The Network Structure of Production

Econometric Methods for Social Spillovers and Networks

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Motivation

Economies consist of larges number of inter-linked firms/sectors.

Firms buy inputs from, and sell inputs to, other firms.

Some firms may supply inputs to *many* downstream firms (e.g., semiconductors, electricity generators etc.)

Some firms may be important buyers (e.g., Walmart, The Home Depot, Boeing)

How do shocks to critical input suppliers (or key buying firms) affect other firms as well as the broader macroeconomy?

How does the buyer-supplier network structure affect the pace and scope of technical change?

(Some) Key References

Atalay, Hortacsu, Roberts and Syverson (2011, *PNAS*)

Acemoglu, Carvalho, Ozdaglar & Tahbaz-Salehi (2012, *Econometrica*)

Carvalho (2014, *Journal of Economic Perspectives*)

Acemoglu, Akcigit, Kerr (2015, *NBER Macroeconomics Annual*)

...and older literature on Input-Output models going back to Leontief in the 1940s.

Nice survey by Carvalho & Tahbaz-Salehi in 2019 issue of the *Annual Review of Economics*

Setup

Production

Perfectly competitive economy with $i = 1, \ldots, N$ industries, each producing a specific good.

Firms sell output to other firms (for use as "intermediate inputs") and also to a representative consumer.

Setup (continued)

Production (continued)

The i^{th} industry's production function is

$$Y_i = A_i L_i^{\beta_i} \prod_{j=1}^{N} X_{ij}^{\gamma_{ij}} \tag{1}$$

with

Y_i - total industry output

A_i - total factor productivity (TFP)

L_i - labor input

X_{ij} - use of good j in production of good i

$\beta_i + \sum_{j=1}^{N} \gamma_{ij} = 1$ - *constant returns to scale*

Setup (continued)

Market clearing:

For all $i = 1, \ldots, N$,

$$Y_i = C_i + \sum_{j=1}^{N} X_{ji} + G_i \tag{2}$$

with

C_i - consumption of i^{th} good by (representative) consumer;

$\sum_{j=1}^{N} X_{ji}$ - total use of i^{th} good as an intermediate input;

G_i - government purchases of i^{th} good.

Setup (continued)

Household preferences:

Representative household with utility

$$u(\mathbf{c}, l) = b(l) \prod_{i=1}^{N} c_i^{\alpha_i} \tag{3}$$

where

$\mathbf{c} = (c_1, \ldots, c_N)'$ is a vector consumption levels for each good;

l is amount of labor supplied by the household.

Setup (continued)

Household preferences (continued):

$b(l) = (\bar{L} - l)^{\delta}$ is decreasing in l and differentiable.

α_i equals weight of good i in preferences; normalized s.t. $\sum_{i=1}^{N} \alpha_i = 1$.

Setup (continued)

Budget constraint

Government imposes a lump sum tax of T to finance purchases s.t.

$$\sum_{i=1}^{N} P_i G_i = T$$

where P_i is the price of good i.

Household therefore maximizes utility subject to the constraint

$$\sum_{i=1}^{N} P_i c_i \leq Wl - T \tag{4}$$

where W is the market wage (chosen as *numeraire*, $W \equiv 1$)

Competitive Equilibrium

Taking prices as given:

1. All firms maximize profits.

2. Representative household maximizes utility.

Market clearing condition for each good holds.

Labor market clears.

Level of government spending (and hence lump sum tax) is exogenous.

Profit Maximization

Profit maximization (& Cobb-Douglas production) imply that for all industries $i = 1, \ldots, N$

$$\beta_i = \frac{WL_i}{P_i Y_i}, \quad \gamma_{ij} = \frac{P_j X_{ij}}{P_i Y_i} \quad j = 1, \ldots, N \tag{5}$$

Let Γ be the matrix of industry-specific output elasticities

$$\Gamma = \begin{pmatrix} \gamma_{11} & \gamma_{12} & \cdots & \gamma_{1N} \\ \gamma_{21} & \gamma_{22} & \cdots & \gamma_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ \gamma_{N1} & \gamma_{N2} & \cdots & \gamma_{NN} \end{pmatrix}$$

