CSCI338 HW3

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1 Context-Free Grammers

1.1 $\{ \mathbf{a}^n\mathbf{b}^m \mid \mathbf{n \neq 2m} \}$

S $\rightarrow$ aaSb | A | B
A $\rightarrow$ aA | a
B $\rightarrow$ bB | b

1.2 $\{ \mathbf{a}^i\ \mathbf{b}^j\ \mathbf{c}^k \mid \mathbf{i, j, k \geq 0\ j = k\ or\ j = i} \}$

S $\rightarrow S_1 \mid S_2$
$S_1 \rightarrow$ abS_1 | A | ϵ
A $\rightarrow$ cA | c | ϵ
$S_2 \rightarrow$ a S_2 | B | ϵ
B $\rightarrow$ Bbc | bc | ϵ

1.3 $\{ \mathbf{a}^n\ \mathbf{b}^m \mid \mathbf{n = 3m} \}$

S $\rightarrow$ aaaSb | ϵ

1.4 $\{ \mathbf{a}^n\ \mathbf{b}^m \mid \mathbf{n \leq m + 3} \}$

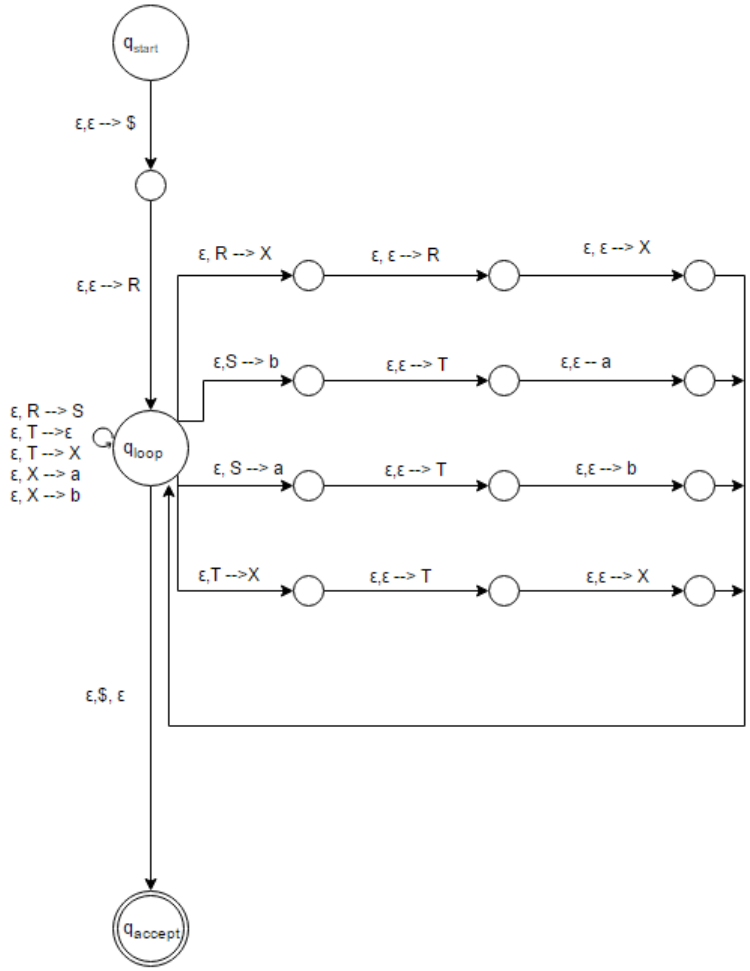
S $\rightarrow$ aSb | A
A $\rightarrow$ a | aa | aaa | B
B $\rightarrow$ bB | ϵ

2 Ambiguous Grammer

Can I construct an identical string using two different paths?
Lets construct the string aab
S $\rightarrow$ aaB $\rightarrow$ b $\rightarrow$ aab

S $\rightarrow$ AB:
A $\rightarrow$ aA $\rightarrow$ aa
B $\rightarrow$ b
$\rightarrow$ aab
This language is ambiguous

3 CFG to PDA



4 Pumping Lemma with Regular Languages

4.1

This langauge accepts some amount(≥ 0) of 0's followed by atleast 1, but no more than 2 #, following by some amount (≥ 0) of 0's or some amount of 0's followed by a # then twice as many 0's as before
{ 0^n # 0^{2n} }

4.2

If G is a regular then there is a number P (Pumping length) such that S $\in$ and |S| $\geq$ P then S can be decomposed into S = XYZ S.T.:

1.$xy^iz \in G$
2. $|y| > 0$
3. $|xy| \leq P$

$S = 0^p \# 0^{2p}$
000#000000
y can only contain either the first set or the second set of 0's. If we pump up y we will have an incorrect amount of 0's on either side. Therefore G is not a regular language.

5 Chomsky Normal Form

A $\rightarrow$ BAB | B | ϵ
B $\rightarrow$ 00 | ϵ
Add new start variable S_1 :
$S_0 \rightarrow$ A
A $\rightarrow$ BAB | B | ϵ
B $\rightarrow$ 00 | ϵ
Remove all ϵ :
$S_0 \rightarrow$ A
A $\rightarrow$ BAB | BB | AB | BA | A | B
B $\rightarrow$ 00
Remove unit rules:
$S_0 \rightarrow$ BAB | BB | AB | BA | 00
A $\rightarrow$ BAB | BB | AB | BA | 00
B $\rightarrow$ 00
Add 'U' :
$S_0 \rightarrow$ BAB | BB | AB | BA | 00
A $\rightarrow$ BAB | BB | AB | BA | 00
B $\rightarrow$ UU
U $\rightarrow$ 0
Simplify:
$S_0 \rightarrow BA_1$ | BB | AB | BA | 00
A $\rightarrow BA_2$ | BB | AB | BA | 00
B $\rightarrow$ UU
U $\rightarrow$ 0
$A_1 \rightarrow$ SB
$A_2 \rightarrow$ SB

6 Pumping Lemma with Context-Free Languages

6.1 $\mathbf{L = \{\ a^n\ b^j\ c^k\ |\ k = nj\ \}}$

Assume L is a context free language. $S = a^p\ b^p\ c^{p^2}$
S can be decomposed into $S = uv^ixy^iz$ such that:
1. $uv^ixy^iz \in L$
2. $|\ uy\ | \quad 0$
3. $|\ uxy\ | \leq P$

Cases:
i. v contains b's or a's and y contains only c's. i = 0 so uv^0xy^0z thus making the string $a^p\ b^p\ c^{p^2-1}$ which is not in the language
ii. v and y both contain a's and b's. i = 0 so uv^0xy^0z thus making the string $a^p\ b^{p-k}\ c^{p^2}$ or $a^{p-k}\ b^p\ c^{p^2}$ which is not in the language
iii. v and y both contain c's i = 0 making $a^p\ b^p\ c^{p^2-k}$ which is not in the language.
iv. v and y contain 2 symbols. i = 2 uv^2xy^2z however the characters will be out of order and not in the language
Thus L is not a CFG

6.2 $\mathbf{L = \{\ a^n\ b^j \mid n \geq (j - 1)^3\ \}}$

Assume L is a context free language. $S = a^p\ b^p$
S can be decomposed into $S = uv^ixy^iz$ such that:
1. $uv^ixy^iz \in L$
2. $|\ uy\ | \quad 0$
3. $|\ uxy\ | \leq P$

Cases:
i. either v or y has more than one symbol. Thus when it is pumped up the characters are out of order and not in the language.
ii. v contains only a's and y contains only b's. i = 2 thus making uv^2xy^2z which is not in the language, there are not enough a's