CSCI338 HW4

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Question 1

Let β be the set of all infinite sequences over {a,b}. Show that β is uncountable, using a proof by diagonalization.
Assume that β is countable, then we can list them as β_1 , β_2

i	f(i)		
1	b_{11}	b_{12}	b_{13}
2	b_{21}	b_{22}	b_{23}
3	b_{31}	b_{32}	b_{33}

Now construct an x such that the i^{th} bit of x is $\neq$ the i^{th} bit of β

i	f(i)		
1	a	a	a...
2	b	a	b...
3	b	b	b...

Lets define x as x = (b, b, a...) thus making x $\neq \beta$ for any i^{th} sequence in the i^{th} bit. Therefore β is uncountable.

Question 2

Let T = {(i,j,k)| i,j,k ϵ N }. Show that T is countable.
By definition, the set T is countable iff:
If there exists an injective function f from T to the natural numbers
If f is surjective
If T has a one-to-one correspondence with natural numbers

Construct a 1-1 and onto function f: T $\rightarrow$ N
We know that A = {(i,j)| i,j ϵ N } is countable
The function g((i,j),k) = (i + j)(i + j + 1)/2 + j is a 1-1 correspondence from T on N
Assume that f(<i,j,k>) = f(<i', j', k'>)
Therefore g(<g(<i,j>,k)>) = g(<g(<i',j'>,k')>)
This makes g a 1-1, therefore f is 1-1

Since n ϵ N, then g(<m,k>) = n for some m,k that is also in N
g(<i,j>) = m for some i,j in N
Therefore f(<i,j,k>) = g(<m,k<) = g(<g(<i,j>),k>)
Therefore T is countable.

Question 3

Show that INFINITE$_{PDA}$ is decidable
To decide INFINITE$_{PDA}$ convert PDA into equivalent CFG, let P be the pumping length. Construct regular language R that accepts strings longer than P. Intersect of CFL and regular languages is a CFL. Test this intersection for emptiness, accept if L of the intersection is empty, reject otherwise.

Question 4

ODD$_{TM}$ = { <M> | M is a TM and L(M) contains only strings of odd length } Prove that ODD$_{TM}$ is undecidable.

Reduce ALL$_{TM}$ to ODD$_{TM}$
Assume that ODD$_{TM}$ is decidable. Let R be a decider for ODD$_{TM}$ and have R decide T on w:
if R accepts, accept
if R rejects, reject

O(T) us a decuder for ODD$_{TM}$. Now build a decider for A$_{TM}$. This is impossible, therefore ODD$_{TM}$ is undecideable

Question 5

Show that EQ$_{CFG}$ is undecidable.
(This can also be proven via rice's theorem) Assume that EQ$_{CFG}$ is decideable
EQ$_{CFG}$ = { <G,H> | G,H are CFG's and L(G) = L(H) }
Reduce ALL$_{CFG}$ to EQ$_{CFG}$ Such That:
ALL$_{CFG}$ = { <G> | G is a CFG's and L(G) = Σ^* }
Let R be a decider for EQ$_{CFG}$ and construct TM S to decide ALL$_{CFG}$.
Construct CFG T such that L(T) = Σ^*

1. Run R on input <G,T$_0$>
2. If R accepts, accept.
3. If R rejects, reject

R decides if L(G) = L(T). S decides ALL$_{CFG}$ but it is undecidable, therefore EQ$_{CFG}$ must also be undecidable

Question 6

Show that EQ$_{CFG}$ is co-Turing-recognizable.

A language is co-turning recognizable if and only if its complement is a turning-recognizable language.

Convert G and H into Chomsky normal form. Begin iterating through the strings in Σ^*. If both G or H can generate or not generate the string a TM will continue on the interation of strings, however, if one CFG accepts a string and the other does not, that means that the CFG's are not equvilent and the TM accepts. Therefore EQ_{CFG} is co-turning recognizeable.

Question 7

Post Correspondence Problem.

$$\left\{ \left[\frac{\mathrm{ab}}{\mathrm{abab}}\right], \left[\frac{\mathrm{b}}{\mathrm{a}}\right], \left[\frac{\mathrm{aba}}{\mathrm{b}}\right], \left[\frac{\mathrm{aa}}{\mathrm{a}}\right] \right\}$$

1 2 3 4

A working string is:
11132124234244
abababababababbaababaaabaaa