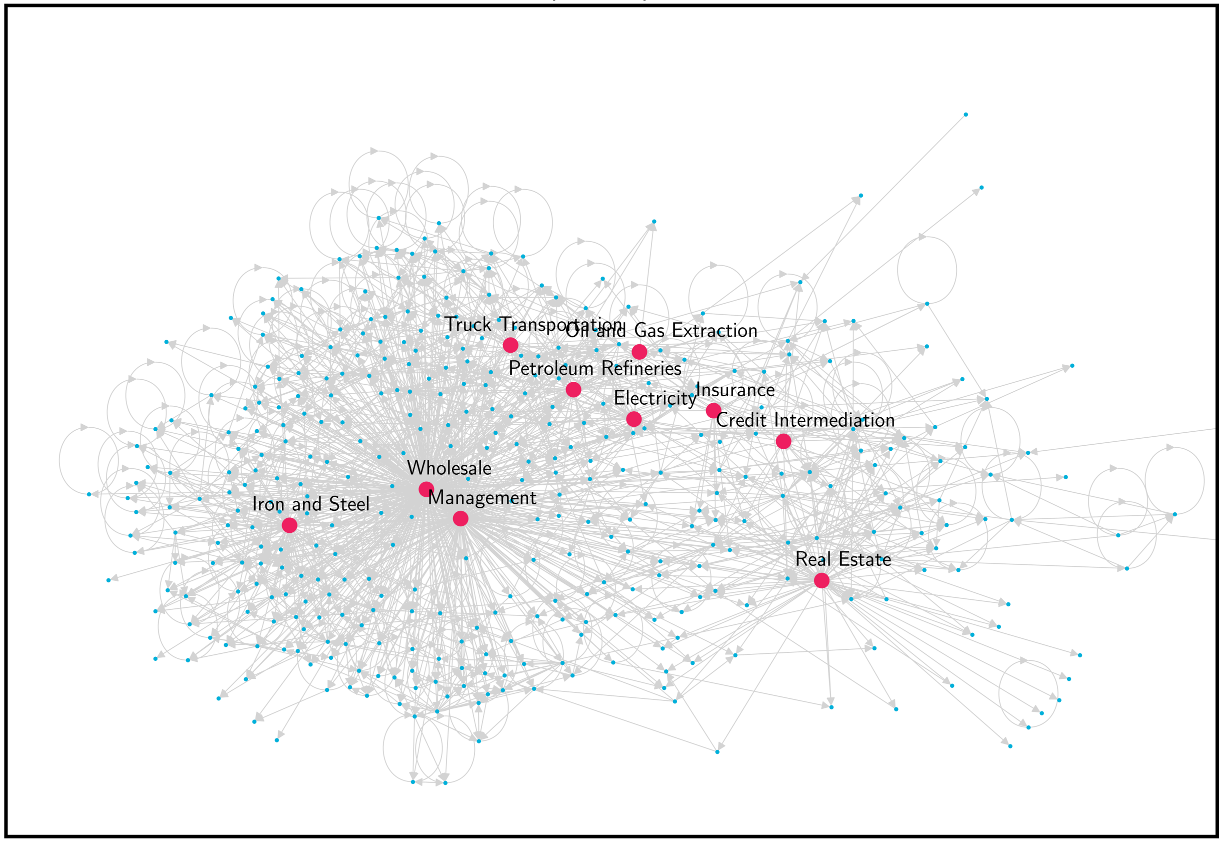
Profit Maximization (continued)

Can construct Γ from BEA "Make" and "Use" input-output tables

$$\gamma_{ij} = \frac{\text{Sales from } j \text{ to } i}{\text{Totals sales of } i}$$

(note: this is a bit *backwards* relative to last lecture!)

United States Input-Output Network, 2007



Source: Bureau of Economic Analysis (BEA) and author's calculations.
Raw data available at https://www.bea.gov/industry/input-output-accounts-data (Accessed September 2018)

Utility Maximization

Solving the household's problem implies that for all i, j

$$\frac{\frac{\partial u(\mathbf{C},L)}{\partial c_i}}{P_i} = \frac{\alpha_i}{P_i C_i} = \frac{\alpha_j}{P_j C_j} = \frac{\frac{\partial u(\mathbf{C},L)}{\partial c_j}}{P_j}. \tag{6}$$

Marginal utility per unit of labor ("money") is equated across goods...

Budget shares for each good equal to

$$\alpha_i = \frac{P_i C_i}{WL - T}. \tag{7}$$

Utility Maximization (continued)

Household labor supply is given by

$$L = \frac{\bar{L} + \delta\left(\sum_{i=1}^{N} P_i G_i\right)}{1 + \delta W} = \bar{L}\left[\frac{1 + \delta W\left(\frac{T}{W\bar{L}}\right)}{1 + \delta W}\right]. \tag{8}$$

Increasing in ratio of lump sum tax to "potential income"...

