Wave Mathematics

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INTRODUCTION

Algebra began with arithmetic such as $1+1=2$, which was invented numerous times by different ancient civilisations for accountancy, exchange of valuables and currencies.

Soon after, angles, shapes, early polar coordinates and pi was discovered due to the requirements of complex construction of buildings and also the observation of space, stars and planets.

Mathematical Physics has traversed vastly since, with exciting new topics such as Relativity and Quantum Physics.

WAVE HARMONICS

During 500 BC, Pythagoras discovered the principles of Harmonics while venturing through Ancient Egypt. He began teaching at the age of 50, although his teachings were not introduced to the western world until Plato's Timeaus, Chapter 6, The Soul of the World in the form of the Tetraktys. His work was translated during the Renaissance era in 15th Century. Later on, the astronomer Kepler rediscovered this work in the 16th Century and soon after Newton took a hold of it in 17th Century. Hermann Helmholz, in the 19th Century applied Harmonics to musical tones which misconvieved many that Micronality and Harmonics were the same discipline. Soon after, the Lambdoma, a Pythagoras construct, was also rediscovered and translated from Latin and Greek extracts by Albert Thmius. Albert's student, Hans Kayser, organised the principles of Harmonics in a series of books such as the Handbook of Harmonics: Zurich 1950 (Lehrbuch der Harmonik). In the 20th Century arose one of the first practioners of harmonics, Harry Partch, he was very familiar with how Pythagoras' work explains the foundations for the science of art and music.

Lambdoma

The main focus of Harmonics is the study of a three dimensional model called the Lambdoma. The Lambdoma is constructed by combining the ratios of the arithmetic, harmonic and geometric series into one form. Ernest G. McClain (USA) in his book The Pythagorean Plato - Nicholas-Hays, York Beach, Maine 1978, confirmed that the Lambdoma was recognized in antiquity by findings that were concealed within the Marriage Allegory of Plato's Republic. (Re write everything before this in this section)

The Lambdoma matrix:

$$LM = \begin{bmatrix} 1 & 1/2 & 1/3 & ... & 1/n \\ 2 & 1 & 2/3 & ... & 2/n \\ 3 & 3/2 & 1 & ... & 3/n \\ ... & ... & ... & ... & ... \\ n & n/2 & n/3 & ... & 1 \end{bmatrix} \tag{1}$$

The Lambdoma matrix is linked to the length of different strings.

Sequential summation of normal distributions

If normal distributions are summed together in sequence they form a continuous wave. If they are summed together where their means are located at positions in a row or coloumn of the Lambdoma then the sequential summation of normal distributions is approximately sinusoidal. (Check this) This provides an insight into the true properties of waves and how they form.

Hence we start with a single quantity. If that quantity duplicates itself with a certain standard deviation error of sigma after an infinite amount of duplication, a distribution is formed. A single quantity from that distribution may shift out of phase by 2 sigma and continue the duplication process in that new space-time. Hence a second identical distribution forms which is connected to the original distribution. This forms a sequence of distributions otherwise known as a wave.

Lambdoma wave

If there exists a sequence of normal distributions with means that produce a row or column from the Lambdoma then the summation can be approximated by a single sinusoidal wave.

Standing wave harmonics (2D)

Let's work out the relationships among the frequencies of these modes. For a wave, the frequency is the ratio of the speed of the wave to the length of the wave: f = v/. Compared with the string length L, you can see that these waves have lengths 2L, L, 2L/3, L/2. We could write this as 2L/n, where n is the number of the harmonic.

The fundamental or first mode has frequency f1 = v/1 = v/2L, The second harmonic has frequency f2 = v/2 = 2v/2L = 2f1 The third harmonic has frequency f3 = v/3 = 3v/2L = 3f1, The fourth harmonic has frequency f4 = v/4 = 4v/2L = 4f1, and, to generalise,

The nth harmonic has frequency fn = v/n = nv/2L = nf1. (Re write everything before this in this section)

Polar wave harmonics (2D)

Polar wave harmonics is the Polar perspective of traditional wave harmonics.

