

Exercises for MA 413 – Statistics for Data Science

This sheet will cover lecture material from the lecture 28/10/2019 and later material.

1. Suppose X_i for $1 \leq i \leq n$ are iid $N(0, 1)$. Use the **theorems** from class to find the **limiting distribution** of

$$\frac{n(X_1X_2 + X_3X_4 + \cdots + X_{2n-1}X_{2n})^2}{\left(\sum_{i=1}^{2n} X_i^2\right)^2}.$$

2. Let X_n have the $\text{Bin}\left(n, \frac{1}{n}\right)$ distribution. Show that X_n **converges in distribution** to X which has a $\text{Poisson}(1)$ distribution.
3. (Hard) Let Y_n have a symmetric Beta distribution with each parameter equal to $1/n$. Determine what **distributional convergence** Y_n has.
4. Suppose U_1, \dots, U_{2m+1} are iid $\text{Unif}(0, 1)$. Recall our notation for the **order statistics** by ordering the sample in increasing size and writing them as $U_{(1)} < U_{(2)} < \cdots < U_{(2m)} < U_{(2m+1)}$. Find the exact distribution of the **median** $U_{(m+1)}$, and then use this to determine the **limiting distribution** of $U_{(m+1)}$. Given our study of the minimum and maximum, what do you expect this to be, and does your calculations support this intuition?
5. Let X_i be iid $\text{Exp}(1)$ random variables. Define

$$Y_n = \frac{1}{n} \begin{pmatrix} \sum_j X_j \\ \sum_j X_j^2 \\ \sum_j X_j^3 \\ \sum_j X_j^4 \end{pmatrix}.$$

Decide on how to **center** and **normalizing** Y_n so that you can determine the **limiting distribution** of this random variable using **Theorems** from class.

6. Take X_i as iid $\text{Poisson}(\mu)$ random variables. Determine the expectation and variance of $\hat{\mu} = \frac{1}{n} \sum_i X_i$. What is the **MSE** of $\hat{\mu}$ as an **estimator** of μ ?
7. Determine the **Cramer-Rao lower bound** on estimating the Bernoulli success probability p from a random sample of this distribution.