



The Balance between Excitation and inhibition in the Brain

Lin Xiaohan

11/17/23



Historical Note



- Rate Coding vs. Temporal Coding
 - **Rate coding:** information is conveyed via the average rate of spikes;
 - Neurons → integrator
 - **Temporal coding:** the precise timing of spikes conveys information.
 - Neurons → coincidence detector

Historical Note



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The Highly Irregular Firing of Cortical Cells Is Inconsistent with Temporal Integration of Random EPSPs

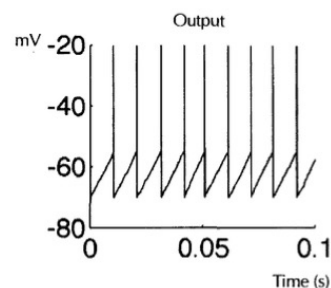
Softky & Koch 1993

William R. Softky^{1,2} and Christof Koch²

¹Division of Physics, Mathematics, and Astronomy and ²Computation and Neural Systems Program, California Institute of Technology, Pasadena, California 91125

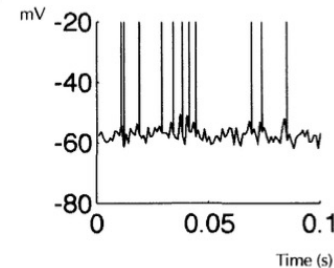
Independent, random EPSPs induce highly regular firing patterns, inconsistent with experimental findings; Concluded neurons were coincidence detectors.

(a) Counts of 300 EPSPs



LIF neuron

(b) Coincidence of 35 EPSPs in 1m



Experimental data

Historical Note

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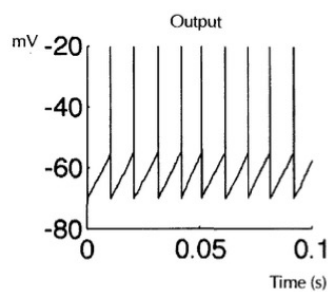
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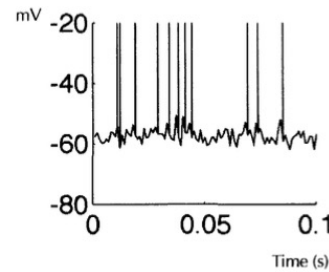
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LIF neurons (and all integrator models) are wrong!

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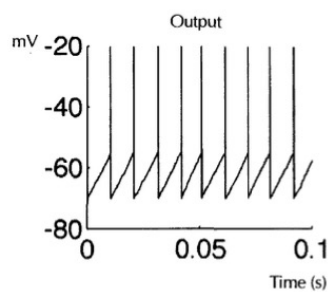
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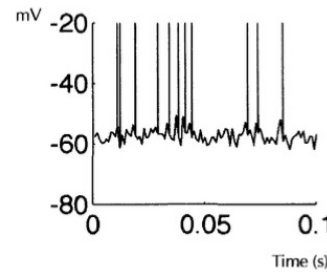
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Or are they?

Historical Note



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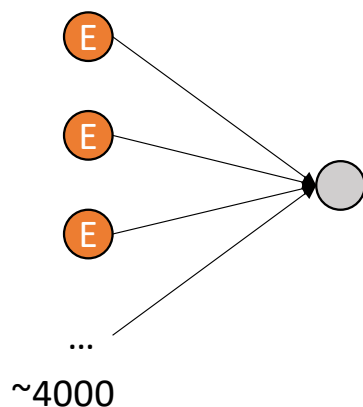
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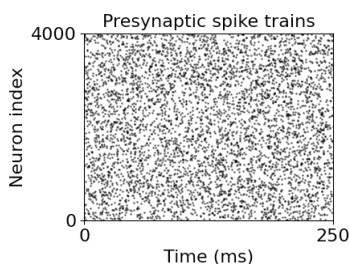
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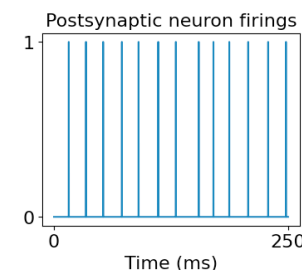
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Central Limit Theorem



(almost)
constant input
current



Historical Note



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- We need to take inhibition into account.

Noise, neural codes and cortical organization

Michael N Shadlen and William T Newsome

Stanford University School of Medicine, Stanford, USA

(Shadlen & Newsome 1994)

Membrane potential undergoes a **random walk** under balanced EPSPs and IPSPs.

Historical Note

- We need to take inhibition into account.

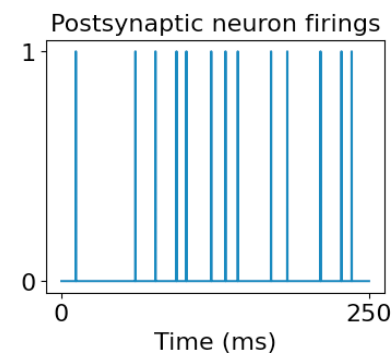
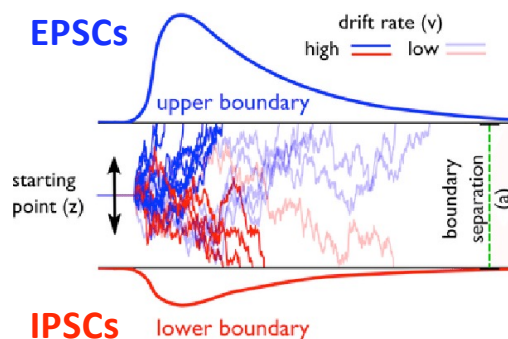
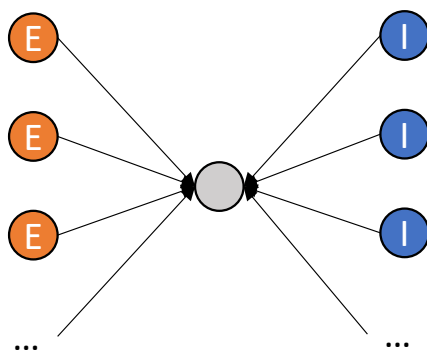
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Sanity Check



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Biological Constraints

Resting membrane potential	$\sim -70\text{mV}$
Spike threshold	$\sim -55\text{mV}$
Average EPSPs	$\sim -0.55\text{mV}$

About 30 EPSPs can initiate an action potential.

Number of cortical neuron contacts	~ 4000
Excitatory synapses ratio	$\sim 85\%$
Spontaneous activity in cortex	$\sim 3\text{ Hz}$
Neuron membrane time constant τ	$\sim 10\text{ ms}$

A neuron receives about 100 EPSPs per τ on average.

Coincidence detector model would need $\tau \sim 1\text{ms}$ to avoid the accumulation of EPSP on membrane potential.

Random-walk model is more biologically plausible.

