Games and Language

Robert van Rooij

aka: strategic game or normal form game

A STATIC GAME with complete information is a triple

to be explained later

$$< N, A_{i \in N}, U_{i \in N} >$$

aka strategies

where N is a set of players, A_i are the actions available to player i, and U_i is player i's utility function.

Each utility function $U_i : \times_{j \in N} A_j \rightarrow \mathbb{R}$ gives a numerical payoff to each possible outcome of the game. An outcome of the game is given by a tuple $< a_1, \ldots, a_n >$ that lists the actions taken by every individual player. These tuples are known as action profiles.

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Prisoner's dilemma:

	Cooperate	Defect
Cooperate	$2, 2$	$0, 3$
Defect	$3, 0$	$1, 1$

So what should each prisoner do?

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In a strategic game, an action $a_i \in A_i$ is strictly dominated by an action $a_i' \in A_i$ if and only if

$$U_i(a_i', a_{-i}) > U_i(a_i, a_{-i})$$

for all action profiles a_{-i}.

	Cooperate	Defect
Cooperate	$2, 2$	$0, 3$
Defect	$3, 0$	$1, 1$

Cooperate is strictly dominated by defect because no matter what your opponent does, defection yields a greater payoff than cooperation.

Stag hunt

	Stag	Hare
Stag	3,3	0,2
Hare	2,0	2,2

What actions should we expect players to choose?
Which action should you choose?

A NASH EQUILIBRIUM (in pure strategies) is an action profile $< a_1, \ldots, a_n >$ such that for all $i \in N$ there is no $a_i' \in A_i$ such that

