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3.

(a) **Answer:***Proof.*

$$\begin{aligned}
& \mathbb{E}[g(X, Y) \mid Y = y] \\
&= \sum_{a \in S_X} \sum_{b \in S_Y} g(a, b) P(X = a, Y = b \mid Y = y) \\
&= \sum_{a \in S_X} g(a, y) P(X = a, Y = y \mid Y = y) \quad \because P(X = a, Y = b \mid Y = y) = 0, \forall b \neq y \\
&= \sum_{a \in S_X} g(a, y) P(X = a \mid Y = y) \\
&= \mathbb{E}[g(X, y) \mid Y = y]
\end{aligned}$$

□

(b) **Answer:***Proof.*

$$\begin{aligned}
& \mathbb{E}[XY \mid Y = y] \\
&= \sum_{a \in S_X} \sum_{b \in S_Y} ab P(X = a, Y = b \mid Y = y) \\
&= \sum_{a \in S_X} ay P(X = a, Y = y \mid Y = y) \quad \because P(X = a, Y = b \mid Y = y) = 0, \forall b \neq y \\
&= y \sum_{a \in S_X} a P(X = a \mid Y = y) \\
&= y \mathbb{E}[X \mid Y = y]
\end{aligned}$$

□

Student Number: 20053722Name: Bryan Hoang(c) **Answer:***Proof.*

$$\begin{aligned} & \mathbb{E}[Y \mathbb{E}[X \mid Y]] \\ &= \sum_{b \in S_Y} b \left(\sum_{a \in S_X} a P(X = a \mid Y = b) \right) P(Y = b) \\ &= \sum_{b \in S_Y} \left(\sum_{a \in S_X} ab P(X = a \mid Y = b) P(Y = b) \right) \\ &= \sum_{a \in S_X} \sum_{b \in S_Y} ab P(X = a, Y = b) \\ &= \mathbb{E}[X \mid Y] \end{aligned}$$

□