

## Exercises 6 for MA 413 – Statistics for Data Science

*This sheet will cover lecture material from the lecture 21/10/2019 and later material.*

1. Let the random sample  $Y_1, \dots, Y_n$  be generated from the distribution

$$f_Y(y) = \begin{cases} \frac{1}{\theta_2 - \theta_1} & \text{if } 0 < \theta_1 < y < \theta_2 \\ 0 & \text{otherwise} \end{cases}$$

Let  $S = \min_i(Y_1, \dots, Y_n)$  and let  $T = \max_i(Y_1, \dots, Y_n)$ .

- \* Are either  $S$  or  $T$  ancillary?
  - \* Derive their density functions.
  - \* Do they become concentrated?
  - \* Set  $\theta_1 = \theta$  and  $\theta_2 = \theta + 1$ . Is  $U = T - S$  ancillary?
2. Let  $Y_i \stackrel{iid}{\sim} N(\mu, \sigma^2 + \mu^2)$ .
    - \* Write down the distribution of the random sample  $\mathbf{Y}$ .
    - \* Determine any sufficient statistics for  $\mu$  and  $\sigma^2$ .
  3. Let  $(X_i, Y_i)$  be iid with common pdf

$$f_{X,Y}(x, y) = \exp(-\theta x - \frac{y}{\theta}) I(x > 0, y > 0).$$

- \* Write down the distribution of the random sample  $(\mathbf{X}, \mathbf{Y})$ .
- \* Determine any ancillary statistics for  $\theta$ .

Hint: consider  $T = \sqrt{\frac{\sum_i Y_i}{\sum_i X_i}}$  and  $U = \sqrt{\sum_i Y_i} \sqrt{\sum_i X_i}$ .

Are they minimally sufficient and/or sufficient?

4. Let  $X_1, \dots, X_n$  be  $\text{Poisson}(\theta)$  generated independently. Write down the joint distribution of  $X$  and  $T = \sum_i X_i$ . Determine the conditional distribution of  $X$  given  $T$ . Is  $T$  sufficient for  $\theta$ ?
5. (Hard) Suppose that  $X_n$  is uniformly distributed on  $\{1/n, 2/n, \dots, 1\}$ . Show that  $X_n$  converges in distribution to a continuous uniform on  $(0, 1)$ .
6. Let  $X_n = 1 + N(0, 1/n)$ . Show that  $X_n$  converges in probability to one.