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3. (4 points) **Answer:**

*Proof.*

**Part 1.** (Irreducibility)

Let $i, j \in S$ where $S$ is the state space of $X_n : n \geq 0$. WLOG, suppose that $i < j$ and let $n = j - i > 0$. Then

$$\begin{aligned} p_{ij}^{(n)} &= P_i(X_n = j) \\ &= \prod_{k=i}^{j-1} p_{k,k+1} \\ &> 0. \end{aligned}$$

Thus, $i \to j$. In the other direction, we have

$$\begin{aligned} p_{ji}^{(n)} &= P_j(X_n = i) \\ &= \prod_{k=i}^{j-1} p_{k+1,k} \\ &> 0. \end{aligned}$$

Thus, $j \to i$. Then for arbitrary $i, j$, we have that $i \leftrightarrow j$. Therefore, the Markov chain is irreducible.

**Part 2.** (Transience)

Suppose that the Markov chain is recurrent. Then

$$P_1(X_n = 0 \text{ for some } n) = 1.$$

But the result from Question 3 in Assignment 2 is that

$$\begin{aligned} &P_1(X_n \geq 1, \forall\, n \geq 0) > 0 \\ \Rightarrow\ &P_1(X_n = 0 \text{ for some } n) < 1 \end{aligned}$$

which leads to a contradiction. Therefore, the Markov chain must be transient.

$\square$