

CSCI338 HW4

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Question 1

Let β be the set of all infinite sequences over $\{a,b\}$. Show that β is uncountable, using a proof by diagonalization.

Assume that β is countable, then we can list them as $\beta_1 \beta_1$

i	f(i)		
1	b ₁₁	b ₁₂	b ₁₃
2	b ₂₁	b ₂₂	b ₂₃
3	b ₃₁	b ₃₂	b ₃₃

Now construct an x such that the i^{th} bit of x is \neq the i^{th} bit of β

i	f(i)		
1	0	0	0...
2	1	0	1...
3	1	1	1...

Lets define x as $x = (1, 1, 0...)$ thus making $x \neq \beta$ for any i^{th} sequence in the i^{th} bit. Therefore β is uncountable.

Question 2

Question 3

Question 4

Question 5

Show that EQ_{CFG} is undecidable.

Assume that EQ_{CFG} is decidable

$EQ_{CFG} = \{ \langle G, H \rangle \mid G, H \text{ are CFG's and } L(G) = L(H) \}$

Reduce ALL_{CFG} to EQ_{CFG} Such That:

$ALL_{CFG} = \{ \langle G \rangle \mid G \text{ is a CFG's and } L(G) = \Sigma^* \}$

Let R be a decider for EQ_{CFG} and construct TM S to decide ALL_{CFG} .

Construct CFG T such that $L(T) = \Sigma^*$

1. Run R on input $\langle G, T_0 \rangle$

2. If R accepts, accept.

3. If R rejects, reject

R decides if $L(G) = L(T)$. S decides ALL_{CFG} but it is undecidable, therefore EQ_{CFG} must also be undecidable

Question 6

Question 7