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2. (4 points) **Answer:**

Proof. For $s \in S$, we have

$$\begin{aligned} P(Y_0 = s) &= P(X_{k(0)} = s) && \text{by the definition of } Y_n \\ &= P(X_0 = s) \\ &= \lambda && \because \{X_n : n \geq 0\} \sim \text{Markov}(\lambda, P). \end{aligned} \tag{1}$$

Therefore, the initial distribution of $\{Y_n\}_{n\geq 0}$ is λ.

Now for $i, j \in S$, consider

$$\begin{aligned} P(Y_{n+1} = j | Y_n = i) &= P(X_{k(n+1)} = j | X_{k(n)} = i) && \text{by the definition of } Y_n \\ &= P(X_{kn+k} = j | X_{kn} = i) \end{aligned}$$

Then by the Markov Property, conditional on $\{X_{kn} = i\}$,

$$\{X_{kn+k}\}_{n\geq 0} = \{\tilde{X}_k\}_{k\geq 0} \sim \text{Markov}(\delta_i, P).$$

Thus, by the CK equation,

$$\begin{aligned} P(Y_{n+1} = j | Y_n = i) &= P(\tilde{X}_k | X_0 = i) \\ &= P_{ij}^k. \end{aligned} \tag{2}$$

Then by (1) and (2), we have proven that $(Y_n)_{n\geq 0} \sim \text{Markov}(\lambda, P^k)$. $\square$