Spherical wave harmonics (3D)

Spherical harmonics was first founded by Laplace

QUANTUM HARMONICS

WAVE AND POLAR WAVE ARITHMETIC

Similar to normal sinusoidal waves, quantum waves will also undergo constructive and destructive interference when summed together which will result in a new wave forming. If two identical waves in phase are summed, then constructive interference will cause the peaks and troughs to increase by a power of two. If two identical waves out of phase are summed, then the wave will undergo complete destructive interference with 0 energy remaining. The following arrows can be used to denote identical waves moving in different directions up $\upsquigarrow$, down $\downsquigarrow$, left $\leftsquigarrow$, right $\rightsquigarrow$, left-right $\leftrightsquigarrow$, up-down $\updownsquigarrow$, polar left spiral $\circlearrowleft$ or right spiral $\circlearrowright$ and spherical polar $\bigstar$. There are three required variables which are amplitude A, wavelength λ and frequency f. To denote these properties of a wave we will use the following notation when conducting equations involving more than one type of wave: ${}^{A}_{\lambda}\updownsquigarrow(f)$.

Notation

The chosen notation has been decided in order to simplify mathematical expressions involving waves in Cartesian, Polar and Spherical Polar scenarios:

$$k = \frac{2\pi}{\lambda} \tag{2}$$

Starting with open wave strings:

$${}^{A}_{\lambda}\rightsquigarrow(f) = A\exp(i(kx - \frac{w}{f})) \tag{3}$$

$${}^{A}_{\lambda}\leftsquigarrow(f) = A\exp(-i(kx - \frac{w}{f})) \tag{4}$$

$${}^{A}_{\lambda}\upsquigarrow(f) = A\exp(i(ky - \frac{w}{f})) \tag{5}$$

$${}^{A}_{\lambda}\downsquigarrow(f) = A\exp(-i(ky - \frac{w}{f})) \tag{6}$$

Open alternating wave strings (traditional sinusoidal waves):

$${}^{A}_{\lambda}\leftrightsquigarrow(f) = A(\exp(i(kx - \frac{w}{f})) + \exp(-i(kx - \frac{w}{f}))) \tag{7}$$

$${}^{A}_{\lambda}\leftrightsquigarrow(f) = 2Acos(kx - \frac{w}{f}) \tag{8}$$

$${}^{A}_{\lambda}\updownsquigarrow(f) = A(\exp(i(ky - \frac{w}{f})) + \exp(-i(ky - \frac{w}{f}))) \tag{9}$$

$${}^{A}_{\lambda}\updownsquigarrow(f) = 2Acos(ky - \frac{w}{f}) \tag{10}$$

Note that $C = 2\pi r$ and so $\lambda = 2\pi r$ as the wavelength covers a full circle with frequency f, otherwise known as a closed string,

$${}^{A}_{\lambda}\circlearrowright(f) = -x^2 - y^2 + r^2 = -x^2 - y^2 + (\lambda/2\pi f)^2 = exp(-iwx) \tag{11}$$

$${}^{A}_{\lambda}\circlearrowleft(f) = x^2 + y^2 - r^2 = x^2 + y^2 - (\lambda/2\pi f)^2 = exp(iwx) \tag{12}$$

$${}^{A}_{\lambda}\circlearrowright(f) = -{}^{A}_{\lambda}\circlearrowleft(f) \tag{13}$$

$${}^{A}_{\lambda}\circlearrowright(f) + {}^{A}_{\lambda}\circlearrowleft(f) = {}^{A}_{\lambda}\bigstar(f) \tag{14}$$

$${}^{A}_{\lambda}\bigstar(f) = (x^2 + y^2 + z^2)/\rho^2 \tag{15}$$

where,

$$\rho = A\Psi(x, y, z) = Ae^{2f(x+y+z)i} \tag{16}$$

$$x = \rho\cos\lambda\theta\sin f\phi \tag{17}$$

$$y = \rho\sin\lambda\theta\sin f\phi \tag{18}$$

$$z = \rho \cos f\phi \tag{19}$$

Note that the Spherical Polar wave has a net zero effect on any Polar wave at the Spherical Polar wave boundary. That is true if the Polar wave is centered with respect to the Spherical Polar wave and has the same frequency and wavelength. This is how equation 13 and 20 can coexist and so,

$$^{A}_{\lambda} \circlearrowright (f) +^{A}_{\lambda} \bigstar(f) =^{A}_{\lambda} \circlearrowright (f) \tag{20}$$

where n is any real number,

$$^{A}_{\lambda} \circlearrowright (f) +^{nA}_{\lambda} \bigstar(f) =^{nA}_{\lambda} \circlearrowright (f) \tag{21}$$

In equation 21 one observes how the Spherical Polar wave acts like an amplification transformation in all directions within a 3D space of size $nA \times nA$.

