Exercises for MA 413 – Statistics for Data Science

This sheet will cover lecture material from the lecture 11/11/2019, overlapping a little with material from the previous week and later material.

1. Assume that each X_i for $i = 1, \dots, n$ has a geometric distribution with $f(x;p) = (1-p)^x p$.
   (a) Write down the likelihood function $L(p)$.
   (b) Determine the log-likelihood function $\ell(p)$.
   (c) Determine the maximum likelihood estimate of p.

2. Let $X_1, X_2, \dots, X_n$ be i.i.d. random variables with density function
$$f(x;\sigma) = \frac{1}{2\sigma}\exp(-\frac{|x|}{\sigma}).$$
   Find the MLE of σ.

3. Let $X_1, X_2, \dots, X_n$ be i.i.d. random variables with density function
$$f(x;\theta) = \begin{cases} \frac{1}{\theta} & \text{if} \quad x \in (-\theta, 0) \\ 0 & \text{o/w} \end{cases}.$$
   Find the MLE of θ.

4. Let $X_1, X_2, \dots, X_n$ be i.i.d. random variables from the gamma density with $\alpha = \alpha$ and $\beta = 1$. Find the MLE of α.

5. The Pareto distribution is popular in economics as a model for a density function with a slowly decaying tail:
$$f(x;\mu,\theta) = \theta\mu^\theta x^{-1-\theta}, \quad x \geq \mu, \theta > 1.$$
   Find the MLE of θ assuming μ known.

6. Let $X_1, \dots, X_n$ be an i.i.d. sample from a Poisson distribution with parameter λ. Determine the MLE of λ.

7. Let $X_1, X_2, \dots, X_n$ be i.i.d. random variables with density function
$$f(x;\theta) = \begin{cases} \theta x^{\theta-1} & \text{if} \quad x \in (0,1) \\ 0 & \text{o/w} \end{cases}.$$
   Find the MLE of θ.

8. Let $X_1, X_2, \dots, X_n$ be i.i.d. random variables with density function
$$f(x;\theta) = \frac{1}{2}e^{-|x-\theta|}, \quad x \in \mathbb{R}.$$
   Find the MLE of θ.