Game Theoretic Pragmatics

L&G Project 2017

Thomas Brochhagen & Robert van Rooij

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- On-line language use as a game between interlocutors
- *Computational* model of back-and-forth reasoning
- Bridge between theoretical descriptive analysis and quantitative experimental data
- Predictions about different types of interlocutor behavior
  - Level of sophistication (bounded rationality)
  - Preferences
  - Choice rule
  - Priors

Gricean Maxims (quick recap)

1. Maxim of Quality
   - Do not say what you believe to be false
   - Do not say that for which you lack adequate evidence
2. Maxim of Quantity
   - Make your contribution as informative as is required
   - Do not make your contribution more informative than is required
3. Maxim of Relevance
4. Maxim of Manner
   (avoid ambiguity & obscurity, be brief & orderly)

· Paul Grice. Logic and conversation.
In *Studies in the Ways of Words*, chapter 2, pages 22–40. Harvard University Press, Cambridge, MA, 1975

Grice

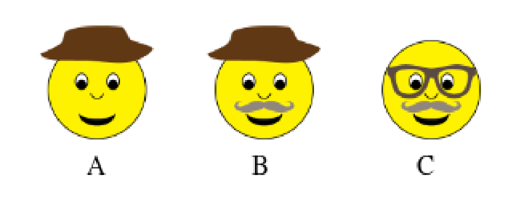
Conjecture about rational foundation of maxims
(+ cooperativity assumption)

Game Theoretic Pragmatics

Can use be accounted for as (approximate) rational behavior?

- Not restricted to cooperation
- No mention or refinement of maxims

The guy with the mustache...



(1) Some students passed the exam.

a. $\rightarrow$ More than one student passed the exam.

b. $\rightsquigarrow$ Some but not all students passed the exam.

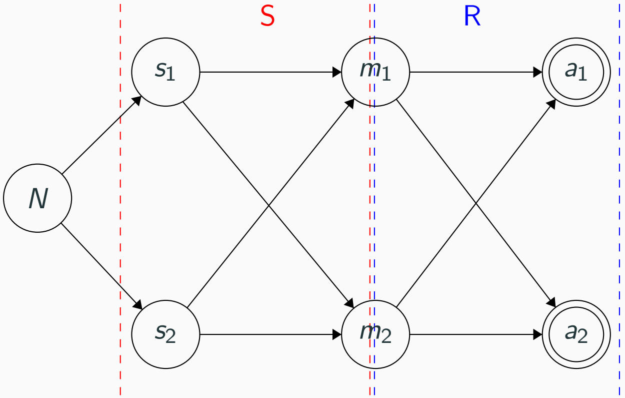
Away from static solution concepts...

- Model agent-internal perspective
- What behavioral predictions follow from which assumptions about agents'...
    - (higher order) beliefs
    - rationality
    - dispositions to act or revise beliefs
    - ...

This session

- Games as conversational *context* models
- Iterated X-Response
  - Iterated Best Response
  - Iterated Quantal Response
- Recent developments & quantitative modelling

Reminder: Lewisian signaling



· David Lewis. *Convention: A Philosophical Study*.
Harvard University Press, Cambridge, 1969

Setup I

- Two players: Sender & Receiver;
- States S
- Messages M
- Acts A

In the following: $S = A$

Interpretation game

Setup II

- Beliefs $\mathbf{p} = \Delta^+(|S|)$,
  $\Delta^+(n) = \{\langle p_0, ...p_{n-1}\rangle \mid \sum p_i = 1 \text{ and } p_i > 0\}$
- Conventional meaning as Boolean matrix L, $(|S|, |M|)$, where L_{ij} is the value of m_j in s_i;
- U_R and U_S are $(|S|, |A|)$-matrices;
- cost vector $\mathbf{c}$ of length $|M|$

In the following: $U_R = U_S = U$

Games as contextual models

- Semantic conventions as a starting point
  Usually: a shared language fragment

- Game represents relevant conversational context

Scalar implicatures

(2) Some students passed the exam.
$\rightsquigarrow$Some but not all students passed the exam

- S?
- M?
- $\mathbf{p}$?
- $\mathbf{c}$?
- ...

