Evolution and conceptual spaces

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Language and Games 2017

1. **Conventional meaning of Predicates: Sim-max games**
2. **Evolution of Vagueness: Bounded Rationality**
3. **Stereotypes: Picking games**

1: Conventional Meaning of Predicates

Simmax games

Philosophical interest

(sparse) Natural properties versus (abundant) Sets

- Natural properties important for
  - Natural Laws (Goodman's new riddle)
  - Meaning (Putnam's paradox)
  - Rules (Kripke's Wittgenstein)
- Sets closed under complementation, union Ptys not
- vFraassen, Stalnaker: Meanings modelled on structured state space
- Gardenfors (2000): natural properties modelled as convex sets
- Why?

Signalling games and Evolution

Lewis' signaling games

- David Lewis (1969): Language use is interactive ⇒ Game theory
- Signalling games: senders and receivers

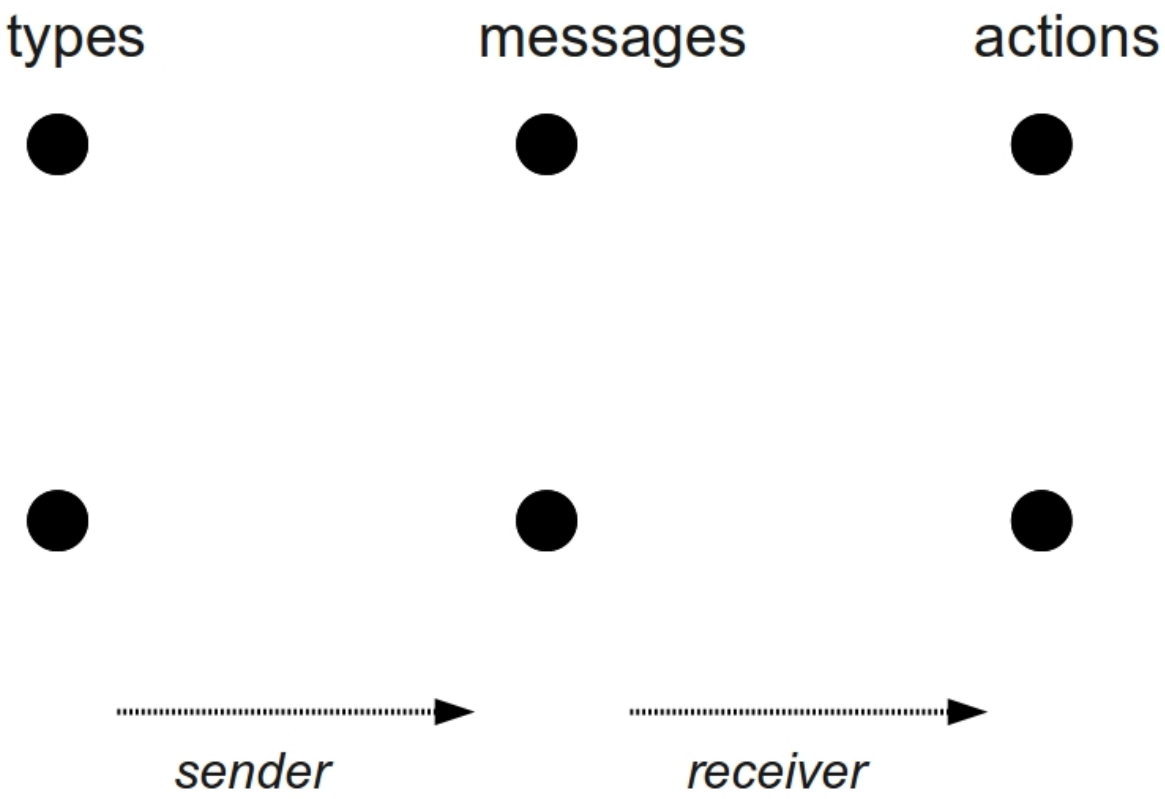
Evolutionary games (Brian Skyrms)

- Lewis requires common knowledge, perfect rationality,
  and does not explain how conventions arise
- Hopeless for us, certainly for animals
  ⇒ Convention due to evolution (inheritance)
  or (best response, or reinforcement) learning

Strategic communication: signaling games

- sequential game:
  1. **nature** chooses a type T
     - out of a pool of possible types T
     - according to a certain probability distribution P
  2. nature shows t to sender **S**
  3. S chooses a message m out of a set of possible signals M
  4. S transmits m to the receiver **R**
  5. R chooses an action a, based on the sent message.
- Both S and R have preferences regarding R's action, depending on t.
- S might also have preferences regarding the choice of m (to minimize signaling costs).

