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2. (5 points)

(a) **Answer:**

Proof. $\{X_n, n \geq 0\}$ has a one-step transition probability matrix

$$P = \begin{bmatrix} p & 1-p & 0 & 0 \\ 0 & 0 & p & 1-p \\ p & 1-p & 0 & 0 \\ 0 & 0 & p & 1-p \end{bmatrix}$$

Therefore, $\{X_n, n \geq 0\}$ is a Markov chain. $\square$

(b) **Answer:**

Let $\pi = (\pi_0, \pi_1, \pi_2, \pi_3)$. We have

$$\pi = \pi P$$

$$\Rightarrow \begin{cases} \pi_0 = p\pi_0 + p\pi_2 \\ \pi_1 = (1-p)\pi_0 + (1-p)\pi_2 \\ \pi_2 = p\pi_1 + p\pi_3 \\ \pi_3 = (1-p)\pi_1 + (1-p)\pi_3 \end{cases}$$

and the additional constraint that

$$\pi_0 + \pi_1 + \pi_2 + \pi_3 = 1$$

Solving the above system of equations gives

$$\boxed{\pi = (p^2, p - p^2, p - p^2, p^2 - 2p + 1)}$$