Scalar implicatures

(3) Some students passed the exam.
$\rightsquigarrow$Some but not all students passed the exam

- $S = \{s_{\exists\neg\forall}, s_{\forall}\}$
- $M = \{m_{\text{some}}, m_{\text{all}}\}$
- $\mathbf{p} = \left\langle \frac{1}{2}, \frac{1}{2} \right\rangle$

$$L = \begin{array}{cc} & \begin{array}{cc} m_{\text{some}} & m_{\text{all}} \end{array} \\ \begin{array}{c} s_{\exists\neg\forall} \\ s_{\forall} \end{array} & \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix} \end{array}$$

Manner implicatures

(4) Mercader killed Trotsky.
$\rightsquigarrow$ Mercader killed Trotsky in a stereotypical way

(5) Mercader caused Trotsky to die.
$\rightsquigarrow$ Mercader killed Trotsky in a non-stereotypical way

Manner implicatures

(6) Mercader killed Trotsky.
$\rightsquigarrow$ Mercader killed Trotsky in a stereotypical way

(7) Mercader caused Trotsky to die.
$\rightsquigarrow$ Mercader killed Trotsky in a non-stereotypical way

- $S = \{s, s^*\} = A$
- $M = \{m, m^*\}$
- $\mathbf{p} = \left\langle \frac{1}{2} + \epsilon, \frac{1}{2} - \epsilon \right\rangle$
- $\mathbf{c} = \langle 0, \delta \rangle, \delta > 0$

$$L = \begin{array}{c c} & \begin{array}{cc} m & m^* \end{array} \\ \begin{array}{c} s \\ s^* \end{array} & \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \end{array}$$

Lifted games for epistemic states

(8) Alice ate some of the cookies.

a. Speaker believes that Alice didn't eat all of the cookies.

b. Speaker is uncertain whether Alice ate all of the cookies.

- $S = \{s_{[\exists\neg\forall]}, s_{[\forall]}, s_{[\exists\neg\forall,\forall]}\}$
- $\mathbf{p} = \langle p_1, p_2, p_3 \rangle$
- Under competence assumption: $p_1, p_2 > p_3$
- Uncertain competence: $p_1 = p_2 = p_3$

$$L_E = \begin{array}{r c} & \begin{array}{cc} m_{\text{some}} & m_{\text{all}} \end{array} \\ \begin{array}{r} s_{\exists\neg\forall} \\ s_{\forall} \\ s_{\exists\neg\forall,\forall} \end{array} & \begin{pmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 0 \end{pmatrix} \end{array}$$

Iterated x-Response

IxR-models: Family of pragmatic reasoning models,
x is *best* (IBR), *cautious* (ICR), or *quantal* (IQR)

- Sophistication as strategic player type
- Strategic types are hierarchically organized: level-0, ..., level-n
- Level-0 players are naïve/literal/honest
- level-$k+1$ agents have a behavioral belief about his interlocutor being of level-l, $l \leq k$

· Michael Franke. *Signal to Act: Game Theoretic Pragmatics.*
PhD thesis, University of Amsterdam, 2009

· Michael Franke and Gerhard Jäger. Pragmatic back-and-forth reasoning.
***Semantics, Pragmatics and the Case of Scalar Implicatures. Ed. by Salvatore Pistoia Reda. Palgrave Studies in Pragmatics Language and Cognition. New York: Palgrave MacMillan*, pages 170–200, 2014**

Preliminaries I

Strategies

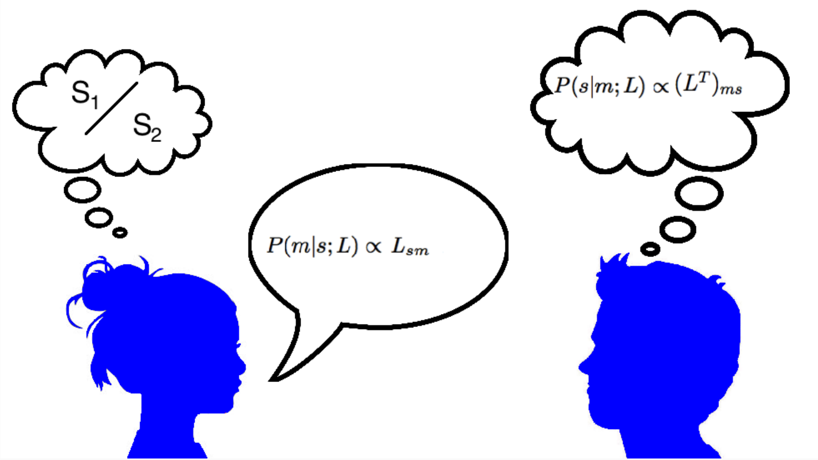
A sender strategy σ is a row-stochastic $(|S|, |M|)$-matrix