$$U_i(a_i', a_{-i}) \geq U_i(a_i, a_{-i})$$

An action profile is a Nash equilibrium if no player can gain by unilaterally switching her strategy.

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	Heads	Tails
Heads	1,0	0,1
Tails	0,1	1,0

Given a strategic game $< N, A_{i \in N}, U_{i \in N} >$, a mixed strategy for player i is a probability distribution $\sigma_i \in \Delta^{|A_i|}$ over player i's possible actions.

$\sigma \in \Delta^{|A_1|} \times \cdots \times \Delta^{|A_n|}$ is called a mixed strategy profile.

A Nash equilibrium is a mixed strategy profile $< \sigma_1, \ldots, \sigma_n >$ such that for all $i \in N$ there is no $\sigma_i' \in \Delta^{|A_i|}$ such that:

$$U_i(\sigma_i', \sigma_{-i}) \geq U_i(\sigma_i, \sigma_{-i})$$

With mixed strategies, we can extend the player's utility functions to the average utility a player earns when the mixed strategies are played:

$$U_i(\sigma_i, \ldots, \sigma_n) = \sum_{<a_1, \ldots, a_n>} \sigma_1(a_1) \times \cdots \times \sigma_n(a_n) \times U_i(< a_i, \ldots, a_n >)$$

	Heads	Tails
Heads	1,0	0,1
Tails	0,1	1,0

q is the probability that player 2 plays Tails.

$$U_1(Heads, q) = 1(1-q) + 0 \cdot q = 1 - q$$
$$U_1(Tails, q) = 0(1-q) + 1 \cdot q = q$$

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$$U_1(Heads, q) = 1(1-q) + 0 \cdot q = 1 - q$$
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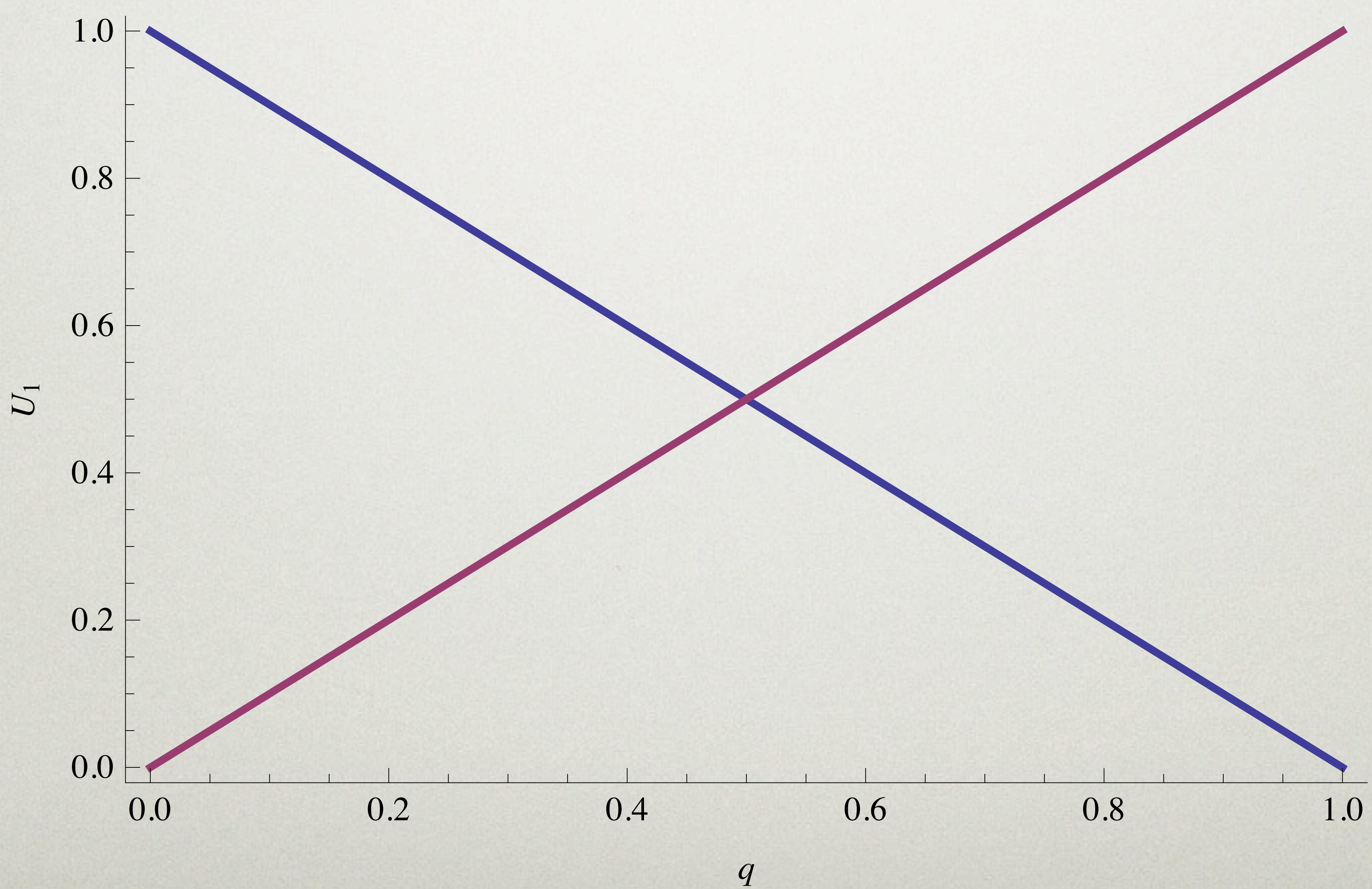
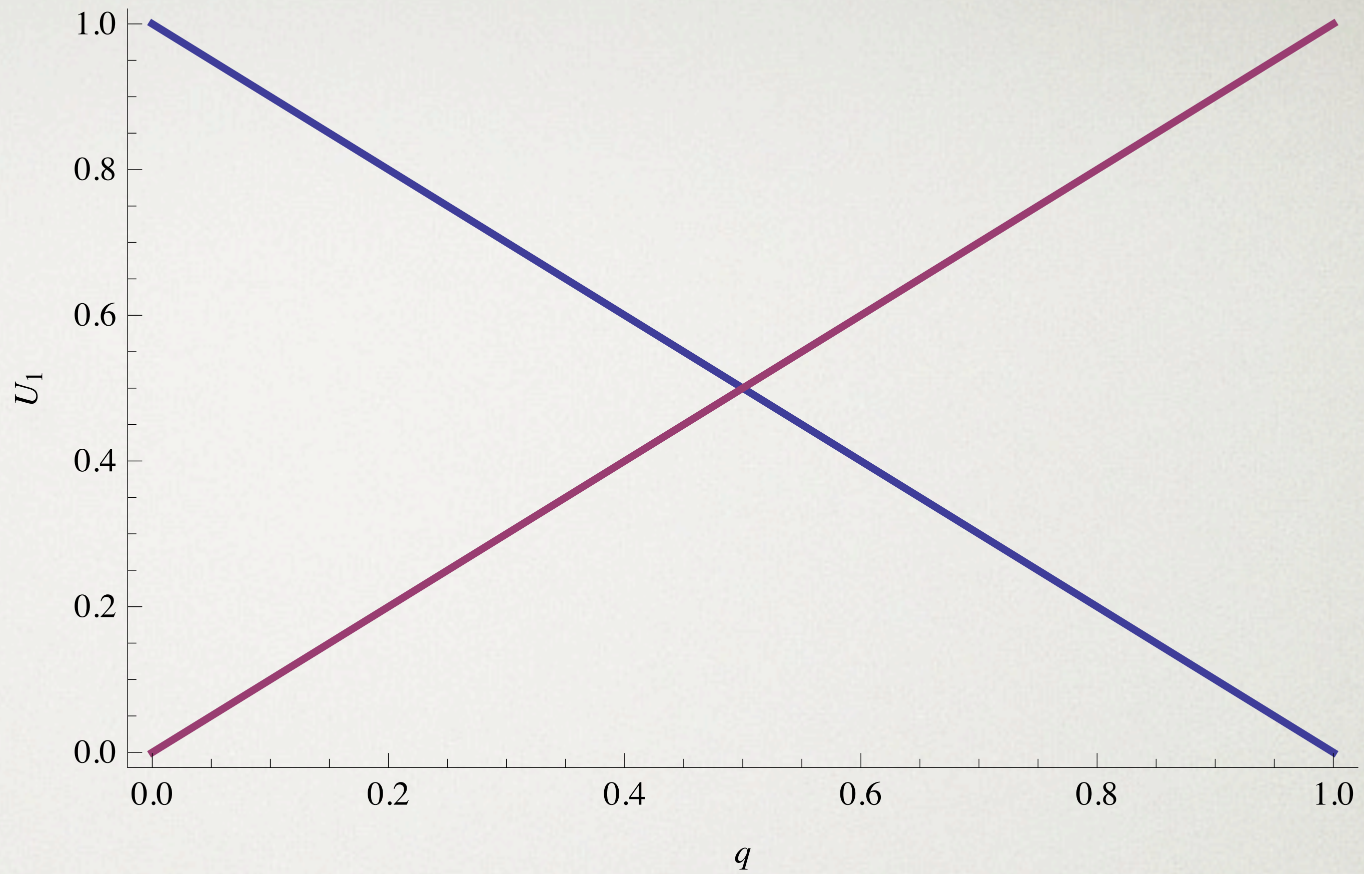
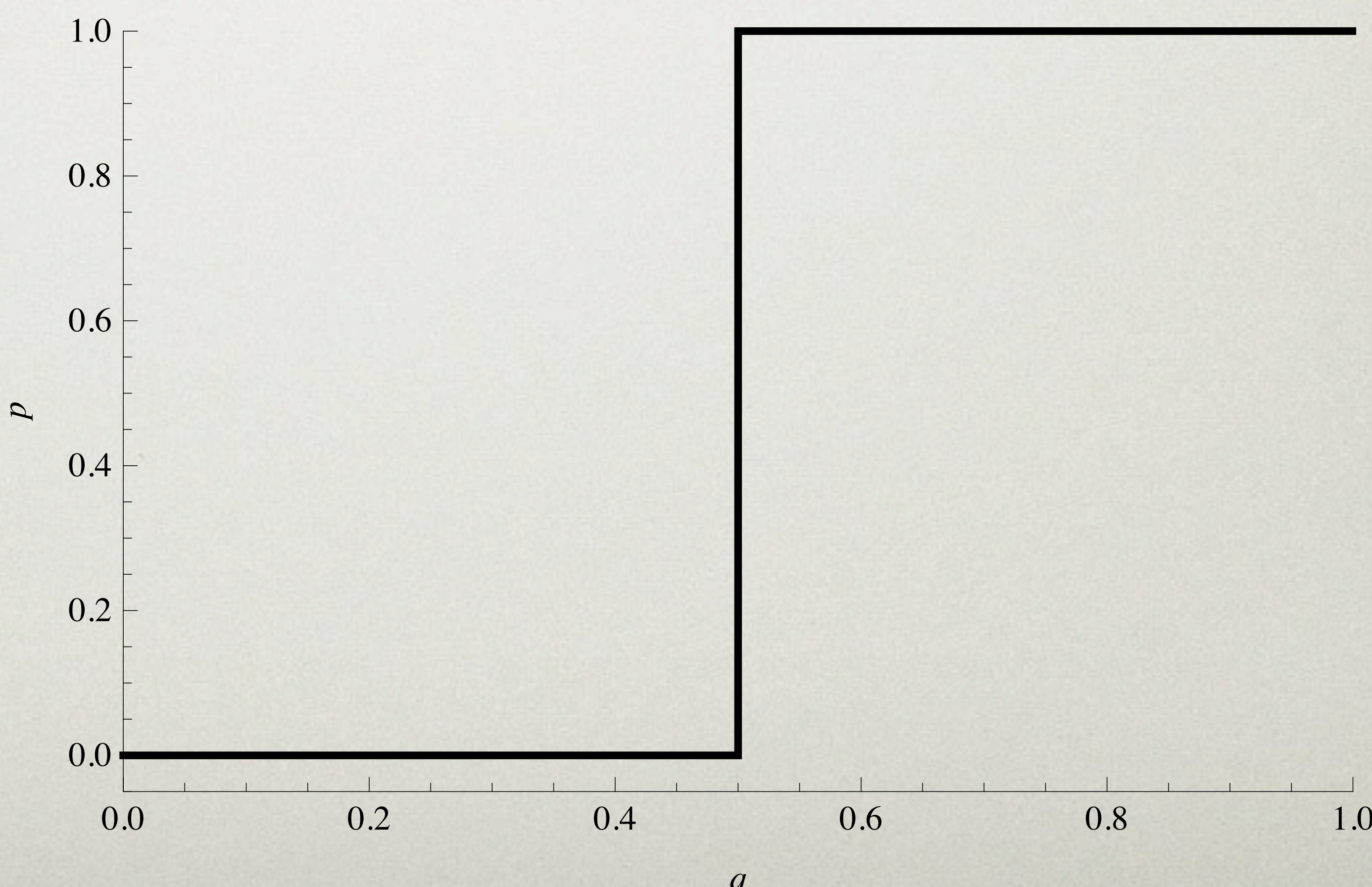