Simulation Time

`irregular_inconsistent_integrator_Softky_Koch_1993.ipynb`

`random_walk_Shadlen_Newsome_1994.ipynb`

Talk is cheap, what about experimental evidence?

Experimental Evidence



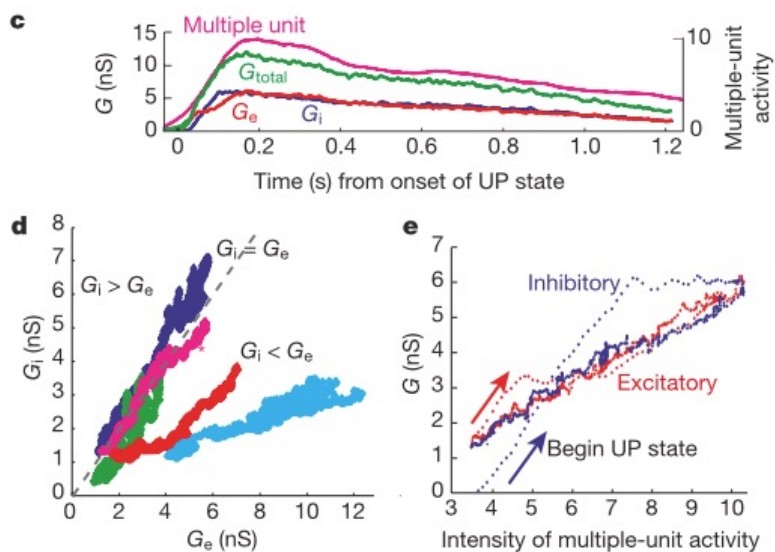
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Turning on and off recurrent balanced cortical activity

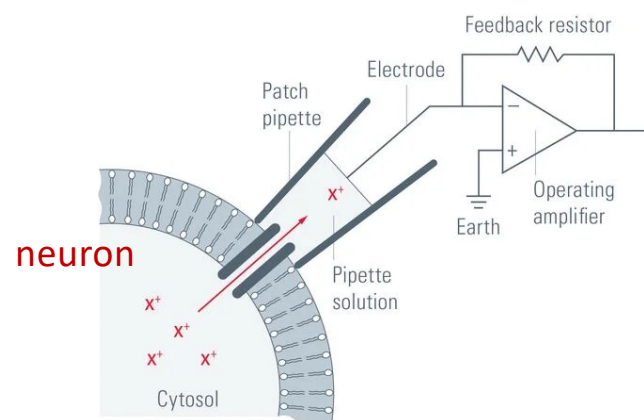
(Shu et al., 2003)

Yousheng Shu, Andrea Hasenstaub & David A. McCormick

Department of Neurobiology, Yale University School of Medicine, 333 Cedar Street,
New Haven, Connecticut 06510, USA



Patch clamp



Experimental Evidence



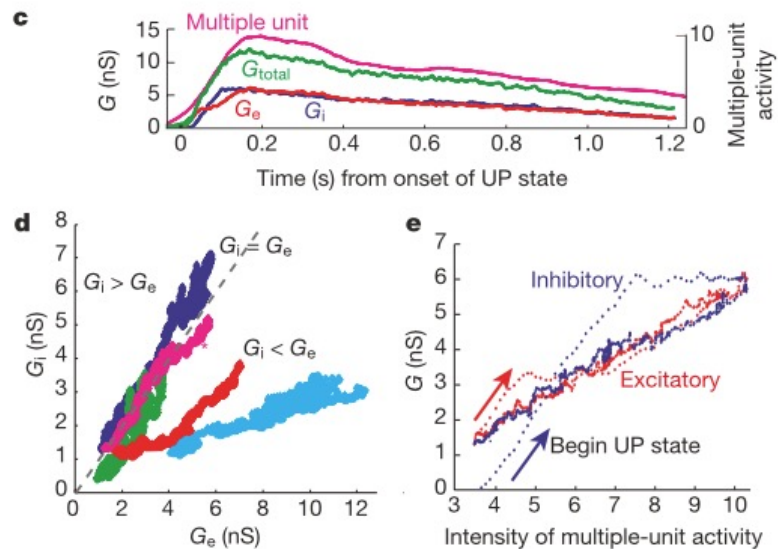
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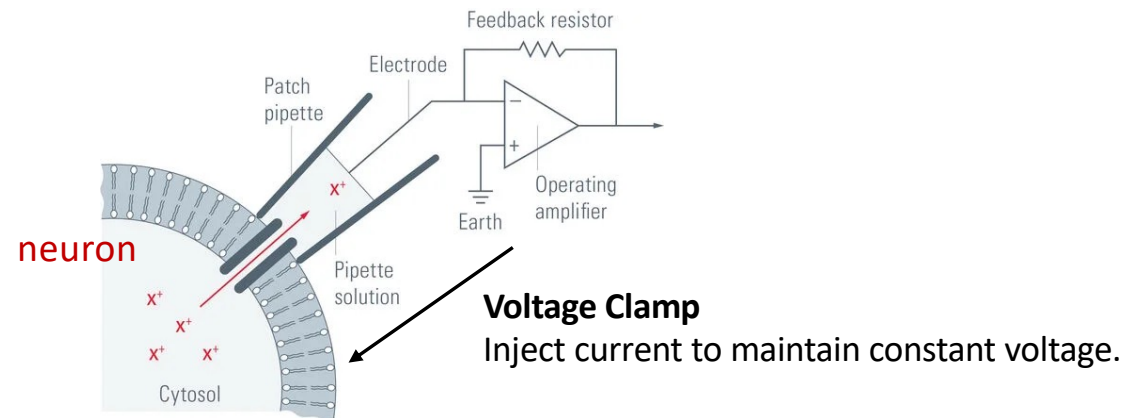
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Patch clamp { Voltage clamp
Current clamp



