CSCI432 HW5

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1 Prove that the Frechet distance is a distance metric

Prove that the Frechet distance is a distance metric.

To be a distance metric, you must satisfy 4 requirements:
Let X = discrete space
A metric is a function d: X x X $\rightarrow \mathbb{R}$ such that:

1. d(x,y) = d(y,x)
2. d(x,y) = 0 $\Leftrightarrow$ x = y
3. d(x,y) + d(y,z) $\geq$ d(x,z)
4. d(x,y) $\geq$ 0

So,

2 Recurrence Relations

2.a T(n) = 2T(n/4) + n^2

Master's Theorem:
T(n) = aT($\frac{n}{b}$) + f(n)
Where, a $\geq$ 1, b > 1, f(n) is asymptotically positive

T(n) = 2T(n/4) + n^2
a = 2, b = 4, f(n) = n^2
$n^{log_b a} \Rightarrow n^{log_4 2} \Rightarrow n^{1/2}$
Case 3:
if f(n) is $\Omega(n^{log_b a+\epsilon})$ for some $\epsilon > 0$ and if f(n/b) $\leq$ f(n), then T(n) = $\theta(f(n))$
Therefore, T(n) = $\theta(n^2)$

2.b T(n) = 4T(n/2) + n

Master's Theorem:
T(n) = aT($\frac{n}{b}$) + f(n)
Where, a $\geq$ 1, b > 1, f(n) is asymptotically positive

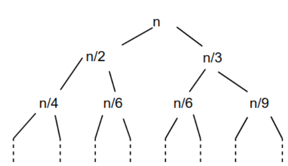
T(n) = 4T(n/2) + n
a = 4, b = 2, f(n) = n
n$^{log_b a}$ $\Rightarrow$ n$^{log_2 4}$ $\Rightarrow$ n^2
Case 1:
if f(n) = O(n$^{log_b a-\epsilon}$) for some $\epsilon > 0$ then T(n) = θ(n^2).
Therefore, T(n) = θ(n^2)

2.c T(n) = 3T(2n/3) + 4n

Master's Theorem:
T(n) = aT($\frac{n}{b}$) + f(n)
Where, a $\geq$ 1, b > 1, f(n) is asymptotically positive

T(n) = 3T(2n/3) + 4n
a = 3, b = 2/3, f(n) = 4n
n$^{log_b a}$ $\Rightarrow$ n$^{log_{3/2} 3}$ $\Rightarrow$ n$^{2.71}$
Case 1:
if f(n) = $O(n^{log_b a-\epsilon})$ for some $\epsilon > 0$ then T(n) = $\theta(n^{2.71})$.
Therefore, T(n) = $\theta(n^{2.71})$

2.d T(n) = T(n/2) + T(n/3)



Our longest path in this tree is the leftmost path, following a sequence: $log_2 n$.
So our initial guess is for this recurrence is $O(nlogn)$.

2.e 2T(n/2) + O(log n)

3 Climbing Stairs Problem