...labor supply sensitive to government expenditures

Aggregation Fluctuations

Acemoglu, Carvalho, Ozdaglar & Tahbaz-Salehi (2012, *Econometrica*) study the "network origins" of aggregate output fluctuations.

They work with a specialized version of the setup so far, additionally assuming that

1. $\beta_i = (1 - \eta)$ and $\sum_j \gamma_{ij} = \eta$ for all $i = 1, \ldots, N$ (cf., Figure 6 of paper);

2. labor is supplied inelastically s.t. $b\,(l) = \bar{b}$ (with $\bar{L} = 1$)

3. and $\alpha_i = N^{-1}$ (i.e., all goods valued equally by the household);

4. $T = 0$ (i.e., no government expenditures).

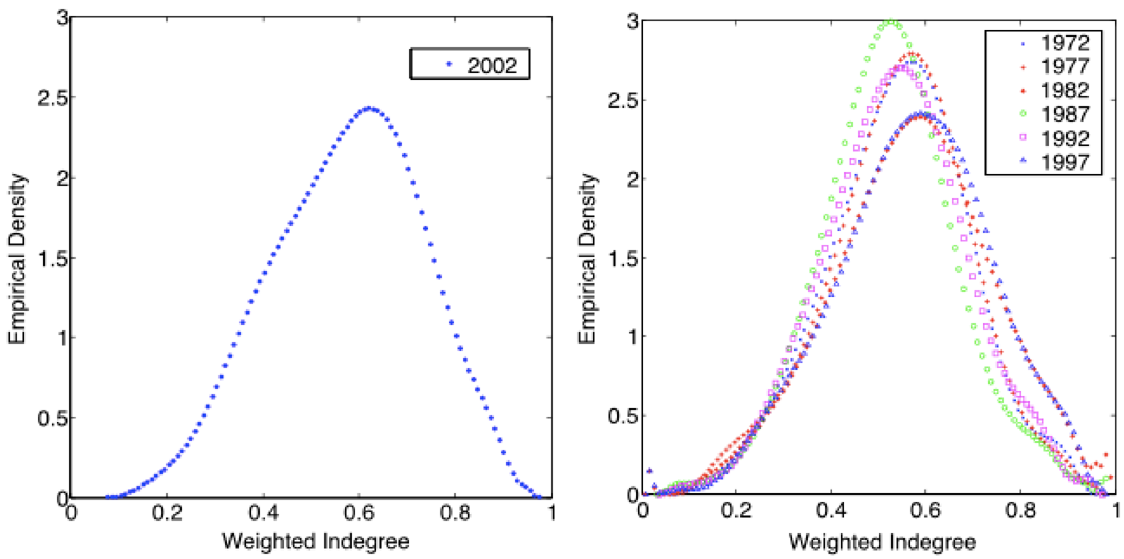


FIGURE 6.—Empirical densities of intermediate input shares (indegrees).

Aggregation Fluctuations (continued)

Log of the production function 1 equals:

$$\ln Y_i = \ln A_i + (1-\eta)\ln L_i + \eta \sum_{j=1}^{N} \pi_{ij} \ln X_{ij}$$

for $\pi_{ij} = \gamma_{ij}/\eta$.

Define *Leontief Centrality* vector as

$$\boxed{\mathbf{c}^{\mathrm{L}}(\Gamma, \eta) = \frac{(1-\eta)}{N}\iota_N' \left[I_N - \eta\Pi\right]^{-1}} \tag{9}$$

for $\Pi = \left[\pi_{ij}\right] = \left[\frac{\gamma_{ij}}{\eta}\right]$ (note: $\mathbf{c}^{\mathrm{L}}(\Gamma, \eta)' \iota_N = 1$).

We will have more to say about $\mathbf{c}^{\mathrm{L}}(\Gamma, \eta)$ shortly.

It should look very familiar!

Aggregation Fluctuations (continued)

Using the firm FOCs (5) to substitute for $\ln L_i$ and $\ln X_{ij}$ for $j = 1, \ldots, N$ yields:

$$\begin{aligned}(1-\eta)\ln W &= \ln A_i + H + \ln P_i \\ &\quad - \eta \sum_{j=1}^{N} \pi_{ij} \ln P_j + \eta \sum_{j=1}^{N} \pi_{ij} \ln \pi_{ij}\end{aligned}$$

for $H = \eta \ln \eta + (1-\eta)\ln(1-\eta)$.

