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2.

(a) **Answer:**

Let T_i be the time to collect the i-th coupon after $i-1$ coupons have been collected. Then $T = \sum_{i=1}^{n} T_i$. Observe that the probability of collecting a new coupon is $p_i = \frac{n-(i-1)}{n} = \frac{n-i+1}{n}$. Therefore, T_i has a geometric distribution with expectation $\frac{1}{p_i} = \frac{n}{n-i+1}$. By the linearity of expectations, we have:

$$\begin{aligned}
\mathrm{E}[T] &= \mathrm{E}[\sum_{i=1}^{n} T_i] \\
&= \sum_{i=1}^{n} \mathrm{E}[T_i] \\
&= \sum_{i=1}^{n} \frac{n}{n-i+1} \\
&= n \sum_{i=1}^{n} \frac{1}{n} \\
&\boxed{= n \cdot H_n}
\end{aligned}$$

where H_n is the n-th harmonic number.

(b) **Answer:**

Let X be the numbers of times the coupon c_1 appears in time T. Then $X \sim \text{Binomial}(\frac{1}{n}, T)$. Thus the probability we're interested in is

$$\boxed{P(X=1) = \binom{T}{1} \frac{1}{n} \left(1 - \frac{1}{n}\right)^{T-1}}$$