Experimental Evidence



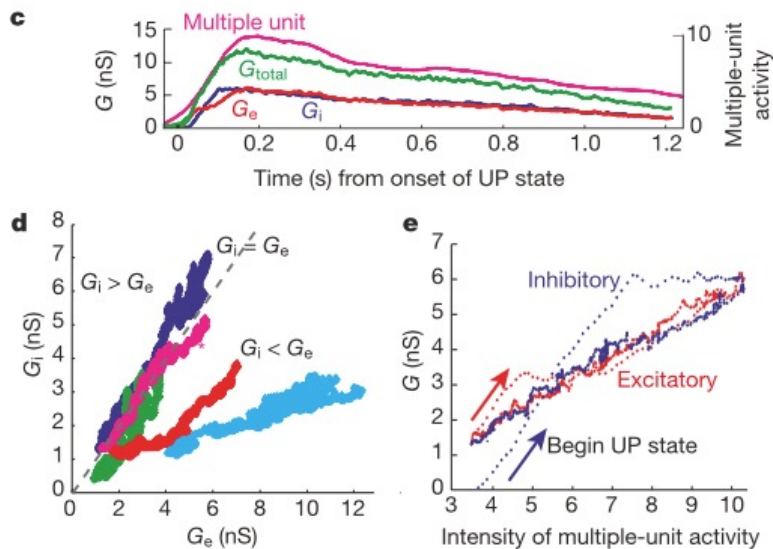
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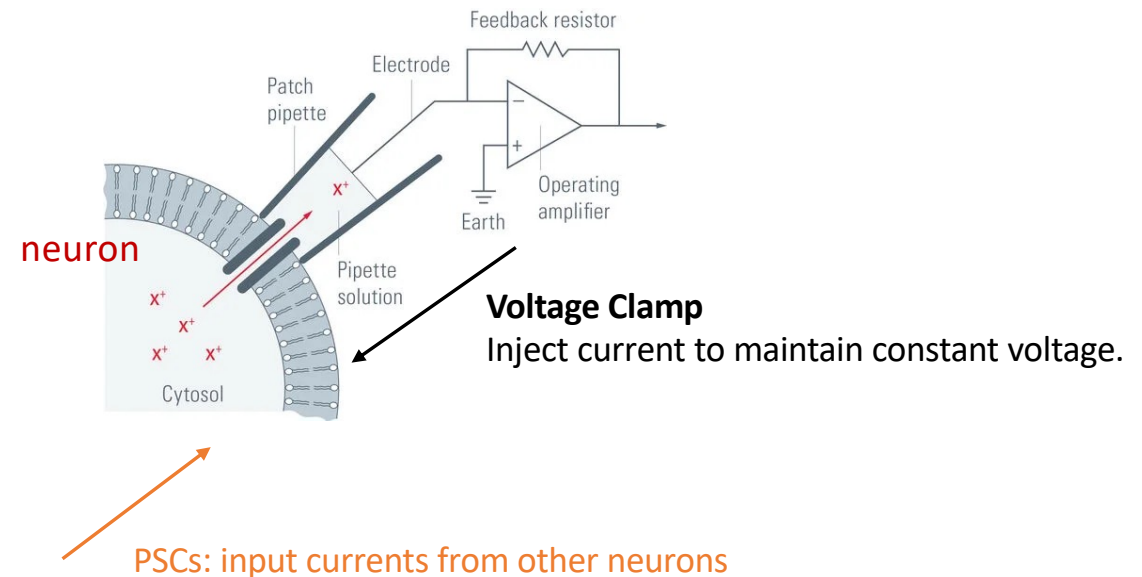
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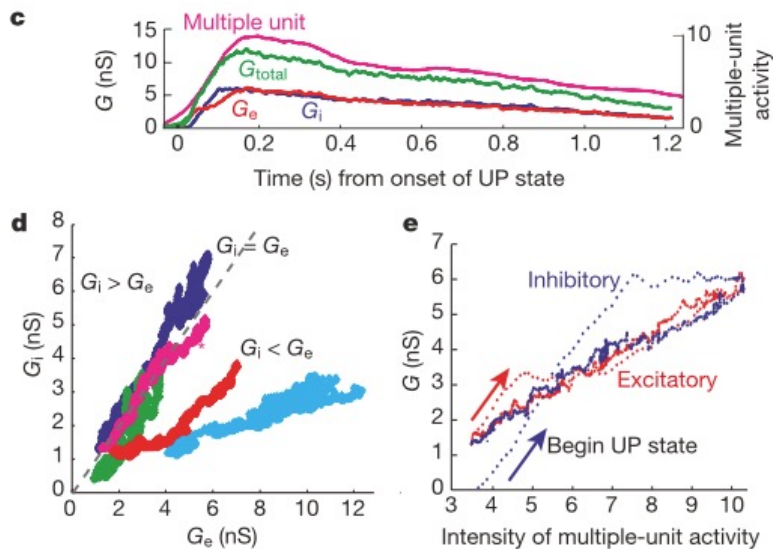
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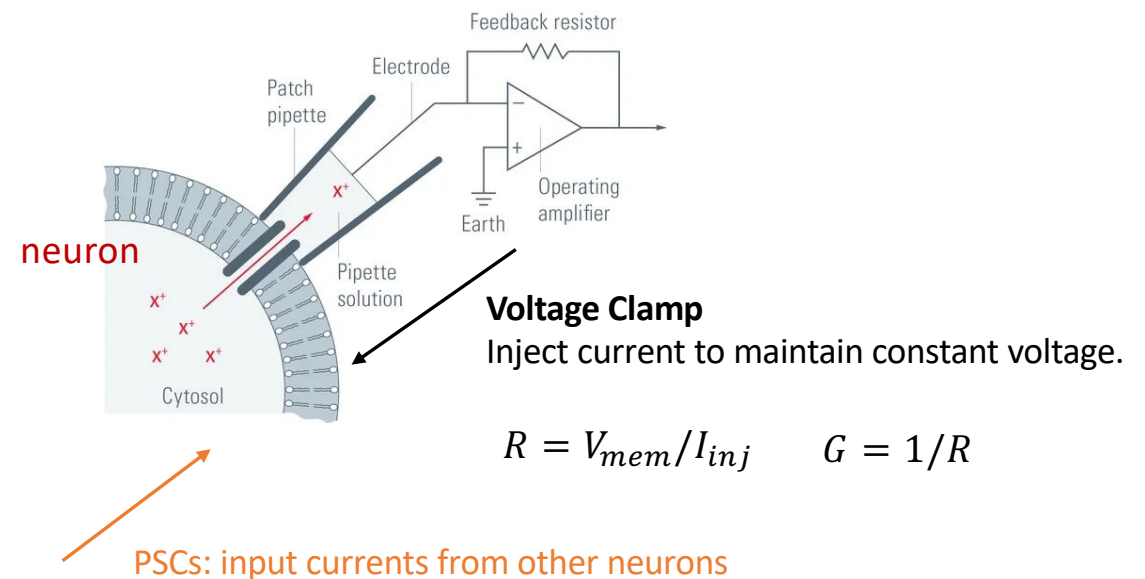
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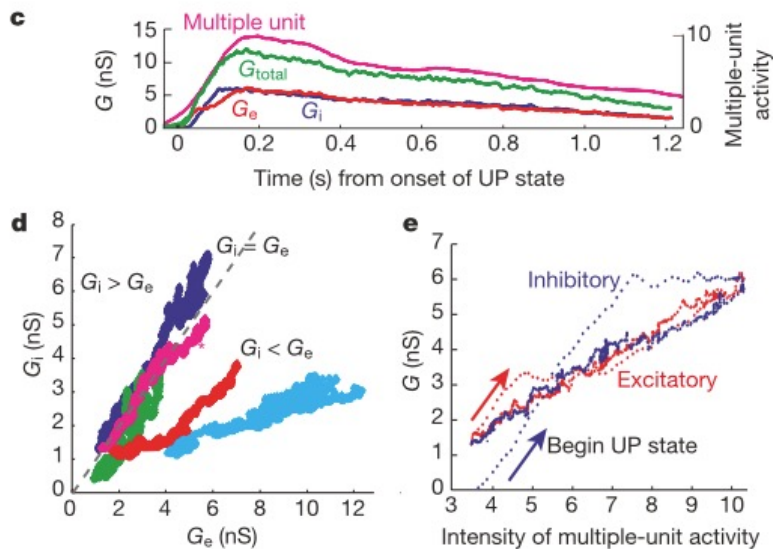
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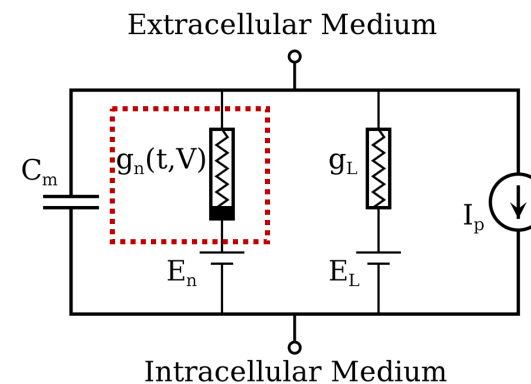
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$$R = V_{mem}/I_{inj} \quad G = 1/R$$