Multiplying by c_i^L and summing over all industries then gives

$$\ln W = (1-\eta)^{-1} \mathbf{c}^{\mathrm{L}} (\Gamma, \eta)' \ln \mathbf{A} + \mu$$

for

$$\mu \overset{def}{\equiv} \frac{1}{N} \sum_{i=1}^{N} \ln P_i + \frac{H}{1-\eta} + \frac{\eta}{1-\eta} \sum_{i=1}^{N} \sum_{j=1}^{N} c_i^L \pi_{ij} \ln \pi_{ij}$$

Aggregation Fluctuations (continued)

Second to last step is to note that W coincides with nominal *value added* since, using the market clearing condition (2) and the household budget constraint (4),

$$\begin{aligned}
\text{Value Added} &= \underbrace{\sum_{i=1}^{N} P_i Y_i}_{\text{Total sales}} - \underbrace{\sum_{i=1}^{N}\sum_{j=1}^{N} P_i X_{ji}}_{\text{Input purchases}} \\
&= \sum_{i=1}^{N} P_i C_i \\
&= W.
\end{aligned}$$

Acemoglu et al (2012) refer to GDP and value added interchangeably in the paper.

Aggregation Fluctuations (continued)

Normalizing utility such that

$$\bar{b} = N \exp\left(-\frac{H}{1-\eta} - \frac{\eta}{1-\eta} \sum_{i=1}^{N} \sum_{j=1}^{N} c_i^L \pi_{ij} \ln \pi_{ij}\right)$$

and using the ideal price index under Cobb-Douglas utility

$$\bar{P}_u = \frac{N}{\bar{b}} \prod_{i=1}^{N} P_i^{\frac{1}{N}},$$

we get *real* valued added (VA) equal to

$$\boxed{\ln \mathrm{VA} = \frac{1}{1-\eta} \mathbf{c}^{\mathrm{L}} \left(\Gamma, \eta\right)' \ln \mathbf{A}} \tag{10}$$

Comments on model (so far)

1. Productivity shock to sectors with high Leontief Centrality generate the largest effects on aggregate valued added.

2. Imagine each sector experiences an increase in $\ln A_i$ of Δ

   (a) This corresponds to a $100\Delta\%$ increase in TFP across all sectors...

   (b) ...but aggregate (real) value added increases by $\frac{100\Delta\%}{1-\eta}$ (feedback/spillovers)!

   (c) This corresponds to an (average) Social Multiplier (cf., Glaeser, Sacercote and Scheinkman, 2003, *JEEA*; Jones, 2011, *AEJ-Macro*) of $\frac{1}{1-\eta} \approx 2$ (BEA)

Aggregate Volatility

Theorem 2 of Acemoglu et al (2012, p. 1990) relates aggregate volatility to the variance of the outdegree distribution/sequence.

Let $D_{+i} = \sum_{j=1}^{N} \pi_{ji}$ equal the *outdegree* of firm i (remember things are set up backwards!).

Outdegree measures the intensity with which product i is used as an input by other sectors.

Assume that $\{\ln A_i\}_{i=1}^{N}$ are i.i.d. random variables with variance σ^2.

We have

$$\mathbb{V}(\ln \mathrm{VA})^{1/2} = \frac{1}{1-\eta} \left\| c^{\mathrm{L}}(\Gamma, \eta) \right\|_2 \sigma$$

Aggregate Volatility (continued)

To understand aggregate volatility we need to understand $\left\| c^{\mathrm{L}}(\Gamma, \eta) \right\|_2$.

Using the infinite series expansion we have

$$
\begin{aligned}
\left\| c^{\mathrm{L}}(\Gamma, \eta) \right\|_2^2 &= \frac{(1-\eta)^2}{N^2} \iota_N' \left[\sum_{k=0}^{\infty} \eta^k \Pi^k \right] \left[\sum_{k=0}^{\infty} \eta^k \Pi^k \right]' \iota_N \\
&\geq \frac{(1-\eta)^2}{N^2} \iota_N' \iota_N + \frac{2\eta (1-\eta)^2}{N^2} \iota_N' \Pi \iota_N \\
&+ \frac{\eta^2 (1-\eta)^2}{N^2} \left\| \Pi' \iota_N \right\|_2^2 \\
&= \frac{(1-\eta)^2 (1+\eta)^2}{N} + \frac{\eta^2 (1-\eta)^2}{N} \mathbb{V}(D_{+i})
\end{aligned}
$$