	Heads	Tails
Heads	1,0	0,1
Tails	0,1	1,0

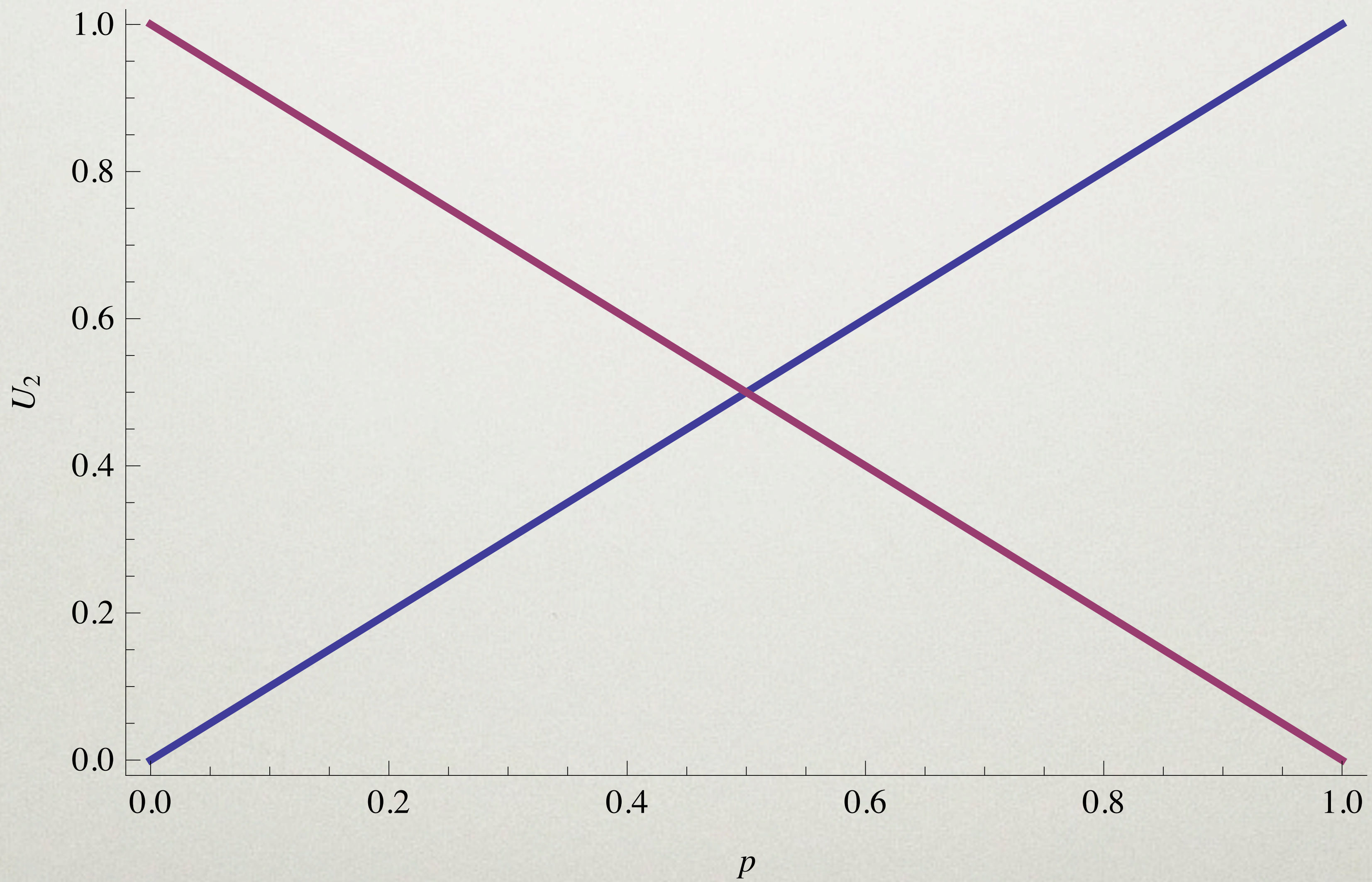




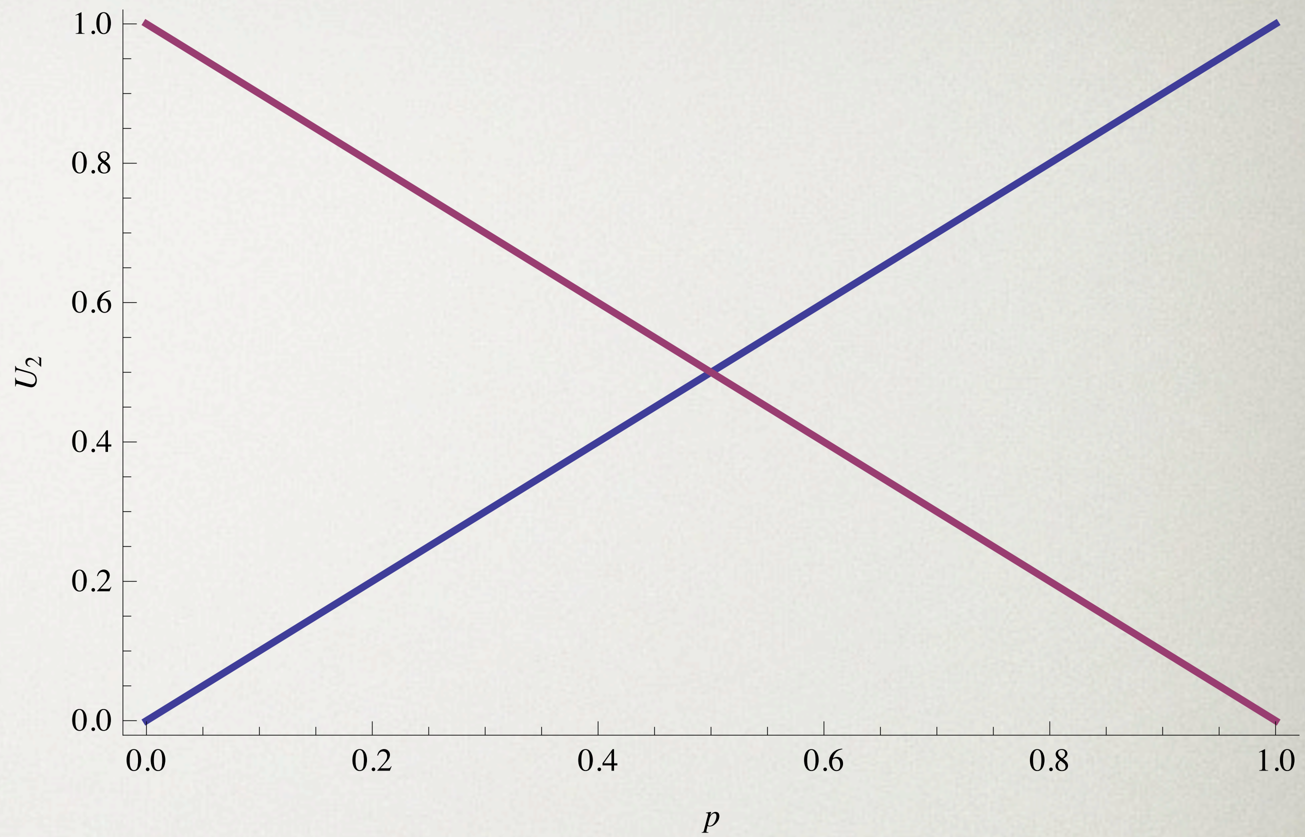
p is the probability that player 1 plays Tails.

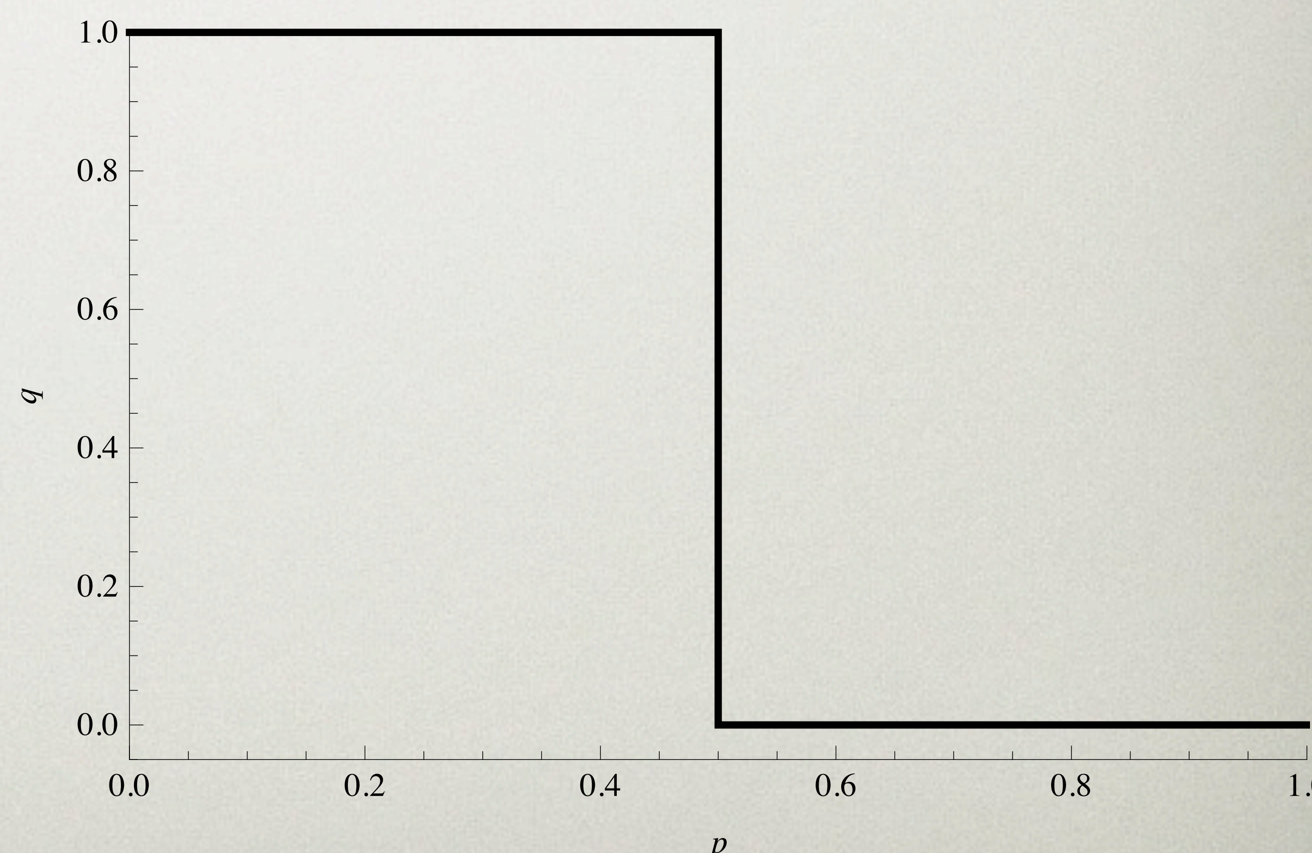
	Heads	Tails
Heads	1,0	0,1
Tails	0,1	1,0

$$U_2(Heads, p) = 0(1-p) + 1 \cdot p = p$$
$$U_2(Tails, p) = 1(1-p) + 0 \cdot p = 1 - p$$

