Context-free languages are closed under the following operations. That is, if L and P are context-free languages, the following languages are context-free as well:

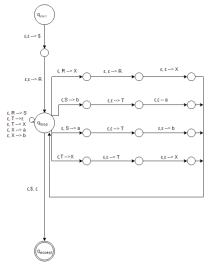
the union L and P
the reversal of L
the concatenation of L and P
the Kleene star of L
the image of L under a homomorphism
the image of L under an inverse homomorphism
the cyclic shift of L

Context-Free Grammers

$\{ \mathbf{a}^n\mathbf{b}^m \mid \mathbf{n} \neq \mathbf{2m} \}$

S → aaSb | A | B
A → aA | a
B → bB | b

CFG to PDA



Chomsky Normal Form

A → BAB | B | ϵ
B → 00 | ϵ
Add new start variable S_1 :
S_0 → A
A → BAB | B | ϵ
B → 00 | ϵ
Remove all ϵ :
S_0 → A
A → BAB | BB | AB | BA | A | B

$B \rightarrow 00$
Remove unit rules:
$S_0 \rightarrow BAB \mid BB \mid AB \mid BA \mid 00$
$A \rightarrow BAB \mid BB \mid AB \mid BA \mid 00$
$B \rightarrow 00$
Add 'U' :
$S_0 \rightarrow BAB \mid BB \mid AB \mid BA \mid 00$
$A \rightarrow BAB \mid BB \mid AB \mid BA \mid 00$
$B \rightarrow UU$
$U \rightarrow 0$
Simplify:
$S_0 \rightarrow BA_1 \mid BB \mid AB \mid BA \mid 00$
$A \rightarrow BA_2 \mid BB \mid AB \mid BA \mid 00$
$B \rightarrow UU$
$U \rightarrow 0$
$A_1 \rightarrow SB$
$A_2 \rightarrow SB$

Pumping Lemma with Context-Free Languages

$\mathbf{L = \{\ a^n\ b^j\ c^k \mid k = nj\ \}}$

Assume L is a context free language. $S = a^p\ b^p\ c^{p^2}$
S can be decomposed into $S = uv^ixy^iz$ such that:
1. $uv^ixy^iz \in L$
2. $\mid uy \mid \ \ 0$
3. $\mid uxy \mid \leq P$

Cases:
i. v contains b's or a's and y contains only c's. i = 0 so uv^0xy^0z thus making the string $a^p\ b^p\ c^{p^2-1}$ which is not in the language
ii. v and y both contain a's and b's. i = 0 so uv^0xy^0z thus making the string $a^p\ b^{p-k}\ c^{p^2}$ or $a^{p-k}\ b^p\ c^{p^2}$ which is not in the language
iii. v and y both contain c's i = 0 making $a^p\ b^p\ c^{p^2-k}$ which is not in the language.
iv. v and y contain 2 symbols. i = 2 uv^2xy^2z however the characters will be out of order and not in the language
Thus L is not a CFG