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5. **Answer:**

Proof. For $n \in \mathbb{Z}_{\geq 1}$, let $A_n = \{X < s_n\}$. Since $\{s_n : n \in \mathbb{Z}_{\geq 1}\}$ is an increasing sequence, then it follows that $\{A_n : n \in \mathbb{Z}_{\geq 1}\}$ is an increasing sequence of events.

Claim: $\cup_{n=1}^{\infty} A_n = \{X < s\}$

Proof. For some $x \in \cup_{n=1}^{\infty} A_n$, then $\exists\, n \in \mathbb{Z}_{\geq 1}$ such that $x < s_n$. But we also have $s_n < s$, which implies that $x < s$, and so $x \in \{X < s\}$

For some $x \in \{X < s\}$, then $x < s \Rightarrow s - x > 0$. Let $\varepsilon = s - x$. Since $s = \lim_{n\to\infty} s_n$, then $\exists\, N \in \mathbb{Z}_{\geq 1}$ such that $|s - s_N| < \varepsilon$. Then $x < s_N$, so $x \in \cup_{n=1}^{\infty} A_n$ $\square$

It follows that

$$\begin{aligned}
\lim_{n\to\infty} F(s_n) &= \lim_{n\to\infty} P(X \leq s_n) \\
&= \lim_{n\to\infty} P(A_n) \\
&= P(\lim_{n\to\infty} A_n) && \text{by the continuity of probability} \\
&= P(X < s) && \text{by the previous claim}
\end{aligned}$$

$\square$