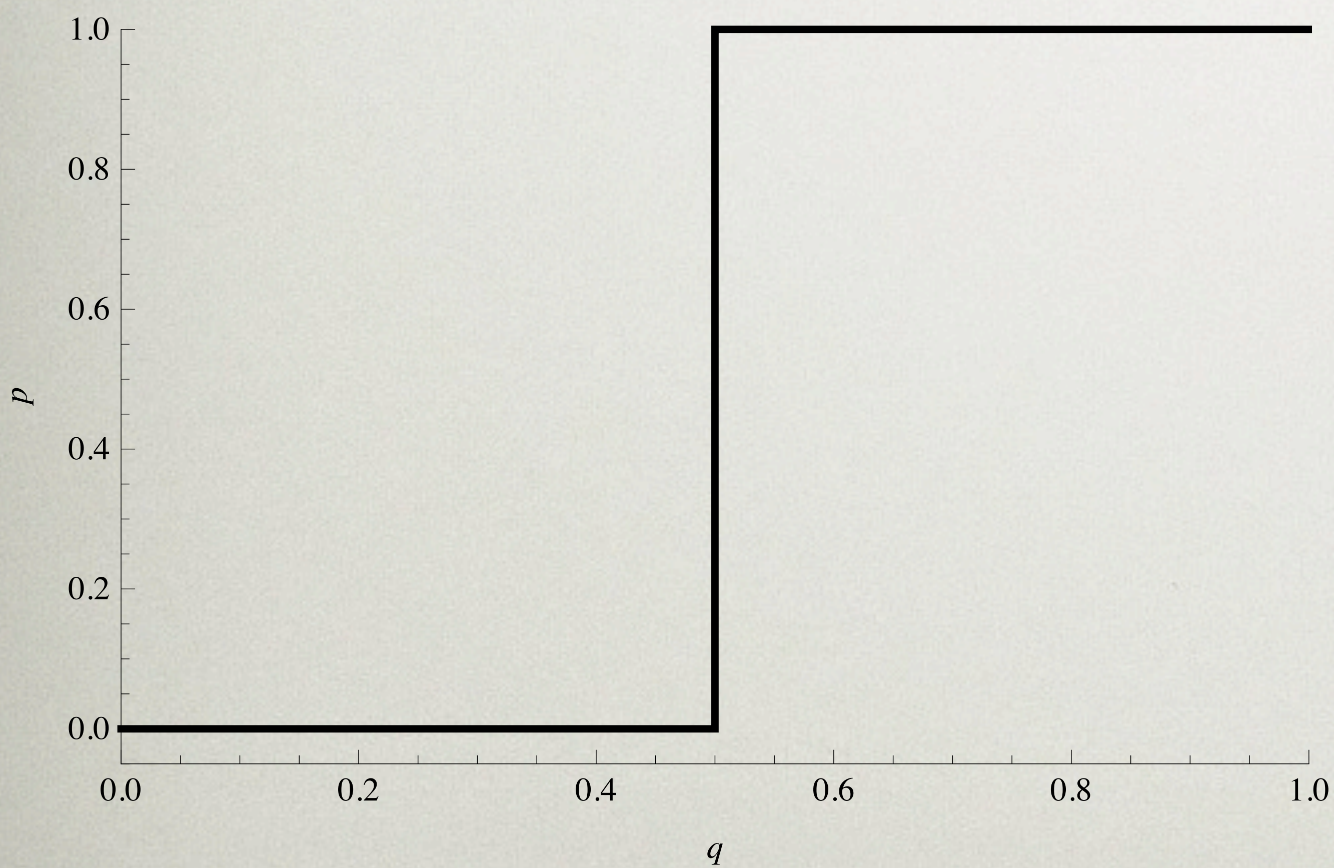


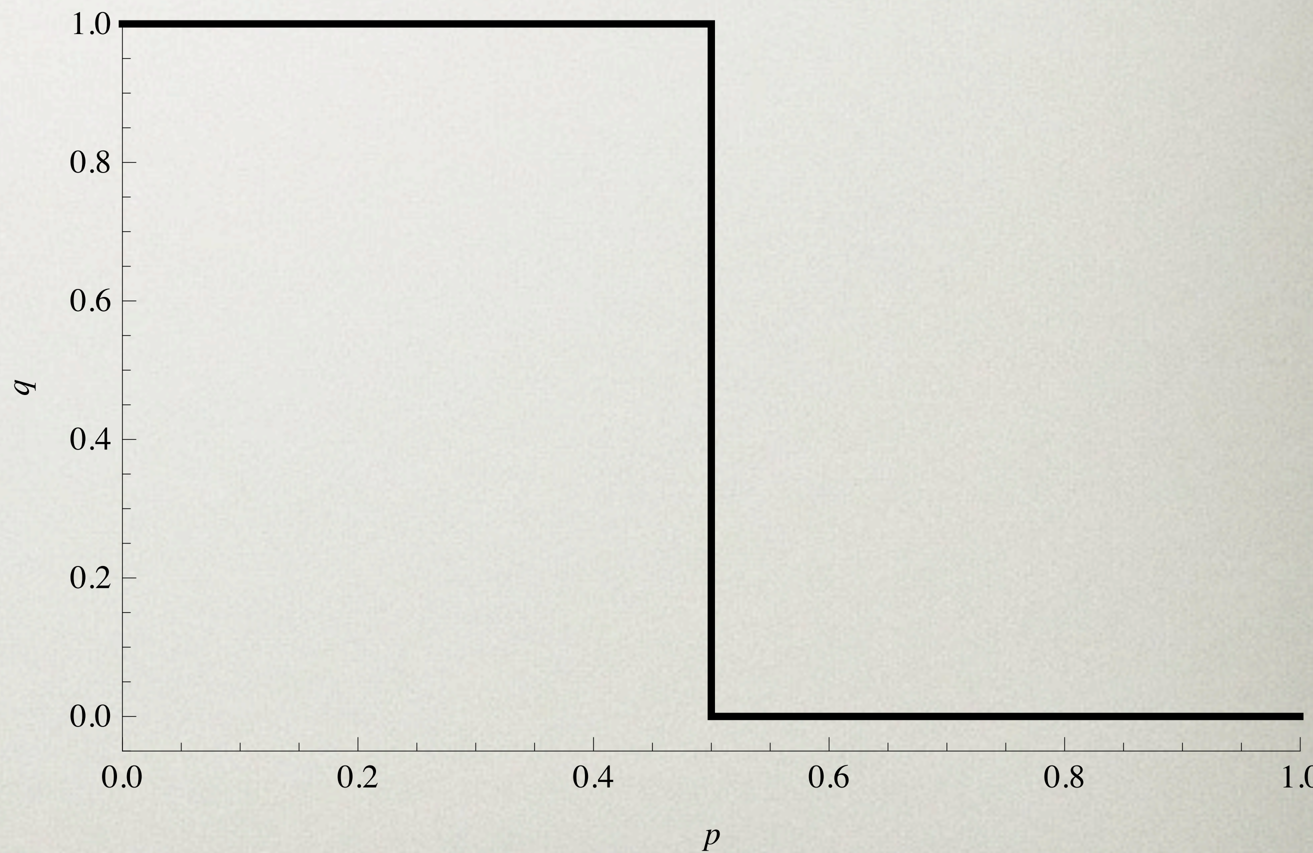
	Heads	Tails
Heads	1,0	0,1
Tails	0,1	1,0