$$\sigma = \begin{array}{c c} & \begin{array}{cc} m_{\text{some}} & m_{\text{all}} \end{array} \\ \begin{array}{c} t_{\exists\neg\forall} \\ t_{\forall} \end{array} & \begin{pmatrix} .7 & .3 \\ .4 & .6 \end{pmatrix} \end{array}$$

A receiver strategy ρ is a row-stochastic $(|M|, |A|)$-matrix

$$\rho = \begin{array}{c c} & \begin{array}{cc} t_{\exists\neg\forall} & t_{\forall} \end{array} \\ \begin{array}{c} m_{\text{some}} \\ m_{\text{all}} \end{array} & \begin{pmatrix} .1 & .9 \\ .5 & .5 \end{pmatrix} \end{array}$$

Strategic types I: Literal behavior

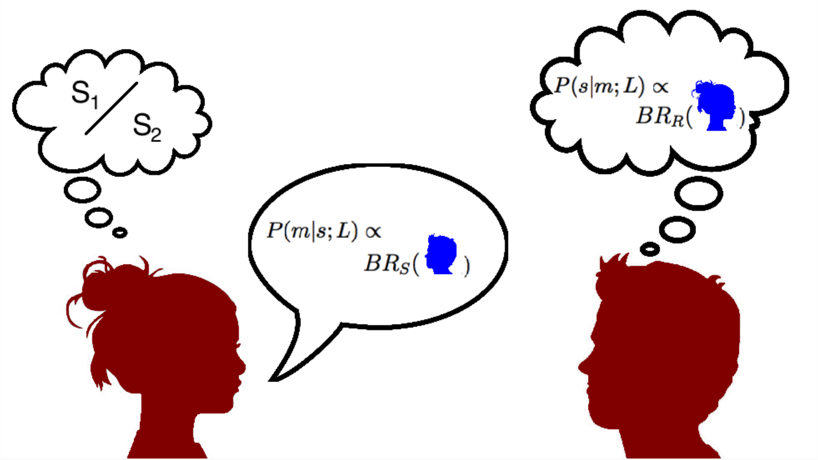


Strategic types I: Literal behavior

$$\sigma_0 = \{\text{Norm}(L)\}$$
$$\rho_0 = \{\text{Norm}(L^T)\},$$

where, for (m,n)-matrix A, Norm(A) maps A to another matrix such that $Norm(A)_i \propto A_i$ if $\sum_j (A_{ij}) > 0$ and $Norm(A)_{ij} = \frac{1}{n}$ otherwise.

Strategic types II: level-1 behavior



Strategic types II: Sophisticated/Gricean behavior

Let X be an ordered set of strategies and $|X| = d$.

Unconstrained belief

$$\Pi(X) = \{\sum_{x_i \in X} p_i x_i | \langle p_1, ..., p_n \rangle \in \Delta(d)\}$$

Cautious belief

$$\Pi^c(X) = \{\sum_{x_i \in X} p_i x_i | \langle p_1, ..., p_n \rangle \in \Delta^+(d)\}$$

Unbiased belief

$$\Pi^u(X) = \{\sum_{x_i \in X} \frac{1}{d} x_i\}$$

Best response

$$BR_S(\rho) = \{s \in S | s_{ij} = 1 \Rightarrow j \in arg_k max(\rho^T - c)_{ik}\}$$

$$S_{k+1} = BR(\Pi^u(R_k)) \quad (\mathtt{IBR}) \qquad \text{or} \qquad S_{k+1} = BR(\Pi^c(R_k)) \quad (\mathtt{ICR})$$

$$\begin{aligned} BR_R(\sigma) = \{r \in R | (r_{ij} = 1 \wedge max\sigma_j^T > 0 \Rightarrow \\ j \in arg_k max(\sigma^T \times \mathbf{p})_{ik}) \wedge \\ (r_{ij} = 1 \wedge max\sigma_j^T = 1 \Rightarrow L_{ij} = 1)\} \end{aligned}$$