Equivalent circuit of a neuron

$$I = C_m \frac{dV_m}{dt} + g_K(V_m - V_K) + g_{Na}(V_m - V_{Na}) + g_l(V_m - V_l)$$

HH model

Experimental Evidence

Turning on and off recurrent balanced cortical activity

(Shu et al., 2003)

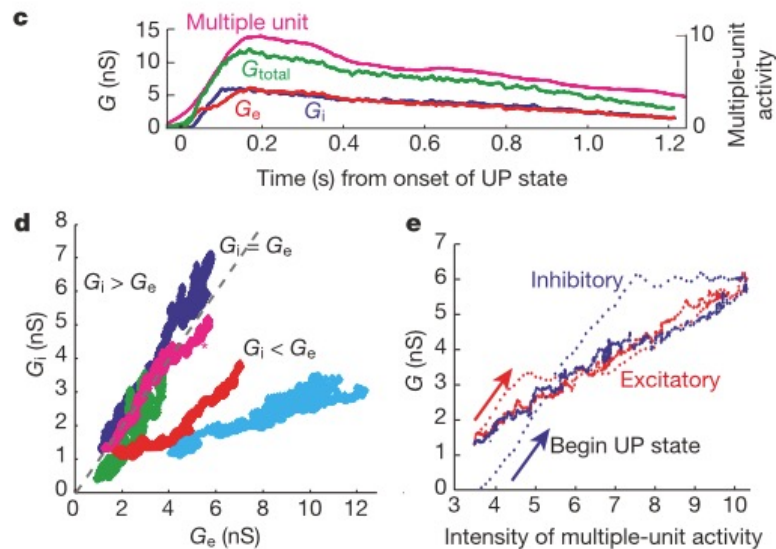
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The conductance of excitatory channels (e.g. Na^+) and inhibitory channels (e.g. K^+ , Cl^-) change together.



EPSCs and IPSCs are highly correlated, and thus balanced, in a single neuron.



Experimental Evidence



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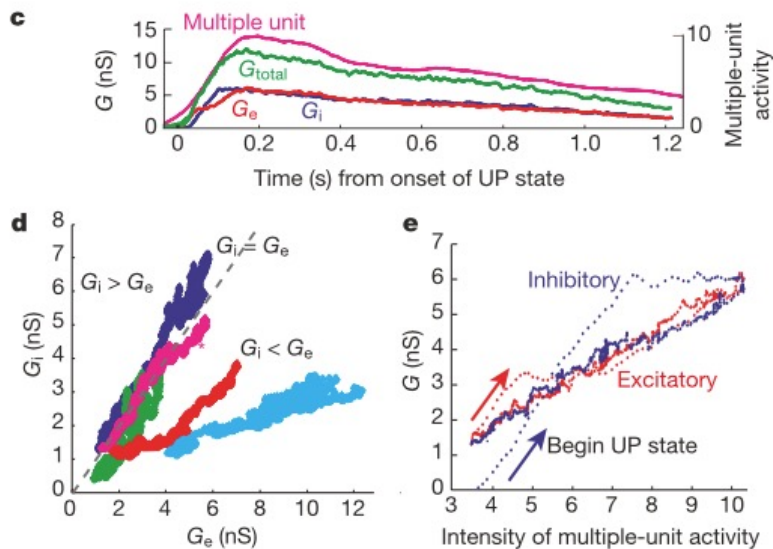
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In vitro
conductance



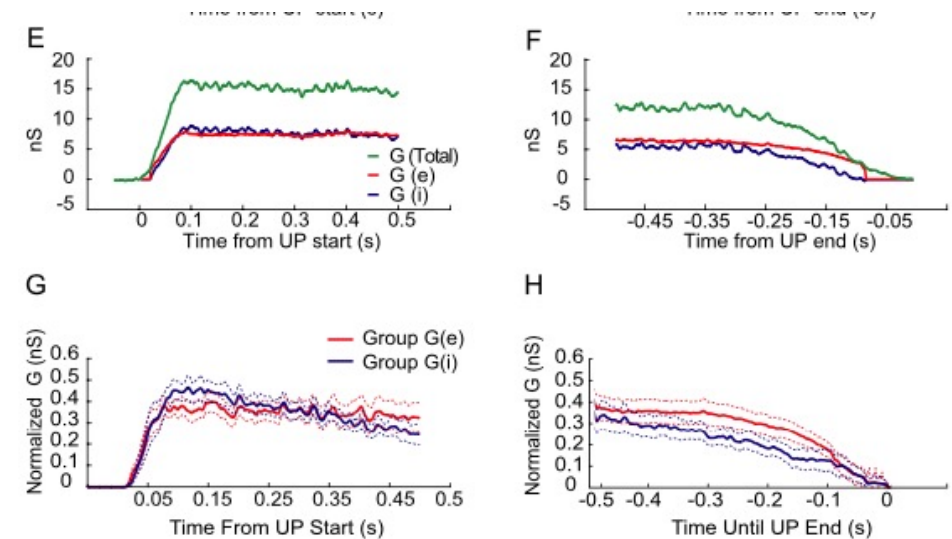
Neocortical Network Activity *In Vivo* Is Generated through a Dynamic Balance of Excitation and Inhibition

Bilal Haider, Alvaro Duque, Andrea R. Hasenstaub, and David A. McCormick

Department of Neurobiology, Kavli Institute for Neuroscience, Yale University School of Medicine, New Haven, Connecticut 06510, USA

In vivo
conductance

(Haider et al., 2006)