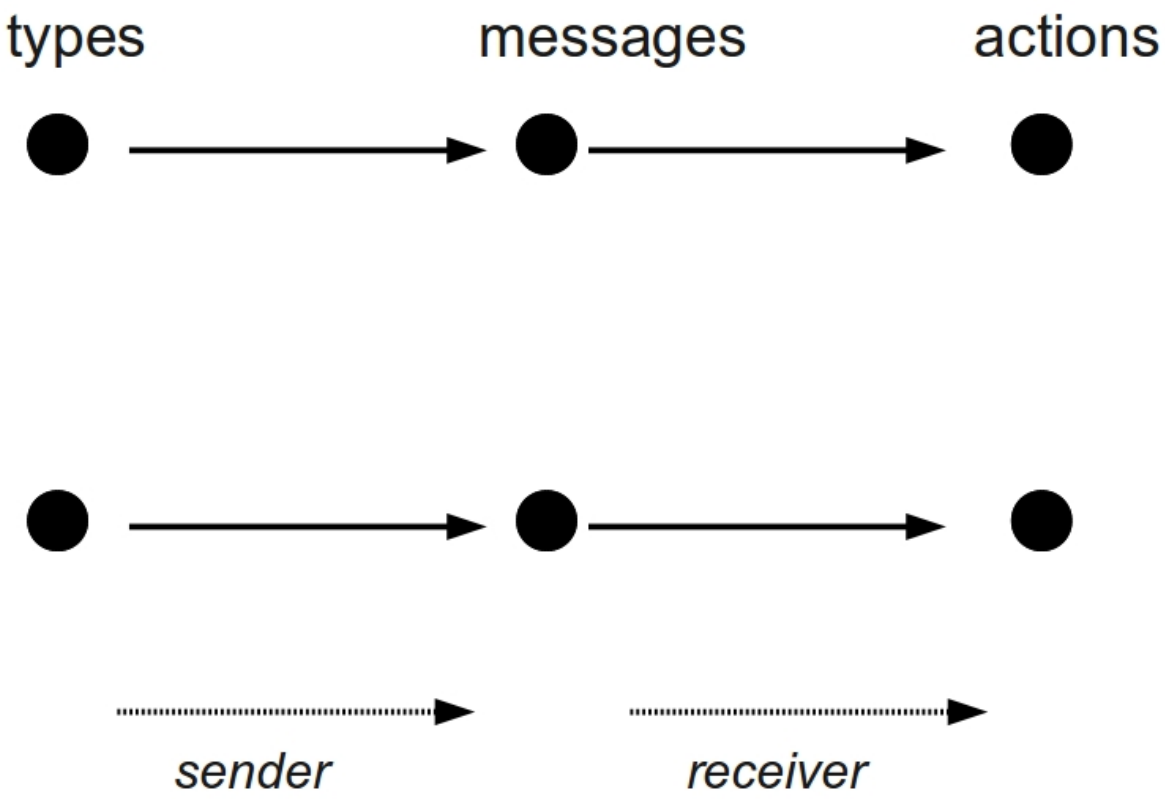
Basic example



utility matrix

	a_1	a_2
t_1	$1, 1$	$0, 0$
t_2	$0, 0$	$1, 1$

Basic example: Equilibrium 1



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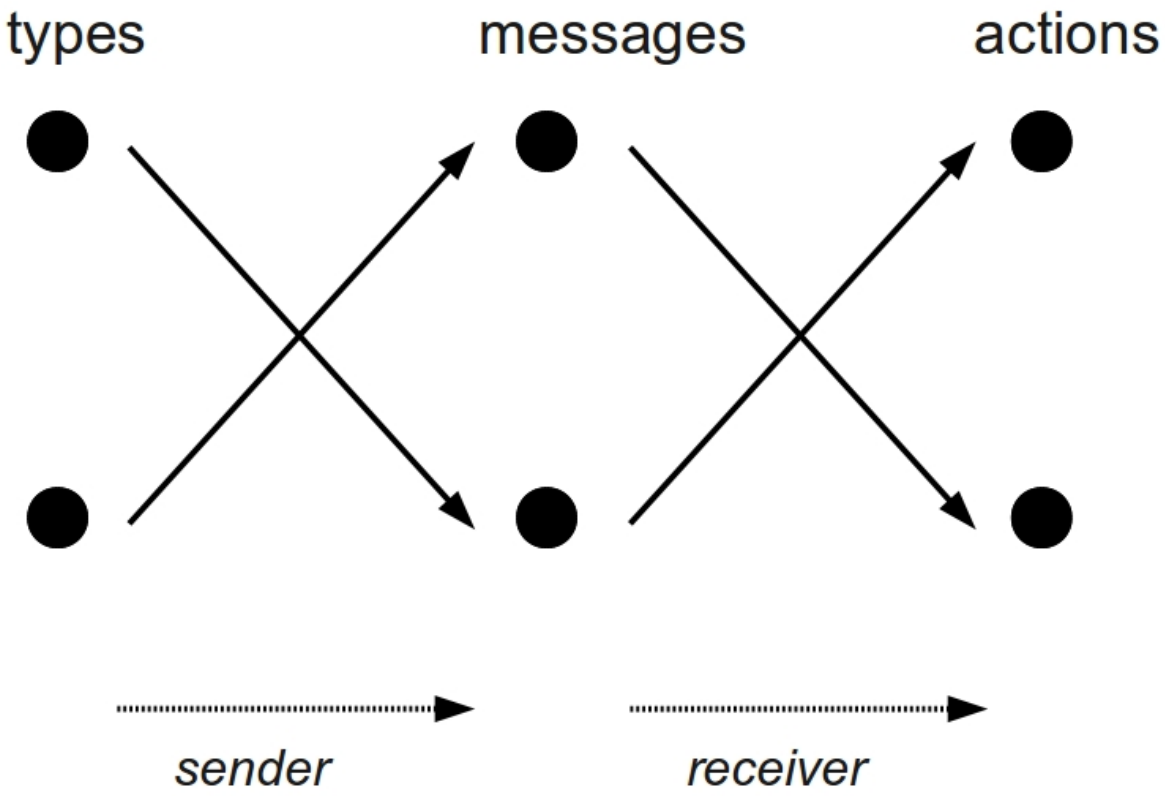
Basic example: Equilibrium 2



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Equilibria and meaning

Equilibria

- two strict Nash equilibria $\langle \sigma, \rho \rangle$
- these are the only 'reasonable' equilibria:
  - they are evolutionarily stable (self-reinforcing under iteration)
  - they are Pareto optimal (cannot be outperformed)

Meaning derived from $\langle \sigma, \rho \rangle$

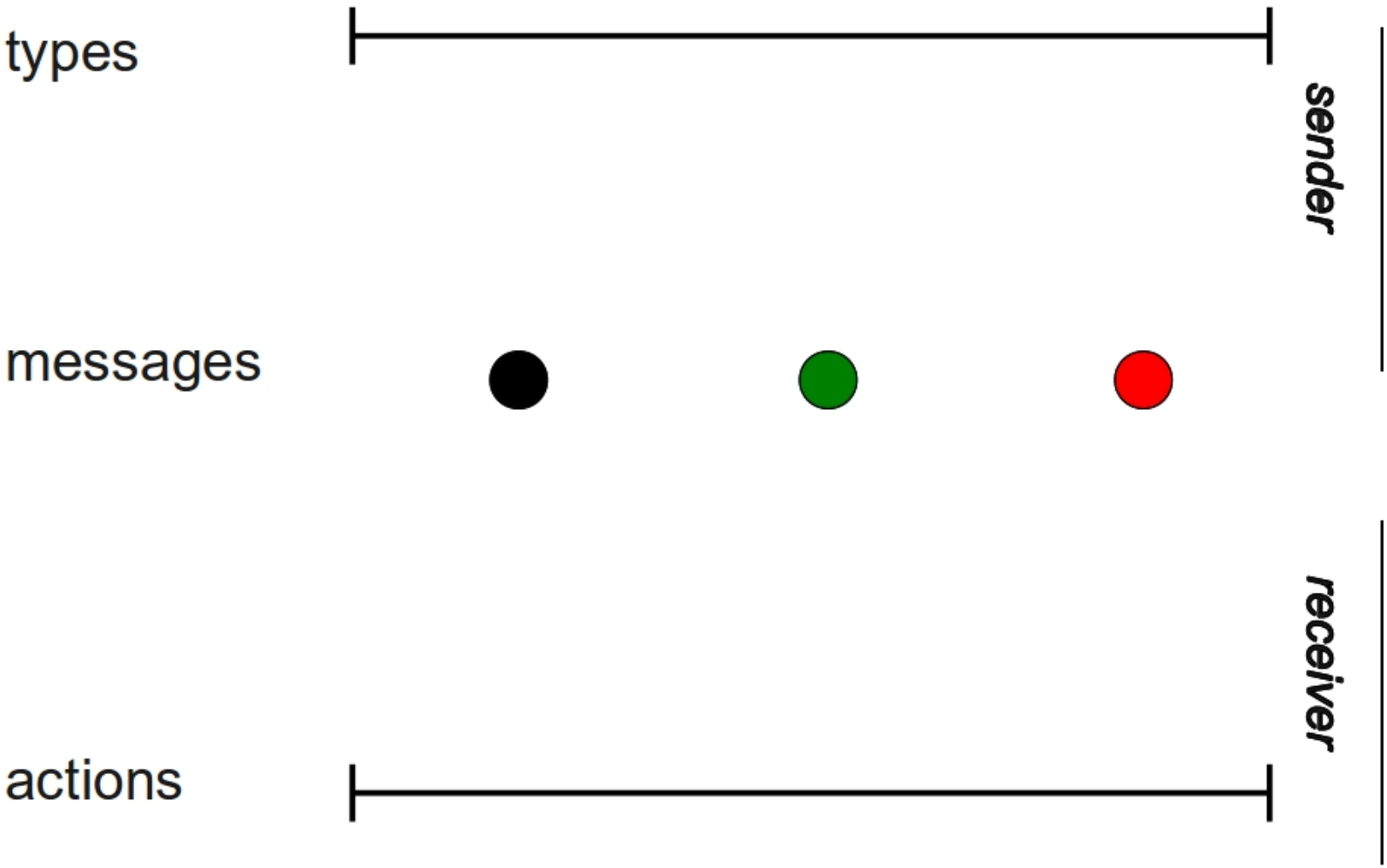
- Pure Speaker strategy $\sigma \in T \times M \to \{0, 1\}$