Example I: Scalar implicatures

- $S = \{s_{\exists\neg\forall}, s_{\forall}\}$
- $M = \{m_{\text{some}}, m_{\text{all}}\}$
- $S = A$
- $U_S = U_R$
- $\mathbf{p} = \left\langle \frac{1}{2}, \frac{1}{2} \right\rangle$

$$L = \begin{array}{c} \\ s_{\exists\neg\forall} \\ s_{\forall} \end{array} \begin{array}{c} \begin{array}{cc} m_{\text{some}} & m_{\text{all}} \end{array} \\ \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix} \end{array}$$

Example I: Scalar implicatures

$$L = \begin{array}{c} \\ t_{\exists\neg\forall} \\ t_{\forall} \end{array} \begin{array}{c} \begin{array}{cc} m_{\text{some}} & m_{\text{all}} \end{array} \\ \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix} \end{array}$$

$$\sigma_0 = \left\{ \begin{pmatrix} 1 & 0 \\ .5 & .5 \end{pmatrix} \right\} \qquad \rho_0 = \left\{ \begin{pmatrix} .5 & .5 \\ 0 & 1 \end{pmatrix} \right\}$$

$$\Pi^u(\sigma_0) = \left\{ \begin{pmatrix} 1 & 0 \\ .5 & .5 \end{pmatrix} \right\} \qquad \Pi^u(\rho_0) = \left\{ \begin{pmatrix} .5 & .5 \\ 0 & 1 \end{pmatrix} \right\}$$

$$\sigma_1 = \left\{ \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \right\} \qquad \rho_1 = \left\{ \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \right\}$$

Example II: Horn games

(9) Mercader killed Trotsky.

$\rightsquigarrow$ Mercader killed Trotsky in a stereotypical way

(10) Mercader caused Trotsky to die.

$\rightsquigarrow$ Mercader killed Trotsky in a non-stereotypical way

- $T = \{t, t^*\} = A$
- $M = \{m, m^*\}$
- $\mathbf{p} = \left\langle \frac{1}{2} + \epsilon, \frac{1}{2} - \epsilon \right\rangle$
- $\mathbf{c} = \langle 0, \delta \rangle$, $\delta > 0$

$$B = \begin{array}{cc} & \begin{array}{cc} m & m^* \end{array} \\ \begin{array}{c} t \\ t^* \end{array} & \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \end{array}$$

Horn game up to divergence

$$\sigma_0 = \left\{ \begin{pmatrix} .5 & .5 \\ .5 & .5 \end{pmatrix} \right\} \qquad \rho_0 = \left\{ \begin{pmatrix} .5 & .5 \\ .5 & .5 \end{pmatrix} \right\}$$

$$\sigma_1 = \left\{ \begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix} \right\} \qquad \rho_1 = \left\{ \begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix} \right\}$$

Under σ_1 m^* is a surprise message. Hence:

$$\sigma_2 = \left\{ \begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix} \right\} \qquad \rho_2 = \left\{ \begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \right\}$$

Unbiased Horn Game (IBR)

$$\Pi^u(\sigma_2) = \left\{ \begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix} \right\} \qquad \Pi^u(\rho_2) = \left\{ \begin{pmatrix} 1 & 0 \\ .5 & .5 \end{pmatrix} \right\}$$

$$\sigma_3 = \left\{ \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \right\} \qquad \rho_3 = \left\{ \begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \right\}$$

$$\sigma_4 = \left\{ \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \right\} \qquad \rho_4 = \left\{ \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \right\}$$

