CSCI432 HW5

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1 Prove that the Frechet distance is a distance metric

Prove that the Frechet distance is a distance metric.

To be a distance metric, you must satisfy 4 requirements:
Let X = discrete space
A metric is a function d: X x X $\rightarrow \mathbb{R}$ such that:

1. d(x,y) = d(y,x)
2. d(x,y) = 0 $\Leftrightarrow$ x = y
3. d(x,y) + d(y,z) $\geq$ d(x,z)
4. d(x,y) $\geq$ 0

So,
Let A, B be two curves in a metric space, and t as a parameter in time. A and B are the infimum of all α and β of [0,1] where t $\in$ [0,1]
Then the Frechet Distance is:
$F(A,B) = inf_{\alpha,\beta} max_{t\in[0,1]}\{d(A(\alpha(t)), B(\beta(t)))\}$.

So, let
$F(A,B) = inf_{\alpha,\beta} max_{t\in[0,1]}\{d(A(\alpha(t)), B(\beta(t)))\}$, and
$F(B,A) = inf_{\beta,\alpha} max_{t\in[0,1]}\{d(B(\beta(t)), A(\alpha(t)))\}$ which is the same value.
Therefore $F(A,B) = F(B,A), andd(x,y) = d(y,x)$

If $F(A,B) = 0$ then $A(\alpha(t)$ and $B(\beta(t)$ then A and B are at the same x and y coordinates at the same t value, hence $A(\alpha(t)$ equals $B(\beta(t), therefored(x,y) = 0$ and $x = y$

Let A, B, C be 3 curves in a metric space, and t as a parameter in time. A and B are the infimum of all α, β, and δ of [0,1] where t $\in$ [0,1]. The distance of A to B and the distance of B to C is always going to be greater than or equal to the distance of A to C by the definition of the triangle inequality. Therefore d(x,y) + d(y,z) $\geq$ d(x,z)

The frechet distance cannot be negitive because distances are calculated using the absolute value, therefore d(x,y) $\geq$ 0

Therefore, the Frechet Distance is a metric distance.

2 Recurrence Relations

2.a T(n) = 2T(n/4) + n^2

Master's Theorem:
T(n) = aT($\frac{n}{b}$) + f(n)
Where, a $\geq$ 1, b > 1, f(n) is asymptotically positive

T(n) = 2T(n/4) + n^2
a = 2, b = 4, f(n) = n^2
n$^{log_b a}$ $\Rightarrow$ n$^{log_4 2}$ $\Rightarrow$ n$^{1/2}$
Case 3:
if f(n) is Ω(n$^{log_b a+\epsilon}$) for some $\epsilon > 0$ and if f(n/b) $\leq$ f(n), then T(n) = $\theta(f(n))$
Therefore, T(n) = $\theta(n^2)$

2.b T(n) = 4T(n/2) + n

Master's Theorem:
T(n) = aT($\frac{n}{b}$) + f(n)
Where, a $\geq$ 1, b > 1, f(n) is asymptotically positive

T(n) = 4T(n/2) + n
a = 4, b = 2, f(n) = n
n$^{log_b a}$ $\Rightarrow$ n$^{log_2 4}$ $\Rightarrow$ n^2
Case 1:
if f(n) = O(n$^{log_b a-\epsilon}$) for some $\epsilon > 0$ then T(n) = θ(n^2).
Therefore, T(n) = θ(n^2)

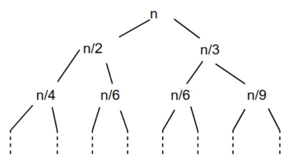
2.c T(n) = 3T(2n/3) + 4n

Master's Theorem:
T(n) = aT($\frac{n}{b}$) + f(n)
Where, a $\geq$ 1, b > 1, f(n) is asymptotically positive

T(n) = 3T(2n/3) + 4n
a = 3, b = 2/3, f(n) = 4n
n$^{log_b a}$ $\Rightarrow$ n$^{log_{3/2} 3}$ $\Rightarrow$ n$^{2.71}$
Case 1:

if f(n) = $O(n^{log_b a - \epsilon})$ for some $\epsilon > 0$ then T(n) = $\theta(n^{2.71})$.
Therefore, T(n) = $\theta(n^{2.71})$

2.d T(n) = T(n/2) + T(n/3)



Our longest path in this tree is the leftmost path, following a sequence: $log_2 n$. So our initial guess is for this recurrence is $O(nlogn)$.

Prove that $T(n) = T(n/2) + T(n/3)$ is in $O(nlogn)$ complexity.
We begin by guessing that $T(n)$ is in $O(nlogn)$
For the base case of n = 1, it's trivial to see that there exists a c where $1 < cnlogn$
For $n > 1$, we have
$T(n) = T(n/2) + T(n/3) \leq c(nlogn/2) + (nlogn/3)$
We can see that this last simplification is less than $cnlogn$ for some positive n and c. Since we are limiting our $n > 1$ already, we meet the requirement that there does exist some c where $T(n) < cnlogn$.
Thus, by induction, $T(n)$ is $O(nlogn)$.

2.e 2T(n/2) + O(log n)

Prove the time complexity of 2T(n/2) + O(log n)
2T(n/2) is a geometric series of $2n/2^i$ so our guess is $O(nlogn)$,
which would make this algorithm approximently $O(nlogn)$

3 Climbing Stairs Problem

Loop Invariant:
The loop invariant is that with each loop iteration, we will take one more step, adding on to our solution by 1 each time.

Algorithm 1 Climbing Stairs

Input: n (number of stairs), k (max steps can advance at once)
Output: Integer (Numbers of ways to reach destination)
Procedure: $CountingStairs(n, k)$
count $\leftarrow$ 0
if n $\leq$ 1 **then**
    return 1
end if
for $i \leftarrow 1...(k)$ **do**
    $count \leftarrow count + COUNTINGSTAIRS(n - i, k)$
end for
return count