$\Rightarrow$ $[[m]]^s = \{t \in T : \sigma(t)(m) = 1\}$ Descriptive meaning

- Pure Hearer strategy $\rho \in M \times A \to \{0, 1\}$

$\Rightarrow$ $[[m]]^h =$ action with highest expected utility Imperative meaning

Euclidean meaning space: Sim-max games

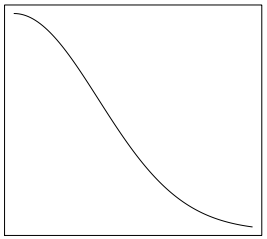


Utility function

General format

$$u_{s/r}(t, m, t') \quad = \quad \mathrm{sim}(t, t')$$

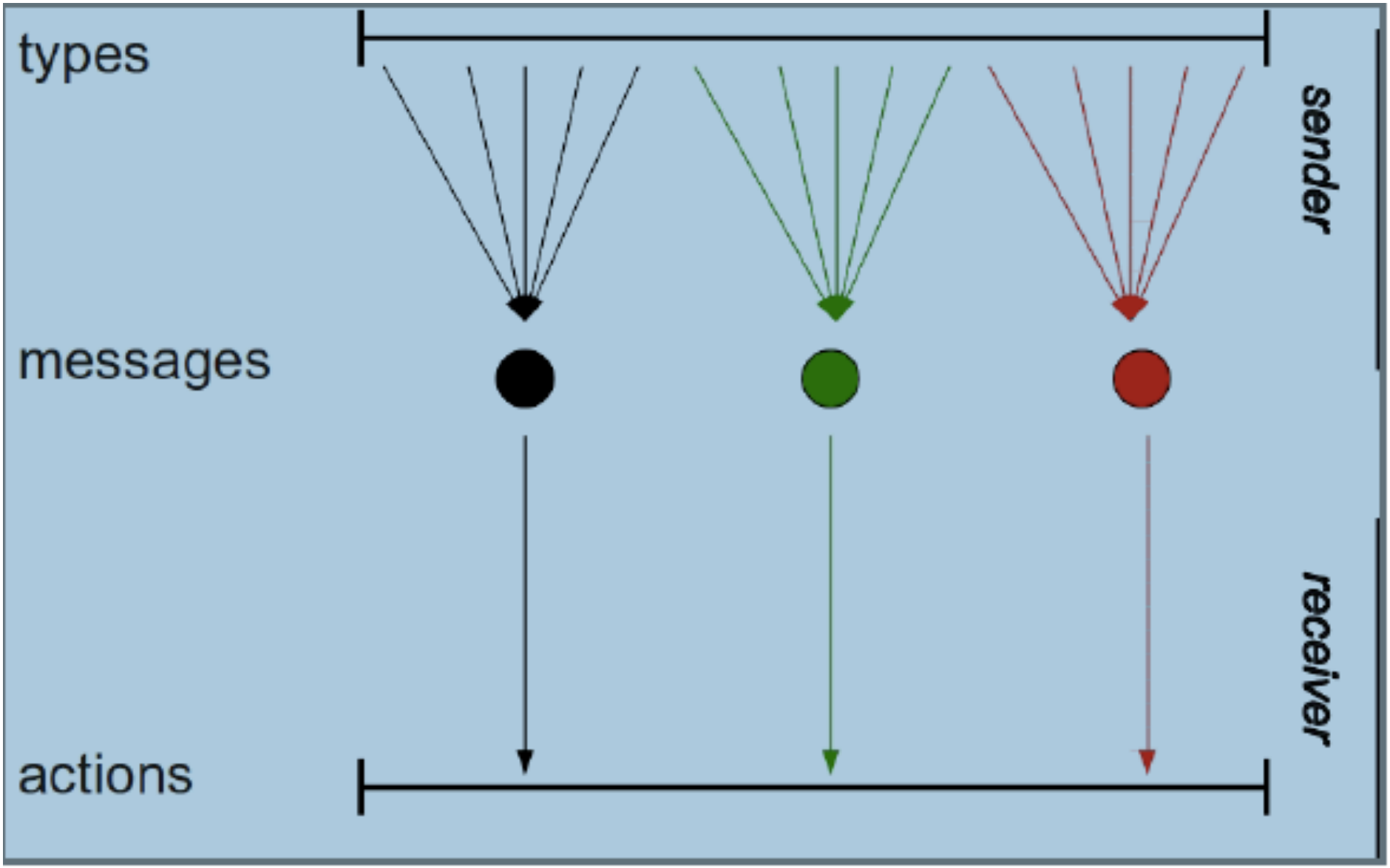
- $\mathrm{sim}(x, y)$ is strictly monotonically decreasing in Euclidean distance $\|x - y\|$



In this talk, we assume a **Gaussian** similarity function

$$\mathrm{sim}(x, y) \doteq \exp(-\frac{\|x - y\|^2}{2\sigma}).$$

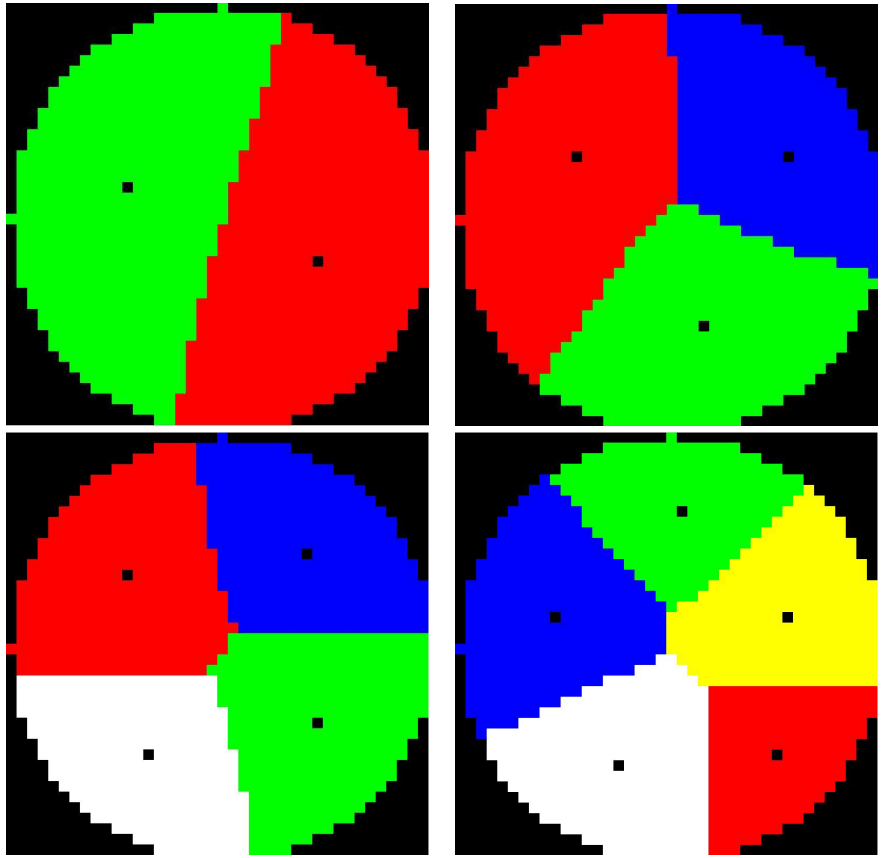
Euclidean meaning space: equilibrium



Simulations

- two-dimensional circular meaning space
- finitely many pixels (meanings)
- uniform distribution over meanings

- Descriptive meanings partition meaning space
- cells are convex sets
- give rise to prototype



2: Evolution of Vagueness: Bounded Rationality

Vagueness

- many evolutionarily stable/Pareto optimal equilibria
- all are strict (except for a null set at category boundaries)
- a *vague* language would be one where the sender plays a mixed strategy

Vagueness is not rational

Rational players will never prefer a vague language over a precise one in a signaling game. (Lipman 2009)

- similar claim can be made with regard to evolutionary stability (as corollary to a more general theorem by Reinhard Selten)

Vagueness is not evolutionarily stable

In a signaling game, a vague language can never be evolutionarily stable.

