timmer et al. 2012). The later, is a fictional 3-country 3-sector data set, which we will use throughout this article to demonstrate the usage of the `decompr` package.

```
# load the package
library(decompr)

# load the data
data(leather)

# list the objects in the data set
ls()

## [1] "countries" "final"      "industries" "inter"      "out"
```

The data is set up in order to illustrate the benefits of the decompositions. We do this by following the flows of intermediate goods through a fictional GVC and showing how the readily available gross trade flows differ from the decomposed value added flows. To this end, we construct the elements of the input-output tables such that we have two countries and two industries that focus on upstream tasks, which means they focus on supplying other industries, and one industry and one country that is specialized in downstream tasks, i.e. it serves mainly final demand. In our example the upstream industries are Agriculture and Textiles while the downstream industry is Transport Equipment. Similarly, Argentina and Turkey represent upstream countries with Germany being located downstream within this specific value chain (see table 1).

The first step of the analytic process is to load the input object and create a **decompr** class object which contains the data structures for the

decompositions. This step is not need when using the **decomp** wrapper function, more on this later.

```
# create the decomp object
decomp_object <- load_tables_vectors( x = inter,
                                     y = final,
                                     k = countries,
                                     i = industries,
                                     o = out      )

# inspect the content of the decomp object
ls(decomp_object)

## [1] "A"      "Ad"     "Am"     "B"      "Bd"
## [6] "bigrownam" "Bm"     "E"      "Efd"    "Eint"
## [11] "ESR"     "Exp"    "G"      "GN"     "k"
## [16] "L"       "N"      "rownam" "tot"    "Vc"
## [21] "Vhat"    "X"      "Y"      "Yd"     "Ym"
## [26] "z"       "z01"    "z02"    "z1"     "z2"
```

As can be seen above, a **decomp** class object is in fact a list containing thirty different objects. Depending on the choice of decomposition, all or some of these objects are used.

### 3 Leontief decomposition

Let us now turn to the algorithms, starting with the **Leontief** decomposition. We shortly describe the theoretical derivation of the method to expose the internal steps of the **decomp** package. After which, we turn to the technical implementation. Finally, we describe the output.

#### 3.1 Theoretical derivation

The tools to derive the source decomposition date back to Leontief (1936) who showed that, with a set of simple calculations, national Input-Output tables based on gross terms give the true value added flows between industries. The idea behind this insight is that the production of industry *i*'s output requires inputs of other industries and *i*'s own value added. The latter is the direct contribution of *i*'s output to domestic value added. The former refers to the first round of *i*'s indirect contribution to domestic value added since the input from other industries that *i* requires for its own production triggers the creation of value added in the supplying industries. As supplying industries usually depend on inputs from other industries, this sets in motion a second round of indirect value added creation in the supplying industries of the suppliers, which is also caused by *i*'s production. This goes on until value added is traced back to the original suppliers and can mathematically be expressed as

$$VB = V + VA + VAA + VAAA + \dots = V(I + A + A^2 + A^3 + \dots), \quad (1)$$

which, as an infinite geometric series with the elements of  $A < 1$ , simplifies to