Iterated Quantal Response

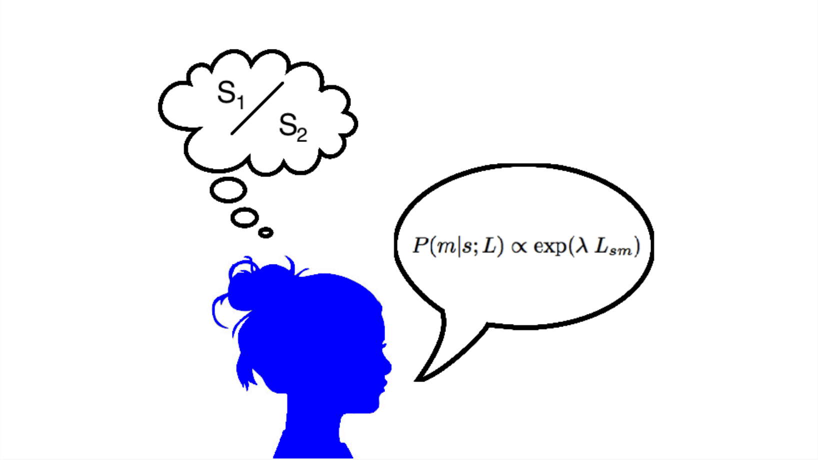
- IBR/ICR maximizes expected utility; response as function from expected utilities to choice probabilities

  For arbitrary expected utility matrix U;
  $BR(U) = Norm(maxrow(U))$

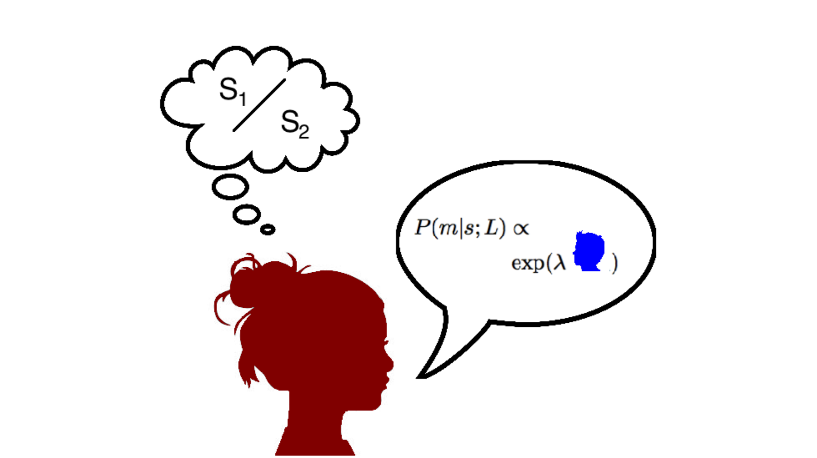
- However, mistakes may happen in calculating U or in implementing $BR(\cdot)$

- IQC as probabilistic member of the IxR family with no assumption of fully rational best responses (cf. RSA models)

IQR: Literal behavior



IQR: Level-1 behavior



Iterated Quantal Response

- $QR_\lambda(U)_{ij} \propto exp(\lambda U_{ij})$, where λ is a rationality parameter

- Higher rationality with higher λ-values
  Random for $\lambda = 0$ and $\lim_{\lambda \to \infty} QR_\lambda(U) = BR(U)$

· Duncan R. Luce. *Individual choice behavior: a theoretical analysis.*
Wiley, 1959

· Richard S. Sutton and Andrew G. Barto. *Introduction to Reinforcement Learning.*
MIT Press, Cambridge, MA, USA, 1998

Some-all IQR

Assuming a single λ for all players and that each player believes his oponent to be quantal, $\lambda \in \{0, 0.5, 1, 5\}$ yields

$$QR_0(L) = \left\{ \begin{pmatrix} .5 & .5 \\ .5 & .5 \end{pmatrix} \right\} \qquad QR_{.5}(L) = \left\{ \begin{pmatrix} .622 & .378 \\ .5 & .5 \end{pmatrix} \right\}$$

$$QR_1(L) = \left\{ \begin{pmatrix} .731 & .269 \\ .5 & .5 \end{pmatrix} \right\} \qquad QR_5(L) = \left\{ \begin{pmatrix} .993 & .007 \\ .5 & .5 \end{pmatrix}$$