Vagueness and bounded rationality

- Lipman's result depends on assumption of perfect rationality
- we experimented (toyed around?) with three deviations from perfect rationality that support vagueness:
  - Learning: players have to make decisions on basis of limited experience
  - Stochastic decision: players are imperfect/non-deterministic decision makers
  - Noisy decision makers
  - Imperfect reinforcement learning (O'Connor)

Stochastic choice (Luce, 1965)

- real people are not perfect utility maximizers
- they make mistakes $\rightsquigarrow$ sub-optimal choices
- still, high utility choices are more likely than low-utility ones

Rational choice: best response

$$P(a_i) = \begin{cases} \frac{1}{|\arg_j \max u_i|} & \text{if } u_i = \max_j u_j \\ 0 & \text{else} \end{cases}$$

Stochastic choice: (logit) quantal response

$$P(a_i) = \frac{\exp(\lambda u_i)}{\sum_j (\lambda \exp u_j)}$$

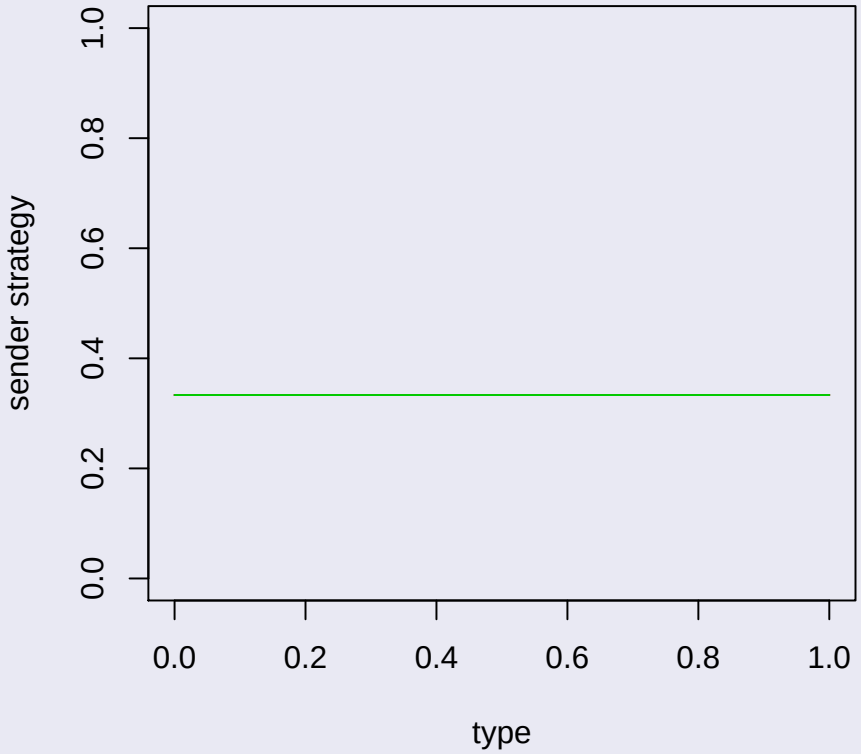
Quantal response

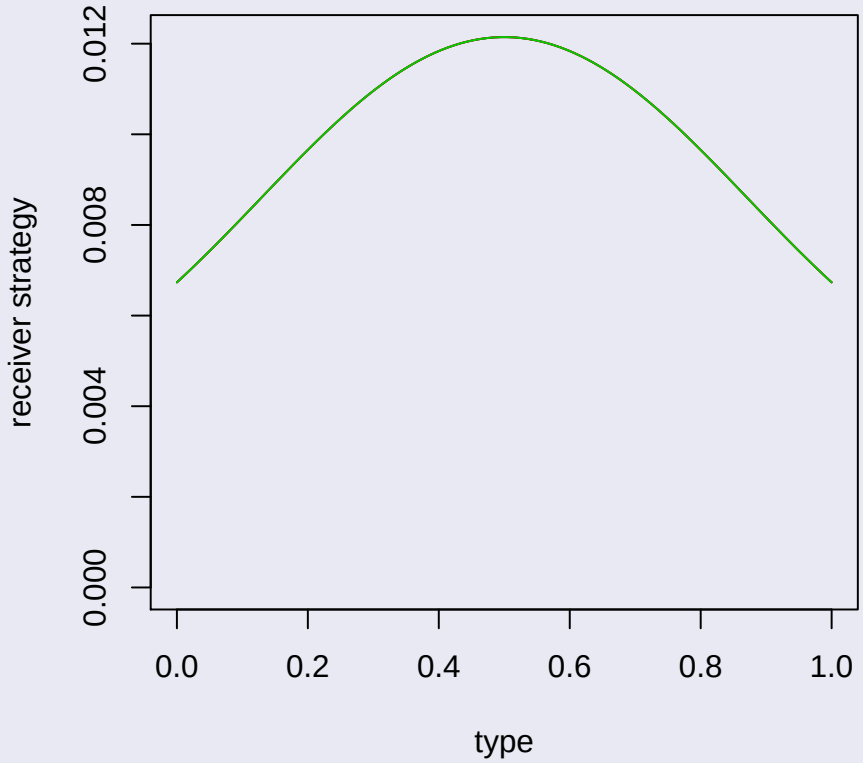
- λ measures degree of rationality
- $\lambda = 0$:
  - completely irrational behavior
  - all actions are equally likely, regardless of expected utility
- $\lambda \to \infty$
  - convergence towards behavior of rational choice
  - probability mass of sub-optimal actions converges to 0
- if everybody plays a quantal response (for fixed λ), play is in **quantal response equilibrium** (QRE)
- asl $\lambda \to \infty$, QREs converge towards Nash equilibria

