CSCI338 HW3

Brock Ellefson

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1 Context-Free Grammers

1.1 $\{ \mathbf{a}^n\mathbf{b}^m \mid \mathbf{n \neq 2m} \}$

S $\rightarrow$ aaSb | A | B
A $\rightarrow$ aA | a
B $\rightarrow$ bB | b

1.2 $\{ \mathbf{a}^i\ \mathbf{b}^j\ \mathbf{c}^k \mid \mathbf{i, j, k \geq 0\ j = k\ or\ j = i} \}$

S $\rightarrow S_1 \mid S_2$
$S_1 \rightarrow$ abS_1 | A | ϵ
A $\rightarrow$ cA | c | ϵ
$S_2 \rightarrow$ a S_2 | B | ϵ
B $\rightarrow$ Bbc | bc | ϵ

1.3 $\{ \mathbf{a}^n\ \mathbf{b}^m \mid \mathbf{n = 3m} \}$

S $\rightarrow$ aaaSb | ϵ

1.4 $\{ \mathbf{a}^n\ \mathbf{b}^m \mid \mathbf{n \leq m + 3} \}$

S $\rightarrow$ aSb | A
A $\rightarrow$ a | aa | aaa | B
B $\rightarrow$ bB | ϵ

2 Ambiguous Grammer

Can I construct an identical string using two different paths?
Lets construct the string aab
S $\rightarrow$ aaB $\rightarrow$ b $\rightarrow$ aab

S $\rightarrow$ AB:
A $\rightarrow$ aA $\rightarrow$ aa
B $\rightarrow$ b
$\rightarrow$ aab
This language is ambiguous

3 CFG to PDA

4 Pumping Lemma with Regular Languages

4.1

This langauge accepts some amount(≥ 0) of 0's followed by atleast 1, but no more than 2 #, following by some amount (≥ 0) of 0's or some amount of 0's followed by a # then twice as many 0's as before
$\{ 0^n \# 0^{2n} \}$

4.2

If G is a regular then there is a number P (Pumping length) such that S $\in$ and |S| $\geq$ P then S can be decomposed into S = XYZ S.T.:
1.$xy^iz \in G$
2. |y| > 0
3. |xy| $\leq$ P

$S = 0^p \# 0^{2p}$
000#000000
y can only contain either the first set or the second set of 0's. If we pump up y we will have an incorrect amount of 0's on either side. Therefore G is not a regular language.

5 Chomsky Normal Form

A $\rightarrow$ BAB | B | ϵ
B $\rightarrow$ 00 | ϵ
Add new start variable S_1 :
$S_0 \rightarrow$ A
A $\rightarrow$ BAB | B | ϵ
B $\rightarrow$ 00 | ϵ
Remove all ϵ :
$S_0 \rightarrow$ A
A $\rightarrow$ BAB | BB | AB | BA | A | B
B $\rightarrow$ 00
Remove unit rules:
$S_0 \rightarrow$ BAB | BB | AB | BA | 00
A $\rightarrow$ BAB | BB | AB | BA | 00
B $\rightarrow$ 00
Add 'U' :
$S_0 \rightarrow$ BAB | BB | AB | BA | 00
A $\rightarrow$ BAB | BB | AB | BA | 00
B $\rightarrow$ UU
U $\rightarrow$ 0
Simplify:
$S_0 \rightarrow BA_1$ | BB | AB | BA | 00
A $\rightarrow BA_2$ | BB | AB | BA | 00

$B \rightarrow UU$
$U \rightarrow 0$
$A_1 \rightarrow SB$
$A_2 \rightarrow SB$

6 Pumping Lemma with Context-Free Languages

6.1 $\mathbf{L = \{\ a^n\ b^j\ c^k \mid k = nj\ \}}$

6.2 $\mathbf{L = \{\ a^n\ b^j \mid n \geq (j - 1)^3\ \}}$