Exercises for MA 413 – Statistics for Data Science

This sheet will cover lecture material from the lecture 28/10/2019 and later material.

1. Suppose X_i for $1 \leq i \leq n$ are iid $N(0,1)$. Use the theorems from class to find the limiting distribution of

$$\frac{n(X_1X_2 + X_3X_4 + \cdots + X_{2n-1}X_{2n})^2}{\left(\sum_{i=1}^{2n} X_i^2\right)^2}.$$

2. Let X_n have the Bin $\left(n, \frac{1}{n}\right)$ distribution. Show that X_n converges in distribution to X which has a Poisson(1) distribution.

3. (Hard) Let Y_n have a symmetric Beta distribution with each parameter equal to $1/n$. Determine what distributional convergence Y_n has.

4. Suppose $U_1, \ldots U_{2m+1}$ are iid Unif$(0,1)$. Recall our notation for the order statistics by ordering the sample in increasing size and writing them as $U_{(1)} < U_{(2)} < \cdots < U_{(2m)} < U_{(2m+1)}$. Find the exact distribution of the median $U_{(m+1)}$, and then use this to determine the limiting distribution of $U_{(m+1)}$. Given our study of the minimum and maximum, what do you expect this to be, and does your calculations support this intuition?

5. Let X_i be iid Exp(1) random variables. Define

$$Y_n = \frac{1}{n}\begin{pmatrix} \sum_j X_j \\ \sum_j X_j^2 \\ \sum_j X_j^3 \\ \sum_j X_j^4 \end{pmatrix}.$$

   Decide on how to center and normalizing Y_n so that you can determine the limiting distribution of this random variable using Theorems from class.

6. Take X_i as iid Poisson(μ) random variables. Determine the expectation and variance of $\hat{\mu} = \frac{1}{n}\sum_i X_i$. What is the MSE of $\hat{\mu}$ as an estimator of μ?

7. Determine the Cramer-Rao lower bound on estimating the Bernoulli success probability p from a random sample of this distribution.