QRE and vagueness

$\lambda \leq 4$

- only babbling equilibrium

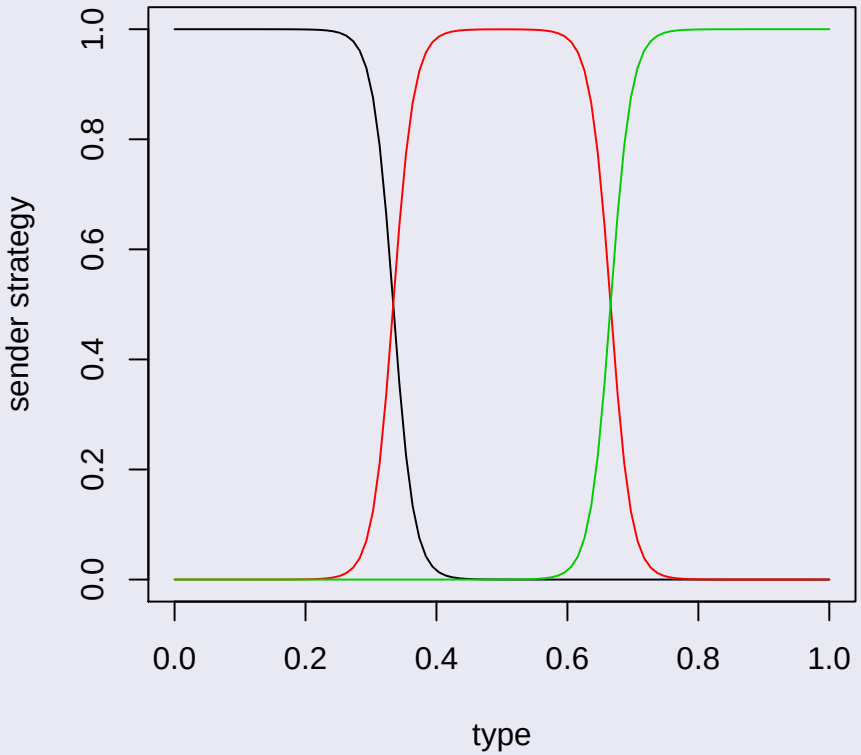


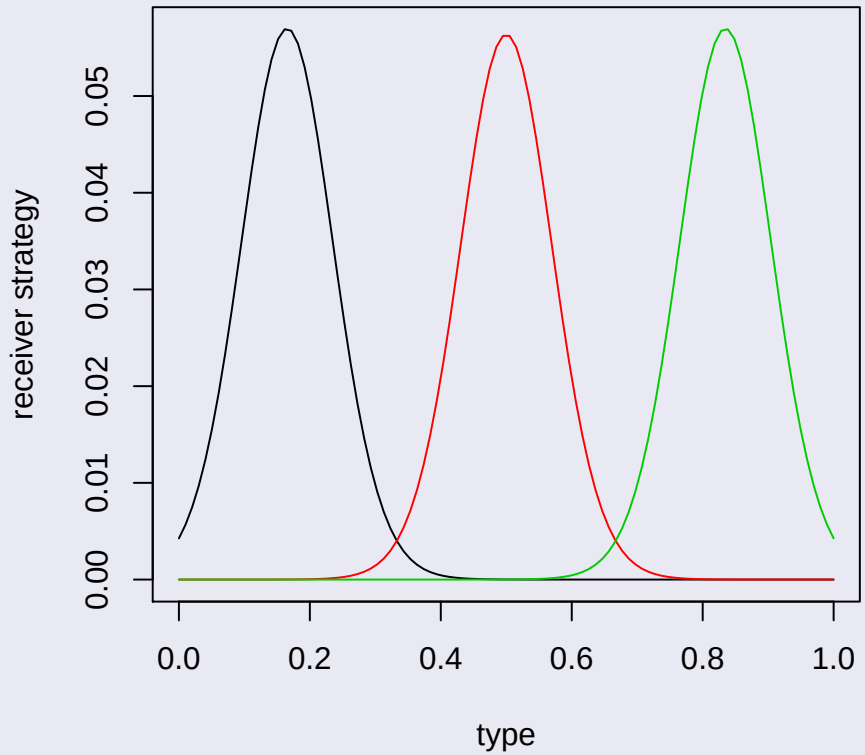


QRE and vagueness

$\lambda > 4$

- separating equilibria
- smooth category boundaries
- prototype locations follow bell-shaped distribution



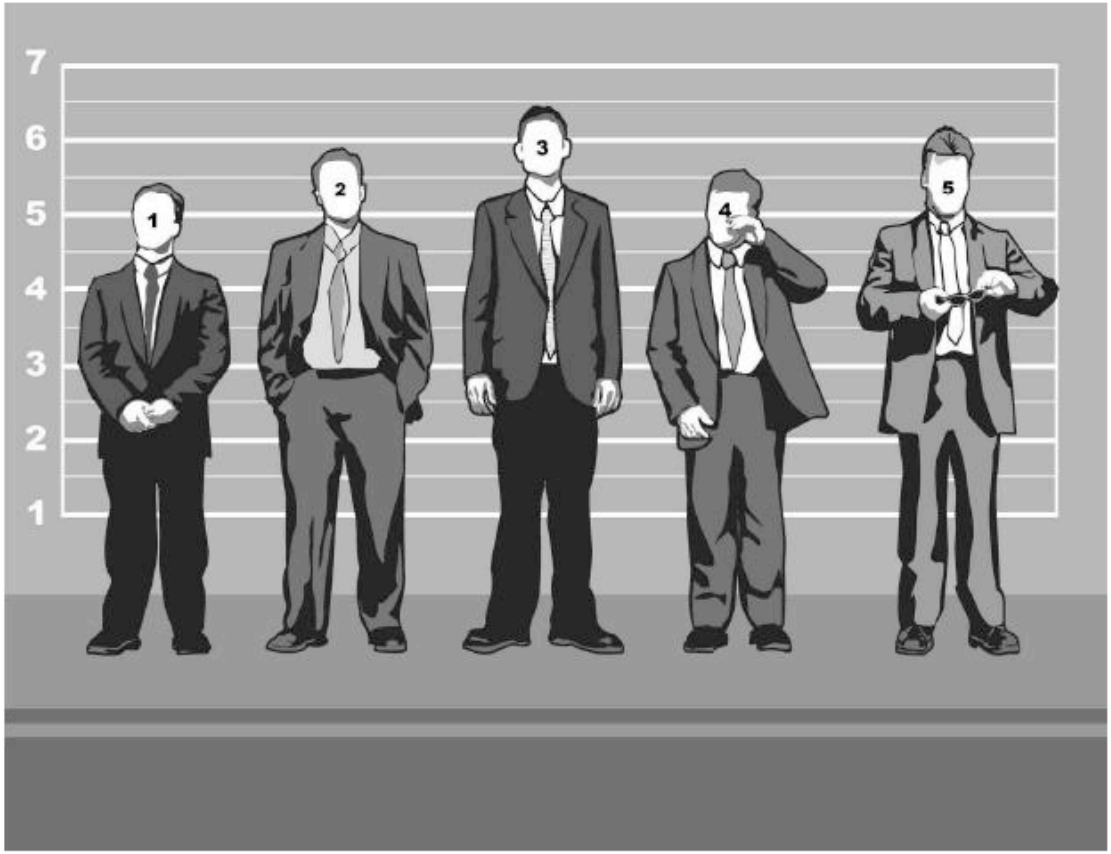


From Language to Thought?

- We don't have to think of signaling as a 2-person game:
  One person observing, representing, and acting of/on world is enough
- Given our non-perfect λ, this suggest that our thoughts/beliefs are vague as well
- $\Rightarrow$ it is not that we have precise thoughts that we only vaguely communicate
  but we have only vague thoughts that we want to communicate in language
- $\Rightarrow$ it is irrational to make our language precise
- That's why language is and should be vague!

- From Wittgenstein to Fodor?

Use of contradictions (Alxatib & Pelletier, Ripley)



