Exercises 5 for MA 413 – Statistics for Data Science

This sheet will cover lecture material from the lecture 14/10/2019 and later material.

1. Let $X_1, \dots X_n$ be inter-arrival times with an exponential distribution with parameter θ. Use the Fisher-Neymann factorization theorem to show $T(X_1, \dots, X_n) = \sum_{i=1}^{n} X_i$ is sufficient.
2. Let $X_1, \dots X_n$ be $U(\alpha, \beta)$.
   (a) Show that $T(x_1, \dots, x_n) = (\min x_i, \max x_i)$ is sufficient for (α, β).
   (b) If α is known then $T = \max x_i$ is sufficient for β.
   (c) If β is known then $T = \min x_i$ is sufficient for α.
3. Let $X_1, \dots X_n$ be $N(\mu, \sigma^2)$. Show that $(\sum X_i, \sum X_i^2)$ is sufficient. Also show $(\bar{X}, s^2)$ is sufficient. Show that $X_1, \dots X_n$ are sufficient.
4. Determine whether 1–3 are minimally sufficient.
5. Rewrite the binomial distribution as exponential family.
6. Rewrite the gamma distribution as exponential family.
7. Using MGFs determine the distriution of $\bar{X}$ for a Gaussian random sample.
8. Using MGFs determine the distriution of $\bar{X}$ for an Exponential random sample.