$$^{m}_{\lambda} \circlearrowright (f) +^{n}_{\lambda} \bigstar(f) =^{nm}_{\lambda} \circlearrowright (f) \tag{22}$$

$$^{A}_{\lambda} \leftrightsquigarrow (f) = A\exp(i(kx - \frac{w}{f})) - A\exp(i(kx - \frac{w}{f})) = \pm^{A}_{\lambda} \circlearrowright (f) \tag{23}$$

$$^{A}_{\lambda} \updownsquigarrow (f) = A\exp(i(ky - \frac{w}{f})) - A\exp(i(ky - \frac{w}{f})) = \pm^{A}_{\lambda} \circlearrowright (f) \tag{24}$$

$$^{A}_{\lambda}\bigstar(f) = A\exp(i(ky - \frac{w}{f}))(x^2 + y^2 + z^2) \tag{25}$$

Arithmetic examples

For example, a wave propagating right summed with a wave propagating right will result in double the amplitude,

$$\rightsquigarrow + \rightsquigarrow =^{2} \rightsquigarrow . \tag{26}$$

A wave propagating right sequenced with a wave propagating right will result in double the wavelength,

$$\rightsquigarrow, \rightsquigarrow =_{2} \rightsquigarrow . \tag{27}$$

A wave propagating right sequenced with a wave propagating left three times will result in alternating waves,

$$3(\rightsquigarrow, \leftsquigarrow) = \rightsquigarrow\leftsquigarrow\rightsquigarrow\leftsquigarrow\rightsquigarrow\leftsquigarrow =_{3} \leftrightsquigarrow =_{3} \circlearrowright . \tag{28}$$

$$3(\leftsquigarrow, \rightsquigarrow) = \leftsquigarrow\rightsquigarrow\leftsquigarrow\rightsquigarrow\leftsquigarrow\rightsquigarrow = -_{3} \leftrightsquigarrow =_{3} \circlearrowleft . \tag{29}$$

A wave propagating right summed with a wave propagating left five times will result in a left-right wave with amplitude 5,

$$5(\rightsquigarrow + \leftsquigarrow) = 5(\leftsquigarrow + \rightsquigarrow) =^{5} \leftrightsquigarrow =^{5} \circlearrowright . \tag{30}$$

A wave propagating up summed with a wave propagating down sequenced five times will result in a up-down wave with wavelength 5,

$$5(\upsquigarrow, \downsquigarrow) =_{5} \updownsquigarrow =_{5} \circlearrowright . \tag{31}$$

$$5(\downsquigarrow, \upsquigarrow) = -_{5} \updownsquigarrow =_{5} \circlearrowleft . \tag{32}$$

A wave propagating right summed with a wave propagating left, up and down six times will result in a polar wave with amplitude 6,

$$6(\rightsquigarrow + \leftsquigarrow + \upsquigarrow + \downsquigarrow) = 6(\leftrightsquigarrow + \updownsquigarrow) =^{6} \bigstar . \tag{33}$$

A wave propagating right summed with a wave propagating up will result in an angled wave,

$$\rightsquigarrow + \upsquigarrow = \upsquigarrow \rightsquigarrow . \tag{34}$$

Geometric examples

For example, a wave propagating right multiplied with a wave propagating right will result in a squared wave,

$$\rightsquigarrow \times \rightsquigarrow = \rightsquigarrow^{2} . \tag{35}$$

A wave propagating right multiplied with a wave propagating left five times will result in a left-right wave dot product of amplitude 5,

$$5(\rightsquigarrow \times \leftsquigarrow) =^{5} \rightsquigarrow . \leftsquigarrow \tag{36}$$

A wave propagating right multiplied with a wave propagating up will have no effect,

$$\rightsquigarrow \times \upsquigarrow = \upsquigarrow \times \rightsquigarrow \tag{37}$$

Geometric transformations

A higher dimension wave acting geometrically on a lower dimensional wave results in a transformation.

