

## Exercises 2 for MA 413 – Statistics for Data Science

*This sheet will cover lecture material from the lecture 23/09/2019 and later material.*

1. Let  $Y \sim U(0, 1)$ . Assume you want to generate a random variable with the distribution for  $\lambda > 0$  of

$$f_X(x) = \begin{cases} \lambda e^{-\lambda x} & \text{if } x > 0 \\ 0 & \text{o/w} \end{cases} \quad (1)$$

Explain how to **generate**  $X$  from  $Y$ .

2. Let the random variables  $X$  and  $Y$  be distributed according for  $C > 0$  to

$$f_{X,Y}(x, y) = C \cdot \begin{cases} xy & \text{if } 0 < x < y < 1 \\ 0 & \text{o/w} \end{cases} \quad (2)$$

- (a) Use the second axiom of probability to deduce the value of  $C$ .  
(b) Are  $X$  and  $Y$  **independent**? Motivate your answer and calculate  $f_X(x)$  as well as  $f_Y(y)$ .  
3. Let  $X$  have the distribution for  $\mathcal{X} = \{0, 1, 2, 3, \dots\} = \mathbb{N}$

$$f_X(x) = C \cdot \begin{cases} \frac{\lambda^x}{x!} & \text{if } x \in \mathbb{N} \\ 0 & \text{o/w} \end{cases} \quad (3)$$

- (a) Use the second axiom of probability to deduce the value of  $C$ .  
(b) Calculate the probability that  $X \leq 5$ . Evaluate it for  $\lambda = 2$ .  
(c) Determine the mean and variance of  $X$ . Calculate the dispersion of  $X$  which is  $\text{Var}(X)/E(X)$ .  
4. Let  $X$  be a Binomial random variable with  $n$  trials and probability of success  $\theta$ . Write down its **expectation**. Compare numerically how its mean and variance compared to a Poisson with mean  $\lambda = n\theta$  when  $n = 10, 50, 100$  and  $\lambda = 1/50$ .  
5. Explain how to **generate** a Poisson random variable from  $Y \sim U(0, 1)$ . Test this numerically in R or matlab.  
6. If  $f_X(x) = e^{-x/2}$  and  $f_Y(y) = \frac{1}{4\Gamma(2)}ye^{-y/2}$  then find the distribution of the ratio of  $X$  to  $Y$ . Is normally an additional assumption required to perform that task? What is it? Find the expectation and variance of  $X$  and  $Y$ .