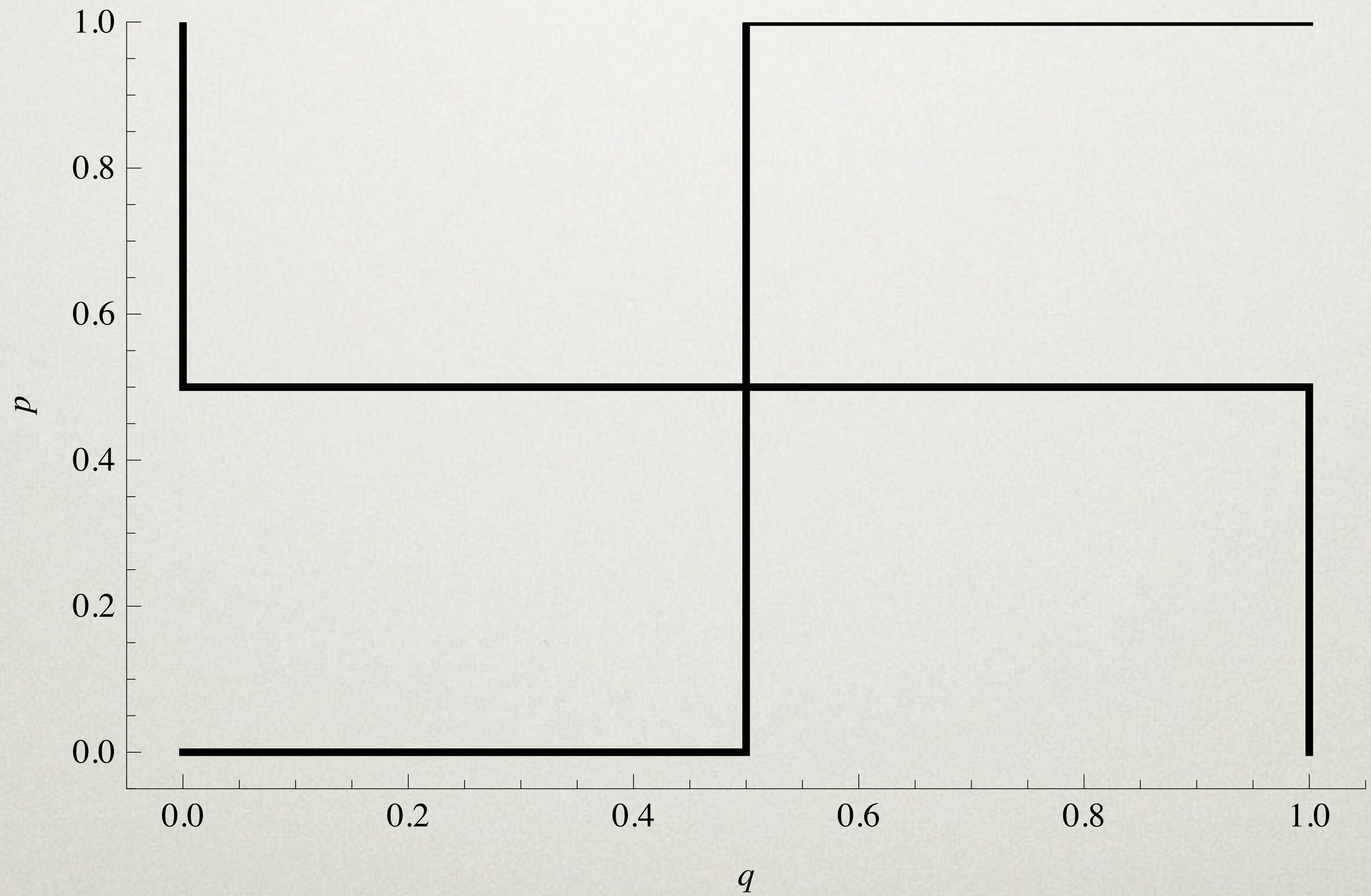


	Heads	Tails
Heads	1,0	0,1
Tails	0,1	1,0



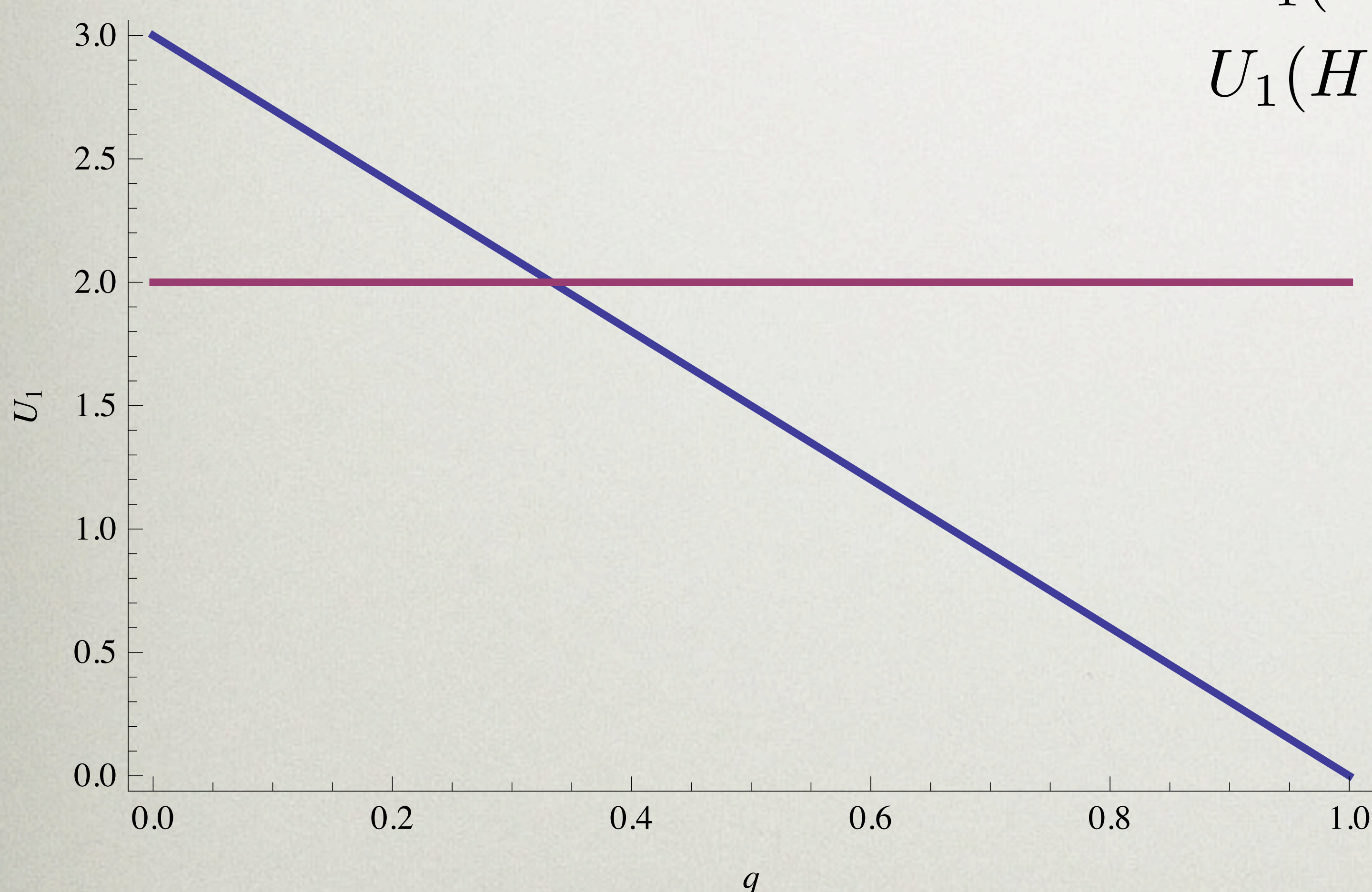


	Heads	Tails
Heads	1,0	0,1
Tails	0,1	1,0



	Stag	Hare
Stag	3,3	0,2
Hare	2,0	2,2

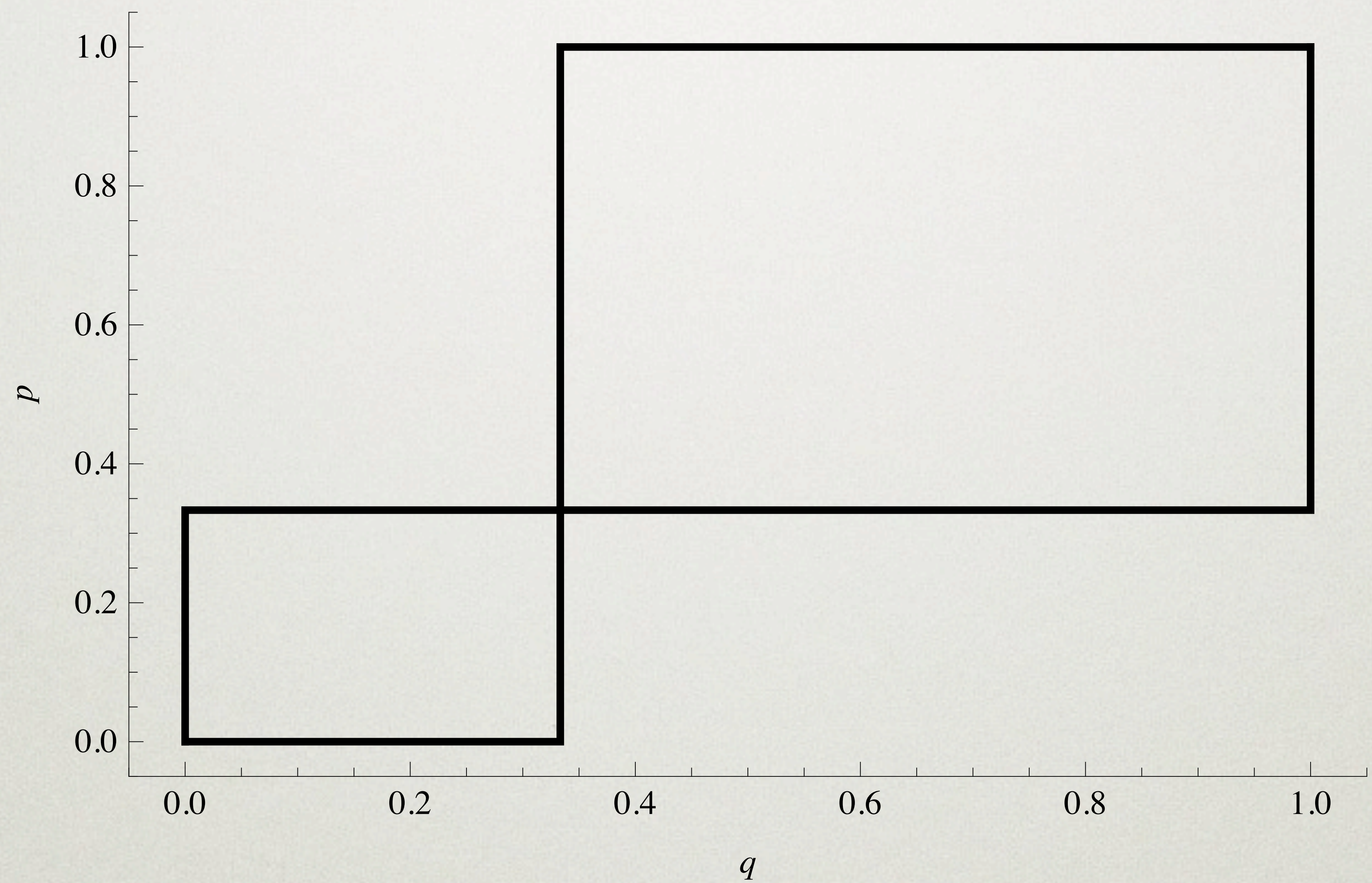
q is the probability that player 2 plays Hare.



$$U_1(Stag, q) = 3(1-q) + 0 \cdot q = 3 - 3q$$
$$U_1(Hare, q) = 2(1-q) + 2 \cdot q = 2$$