$$VB = V(I - A)^{-1}, \quad (2)$$

Table 1: Example Input-Output Table: Leather

Country	Industry	Argentina				Turkey				Germany				Final Demand		Output
		Agriculture	Textile and Leather	Transport Equipment	Agriculture	Textile and Leather	Transport Equipment	Agriculture	Textile and Leather	Transport Equipment	Argentina	Turkey	Germany			
Argentina	Agriculture	16.1	5.1	1.8	3.2	4.3	0.4	3.1	2.8	4.9	21.5	6.1	8.4	77.7		
Argentina	Textile and Leather	2.4	8.0	3.2	0.1	3.2	1.6	1.2	3.9	11.5	16.2	1.9	5.1	58.3		
Argentina	Transport Equipment	0.9	0.5	4.0	0.0	0.1	0.3	0.0	0.4	0.5	11	0.5	0.8	19.0		
Turkey	Agriculture	1.1	1.9	0.2	18.0	13.2	6.1	9.0	3.1	8.9	7.5	29.5	14.2	112.7		
Turkey	Textile and Leather	0.3	2.8	0.1	6.1	28.1	6.3	2.1	2.5	25.6	8.9	24.9	16.9	124.6		
Turkey	Transport Equipment	0.0	0.1	0.3	4.1	3.2	8.9	0.2	0.0	1.8	1.2	18.5	4.9	43.2		
Germany	Agriculture	1.2	4.2	0.3	4.1	1.2	0.6	29.0	19.5	17.9	9.2	17.9	51.2	156.3		
Germany	Textile and Leather	1.3	1.1	0.0	3.2	4.8	2.6	5.1	29.1	24.1	7.9	10.1	38.5	127.8		
Germany	Transport Equipment	2.1	1.4	3.0	4.1	3.1	3.9	11.3	8.1	51.3	25.1	35.2	68.4	217.0		

Table 2: Non-decomposed Values

	Argentina. Agriculture	Argentina. Textile and. Leather	Argentina. Transport. Equipment	Turkey. Agriculture	Turkey. Textile and. Leather	Turkey. Transport. Equipment	Germany. Agriculture	Germany. Textile and. Leather	Germany. Transport. Equipment
Argentina. Agriculture	6.88	2.49	0.25	1.30	2.04	0.08	0.77	0.68	1.76
Argentina. Textile and. Leather	1.03	3.91	0.44	0.04	1.52	0.31	0.30	0.95	4.13
Argentina. Transport. Equipment	0.38	0.24	0.55	0.05	0.05	0.06	0.00	0.10	0.18
Turkey. Agriculture	0.47	0.93	0.03	7.33	6.27	1.20	2.23	0.75	3.19
Turkey. Textile and. Leather	0.13	1.37	0.01	2.48	13.35	1.24	0.52	0.61	9.19
Turkey. Transport. Equipment	0.00	0.05	0.04	1.67	1.52	1.75	0.05	0.00	0.65
Germany. Agriculture	0.51	2.05	0.04	1.67	0.57	0.12	7.18	4.73	6.43
Germany. Textile and. Leather	0.56	0.54	0.00	1.30	2.28	0.51	1.26	7.06	8.65
Germany. Transport. Equipment	0.90	0.68	0.41	1.67	1.47	0.77	2.80	1.96	18.42

Table 3: Leontief Decomposition

	Argentina. Agriculture	Argentina. Textile and. Leather	Argentina. Transport. Equipment	Turkey. Agriculture	Turkey. Textile and. Leather	Turkey. Transport. Equipment	Germany. Agriculture	Germany. Textile and. Leather	Germany. Transport. Equipment
Argentina. Agriculture	28.52	2.79	0.36	1.81	3.12	0.36	1.24	1.30	4.12
Argentina. Textile and. Leather	1.06	19.12	0.42	0.48	1.83	0.43	0.59	1.15	4.75
Argentina. Transport. Equipment	0.21	0.14	1.06	0.03	0.08	0.04	0.02	0.07	0.19
Turkey. Agriculture	0.72	1.34	0.12	34.93	7.00	1.48	2.55	1.52	6.18
Turkey. Textile and. Leather	0.41	1.39	0.12	2.69	40.17	1.32	1.11	1.15	9.51
Turkey. Transport. Equipment	0.03	0.09	0.03	0.81	0.91	3.16	0.12	0.07	0.65
Germany. Agriculture	0.93	2.25	0.16	2.31	2.06	0.51	29.88	5.25	9.60
Germany. Textile and. Leather	0.65	0.73	0.08	1.54	2.55	0.63	1.46	18.96	8.16
Germany. Transport. Equipment	0.67	0.65	0.26	1.29	1.49	0.57	1.73	1.51	34.74

where  $\mathbf{V}$  is a  $N \times N$  matrix with the diagonal representing the direct value added contribution of the industries,  $\mathbf{A}$  is the Input-Output coefficient matrix with dimension  $N \times N$ , i.e. it gives the direct input flows between industries required for 1\$ of output,  $B = (I - A)^{-1}$  is then the so called Leontief inverse, and  $\mathbf{T}$  indicates a matrix transpose operation.  $\mathbf{VB}$  gives thus a  $N \times N$  matrix of so called value added multipliers, which denote the amount of value added that the production of an industry's 1\$ of output or exports brings about in all other industries. Looking from the perspective of the supplying industries, the matrix gives the value added that they contribute to the using industry's production. If we multiply it with a  $N \times N$  matrix whose diagonal specifies each industry's total output, we get value added origins as absolute values instead of shares.

