

$$S_n = \sum_{i=1}^n X_i \quad X_i \sim N(\mu_i, \sigma_i^2)$$

$$M_{X_i}(t) = \exp(t\mu_i + t^2\sigma_i^2/2)$$

$$M_{S_n}(t) = \prod_{i=1}^n M_{X_i}(t)$$

$$= \prod_{i=1}^n \exp(t\mu_i + t^2\sigma_i^2/2)$$

$$= \exp\left(t \sum_i \mu_i + t^2 \sum_i \sigma_i^2 / 2\right)$$

$$\Rightarrow S_n \sim N\left(\sum_{i=1}^n \mu_i, \sum_{i=1}^n \sigma_i^2\right)$$