Experimental Evidence



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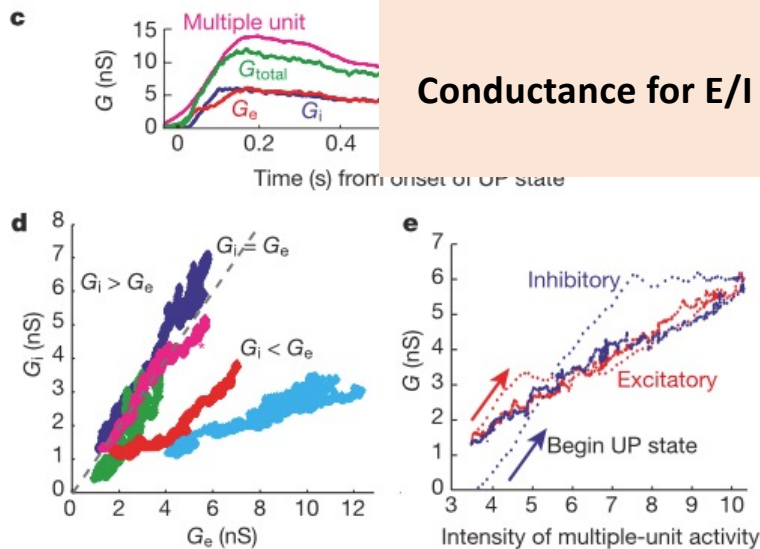
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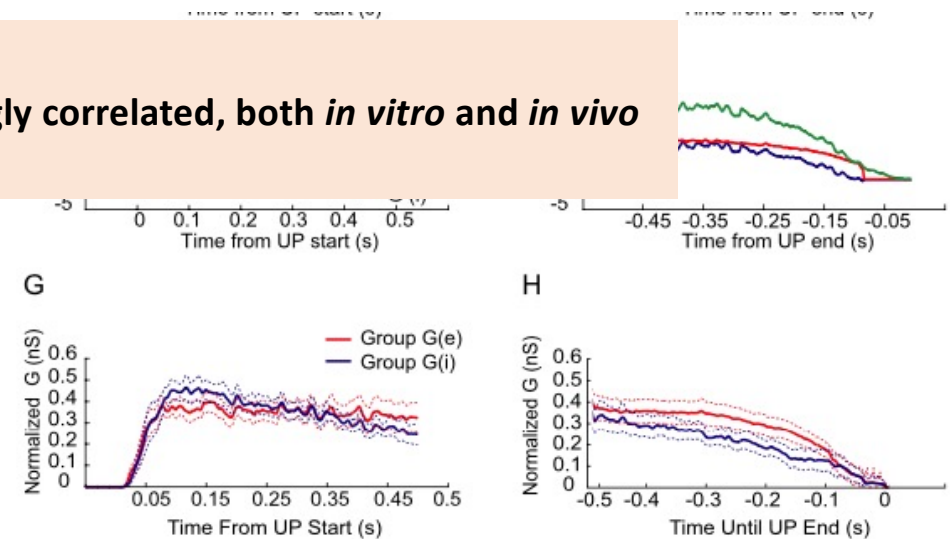
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(Haider et al., 2006)

In vivo
conductance

Conductance for E/I currents is strongly correlated, both *in vitro* and *in vivo*

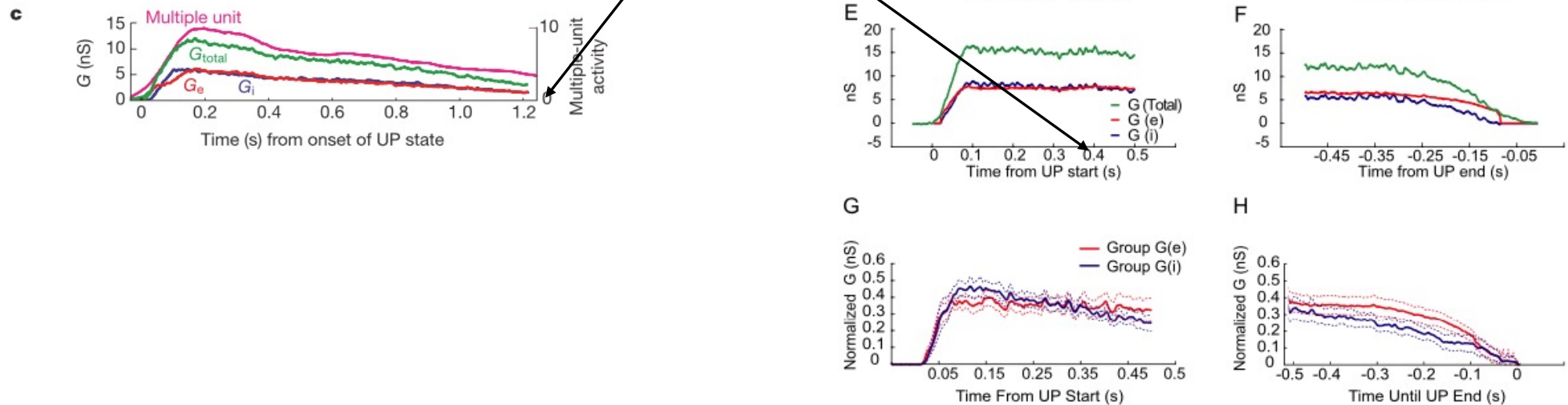


Experimental Evidence

Unit: Seconds

But neuronal dynamics operate at the time scale of ms.

Does the balance exist at finer time scales?



Experimental Evidence



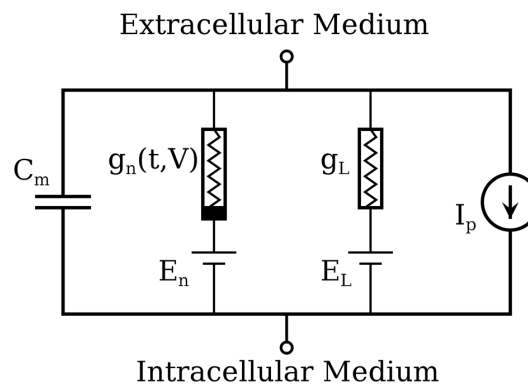
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Does the balance exist at finer time scales?

$$R = V_{mem}/I_{inj} \quad G = 1/R$$



Equivalent circuit of a neuron

Experimental Evidence

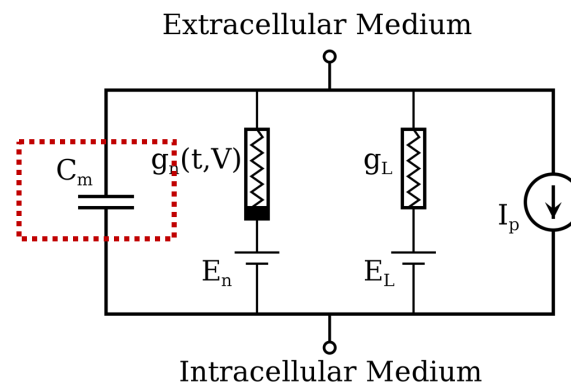
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But neuronal dynamics operate at the time scale of ms.