The application of the Leontief insight to ICIOs as opposed to national Input-Output tables for our source decomposition is straightforward.  $\mathbf{V}$  refers now to a vector of direct value added contribution of all industries across the different countries. Its dimension is correspondingly  $1 \times GN$ .  $\mathbf{A}$  is now of dimension  $GN \times GN$  and gives the industry flows including cross border relationships. Since we are interested in the value added origins of exports we multiply these two matrices with a  $GN \times GN$  matrix whose diagonal we fill with each industry's exports,  $E$ , such that the basic equation behind the source decomposition is given by  $V(I - A)^{-1}E$ . In a simple example with two countries ( $\mathbf{k}$  and  $\mathbf{l}$ ) and industries ( $\mathbf{i}$  and  $\mathbf{j}$ ) we can zoom in to see the matrices' content:

$$V(I - A)^{-1}E = \begin{pmatrix} v_k^i & 0 & 0 & 0 \\ 0 & v_k^j & 0 & 0 \\ 0 & 0 & v_l^i & 0 \\ 0 & 0 & 0 & v_l^j \end{pmatrix} * \begin{pmatrix} b_{kk}^{ii} & b_{kk}^{ij} & b_{kl}^{ii} & b_{kl}^{ij} \\ b_{kk}^{ji} & b_{kk}^{jj} & b_{kl}^{ji} & b_{kl}^{jj} \\ b_{lk}^{ii} & b_{lk}^{ij} & b_{ll}^{ii} & b_{ll}^{ij} \\ b_{lk}^{ji} & b_{lk}^{jj} & b_{ll}^{ji} & b_{ll}^{jj} \end{pmatrix} * \begin{pmatrix} e_k^i & 0 & 0 & 0 \\ 0 & e_k^j & 0 & 0 \\ 0 & 0 & e_l^i & 0 \\ 0 & 0 & 0 & e_l^j \end{pmatrix} =$$

$$\begin{pmatrix} v_k^i b_{kk}^{ii} e_k^i & v_k^i b_{kk}^{ij} e_k^j & v_k^i b_{kl}^{ii} e_l^i & v_k^i b_{kl}^{ij} e_l^j \\ v_k^j b_{kk}^{ji} e_k^i & v_k^j b_{kk}^{jj} e_k^j & v_k^j b_{kl}^{ji} e_l^i & v_k^j b_{kl}^{jj} e_l^j \\ v_l^i b_{lk}^{ii} e_k^i & v_l^i b_{lk}^{ij} e_k^j & v_l^i b_{ll}^{ii} e_l^i & v_l^i b_{ll}^{ij} e_l^j \\ v_l^j b_{lk}^{ji} e_k^i & v_l^j b_{lk}^{jj} e_k^j & v_l^j b_{ll}^{ji} e_l^i & v_l^j b_{ll}^{jj} e_l^j \end{pmatrix} = \begin{pmatrix} vae_{kk}^{ii} & vae_{kk}^{ij} & vae_{kl}^{ii} & vae_{kl}^{ij} \\ vae_{kk}^{ji} & vae_{kk}^{jj} & vae_{kl}^{ji} & vae_{kl}^{jj} \\ vae_{lk}^{ii} & vae_{lk}^{ij} & vae_{ll}^{ii} & vae_{ll}^{ij} \\ vae_{lk}^{ji} & vae_{lk}^{jj} & vae_{ll}^{ji} & vae_{ll}^{jj} \end{pmatrix} \quad (3)$$