A polar wave multiplied with a wave will rotate the wave in the same direction,

$$\rightsquigarrow \times \circlearrowleft = \rotatebox{90}{$\rightsquigarrow$} \tag{38}$$

A spherical polar wave multiplied with a wave will stretch or compress the wave,

$${}^{4}_{7}\rightsquigarrow \times {}^{3}_{2}\bigstar = {}^{12}_{14}\rightsquigarrow \tag{39}$$

A spherical polar wave multiplied with a polar wave will stretch or compress the polar wave,

$${}^{4}_{7}\circlearrowleft \times {}^{3}_{2}\bigstar = {}^{12}_{14}\circlearrowleft \tag{40}$$

LINEAR ALGEBRA

Scalar Multiplication

$$n\begin{pmatrix} \leftsquigarrow \\ \circlearrowleft \\ \rightsquigarrow \end{pmatrix} = \begin{pmatrix} {}^{n}\leftsquigarrow \\ {}^{n}\circlearrowleft \\ {}^{n}\rightsquigarrow \end{pmatrix} \tag{41}$$

Matrix Addition

$$\begin{pmatrix} \leftsquigarrow & \circlearrowleft & \leftsquigarrow \end{pmatrix} + \begin{pmatrix} \leftsquigarrow \\ \circlearrowleft \\ \rightsquigarrow \end{pmatrix} = {}^{2}\leftsquigarrow + {}^{2}\bigstar + {}^{2}\leftrightsquigarrow \tag{42}$$

$${}^{2}\leftsquigarrow + {}^{2}\bigstar + {}^{2}\leftrightsquigarrow = {}^{2}\leftsquigarrow + {}^{2}\bigstar \pm {}^{2}\circlearrowleft \tag{43}$$

$${}^{2}\leftsquigarrow + {}^{2}\bigstar + {}^{2}\leftrightsquigarrow = {}^{4}\leftsquigarrow \pm {}^{4}\circlearrowleft \tag{44}$$

Matrix Multiplication

$$\begin{pmatrix} \leftsquigarrow & \circlearrowleft & \leftsquigarrow \end{pmatrix} . \begin{pmatrix} \leftsquigarrow \\ \circlearrowleft \\ \rightsquigarrow \end{pmatrix} = \leftsquigarrow^{2} \pm \circlearrowleft^{2} + \leftrightsquigarrow^{2} \tag{45}$$

$$\leftsquigarrow^{2} \pm \circlearrowleft^{2} + \leftrightsquigarrow^{2} = \leftsquigarrow^{2} \pm 2\circlearrowleft^{2} \tag{46}$$

WAVE CALCULUS

Limits of Cartesian waves

Infinitely very small amplitude of waves become infinitely thin lines the length of a wavelength λ hence they can be approximated to be continuous lines of zero space. If wavelength also tends to zero then it becomes a singularity otherwise known as a single point.

$$lim_{A\to 0,\lambda\to 0}({}^{A}_{\lambda}\rightsquigarrow(f)) = 0 \tag{47}$$

Limits of polar waves

Infinitely very small amplitude of polar waves or spherical polar waves become infinitely small points hence they can be approximated to be singularities or areas of zero space otherwise known as a single point.

$$lim_{A\to 0}({}^{A}_{\lambda}\circlearrowleft(f)) = 0 \tag{48}$$

$$lim_{A\to 0}({}^{A}_{\lambda}\bigstar(f)) = 0 \tag{49}$$

Very large amplitude polar waves can be approximated as either x = 0 for all y and y = 0 for all x when frequency is 2 or tend to all of space as frequency tends to infinity. Whenever frequency tends to 0 or infinity the polar wave tends to the shape of a circle.

$$lim_{A\to\infty}({}^{A}_{\lambda}\bigstar(2)) = \begin{cases} y = 0, z = 0 & \forall x\epsilon\mathbb{R} \\ x = 0, z = 0 & \forall y\epsilon\mathbb{R} \end{cases} \tag{50}$$

Polar wave is an area of space which does not fit a traditional Cartesian graph as within a polar wave, space becomes very wave-like and will be very similar to polar coordinates. Hence a new type of graph is required which incorporates wave like but also Cartesian behaviour.

A polar wave observed from the outside is similar to zero as anything that goes towards it compresses and disappears, as visualised in a polar plot as radius decreases. However when observed from the inside a polar wave appears to be like infinity as anything that proceeds away expands radially outwards.

Hence by using mathematical limits we obtain the following:

$$lim_{f\to 1\ or\,\infty}({}^{A}_{\lambda}\bigstar(f)) = (x^2 + y^2 + z^2)/A^2 \tag{51}$$

A special case is when $f = 2$:

Integral wave Calculus

Polar wave integration

Spherical polar wave integration

NETWORK WAVE ARITHMETIC

A network is formed when at least two nodes are connected by an edge. This section will analyse how polar and spherical polar waves can be modelled as nodes while waves can be modelled as edges within a network graph.

CONCLUSION

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