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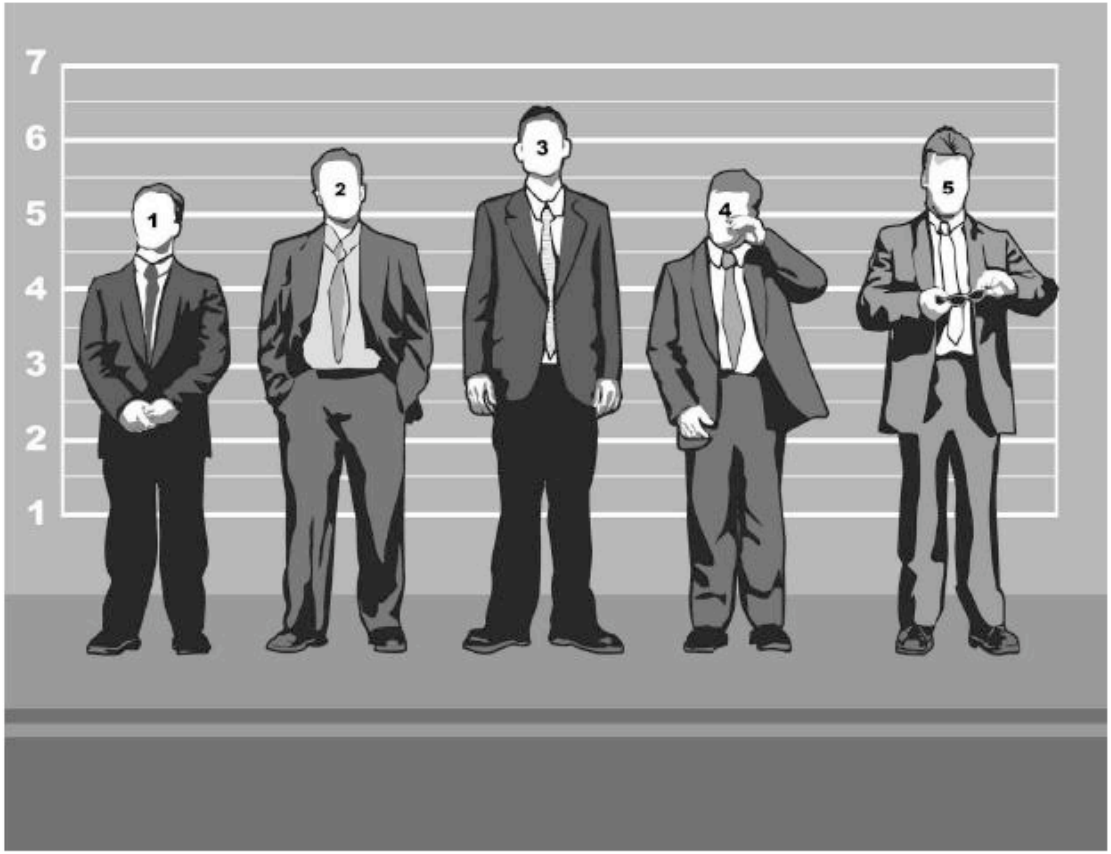

- V: Is 2 tall? A: no. V: Is 2 not tall? A: no.

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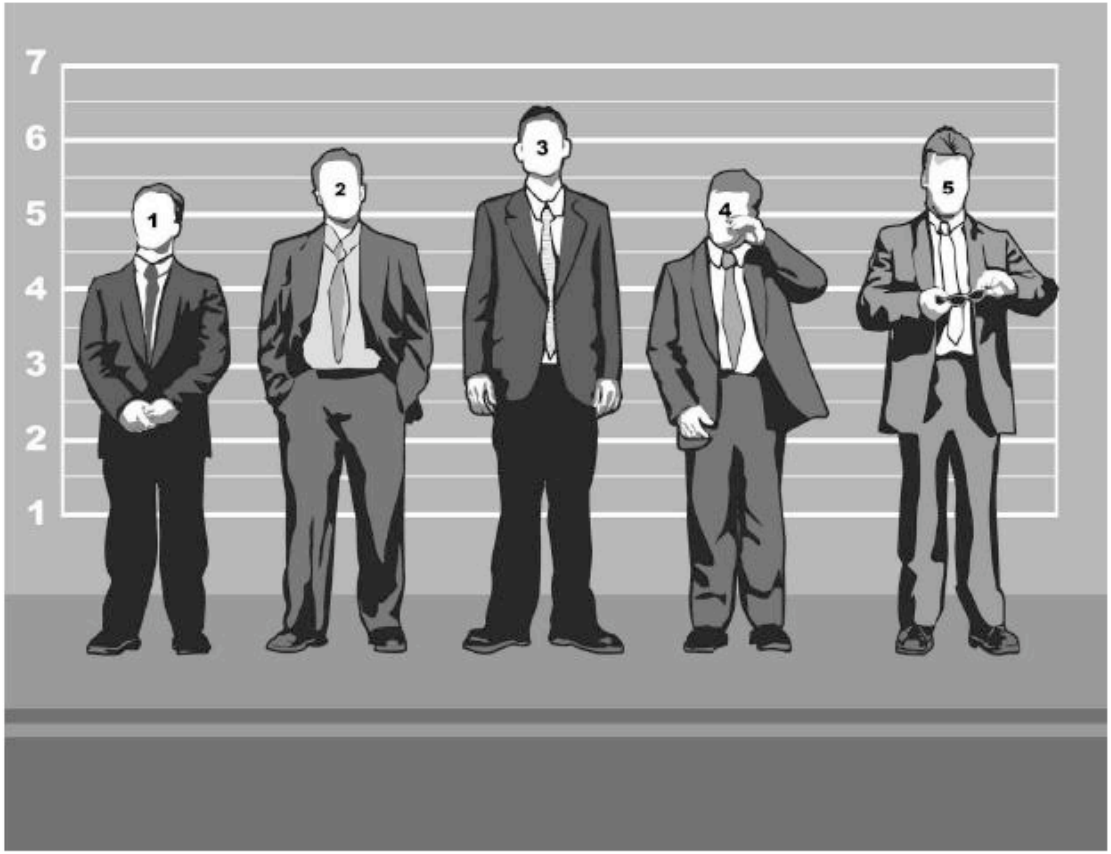


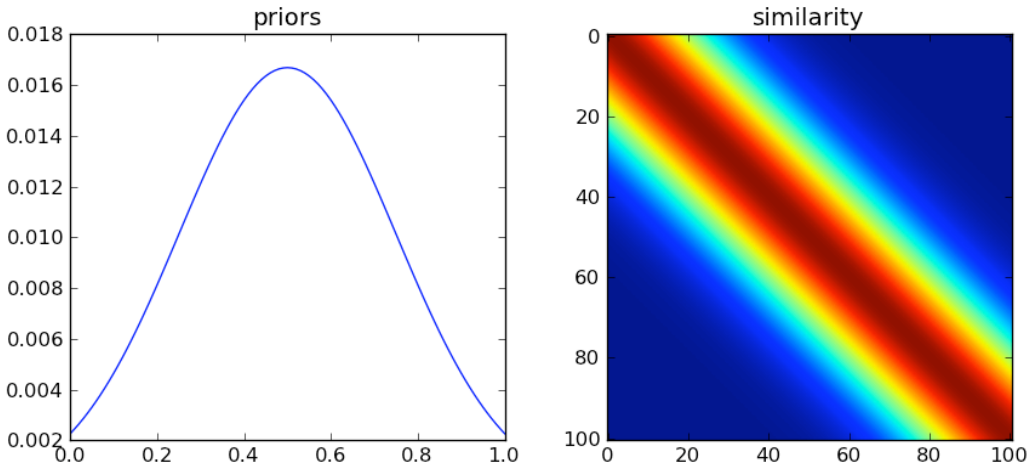
- V: Is 2 tall? A: no. V: Is 2 not tall? A: no.
- V: is 2 tall and not tall? A: yes.

From (equilibrium) use to compositional meaning

- Derive meaning in dynamic process of evolution of basic messages
- $V_s(m) = \frac{\sigma^{-1}(s)(m)}{max_{t \in T} \sigma^{-1}(t)(m)}$
- Derive meaning in process of evolution of complex messages via truth rules
  - $V_s(\neg\phi) = 1 - V_s(\phi)$
  - $V_s(\phi \wedge \psi) = min\{V_s(\phi), V_s(\psi)\}$
  - $V_s(\phi \vee \psi) = max\{V_s(\phi), V_s(\psi)\}$
- Look at meanings in evolved equilibrium state
- Standard opinion: fuzzy logic not good, because we say $T(x) \wedge \neg T(x)$ for borderline case, even though this can have at most value $\frac{1}{2}$. Cannot be!