$$v_c^s = \frac{va_c^s}{y_c^s} = 1 - a_{kc}^{is} - a_{lc}^{js} - a_{lc}^{js} - a_{lc}^{is} \quad (c \in k, l \quad s \in i, j),$$

$$\begin{pmatrix} b_{kk}^{ii} & b_{kk}^{ij} & b_{kl}^{ii} & b_{kl}^{ij} \\ b_{kk}^{ji} & b_{kk}^{jj} & b_{kl}^{ji} & b_{kl}^{jj} \\ b_{lk}^{ii} & b_{lk}^{ij} & b_{ll}^{ii} & b_{ll}^{ij} \\ b_{lk}^{ji} & b_{lk}^{jj} & b_{ll}^{ji} & b_{ll}^{jj} \end{pmatrix} = \begin{pmatrix} 1 - a_{kk}^{ii} & -a_{kk}^{ij} & -a_{kl}^{ii} & -a_{kl}^{ij} \\ -a_{kk}^{ji} & 1 - a_{kk}^{jj} & -a_{kl}^{ji} & -a_{kl}^{jj} \\ -a_{lk}^{ii} & -a_{lk}^{ij} & 1 - a_{ll}^{ii} & -a_{ll}^{ij} \\ -a_{lk}^{ji} & -a_{lk}^{jj} & -a_{ll}^{ji} & 1 - a_{ll}^{jj} \end{pmatrix}^{-1},$$

and

$$a_{cf}^{su} = \frac{inp_{cf}^{su}}{y_f^u} \quad (c, f \in k, l \quad s, u \in i, j),$$

where  $v_c^s$  gives the share of industry  $s$ 's value added,  $va_c^s$ , in output,  $y_c^s$ , and  $e_k^i$  indicates gross exports. Finally,  $a_{su}^{cf}$  denotes the share of inputs,  $inp_{su}^{cf}$ , in output. The elements of the  $V(I - A)^{-1}E$  or  $vae$  matrix are our estimates for the country-industry level value added origins of each country-industry's exports. **decomp** implements this algorithm into R to automate the process of deriving the matrix. Equipped with it, researchers can calculate standard GVC indicators. Examples include Hummels, Ishii, and Yi (2001)'s Vertical Specialisation ratio at the industry-level which

can be derived by summing for each country and industry across the value added of all foreign countries and industries. Alternatively, the four dimensions of the matrix (source country, source industry, using country, using industry) allow for industry-level gravity-type estimations of value added trade flows.

## 3.2 Implementation

As described, in section 2, the first step of our analytic process is to construct a **decompr** object using the **load\_tables\_vectors** function. After this, we can use the **leontief** function to apply to Leontief decomposition.

```
lt <- leontief( decompr_object )
```

In addition, a wrapper function called **decomp** is provided which integrates both elements of the workflow into a single function. We recommended that the atomic functions be used for large data sets, however, for small data sets this is an easy way to derive the results immediately. The **decomp** function requires a method to be specified (see **help("decomp for details")**), if none is provided, the function will default to **leontief**.

```
lt2 <- decomp( x = inter,
               y = final,
               k = countries,
               i = industries,
               o = out,
               method = "leontief" )
```

Note that the output produced by these two different processes is identical.

## 3.3 Output

We can now analyse the output of the Leontief decomposition, which consists of a  $GN \times GN$  matrix that gives for each country and industry the value added origins of its exports by country and industry. To this end, we look at the results of the Leontief decomposition for our example data set (table 3). In the first column we find the source countries and industries while the first row contains the using countries and industries. The first element, 28.52, thus gives the amount of value added that the Argentinian Agriculture industry has contributed to the exports of the Argentinian Agriculture industry. Similarly, the last element of this row, 4.12, gives the amount of value added that the Argentinian Agriculture industry has contributed to the exports of the German Transport Equipment industry.

