CSCI432 HW4

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1 Describe one thing that you already do well in your academic writing, and one thing that you will work on improving throughout the remainder of this class

When it comes to writing proof, I think I have one major crux. I rush through it. Often times I have the concept correct, and show that I understand the problem being presented, however, I fail to fully express my thoughts and process of approaching the proof. Towards the beginning of the semester, I was getting frustrated whenever I got the proof 'correct' but was getting docked points. I understand now that I was making proofs very very similar to this:

Statement: Let a, b be integers. If $a|b$ and $2|a$, then $2|b$.

Not-so-good proof: Let a, b be integers.

$$\left.\begin{array}{l} a|b \Rightarrow b = ak \\ 2|a \Rightarrow a = 2j \end{array}\right\} \Rightarrow b = 2jk.$$

So $2|b$.

I would often forget to write out hypothesis and conclusions, even not defining new variables. After realizing these flaws in my proofs, I've been trying to improve my formality. Its been a slow process but I believe I'm improving in comparison to the first few proofs we did in class.

However, I think a strength I possess in academic writing is my step by step process to proving my proof. For example, in inductive proofs, I often forget my conclusion and problem statement, but I always believed my inductive hypothesis and inductive steps where always pretty solid.

2 Prove that the L$_p$ distance is a distance metric.

Prove that the L$_p$ distance is a distance metric.
L$_p$ $= d_p(x, y) = \|(x_1 - y_1)^p + (x_2 - y_2)^p\|^{\frac{1}{p}}$
To be a distance metric, you must satisfy 4 requirements:
Let X = discrete space
A metric is a function d: X x X $\rightarrow \mathbb{R}$ such that:

1. d(x,y) = d(y,x)
2. d(x,y) = 0 $\Leftrightarrow$ x = y
3. d(x,y) + d(y,z) $\geq$ d(x,z)
4. d(x,y) $\geq$ 0

So,
Let x, y in $\mathbb{R}^n$. Then $d(x, y) = max_i\|x_i - y_i\|$
Then $d(y, x) = max_i\|y_i - x_i\|$
$max_i\|x_i - y_i\| = max_i\|y_i - x_i\|$
Therefore $d(x, y) = d(y, x)$

For all x,y in $\mathbb{R}$, Then $d(x, y) = max_i\|x_i - y_i\|$, therefore by definition of absolute value, the distance can never be negative.
Therefore, d(x,y) $\geq$ 0

Given $d(x, z) = max_i\|x_i - z_i\|$
Then, $max_i\|x_i - z_i\|$ less than or equal to $max_i\|x_i - y_i + max_i\|y_i - z_i\|$ by the definition of the triangle inequality
Therefore, d(x,y) + d(y,z) $\geq$ d(x,z)

For all $X = [x_1, x_2, ...x_n]$ and $Y = [y_1, y_2, ...y_n]$ in $\mathbb{R}$ if $max\|x_i - y_i\| = min\|x_i - y_i\|$, then $X = Y$ for all x_i in X and y_i in Y. Therefore, if and only if X = Y then d(x,y) = 0
Therefore d(x,y) = 0 $\Leftrightarrow$ x = y

L$_p$ fulfills all these conditions, therefore the L$_p$ distance is a metric distance.

3 Recall the randomized quicksort algorithm

3.a Give pseudocode for a version of this algorithm that does not use recursion

Algorithm 1 Iterative Quicksort

```
INPUT: Array
OUTPUT: Sorted Array
newStack
push($)
push(A.length() − 1)
while !Stack.isEmpty do
    head = pop()
    tail = pop()
    if tail − head ≥ 3 then
        partition = head + ((tail − head)/2)
        partition = partition(A, p, head, tail)

        push(partition + 1)
        push(tail)
        push(head)
        push(partition)
    end if
end while
```

3.b Loop Invariant

All elements less than head and greater than tail are smaller steadily increasing
Referencing https://en.wikibooks.org/wiki/Algorithm_Implementation/Sorting/Quicksort