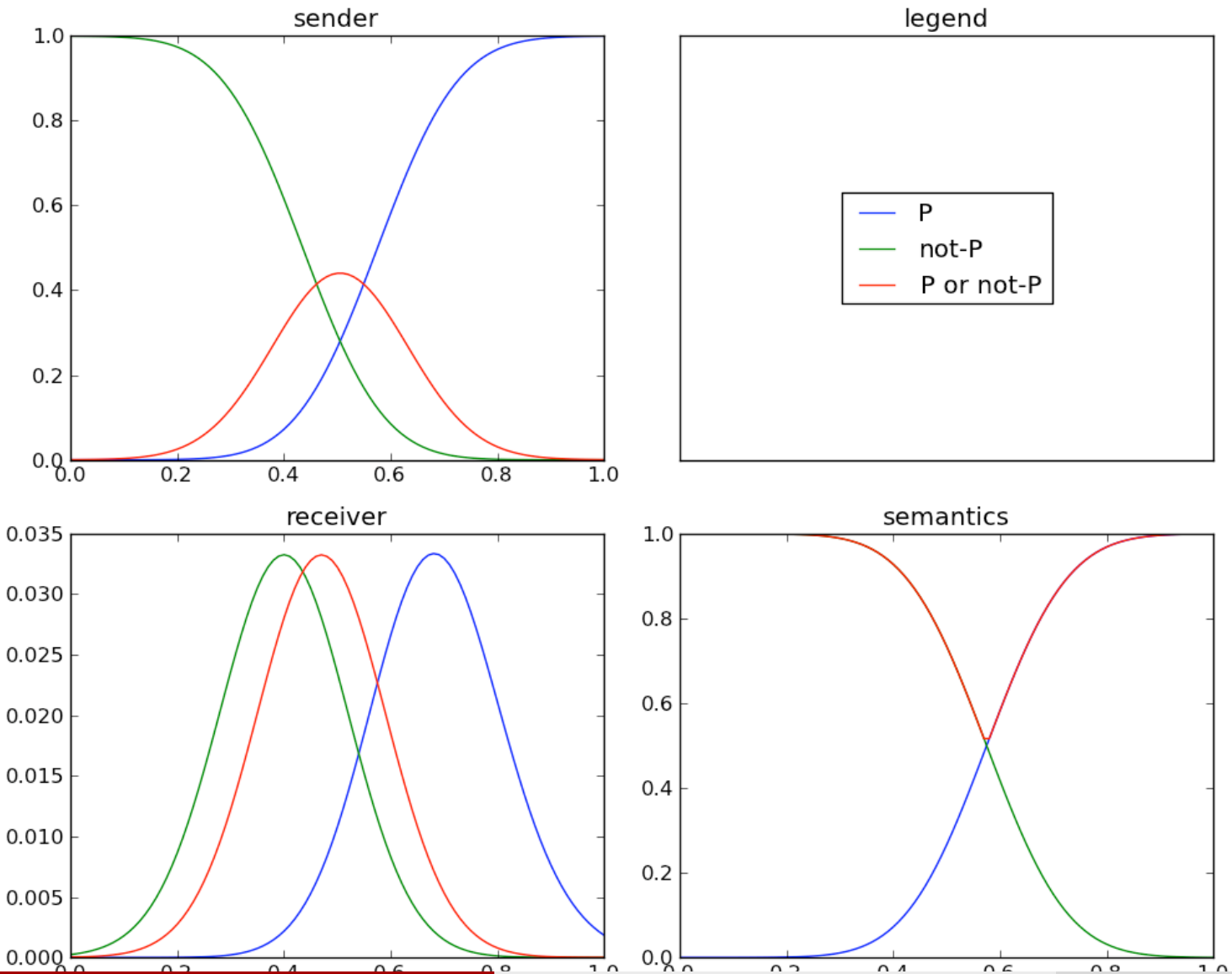
Set up

- Start with set of states (P is prior) + utility based on similarity

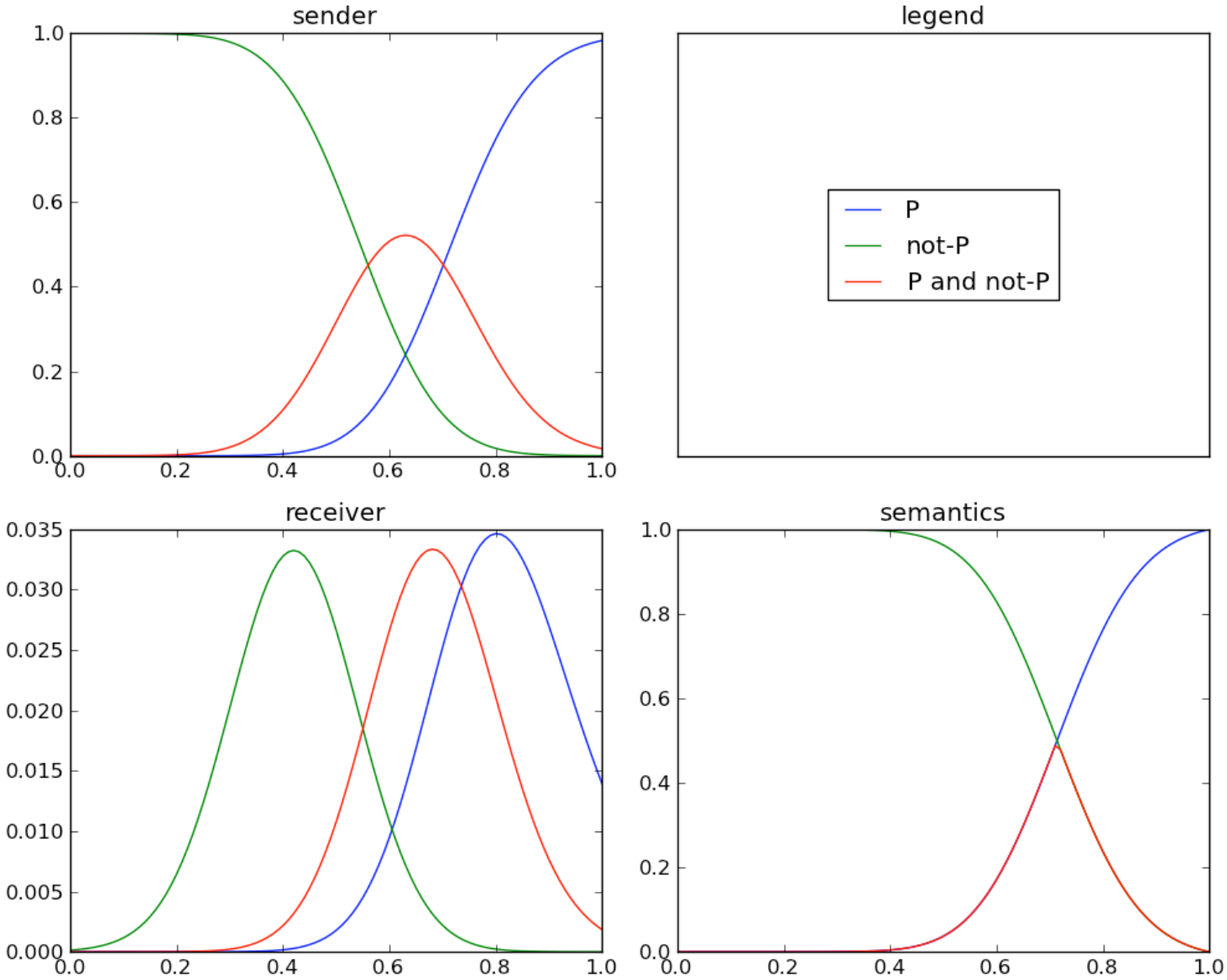


- Set of messages: 'Tall(x)' and '¬ Tall(x)' and ....
- Best response dynamics, constrained by semantic meanings
- Not Quantal, but Noisy best response

Only 'Tall' with tautology (als dummy)



Only 'Tall' with contradiction (semantic impact)



Conclusions

- Natural Properties: evolve in structured meaning space
- Vagueness evolves if learning, observation, decision making ... non-perfect
- Contradictions evolve as well....