A key advantage of the decomposition becomes clear when we compare the decomposed values with the intermediate trade values of the non-decomposed IO table when multiplied with the exports over output ratio to create comparability (table 2). We see for instance that Argentina's Agriculture industry contributes significantly more value added to the German Transport Equipment industry than suggested by the non-decomposed IO table. The reason is that Argentina's Agriculture industry is an important supplier to Turkey's Textile and Leather industry which is in turn an important supplier for the German Transport Equipment industry. The decomposition thus allows us to see how the value added flows along this Global Value Chain.

We can also take look at specific industries. For instance, we find that the non-decomposed values of the Transport Equipment are for many elements larger than the value added elements while the opposite holds for Agriculture. This emphasises the fact that Transport Equipment is a downstream industry that produces mostly final goods. Agriculture on the other hand qualifies as an upstream industry that produces also many intermediate goods so that its value added in other industries is typically large.

Finally let's consider the countries of our specific example. We see that Germany has more instances in which the non-decomposed values are above the value added flows than Argentina and Turkey combined. Along the lines of the industry analysis, this shows that Germany focuses within this GVC on downstream tasks producing mostly final goods that contain value added from countries located more upstream. In our example these are Turkey and Argentina.

## 4 Wang-Wei-Zhu decomposition

The Wang-Wei-Zhu decomposition builds upon the Leontief insight but uses, in addition, further valuable information provided in ICIOS. More specifically, the Leontief decomposition traces the value added back to where it originates but ICIOS also contain data on how the value added is subsequently used. This information is extracted by the Wang-Wei-Zhu decomposition, which thereby allows a much more detailed look at the structures of international production networks and the respective positions of countries and industries within them.

### 4.1 Theoretical derivation

The derivation of the Wang-Wei-Zhu decomposition is significantly more technical than the source decomposition since it splits gross exports up more finely. This is why we present here only the final equation for a two country one industry model (equation 22 in WWZ) and refer the interested reader to the original paper by Wang, Wei, and Zhu (2014). The key idea is to use the Leontief insight and extend it using additional information from ICIOs on the final usage and destination of the exports



(e.g. re-imported vs. absorbed abroad).

$$\begin{aligned}
E^{kl} = & \left( V^k B^{kk} \right)^T * F^{kl} + \left( V^k L^{kk} \right)^T * \left( A^{kl} B^{ll} F^{ll} \right) + \left( V^k L^{kk} \right)^T * \left( A^{kl} \sum_{t \neq k, l}^G B^{lt} F^{tt} \right) \\
& + \left( V^k L^{kk} \right)^T * \left( A^{kl} B^{ll} \sum_{t \neq k, l}^G F^{lt} \right) + \left( V^k L^{kk} \right)^T * \left( A^{kl} \sum_{t \neq k}^G \sum_{l, u \neq k, t}^G B^{lt} F^{tu} \right) \\
& + \left( V^k L^{kk} \right)^T * \left( A^{kl} B^{ll} F^{lk} \right) + \left( V^k L^{kk} \right)^T * \left( A^{kl} \sum_{t \neq k, l}^G B^{lt} F^{tk} \right) + \left( V^k L^{kk} \right)^T * \left( A^{kl} B^{lk} F^{kk} \right) \\
& + \left( V^k L^{kk} \right)^T * \left( A^{kl} \sum_{t \neq k}^G B^{lk} F^{kt} \right) + \left( V^k B^{kk} - V^k L^{kk} \right)^T * \left( A^{kl} X^l \right) + \left( V^l B^{lk} \right)^T * F^{kl} \\
& + \left( V^l B^{lk} \right)^T * \left( A^{kl} L^{ll} F^{ll} \right) + \left( V^l B^{lk} \right)^T * \left( A^{kl} L^{ll} E^{l*} \right) + \left( \sum_{t \neq k, l}^G V^t B^{tk} \right)^T * F^{kl} \\
& + \left( \sum_{t \neq k, l}^G V^t B^{tk} \right)^T * \left( A^{kl} L^{ll} F^{ll} \right) + \left( \sum_{t \neq k, l}^G V^t B^{tk} \right)^T * \left( A^{kl} L^{ll} E^{l*} \right),
\end{aligned} \tag{4}$$

where  $F^{kl}$  is the final demand in  $l$  for goods of  $k$  and  $L^{ll}$  refers to the national Leontief inverse as opposed to the Inter-Country inverse  $B$ . As can be seen from equation (4), the Wang-Wei-Zhu decomposition splits gross exports into 16 terms with three main categories given by domestic value added in exports (**DVIX.B**), foreign value added in exports (**FVA**), and purely double counted terms (**PDC**). The main categories are further divided according to their final destination so that the final decomposition is given by:

- Domestic Value Added absorbed abroad (**VAX\_G**, T1-5)
  - Domestic Value added in final exports (**DVA\_FIN**, T1)
  - Domestic Value added in intermediate exports
    - \* Domestic Value added in intermediate exports absorbed by direct importers (**DVA\_INT**, T2)
    - \* Domestic Value added in intermediate exports re-exported to third countries (**DVA\_INTrex**, T3-5)
      - Domestic Value added in intermediate exports re-exported to third countries as intermediate goods to produce domestic final goods (**DVA\_INTrexI1**, T3)
      - Domestic Value added in intermediate exports re-exported to third countries as final goods (**DVA\_INTrexF**, T4)
      - Domestic Value added in intermediate exports re-exported to third countries as intermediate goods to produce exports (**DVA\_INTrexI2**, T5)
- Domestic Value Added returning home (**RDV\_B**, T6-8)
  - Domestic Value Added returning home as final goods (**RDV\_FIN**, T6)
  - Domestic Value Added returning home as final goods through third countries (**RDV\_FIN2**, T7)
  - Domestic Value Added returning home as intermediate goods (**RDV\_INT**, T8)

- Foreign Value added (**FVA**, T11-12/14-15 )
  - Foreign Value added in final good exports (**FVA\_FIN**, T11/14)
    - \* Foreign Value added in final good exports sourced from direct importer (**MVA\_FIN**, T11)
    - \* Foreign Value added in final good exports sourced from other countries (**OVA\_FIN**, T14)
  - Foreign Value added in intermediate good exports (**FVA\_INT**, T12/15)
    - \* Foreign Value added in intermediate good exports sourced from direct importer (**MVA\_INT**, T12)
    - \* Foreign Value added in intermediate good exports sourced from other countries (**OVA\_INT**, T15)
- Pure Double Counting (**PDC**, T9-10/13/16)
  - Pure double counting from domestic source (**DDC**, T9-10)
    - \* Due to final goods exports production (**DDF**, T9)
    - \* Due to intermediate goods exports production (**DDI**, T10)
  - Pure double counting from foreign source (**FDC**, T13/16)
    - \* Due to direct importer exports production (**FDF**, T13)
    - \* Due to other countries' exports production (**FDI**, T16)

The higher resolution of the WWZ decomposition comes at the cost of a lower dimension (source country, using country, using industry) since the current, highly aggregated, ICIOs render a four-dimensional decomposition unfeasible. This means that the two methods are complementary and imply a trade-off between detail and disaggregation.

## 4.2 Implementation

As with the **leontief** function, the **wwz** function also takes a **decompr** class object as its input, the procedure for this is described in section 2. After having created this **decompr** object, we can apply the Wang-Wei-Zhu decomposition using the **wwz** function.

```
w <- wwz(decompr_object)
```

Furthermore, it is also possible to derive the results of the Wang-Wei-Zhu decomposition directly, using the **decomp** function.

```
w2 <- decomp( x = inter,
              y = final,
              k = countries,
              i = industries,
              o = out,
              method = "wwz" )
```

Both these processes will yield the same results.

## 4.3 Output

The output when using the WWZ algorithm is a matrix with dimensions GNGx19 whereby 19 is the 16 objects the WWZ algorithm decomposes exports into, plus three checksums. GNG represents source country, source industry and using country whereas these terms are slightly ambiguous

here due to the complex nature of the decomposition. More specifically, the using country can also be the origin of the foreign value added in the exports of the source country to the using country (see for example T11 and T12). Therefore we use the terms exporter, exporting industry, and direct importer instead. This becomes clearer when we take a look at specific examples.