Some IQR-sequences

Keeping track of the diagonal, we get

$\lambda = .5$	$\lambda = 5$
$\sigma_0 \approx \langle .622, .5 \rangle$	$\sigma_0 \approx \langle .993, .5 \rangle$
$\rho_1 \approx \langle .514, .517 \rangle$	$\rho_1 \approx \langle .839, .992 \rangle$
$\sigma_2 \approx \langle .503, .504 \rangle$	$\sigma_2 \approx \langle .984, .984 \rangle$
$\rho_3 \approx \langle .5, .5 \rangle$	$\rho_3 \approx \langle .992, .992 \rangle$

That is, IQR gives a parametrized prediction of the probability of scalar inferences, inter alia

Three experiments using MT

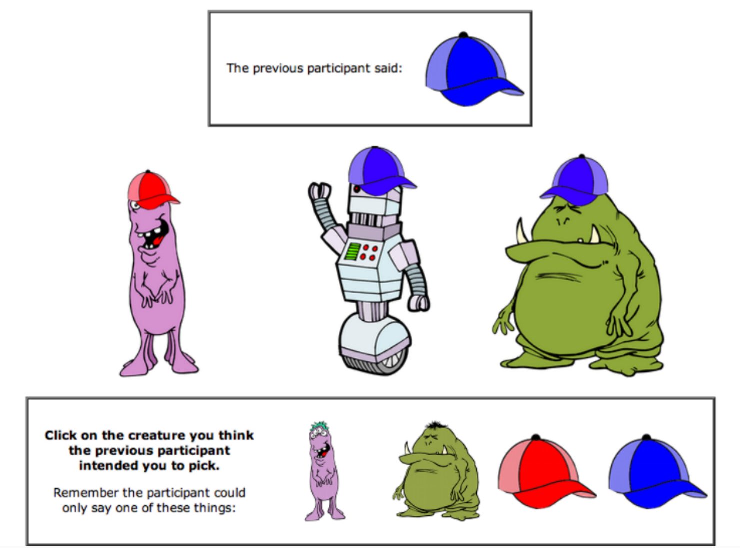
- Experiment I: Comprehension
- Experiment II: Production
- Experiment III: Comprehension (with cost)

Trials

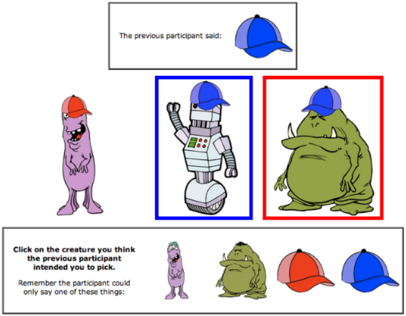
- Simple (single step) implicatures
- Complex (two step) implicatures
- Ambiguous fillers
- Unambiguous fillers

Judith Degen, Michael Franke, and Gerhard Jäger. Cost-based pragmatic inference about referential expressions. In *Proceedings of the 35th Annual Conference of the Cognitive Science Society*, pages 376–381, 2013

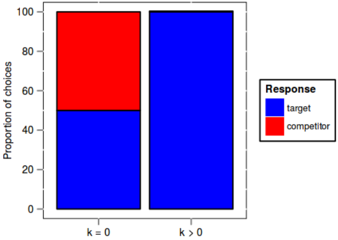
Comprehension trial: simple implicature



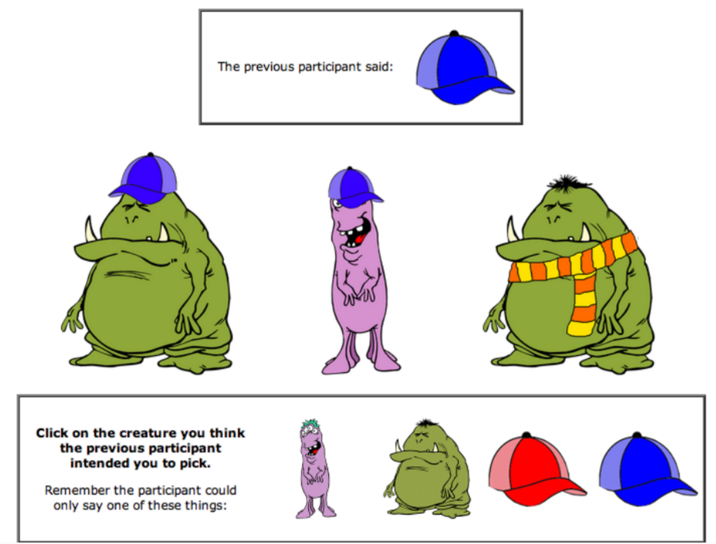
Comprehension prediction: simple implicature