3: Stereotypes: Picking games

Stereotypes or prototypes?

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- How to model, or what are concepts or properties?
- Age old discussion in Philosophy (Universals) and Psychology
- How do we best represent them?

Psychological and Linguistic interest

Efficient representation: average or ideal?

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- Others: extreme representatives important if categories are defined in terms of contrasts with others.
- Stereotypes important also for interpretation of generic statements and interpretation of absolute adjectives
- Why extreme values?

Goal of this talk: answer Why-questions

Why convex meanings?

- Sim-max games
- Picking games

Why pick out typical representative?

- Utility in terms of similarity
- Utility in terms of highest chance of successful communication

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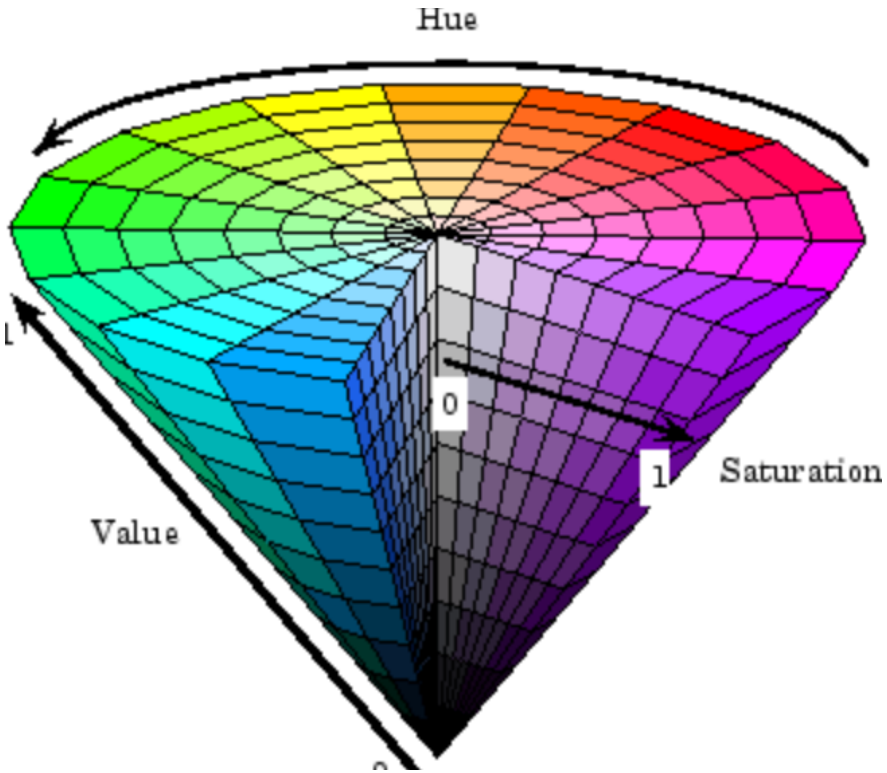

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- What are descriptive and imperative meanings of messages?

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- Imper. meaning: $I(m) = \rho(m) \quad \forall c$ picking the ith largest man in c

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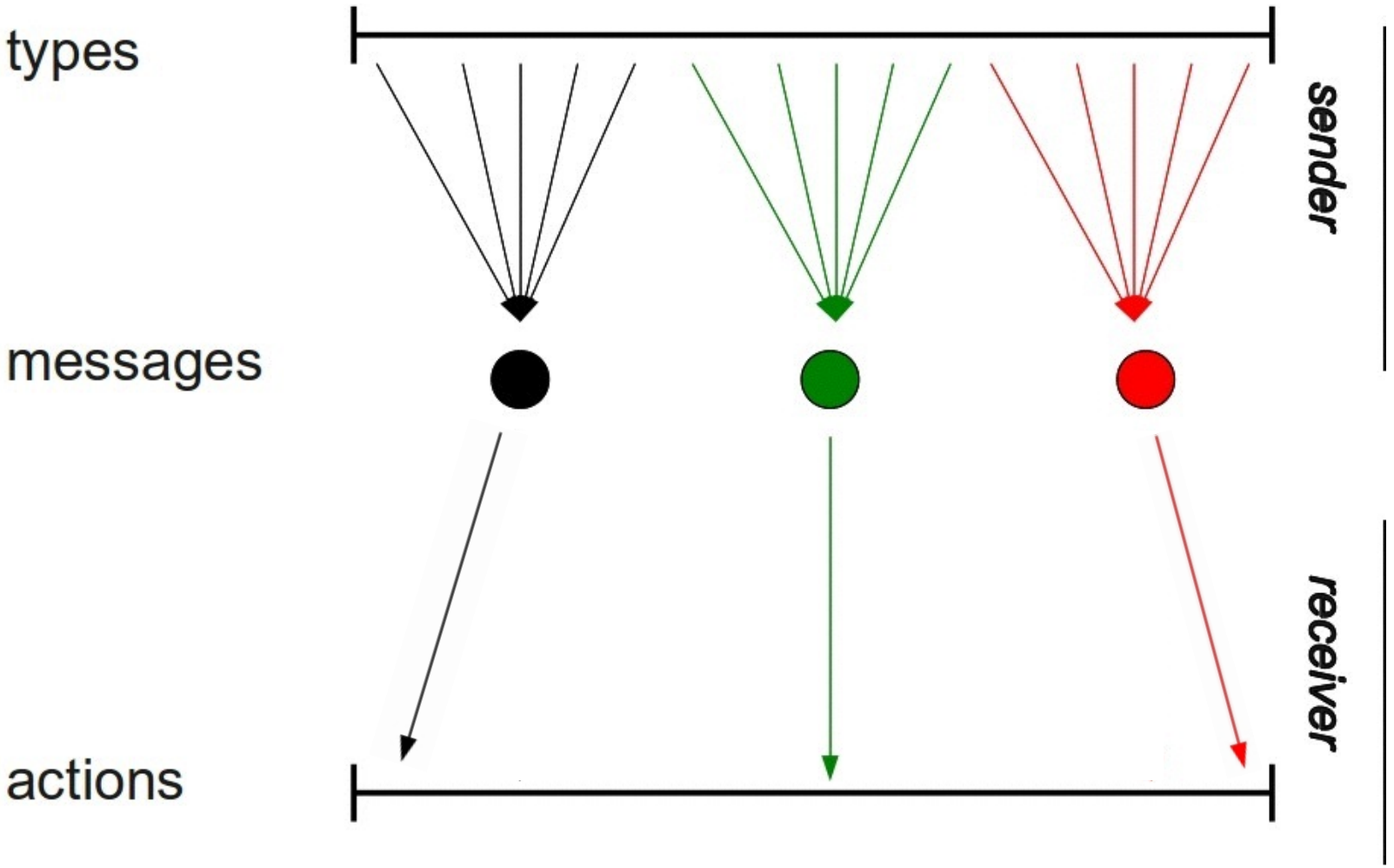
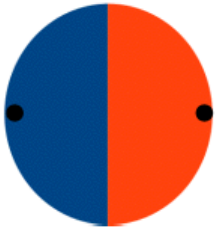
$\Rightarrow$ Imperative meaning is stereotype: extreme representative.

Generalizations I



Generalizations II

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  $\cdots$ also for colours