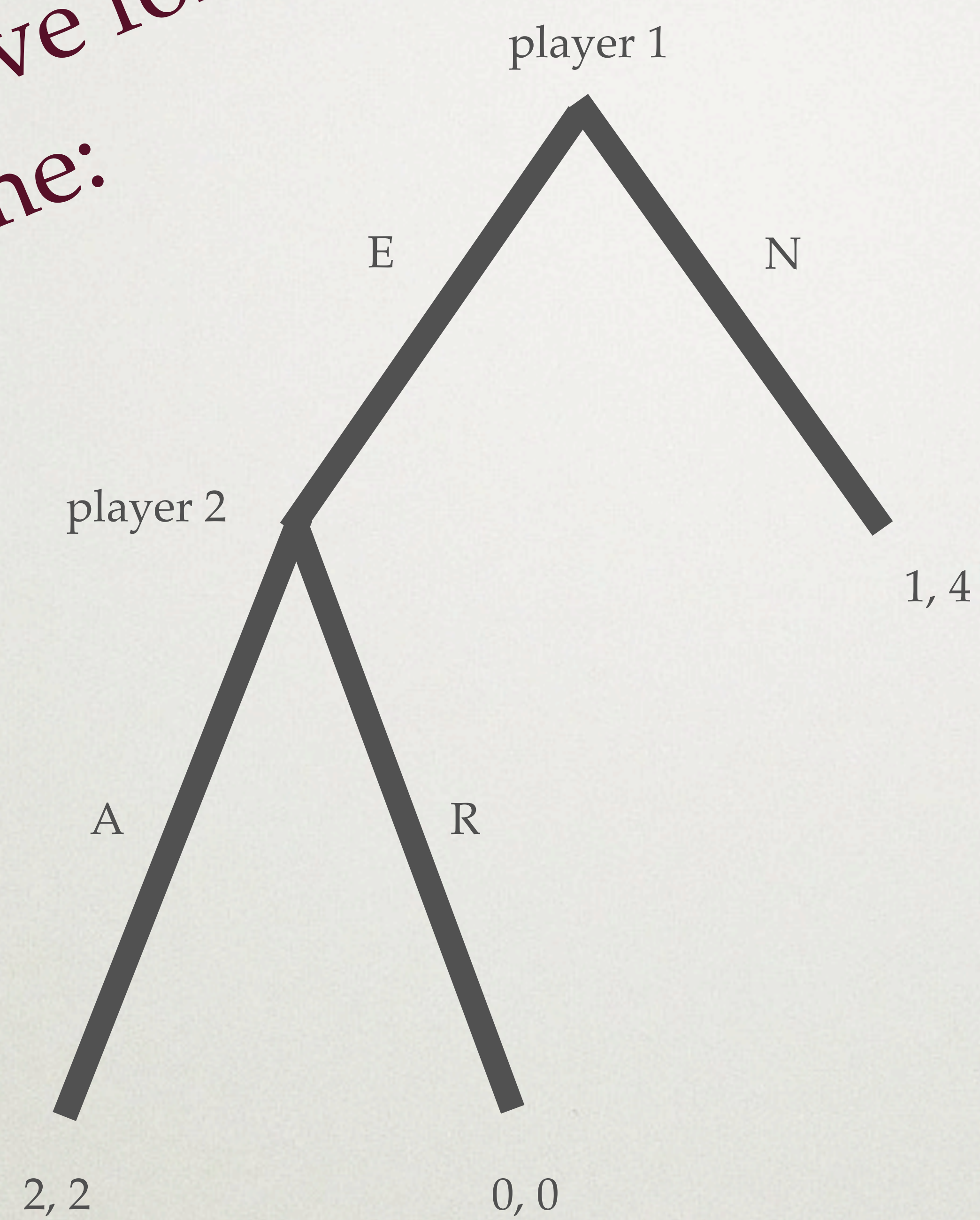
$$U_1(Stag, q) = U_1(Hare, q)$$
$$3 - 3q = 2$$
$$q = \frac{1}{3}$$

	Stag	Hare
Stag	3,3	0,2
Hare	2,0	2,2



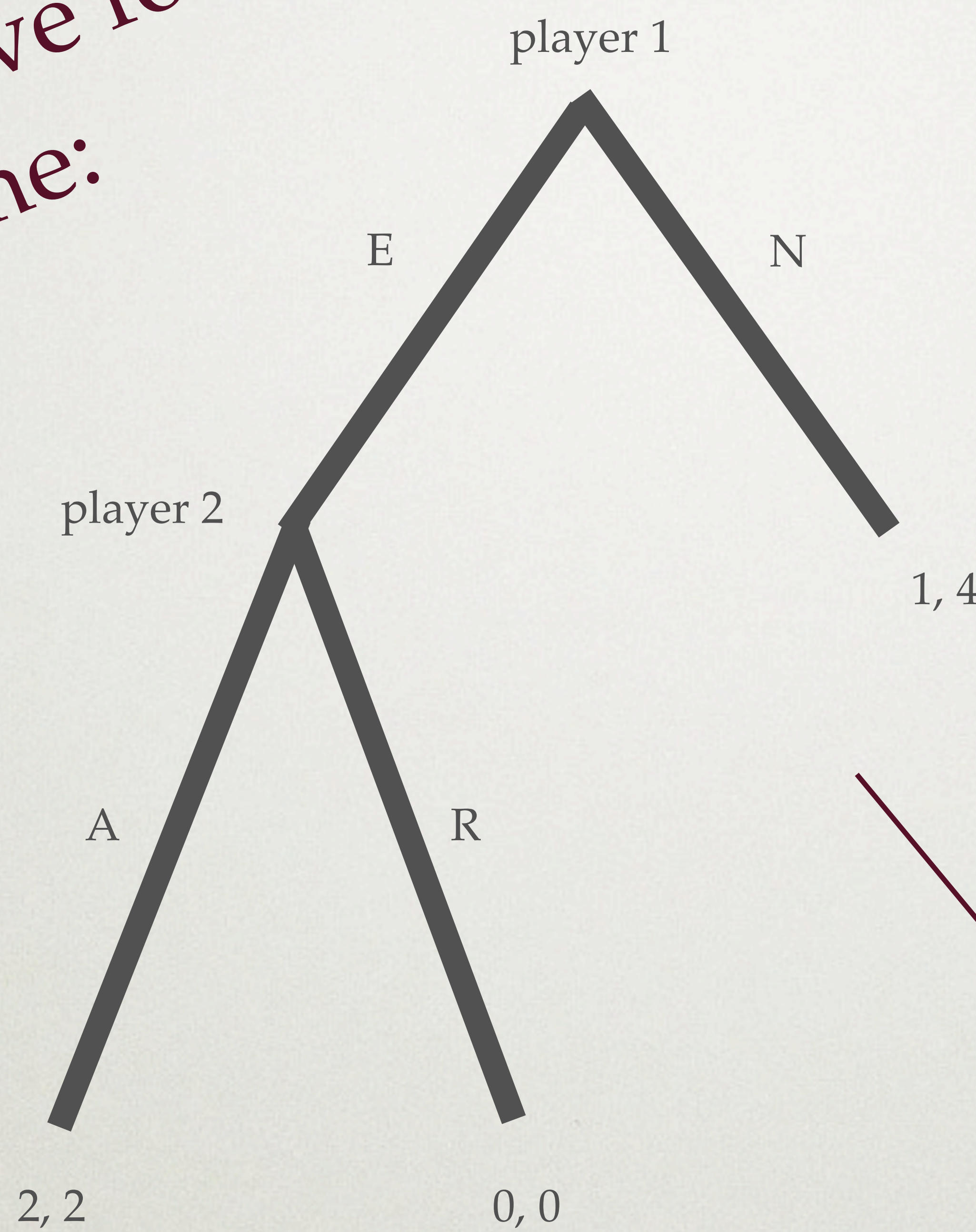
Chain store game

An extensive form game:



A strategy is a rule that stipulate what a player does at every one of her choice points in the tree.

An extensive form game:

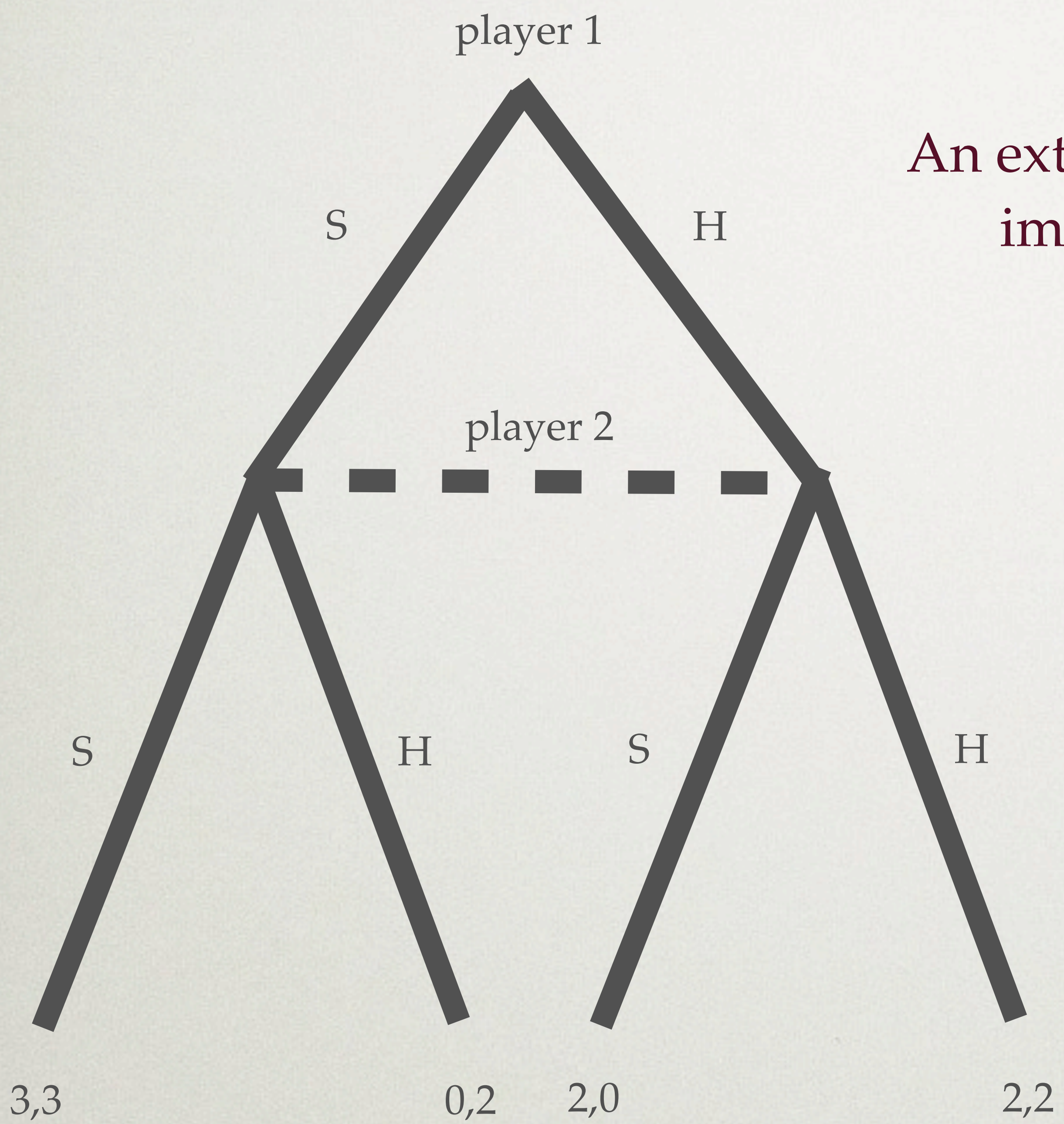


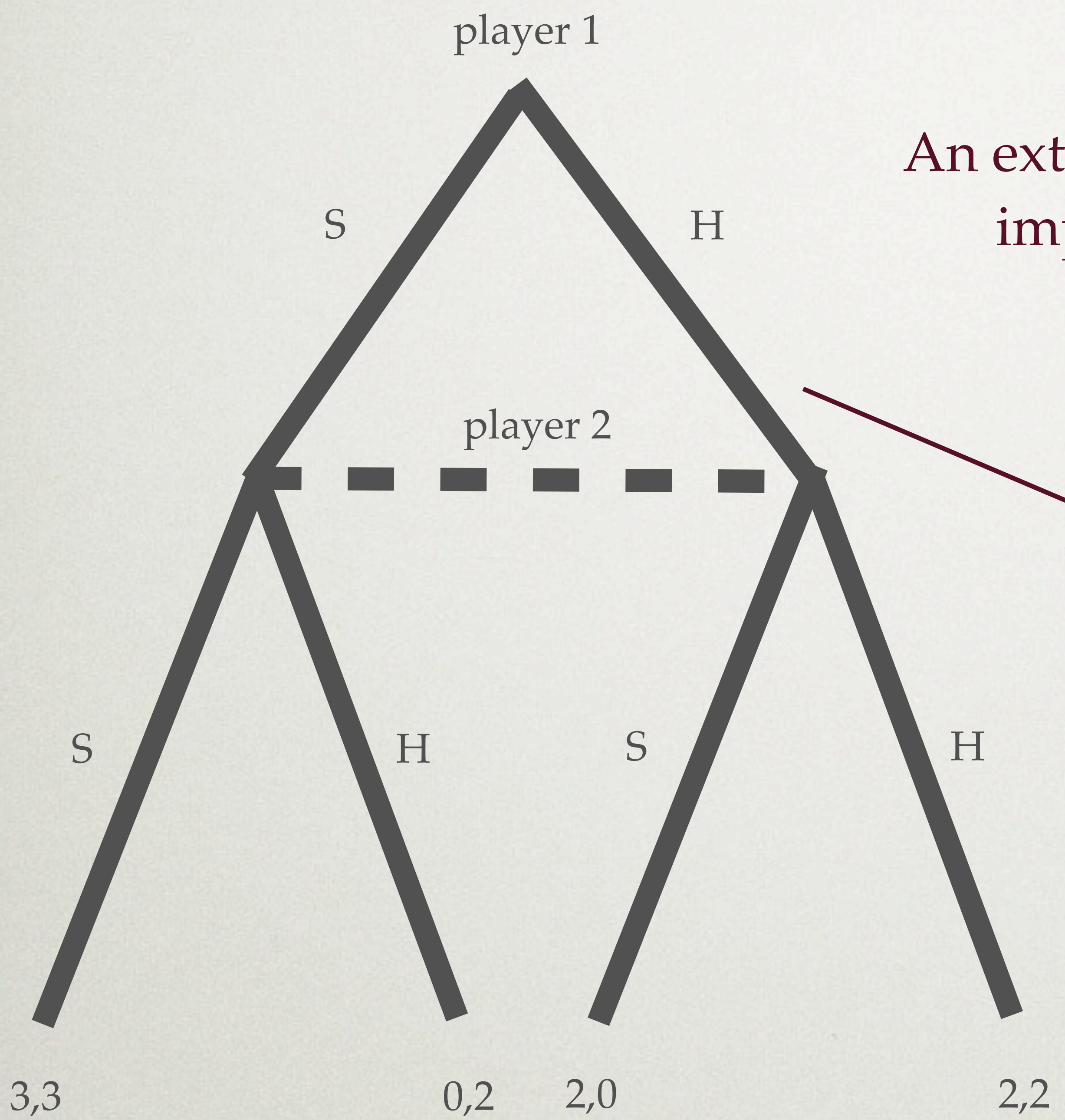
A strategy is a rule that stipulate what a player does at every one of her choice points in the tree.

Transform an extensive form game into a normal form game determining the payoffs for every possible strategy combination.

	A	R
E	2,2	0,0
N	1,4	1,4

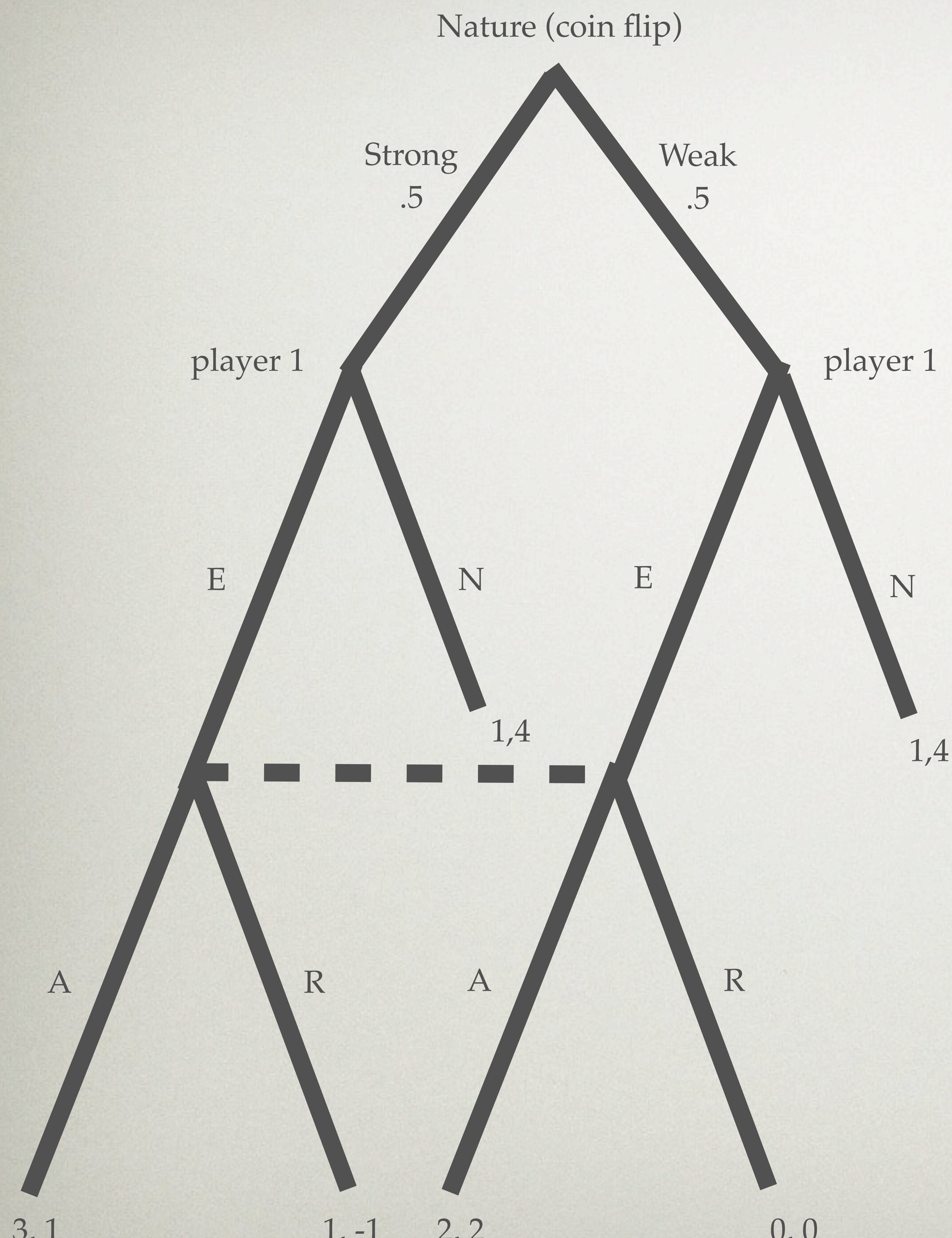
An extensive form game with imperfect information