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$$R = V_{mem}/I_{inj} \quad G = 1/R$$

- Capacitors (of the membrane) prevent measuring conductances at finer time scales.



Equivalent circuit of a neuron

Experimental Evidence



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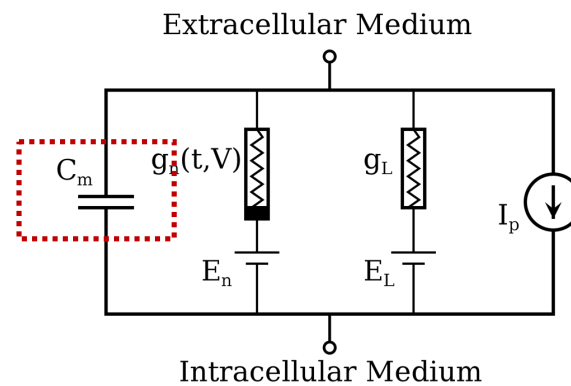
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But neuronal dynamics operate at the time scale of ms.

Does the balance exist at finer time scales?

$$R = V_{mem}/I_{inj} \quad G = 1/R$$

- Capacitors (of the membrane) prevent measuring conductances at finer time scales.
- Cannot simultaneously measure EPSCs and IPSCs.



Equivalent circuit of a neuron

Experimental Evidence



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Published: 30 March 2008

Instantaneous correlation of excitation and inhibition during ongoing and sensory-evoked activities

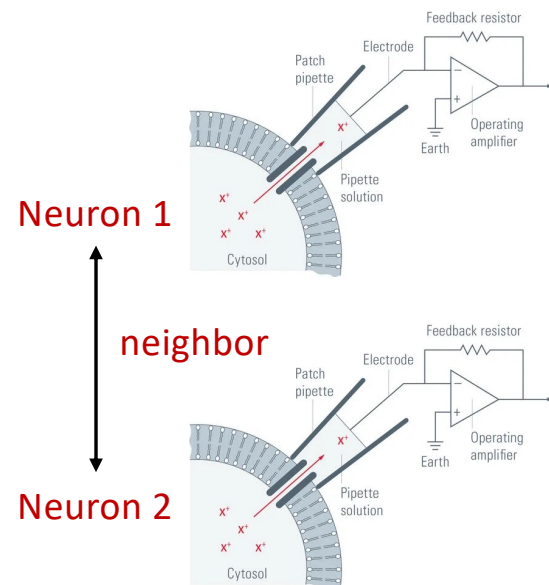
Michael Okun & Ilan Lampl

[Nature Neuroscience](#) 11, 535–537 (2008) | [Cite this article](#)

in vivo

Paired patch clamp recording

Neighboring neurons receive similar inputs.



Exp. 1

Neuron 1

EPSCs

Neuron 2

IPSCs

Exp. 2

IPSCs

EPSCs

Experimental Evidence



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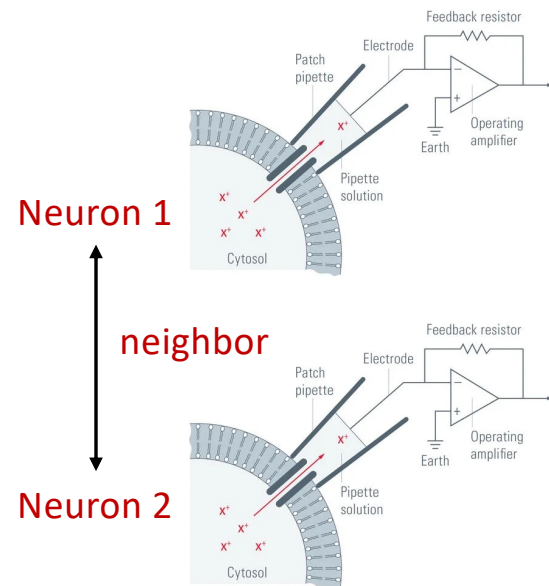
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in vivo

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Neighboring neurons receive similar inputs.



Clamp Voltage

Membrane potential

reversal potential of inhibition $\sim -75mV$

Increase reflect EPSCs

Neuron 2

reversal potential of depolarization
QX-314 added to prevent firing

Decrease reflect IPSCs

Experimental Evidence



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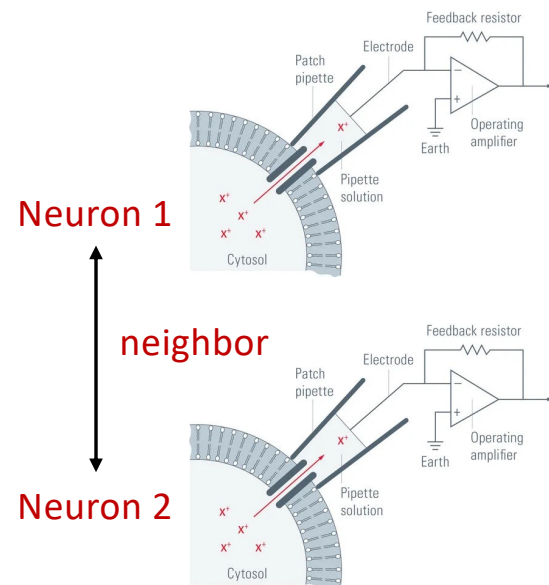
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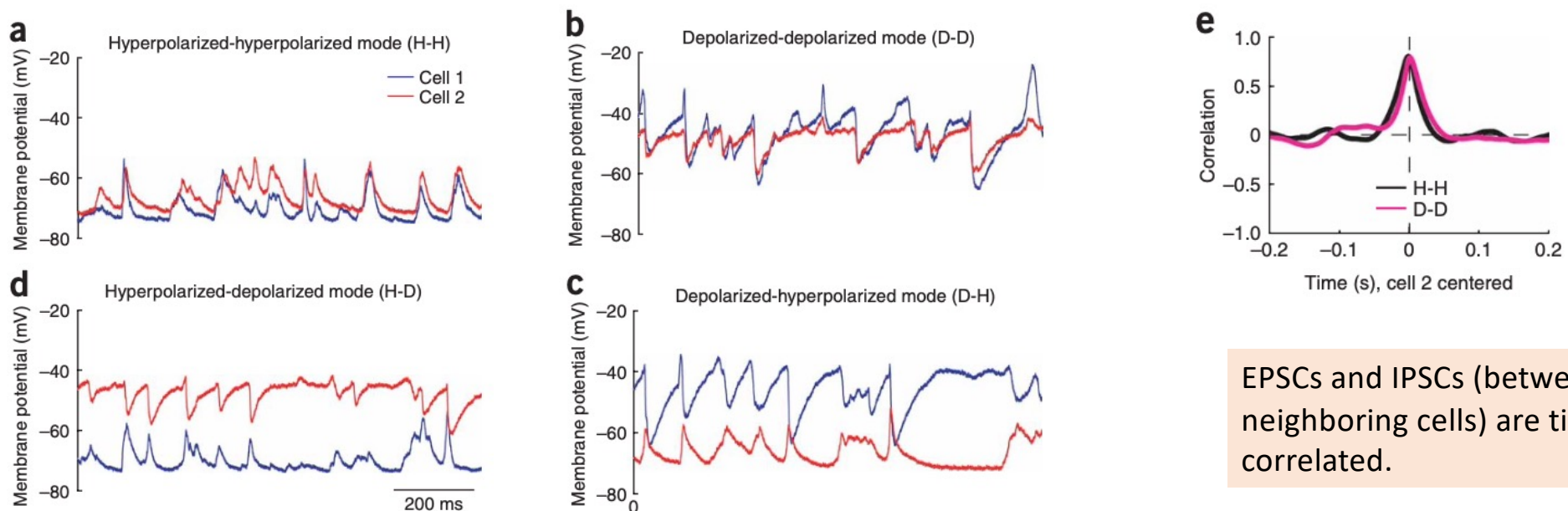
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EPSCs and IPSCs (between neighboring cells) are tightly correlated.

