$$Var(X) = E(X^2) - E^2(X)$$

$$= E(X(X-1)) + E(X) - E^2(X)$$

$$= \sum_{x=0}^{\infty} \frac{x(x-1)\, e^{-\lambda} \lambda^x}{x!} + \lambda - \lambda^2$$

$$= \sum_{x=2}^{\infty} \frac{e^{-\lambda} \lambda^x}{(x-2)!} + \lambda - \lambda^2$$

$$= \lambda^2 \sum_{x=2}^{\infty} \frac{e^{-\lambda} \lambda^{x-2}}{(x-2)!} + \lambda - \lambda^2$$

$$= \lambda^2 + \lambda - \lambda^2 = \lambda$$