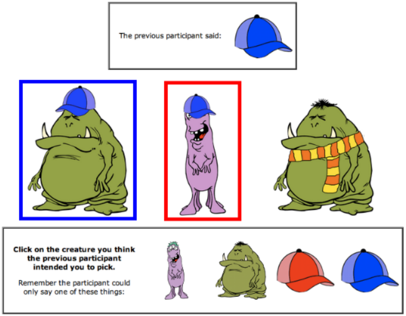
- IBR predictions for distribution of responses over target and competitor:



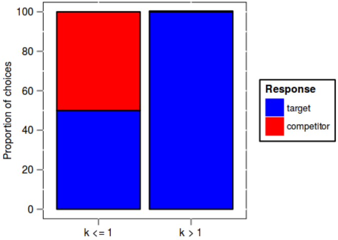
Comprehension trial: complex implicature



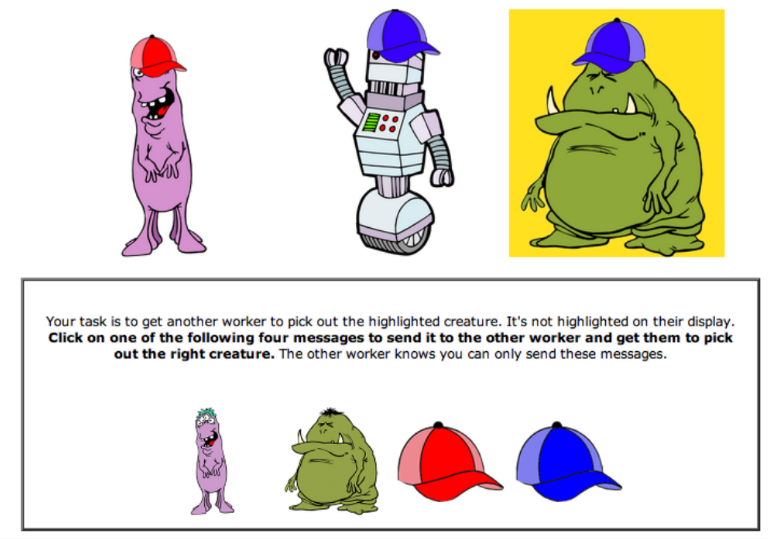
Comprehension prediction: complex implicature



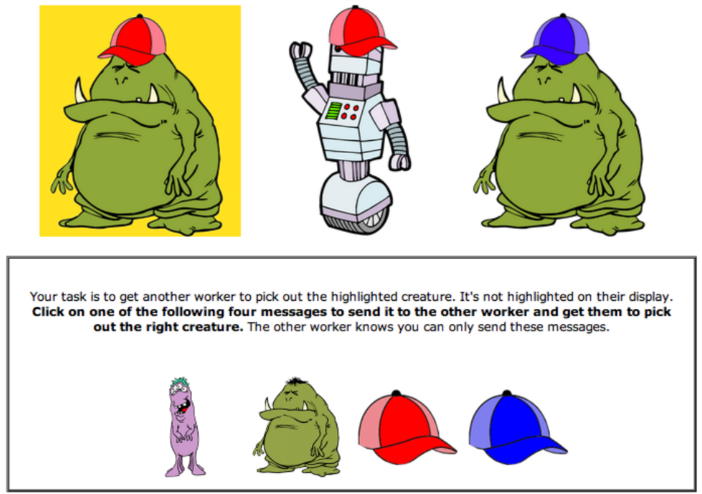
- IBR predictions for distribution of responses over target and competitor:



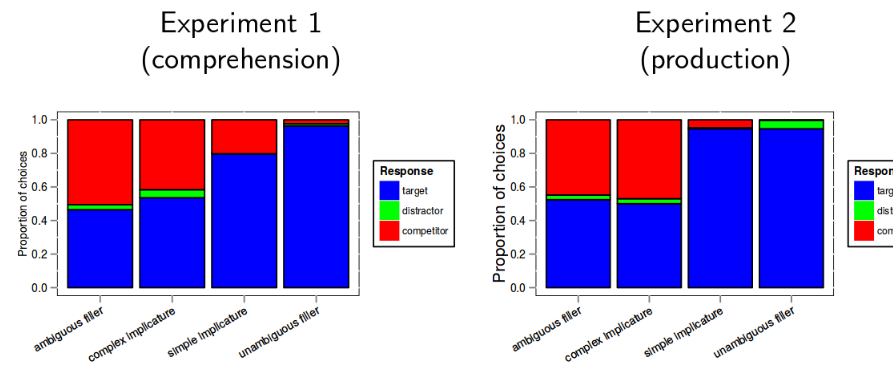
Production trial: simple implicature



Production trial: complex implicature



Experiments I and II: Results



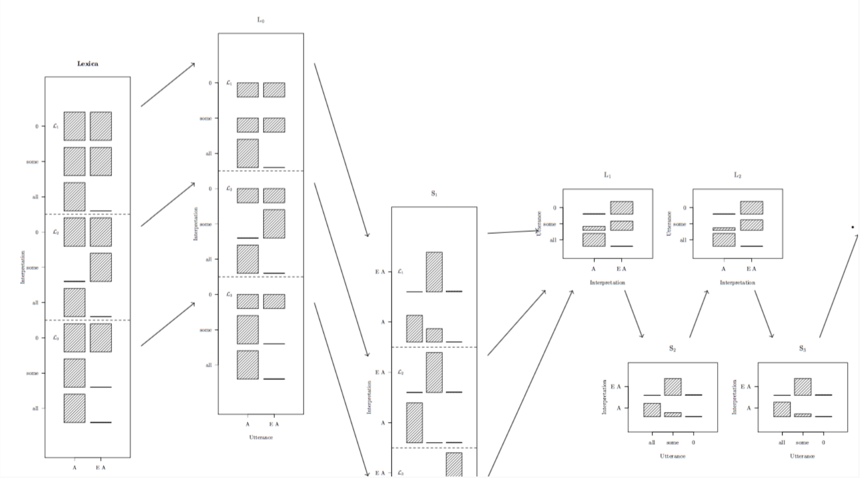
Discussion

- IBR does not provide a good fit
  (particularly for complex implicatures)

- Asymmetry between production and comprehension

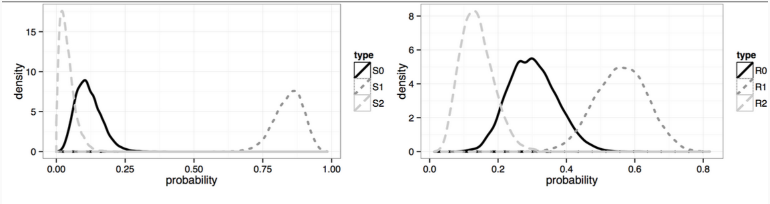
- IQR provides a means to fit the data

What's new in probabilistic rational models of language use



· Leon Bergen, Roger Levy, and Noah D Goodman. Pragmatic reasoning through semantic inference. ***Semantics and Pragmatics*, 2016**

What's new in probabilistic rational models of language use



(a) speaker **(b)** listener

· Michael Franke and Judith Degen. Reasoning in reference games: Individual-vs. population-level probabilistic modeling.
***PloS one*, 11(5), 2016**

What's new in probabilistic rational models of language use

- Lexical uncertainty
- Compositional signaling games
- Homogeneous populations
- Lack of common priors
- Machine learning – IQR/RSA as a trained classifier
- Applications to other implicatures, irony, metaphors, politeness, ...

Some open issues

- Modeler degrees of freedom for RSA/IQR-style models
- Level of description?
- Choice rules?
- Which implicatures should be derived online? What about fossilization?
- What is the right representation of the game being played? (Alternatives matter - a lot)

Final remarks

- IxR-models target reasoning under goal-oriented rational agency
- Inferences not derived from maxims but from behavioral principles
- Potentially interesting for predictions and fit of experiments
- Allow for variation in terms of rationality, preferences, cooperativity, ...
- Straightforward to plug into other dynamics (see tomorrow's session)

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