Exercises 2 for MA 413 – Statistics for Data Science

This sheet will cover lecture material from the lecture 23/09/2019 and later material.

1. Let $Y \sim U(0,1)$. Assume you want to generate a random variable with the distribution for $\lambda > 0$ of

$$f_X(x) = \begin{cases} \lambda e^{-\lambda x} & \text{if} \quad x > 0 \\ 0 & \text{o/w} \end{cases} \tag{1}$$

   Explain how to generate X from Y.

2. Let the random variables X and Y be distributed according for $C > 0$ to

$$f_{X,Y}(x,y) = C \cdot \begin{cases} xy & \text{if} \quad 0 < x < y < 1 \\ 0 & \text{o/w} \end{cases} \tag{2}$$

   (a) Use the second axiom of probability to deduce the value of C.

   (b) Are X and Y independent? Motivate your answer and calculate $f_X(x)$ as well as $f_Y(y)$.

3. Let X have the distribution for $\mathcal{X} = \{0, 1, 2, 3, \dots\} = \mathbb{N}$

$$f_X(x) = C \cdot \begin{cases} \frac{\lambda^x}{x!} & \text{if} \quad x \in \mathbb{N} \\ 0 & \text{o/w} \end{cases} \tag{3}$$

   (a) Use the second axiom of probability to deduce the value of C.

   (b) Calculate the probability that $X \leq 5$. Evaluate it for $\lambda = 2$.

   (c) Determine the mean and variance of X. Calculate the dispersion of X which is $\mathrm{Var}(X)/\mathrm{E}(X)$.

4. Let X be a Binomial random variable with n trials and probability of success θ. Write down its expectation. Compare numerically how its mean and variance compared to a Poisson with mean $\lambda = n\theta$ when $n = 10, 50, 100$ and $\lambda = 1/50$.

5. Explain how to generate a Poisson random variable from $Y \sim U(0,1)$. Test this numerically in R or matlab.

6. If $f_X(x) = e^{-x/2}$ and $f_Y(y) = \frac{1}{4\Gamma(2)} y e^{-y/2}$ then find the distribution of the ratio of X to Y. Is normally an additional assumption required to perform that task? What is it? Find the expectation and variance of X and Y.