National College of Ireland

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BSc. (Hons) in Computing, Year 1 (BSHC1)
BSc. (Hons) in Computing, Evening, Year 1 (BSHCE1)
BSc. (Hons) in Business Information Systems, Year 1 (BSHBIS1)
BSc. (Hons) in Business Information Systems, Evening, Year 1 (BSHBISE1)
Higher Certificate in Computing Applications and Support, Year 1 (HCC1)
Higher Certificate in Computing Applications and Support, Evening, Year 1 (HCCE1)

Semester 1 Examinations – 2011/2012

Wednesday 11th January, 2012
2.00pm – 3.30pm

Introduction to Mathematics for Computing

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Full marks will be awarded to complete answers to **FIVE** questions.

Duration of exam: 90 minutes
Attachments: N/A

1. Sets

a) Let $A = \{1, 2, 3, 4, 5\}$, $B = \{3, 5\}$ and $C = \{2, 6\}$.
   (i) Write down the elements of $A \cap B$.
   (ii) Write down the elements of $A \cup C$.
   (iii) Is B a subset of A? Give a reason for your answer.
   (iv) Is C a subset of A? Give a reason for your answer.
   (v) Compute $|A \cup B \cup C|$. [5 x 1 mark]

b) Let A be the set of letters in the word MISSISSIPPI.
Let B be the set of letters in the word MASSACHUSETTS.
List the elements in:
   (i) $A \cup B$
   (ii) $A \cap B$
   (iii) $A - B$
   (iv) $B - A$. [4 x 1 mark]

c) Of 160 students of computing, 100 studied Java, 50 studied C++ and 30 studied both languages.
   (i) Draw a Venn diagram to represent the situation above. [2 marks]
   (ii) How many students studied Java but not C++? [3 marks]
   (iii) How many students studied neither language? [3 marks]
   (iv) How many studied *only one* language? [3 marks]

2. Relations and functions

a) Let $f(x) = x + 1$ and $g(x) = x^2 - 1$, where x is a real number.
   (i) Compute $f(3)$ and $g(4)$. [2 marks]
   (ii) Compute $g(1)$ and $g(-1)$. Is $g(x)$ injective? Give a reason for your answer. [3 marks]
   (iii) What is the range of $f(x)$? Is $f(x)$ surjective? Give a reason for your answer. [3 marks]
   (iv) Calculate the composite function $g \circ f(x)$. [2 marks]

b) Suppose L is the set of lecturers {Tom, Dick, Harriet}. Suppose M is a set of modules {Databases, Java, Mathematics, XML} and S is the set of semesters {1, 2}.
We have the relation "teaches", $R: L \rightarrow M$ with
$R = \{$(Tom, Databases), (Dick, Java), (Dick, XML), (Harriet, Databases), (Harriet,Mathematics)$\}$.
We also have the relation "is taught during", $T: M \rightarrow S$ with
$T = \{$(Databases, 1), (Java, 1), (Java, 2), (Mathematics, 1), (XML, 2)$\}$.

   (i) Draw a digraph of the relation R. [3 marks]
   (ii) Write down the elements of R^{-1}. [2 marks]
   (iii) Draw a digraph of the composite relation $S \circ R$. [3 marks]
   (iv) Does Dick teach during semester 2 ? Give a reason for your answer. [2 marks]

3. Logic

a)

(i) Explain in one sentence what is meant by a *tautology*. [2 marks]

(ii) Give an example of a logical expression that is a tautology. [3 marks]

b) Construct the truth table for the following logical expressions:

(i) $p \wedge q$ [3 marks]

(ii) $\sim(\sim p \vee \sim q)$ [3 marks]

Decide whether or not the in expressions in (i) and (ii) are logically equivalent, giving reasons for your answer. [2 marks]

c) Consider the following logical propositions:

P: *x* is an integer greater than *8*.

Q: *x* is an integer greater than *3*.

(i) Show that *P* => *Q* has value TRUE. [3 marks]

(ii) What is the converse of *P* => *Q* ? Decide if the converse is TRUE, showing your workings. [2 marks]

(iii) What is the contrapositive of *P* => *Q* ? Decide if the contrapositive is TRUE, showing your workings. [2 marks]

4. Calculus

a) A bus leaves a bus stop and accelerates linearly for 30 seconds up to a speed of 10m/s. It then cruises at 10 m/s for one minute. Finally, it decelerates linearly for 10 seconds and comes to a halt at the next bus stop. Calculate the acceleration of the bus during:

(i) the first 30 seconds

(ii) the final 10 seconds. [2 x 4 marks]

b) Calculate dy/dx for the following functions:

(i) $y = f(x) = (x^3 - 2x)(2x^2 + 9)$

(ii) $y = (x^3 + 2)^4$. [2 x 3 marks]

c) Find the slope of the equation of a tangent to the curve $y = 3x^2 - 12x + 6$ at the point $(4, 2)$. Also, find the turning point and state if it is a maximum or a minimum. [6 marks]

5. Probability and statistics

a) Give one example of qualitative data and one example of quantitative data. [2 marks]

b) For the data set

2, 2, 2, 3, 3, 4, 5, 6, 6, 9, 9, 9

calculate the following statistics:

(i) mean
(ii) mode
(iii) median
(iv) standard deviation. [4 x 2 marks]

c) The country *Ruralia* has ten regions. The population density has the following distribution:

Density (per km^2)	**Frequency**
0 – 20	4
20 – 50	2
50 – 100	3
100 – 200	1

i.e., four regions have a density of 0–20 people per square km,
two regions have density of 20–50 people per square km, etc.
Calculate:

(i) the mean population density [4 marks]
(ii) the variance of the distribution [4 marks]
(iii) the standard deviation of the distribution. [2 marks]

6. Counting

a) In how many ways can the letters of the following words be arranged?
(i) ANSI (ii) HTTP (iii) BANANA [3 x 2 marks]

b) Given 5 black skittles and 6 red, in how many ways can the 11 skittles be placed in a line if:

(i) any colour skittle may be next to any other?
(ii) two skittles of the same colour may not be next to each other?
(iii) all 6 red skittles must be next to each other? [3 x 2 marks]

c) From a class of 4 women and 6 men, how many possible project teams can be made if each team requires:

(i) 5 people [2 marks]
(ii) 5 people, at least 1 of whom must be a woman [3 marks]
(iii) 5 people, at least 2 of whom must be men. [3 marks]