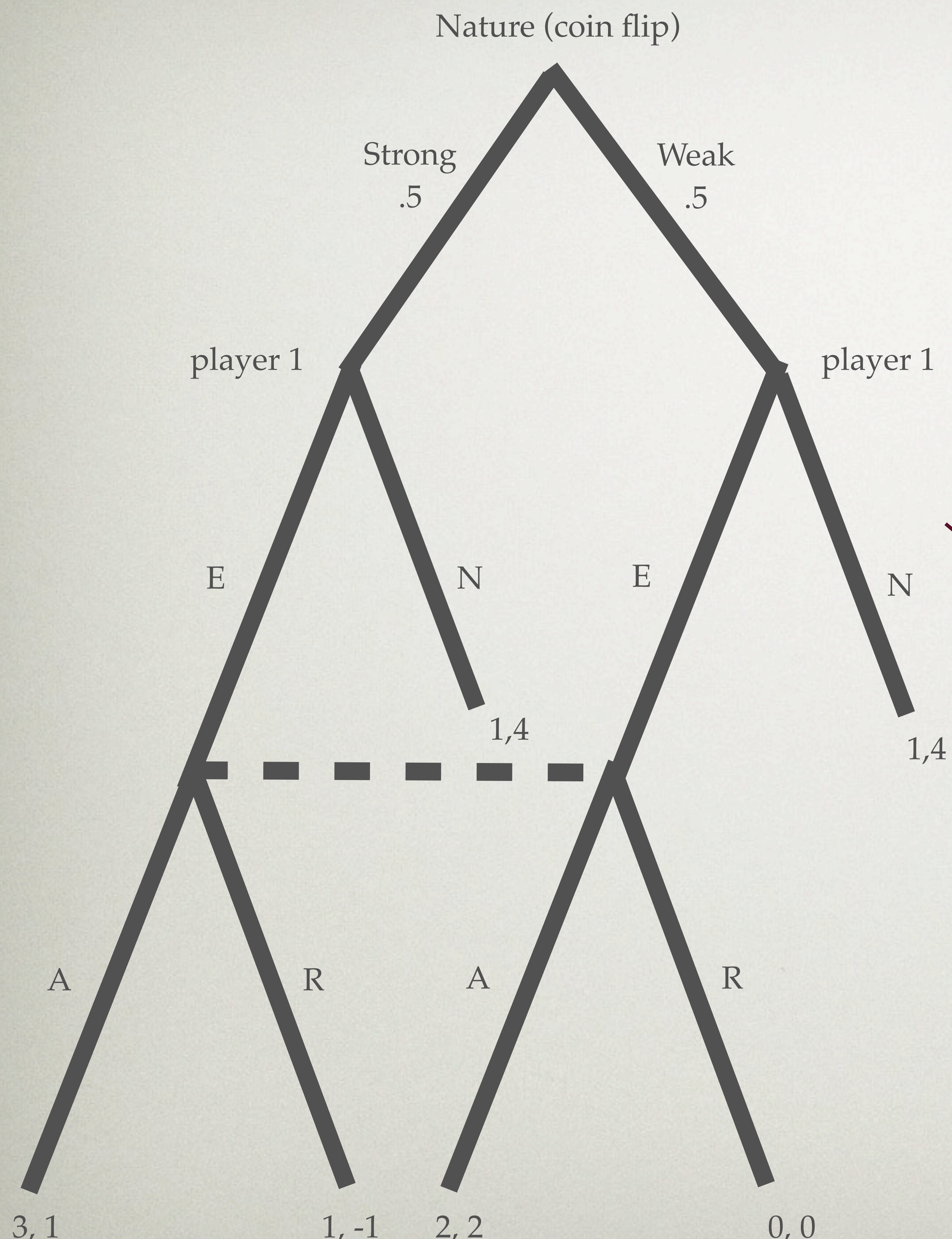




An extensive form game with imperfect information

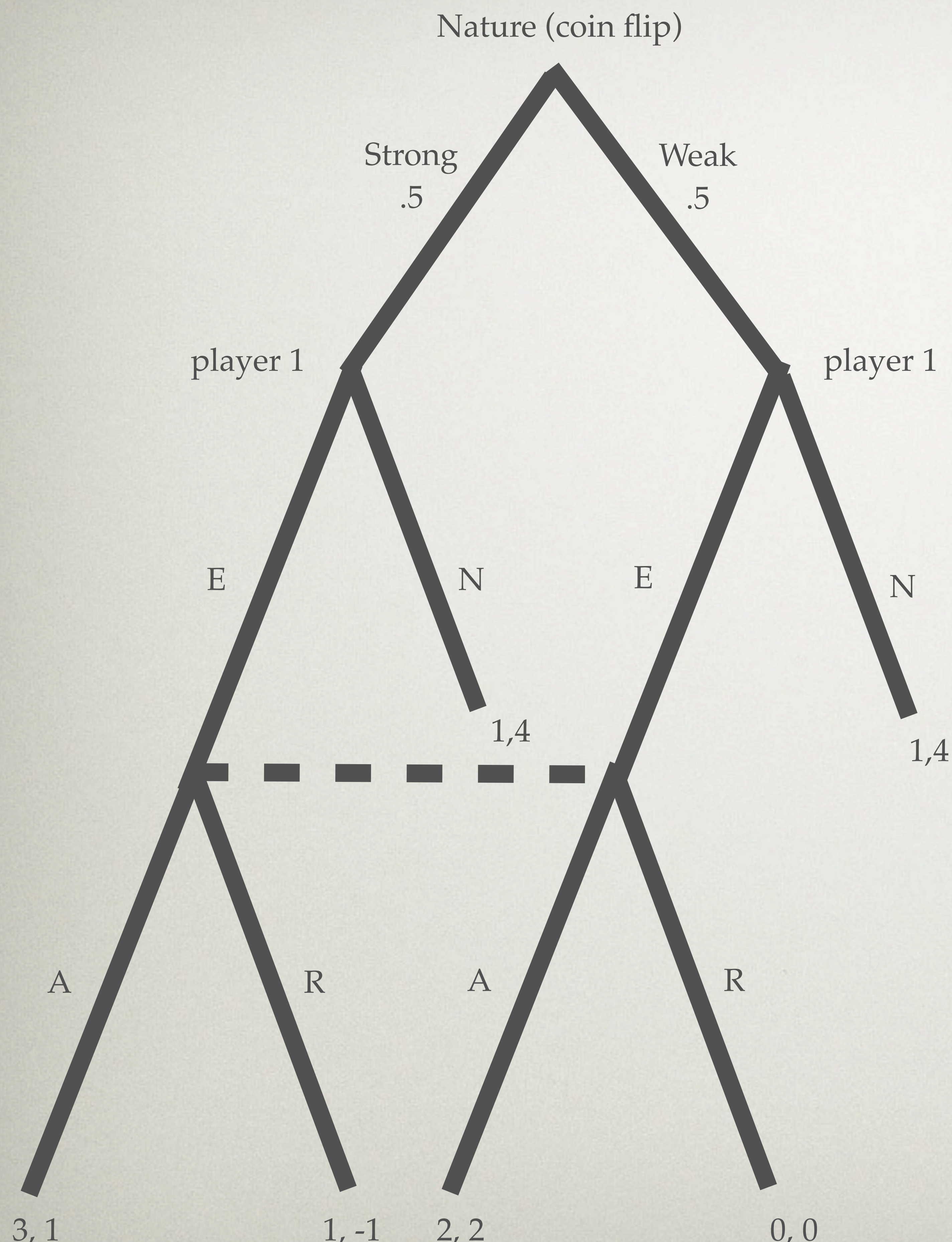
	Stag	Hare
Stag	3,3	0,2
Hare	2,0	2,2


Nature (coin flip)
Strong
.5
Weak
.5
player 1
player 1
E
N
E
N
1,4
1,4
A
R
A
R
3, 1
1, -1
2, 2
0, 0



Strategies stipulate choices at every info set. Payoffs in the strategic form version are expected payoffs from the extensive form.

	A	R
EE	2.5, 1.5	.5, -.5
EN	2, 2.5	1, 1.5
NE	1.5, 3	.5, 2
NN	1, 4	1, 4



In many real life interactions, we don't know each other's utility functions (i.e., their types).

Games like this are referred to as games of incomplete information.

The standard way to deal with incomplete information is to assume that the two players have common priors over the possible types.

These common priors transform games of incomplete information to games of complete but imperfect information.

An EXTENSIVE FORM GAME consists of:

- A set of players
- A rooted tree
- An n-tupe of payoffs for each terminal node of the tree
- A partition of the non-terminal nodes into N+1 subsets, one of which for each player plus one for Nature
- A probability distribution for each of Nature's nodes
- Each player's set of nodes is partitioned into information sets that make choices indistinguishable for the player
- Every directed path from the root to a terminal node crosses each information set at most once