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1. (5 points) **Answer:**

Let X_n be the vertex the particle is on at step n. Then $\{X_n : n \geq 0\}$ is a Markov chain with the state space $S = \{A, B, C, D, E\}$ and the transition matrix

$$P = \begin{bmatrix} 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 \end{bmatrix}$$

Denote by F the event that the Markov chain hits the state E before it hits B, and denote

$$f_i = \mathbb{P}_i(F), \qquad \text{for } i = A, B, C, D, E.$$

Then conditional on X_0, we have

$$\begin{aligned} f_A &= \frac{1}{2} f_B + \frac{1}{2} f_C \\ f_B &= 0 \\ f_C &= \frac{1}{4} f_A + \frac{1}{4} f_B + \frac{1}{4} f_D + \frac{1}{4} f_E \\ f_D &= \frac{1}{2} f_C + \frac{1}{2} f_E \\ f_E &= 1 \end{aligned}$$

Thus, by solving the system of equations, we have $f_A = \frac{1}{4}$, $f_C = \frac{1}{2}$, and $f_D = \frac{3}{4}$.

In particular, if $X_0 = A$, the probability that the particle hits the state E before it hits B is $\boxed{\frac{1}{4}}$.