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Unit: milliseconds

27

Experimental Evidence



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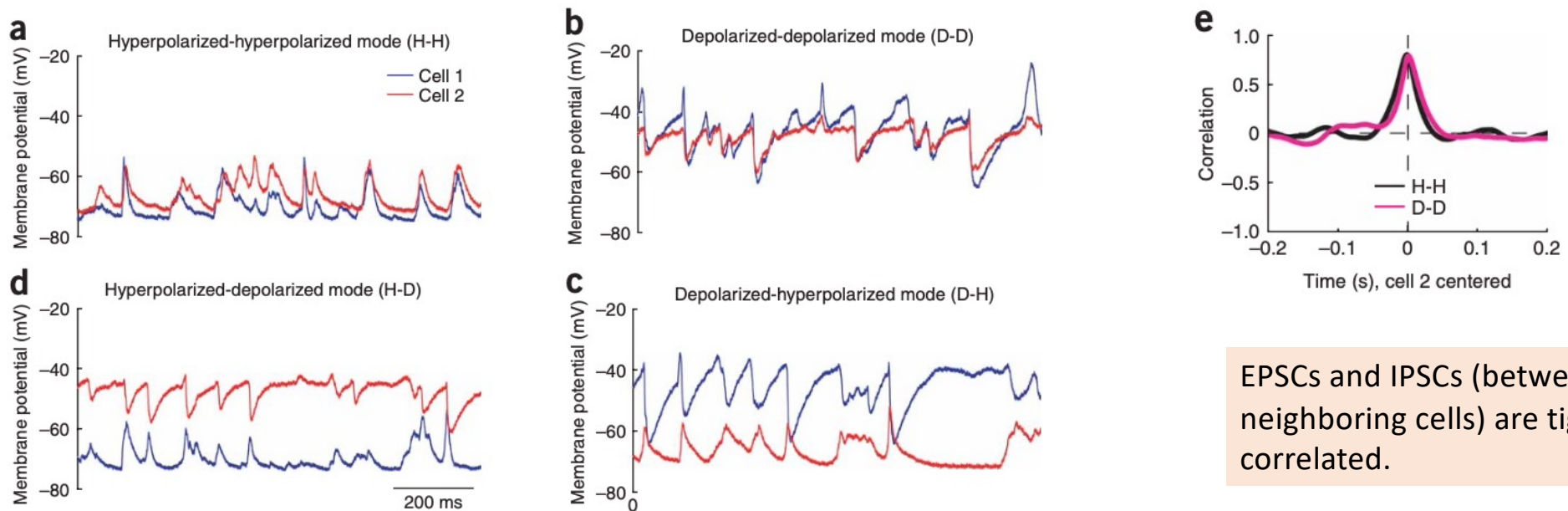
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E/I currents are tightly correlated with short delay.



Tight balance



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Unit: milliseconds

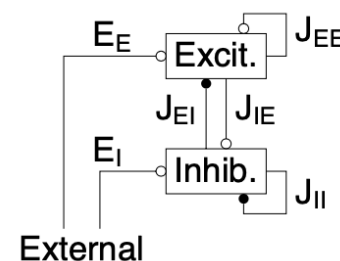
EPSCs and IPSCs (between neighboring cells) are tightly correlated.

Take a break.

E-I Balance Model

Chaos in Neuronal Networks with Balanced Excitatory and Inhibitory Activity

C. van Vreeswijk and H. Sompolinsky (Science, 1996)

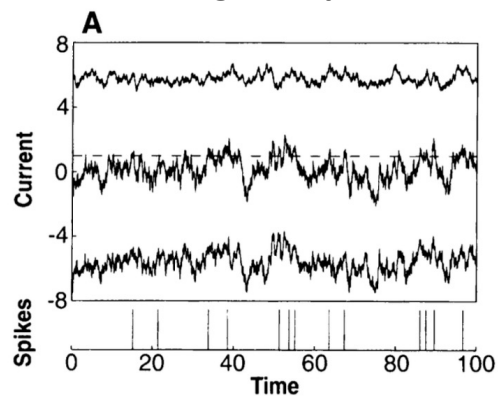


Sparse connectivity $1 \ll K/N \ll 1$

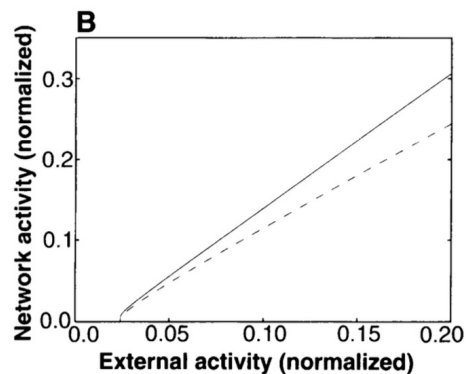
Strong connection strength $J_{kl}^{ij} \sim O(1)$

Threshold $\sim O(\sqrt{K})$

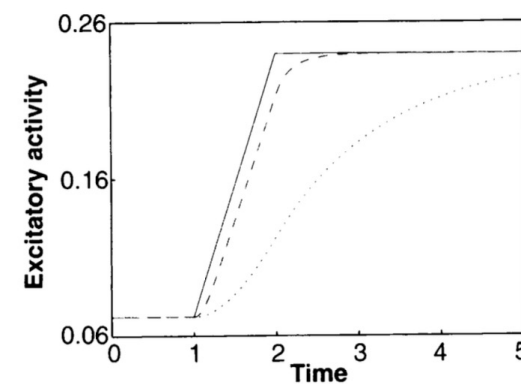
Irregular spike



Linear encoding



Fast response



EI Balance Model

Linear Encoding



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Chaotic Balanced State in a Model of Cortical Circuits

