CS432 HW2

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September 13, 2017

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1 Prove that the while loop in searchFirstOfK in EPI 11.1 (Search a Sorted Array for First Occurrence of k) is correct.

1.1 What is the pre-condition and post-condition to the while loop?

P: left = 0 ; right = A.size() - 1 ; result = -1
Q: left ¿ right ; return result

1.2 What is the loop guard to the while loop?

X: left <= right

1.3 What is the loop invariant?

L: K $\leq$ A[0] to left and and K $\geq$ A.size - 1

1.4 Show that P $\Rightarrow$ L

P $\Rightarrow$ L because if we start with left being the first element and right being the last than K will be always be in between them.

1.5 Show that if L is true when you begin an iteration of the while loop, then L is true at the end of that iteration.

If a loop invariant is true it must hold true to three conditions:
1. P $\Rightarrow$ L
2. L $\wedge$ X $\Rightarrow$ L

3. L $\wedge \sim$ X $\Rightarrow$ Q

A base case is when the initial mid is K, because we do not need to iterate through the loop again. L is true here.

If K is not mid, we will iterate through the array, decreasing right or increasing left, making the distance between left and right smaller and smaller, however k is still inbetween the left and high. So no matter what our L will always be true.

1.5.1 Say, in words, what the following statement means: $\sim$ X $\bigwedge$ L $\rightarrow$ Q

The loop terminated, and if L, our loop invariant is true, then the algorithm works.

1.6 Prove partial correctness

Since the loop invariant was proved to be true, we proved partial correctness.

1.7 A decrementing function is a function such that Y is a well-ordered set, D(i) is strictly decreasing, and D(i) can be interpreted as a function value on the i th iteration of a loop. What are Y and D in searchFirstOfK?

Y: is the entity of the array D: right - left

1.8 What is the minimum value of Y

It will be 0, because right - left will get smaller and smaller as the distance between them will get shorter

1.9 Show that the loop will terminate in iteration i, where D(i) is the minimum value of Y

Well when it right-left hits zero that will be the last iteration of the loop, because our loop guard (left <= right) will break the loop and not allow another iteration.