

CSCI432 HW5

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1 Prove that the Frechet distance is a distance metric

Prove that the Frechet distance is a distance metric.

To be a distance metric, you must satisfy 4 requirements:

Let X = discrete space

A metric is a function $d: X \times X \rightarrow \mathbb{R}$ such that:

1. $d(x,y) = d(y,x)$
2. $d(x,y) = 0 \Leftrightarrow x = y$
3. $d(x,y) + d(y,z) \geq d(x,z)$
4. $d(x,y) \geq 0$

So,

2 Recurrence Relations

2.a $T(n) = 2T(n/4) + n^2$

Master's Theorem:

$$T(n) = aT\left(\frac{n}{b}\right) + f(n)$$

Where, $a \geq 1$, $b > 1$, $f(n)$ is asymptotically positive

$$T(n) = 2T(n/4) + n^2$$

$$a = 2, b = 4, f(n) = n^2$$

$$n^{\log_b a} \Rightarrow n^{\log_4 2} \Rightarrow n^{1/2}$$

Case 3:

if $f(n)$ is $\Omega(n^{\log_b a + \epsilon})$ for some $\epsilon > 0$ and if $f(n/b) \leq f(n)$, then $T(n) = \theta(f(n))$

Therefore, $T(n) = \theta(n^2)$

2.b $T(n) = 4T(n/2) + n$

Master's Theorem:

$$T(n) = aT\left(\frac{n}{b}\right) + f(n)$$

Where, $a \geq 1$, $b > 1$, $f(n)$ is asymptotically positive

$$T(n) = 4T(n/2) + n$$

$$a = 4, b = 2, f(n) = n$$

$$n^{\log_b a} \Rightarrow n^{\log_2 4} \Rightarrow n^2$$

Case 1:

if $f(n) = O(n^{\log_b a - \epsilon})$ for some $\epsilon > 0$ then $T(n) = \Theta(n^2)$.

Therefore, $T(n) = \Theta(n^2)$

2.c $T(n) = 3T(2n/3) + 4n$

2.d $T(n) = T(n/2) + T(n/3)$

2.e $2T(n/2) + O(\log n)$

3 Climbing Stairs Problem