C. van Vreeswijk

(neural comp., 1998)

H. Sompolinsky

Racah Institute of Physics and Center for Neural Computation, Hebrew University,
Jerusalem, 91904 Israel

Neuron dynamics

$$\sigma_k^i(t) = \Theta(u_k^i(t)) \quad \Theta(.) \text{ Heaviside function}$$

$$u_k^i(t) = \sum_{l=1}^2 \sum_{j=1}^{N_l} J_{kl}^{ij} \sigma_l^j(t) + u_k^0 - \theta_k, \quad k = E, I$$

$$J_{EE} = J_{IE} = 1 \quad J_E \equiv -J_{EI}; J_I \equiv -J_{II}$$

Mean activity $m_k^i(t)$

$$m_k^i(t) \equiv \langle \sigma_k^i(t) \rangle$$

Mean-field analysis



$$u_E = (Em_0 + m_E - J_E m_I) \sqrt{K} - \theta_E$$

$$u_I = (Im_0 + m_E - J_I m_I) \sqrt{K} - \theta_I$$

EI Balance Model

Linear Encoding



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Mean-field results

$$u_E = (Em_0 + m_E - J_E m_I)\sqrt{K} - \theta_E$$

$$u_I = (Im_0 + m_E - J_I m_I)\sqrt{K} - \theta_I$$

Balanced state **necessary** condition: $0 < m < 1$ even when K is large

$$\begin{aligned} Em_0 + m_E - J_E m_I &= O(1/\sqrt{K}) \\ Im_0 + m_E - J_I m_I &= O(1/\sqrt{K}) \end{aligned} \xrightarrow{K \rightarrow \infty} \begin{aligned} m_E &= \frac{J_I E - J_E I}{J_E - J_I} m_0 \equiv A_E m_0 \\ m_I &= \frac{E - I}{J_E - J_I} m_0 \equiv A_I m_0 \end{aligned} \quad \text{Linear encoding}$$

Linear encoding property resulted from **linear** summation of E&I current, regardless of channel and activation non-linearity. (i.e. works for Hodgkin–Huxley model as well)

EI Balance Model

Linear Encoding



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Mean-field results

$$u_E = (Em_0 + m_E - J_E m_I)\sqrt{K} - \theta_E$$

$$u_I = (Im_0 + m_E - J_I m_I)\sqrt{K} - \theta_I$$

Balanced state **necessary** condition: $0 < m < 1$ even when K is large

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s.t. $m_E > 0, m_I > 0$

$$\begin{aligned} \frac{E}{I} &> \frac{J_E}{J_I} > 1. \\ J_E &> 1 \end{aligned}$$

Necessary condition for EI balance

El Balance Model

Fast Response



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Dynamics of $m_k^i(t)$ (Ginzburg & Sompolinsky, 1994)

$$\tau_k \frac{d}{dt} m_k^i(t) = -m_k^i(t) + \Theta(u_k^i(t))$$

Rewriting $u_k^i(t)$

$$\Theta(u_k^i(t)) = F_k(m_E, m_I) = \sum_{n_E, n_I=0}^{\infty} p_E(n_E) p_I(n_I) \Theta \left(\boxed{\overset{\mathcal{O}(\sqrt{K})}{\sqrt{K} J_{k0} m_0}} + \boxed{\overset{\mathcal{O}(\sqrt{K})}{\sum_l \frac{J_{kl}}{\sqrt{K}} n_l}} - \theta_k \right)$$

$p_l(n_l)$ the prob. of receiving n_l spikes from population l .

$$p_l(n) = \sum_{s=n}^{\infty} \underbrace{\frac{K^s}{s!} e^{-K}}_{\text{The prob. of } s \text{ contacts with pop. } l} \underbrace{\binom{s}{n} m_l^n (1 - m_l)^{s-n}}_{\text{Out of } s \text{ contacts, only } n \text{ are active}} = \frac{(m_l K)^n}{n!} e^{-m_l K}$$

The prob. of s
contacts with
pop. l

Out of s
contacts, only
 n are active

Poisson distribution with rate $m_l K$



Gaussian distribution $\mathcal{N}(m_l K; m_l K)$

El Balance Model

Fast Response



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Perturbation analysis

Linearize around the fixed point m_k

$$\delta m_k(t) = m_k(t) - m_k$$

$$\tau_k \frac{d}{dt} \delta m_k(t) = -\delta m_k(t) + \sqrt{K} \sum_{l=1,2} f_{kl} \delta m_l(t)$$

$$\text{Sol.} \quad \delta m_k(t) = \delta m_{k,1} \exp(\lambda_1 t) + \delta m_{k,2} \exp(\lambda_2 t) \quad \lambda_1, \lambda_2 \sim \mathcal{O}(\sqrt{K})$$

Setting $\text{Re}(\lambda_1) < 0, \text{Re}(\lambda_2) < 0$

Gives $\tau_k < \tau$ The precise value of τ has a complicated dependence on the system parameters.

When $\tau_k < \tau$, perturbation will decay extremely fast on the order of $\mathcal{O}(1/\sqrt{K})$

EI Balance Model

Fast Response



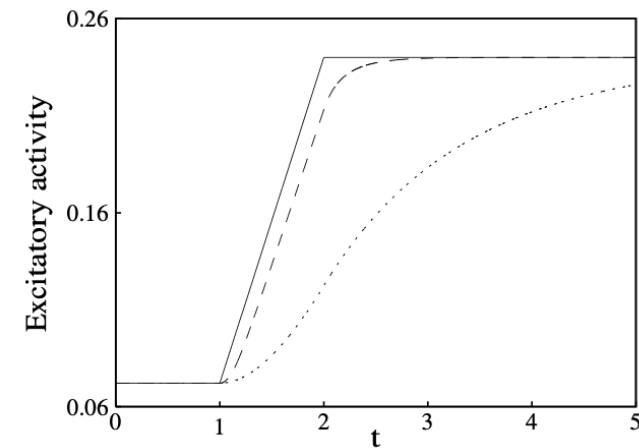
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Perturbation analysis

$\tau_k < \tau$ The precise value of τ has a complicated dependence on the system parameters.

When $\tau_k < \tau$, perturbation will decay extremely fast on the time scale of $\mathcal{O}(1/\sqrt{K})$

➡ Fast response to external changing stimulus





Summary

- The historical background of the EI balance model.
 - Rate coding vs. temporal coding;
 - Neurons as integrator vs. as coincidence detector.
- Experimental evidence
 - Patch clamp experiments, both *in vitro* and *in vivo*.
 - Paired patch clamp → tight balance
- Theoretical EI balance model
 - Irregular spike pattern
 - Linear encoding
 - Fast response