Table 4 shows the results for the example data. The first column lists exporter, exporting industry, and direct importer. Note that the value added is domestic but not necessarily created in the exporting industry. When exporter and importer are identical, the values are zero since there are no exports. The first row lists the 16 components of bilateral exports at the sector level and three checksums.

The first eight components relate to domestic value added of the exporting country contained in the sectoral exports of the exporting industry to the direct importer. For instance, the first non-zero element in table 4 refers to *DVA\_FIN* or domestic value added in final good exports. It shows that there are 5.47 units of domestic value added in the exports of final goods from Argentina's Agriculture industry to Turkey. In the same row the third term, *DVA\_INTrexI1*, is slightly more complicated. As mentioned above it gives the amount of domestic Value added in intermediate exports re-exported to third countries as intermediate goods to produce domestic final goods. In our example this means that there are 1.14 units of domestic value added in the intermediate exports of Argentina's Agriculture industry to Turkey that are re-exported by Turkey as intermediates to a third country which produces final goods with it. Terms six to eight concern domestic value added that eventually returns home. *RDV\_FIN2* reveals for example that there are 0.35 units of domestic value added in the intermediate exports of Argentina's Agriculture industry to Turkey that Turkey re-exports as intermediates to Argentina for the latter's final good production.

The following four terms apply to foreign value added in exports and separate on the one hand between the origin of the foreign value added (*MVA* vs *OVA*) and on the other hand between the type of export (intermediate vs final good). *MVA* describes hereby foreign value added sourced by the exporting country from the direct importer. From the perspective of the latter, these terms are thus part of the *RDV* (value added returning home) share. *OVA* in contrast sums over the foreign value added sourced from all other countries. Going back to the example, this means that there 0.21 units of Turkish value added in the final goods exports of Argentina's Agriculture industry to Turkey.

Terms 13 to 16 collect the double counting of gross trade statistics that occurs when goods cross borders multiple times. *DDC* captures double counting due to domestic value added, which is further classified according to the type of the ultimate export (final vs intermediate good). *MDC* and *ODC* on the other hand capture double counting due to foreign value added from either the direct importer or other countries. For the Argentina-Turkey case this implies for instance that there are 0.18 units of value added in the intermediate exports of Turkey to Argentina which are re-exported by Argentina's Agriculture industry to Turkey as intermediates and then again re-exported. Since they would be part of *MVA* twice, they are now counted once as double-counted term.

Finally, the three checksums give total exports, total final goods exports, and total intermediate exports. The difference between the first and the latter two should be zero.

