Exercises 6 for MA 413 – Statistics for Data Science

This sheet will cover lecture material from the lecture 21/10/2019 and later material.

1. Let the random sample $Y_1, \ldots, Y_n$ be generated from the distribution

$$f_Y(y) = \begin{cases} \frac{1}{\theta_2-\theta_1} & \text{if} \quad 0 < \theta_1 < y < \theta_2 \\ 0 & \text{otherwise} \end{cases}$$

   Let $S = \min_i(Y_1, \ldots, Y_n)$ and let $T = \max_i(Y_1, \ldots, Y_n)$.

   * Are either S or T ancillary?
   * Derive their density functions.
   * Do they become concentrated?
   * Set $\theta_1 = \theta$ and $\theta_2 = \theta + 1$. Is $U = T - S$ ancillary?

2. Let $Y_i \overset{iid}{\sim} N(\mu, \sigma^2 + \mu^2)$.

   * Write down the distribution of the random sample $\mathbf{Y}$.
   * Determine any sufficient statistics for μ and σ^2.

3. Let (X_i, Y_i) be iid with common pdf

$$f_{X,Y}(x, y) = \exp(-\theta x - \frac{y}{\theta})\mathrm{I}(x > 0, y > 0).$$

   * Write down the distribution of the random sample $(\mathbf{X}, \mathbf{Y})$.
   * Determine any ancillary statistics for θ.

   Hint: consider $T = \sqrt{\frac{\sum_i Y_i}{\sum_i X_i}}$ and $U = \sqrt{\sum_i Y_i}\sqrt{\sum_i X_i}$.

   Are they minimally sufficient and/or sufficient?

4. Let $X_1, \ldots, X_n$ be Poisson(θ) generated independently. Write down the joint distribbution of X and $T = \sum_i X_i$. Determine the conditional distriution of X given T. Is T suffficient for θ?

5. (Hard) Suppose that X_n is uniformly distributed on $\{1/n, 2/n, \ldots, 1\}$. Show that X_n converges in distribution to a continuous uniform on $(0, 1)$.

6. Let $X_n = 1 + N(0, 1/n)$. Show that X_n converges in probability to one.