Table 4: WWZ Decomposition

exporter.exportingind.importer	DVA.FIN	DVA.INT	DVA.INTx1	DVA.INTx2	RDV.INT	RDV.FIN	RDV.FIN2	OVA.FIN	MVA.FIN	OVA.INT	MVA.INT	DVC.FIN	DVC.INT	ODC
Argentina.Agriculture.Argentina	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Argentina.Agriculture.Turkey	5.47	2.68	1.14	1.41	0.17	0.71	0.35	0.41	0.21	0.20	0.10	0.06	0.07	0.34
Argentina.Agriculture.Germany	7.54	5.11	0.41	2.07	0.18	0.24	1.41	0.08	0.30	0.57	0.19	0.37	0.09	0.18
sub.TOTAL	13.01	7.79	1.55	3.48	0.69	0.41	2.11	0.43	0.71	0.78	0.39	0.48	0.15	0.52
Argentina.Textile.and.Leather.Argentina	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Argentina.Textile.and.Leather.Turkey	1.47	1.61	0.52	0.74	0.24	0.08	0.33	0.17	0.24	0.19	0.26	0.03	0.08	0.36
Argentina.Textile.and.Leather.Germany	3.95	6.45	0.54	2.82	0.24	0.32	1.98	0.11	0.50	0.65	0.81	1.05	0.11	0.28
sub.TOTAL	5.42	8.06	1.06	3.56	0.48	0.40	2.32	0.27	0.75	0.84	1.07	1.25	0.13	0.37
Argentina.Transport.Equipment.Argentina	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Argentina.Transport.Equipment.Turkey	0.35	0.15	0.03	0.05	0.01	0.00	0.02	0.01	0.10	0.05	0.04	0.02	0.00	0.04
Argentina.Transport.Equipment.Germany	0.57	0.32	0.03	0.13	0.01	0.09	0.01	0.08	0.15	0.05	0.09	0.01	0.03	0.04
sub.TOTAL	0.92	0.47	0.05	0.18	0.02	0.11	0.01	0.18	0.20	0.09	0.11	0.01	0.04	0.08
Turkey.Agriculture.Argentina	6.28	1.12	0.42	0.32	0.13	0.15	0.17	0.18	0.84	0.38	0.15	0.07	0.11	0.21
Turkey.Agriculture.Turkey	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Turkey.Agriculture.Germany	11.89	9.19	0.44	2.46	0.10	0.69	3.74	0.06	0.72	1.59	0.55	1.22	0.45	0.51
sub.TOTAL	18.17	10.31	0.87	2.79	0.23	0.85	3.91	0.24	1.56	1.97	0.70	1.28	0.56	0.72
Turkey.Textile.and.Leather.Argentina	7.23	1.05	0.46	0.30	0.14	0.15	0.13	0.20	0.92	0.76	0.13	0.11	0.10	0.20
Turkey.Textile.and.Leather.Turkey	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Turkey.Textile.and.Leather.Germany	13.72	12.01	0.63	3.91	0.13	0.95	5.58	0.07	1.44	1.74	1.25	1.51	0.60	1.32
sub.TOTAL	20.95	13.05	1.09	4.20	0.27	1.10	5.71	0.28	2.35	2.50	1.38	1.62	0.70	1.51
Turkey.Transport.Equipment.Argentina	0.84	0.18	0.02	0.02	0.01	0.01	0.01	0.01	0.24	0.12	0.05	0.03	0.01	0.02
Turkey.Transport.Equipment.Turkey	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Turkey.Transport.Equipment.Germany	3.43	0.65	0.04	0.22	0.01	0.05	0.31	0.00	0.48	0.99	0.09	0.20	0.03	0.09
sub.TOTAL	4.27	0.83	0.06	0.24	0.01	0.06	0.32	0.01	0.72	1.11	0.15	0.22	0.04	0.11
Germany.Agriculture.Argentina	7.86	2.02	0.28	0.28	0.06	0.82	0.57	0.13	0.90	0.44	0.23	0.11	0.61	0.10
Germany.Agriculture.Turkey	15.29	2.06	0.12	0.48	0.02	0.74	0.97	0.03	0.86	1.75	0.11	0.23	0.53	0.10
Germany.Agriculture.Germany	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
sub.TOTAL	23.16	4.08	0.40	0.76	0.08	1.56	1.54	0.16	1.76	2.19	0.34	0.35	1.14	0.20
Germany.Textile.and.Leather.Argentina	6.55	0.79	0.12	0.15	0.03	0.31	0.26	0.06	0.70	0.65	0.08	0.22	0.05	0.13
Germany.Textile.and.Leather.Turkey	8.38	3.69	0.19	0.79	0.02	1.22	1.70	0.05	0.82	0.90	0.36	0.39	0.92	0.50
Germany.Textile.and.Leather.Germany	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
sub.TOTAL	14.93	4.48	0.31	0.94	0.05	1.53	1.96	0.10	1.53	1.54	0.45	0.47	1.15	0.27
Germany.Transport.Equipment.Argentina	16.92	2.37	0.18	0.26	0.04	0.44	0.43	0.08	5.26	2.92	0.78	0.43	0.31	0.26
Germany.Transport.Equipment.Turkey	23.72	3.27	0.15	0.61	0.02	0.91	1.37	0.04	4.10	7.38	0.59	1.06	0.67	0.45
Germany.Transport.Equipment.Germany	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
sub.TOTAL	40.64	5.64	0.33	0.87	0.06	1.34	1.80	0.12	9.36	10.30	1.36	1.49	0.99	0.71

## 5 Conclusion

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