Aggregate Volatility (continued)

Plugging our approximation back into our expression for the standard deviation of value-added yields

$$\boxed{\mathbb{V}\left(\ln \mathrm{VA}\right)^{1/2} \geq \frac{1}{\sqrt{N}}\sqrt{\left(1+\eta\right)^{2}+\eta^{2}\mathbb{V}\left(D_{+i}\right)}\sigma}$$

...if the outdegree sequence has thick tails, then the effects of sector-specific shocks on aggregate volatility may be large.

...especially in economies with high intermediate input use shares, η (i.e., lots of "backward and forward linkages").

TABLE I

OLS ESTIMATES OF β AND ζ[a]

	1972	1977	1982	1987	1992	1997	2002
$\widehat{\beta}$	1.38	1.38	1.35	1.37	1.32	1.43	1.46
	(0.20; 97)	(0.19; 105)	(0.18; 106)	(0.19; 102)	(0.19; 95)	(0.21; 95)	(0.23; 83)
$\widehat{\zeta}$	1.14	1.15	1.10	1.14	1.15	1.27	1.30
	(0.16; 97)	(0.16; 105)	(0.15; 106)	(0.16; 102)	(0.17; 95)	(0.18; 95)	(0.20; 83)
n	483	524	529	510	476	474	417

[a]The numbers in parentheses denote the associated standard errors (using Gabaix and Ibragimov (2011) correction) and the number of observations used in the estimation of the shape parameter (corresponding to the top 20% of sectors). The last row shows the total number of sectors for that year.

TABLE II

ESTIMATES FOR $\|v_n\|_2$ [a]

	1972	1977	1982	1987	1992	1997	2002
$\|v_{n_d}\|_2$	0.098 ($n_d = 483$)	0.091 ($n_d = 524$)	0.088 ($n_d = 529$)	0.088 ($n_d = 510$)	0.093 ($n_d = 476$)	0.090 ($n_d = 474$)	0.094 ($n_d = 417$)
$\|v_{n_s}\|_2$	0.139 ($n_s = 84$)	0.137 ($n_s = 84$)	0.149 ($n_s = 80$)	0.133 ($n_s = 89$)	0.137 ($n_s = 89$)	0.115 ($n_s = 127$)	0.119 ($n_s = 128$)
$\frac{\|v_{n_d}\|_2}{\|v_{n_s}\|_2}$	0.705	0.664	0.591	0.662	0.679	0.783	0.790
$\frac{1/\sqrt{n_d}}{1/\sqrt{n_s}}$	0.417	0.400	0.399	0.418	0.432	0.518	0.554

[a] $\|v_{n_d}\|_2$ denotes estimates obtained from the detailed level input–output BEA data. $\|v_{n_s}\|_2$ denotes estimates obtained from the summary input–output BEA data. The numbers in parentheses denote the total number of sectors implied by each level of disaggregation.

Aggregate Volatility Wrap-Up

1. Economies with a few critical general purpose inputs (and inputs to such inputs) are vulnerable to shocks to these input-producing sectors.

2. Think about the economy-wide productivity benefits of technical change in electricity generation and transmission.

3. Implications for industrial policy, R&D subsidies etc.

4. Comment: what really matters is the variance of Leontief Centrality (Table II of paper).

Empirics

Acemoglu, Akcigit, Kerr (2015, *NBER Macroeconomics Annual*) test some empirical implications of the model.

They are interested in tracing out differences in the effects of *supply-side* TFP shocks versus those of *demand-side* (government spending) shocks.

Key result: Supply-side shocks move *downstream*, while demand-side shocks move *upstream*.

Framework for thinking about the different roles of central suppliers vs. central buyers in an inter-linked economy.

Supply-Side TFP Shocks

Continue to assume that $T = 0$ (no government expenditures).

Revert to all other baseline assumptions.

Maintain focus on sector-level output (as opposed to aggregate value-added).

How does output across different sectors co-vary?

Supply-Side TFP Shocks (continued)

Use the firm and household FOCs (5), (6) and (7) to substitute into the production function for

1. $\ln L_i$ and $\ln X_{ij}$ for $j = 1, \ldots, N$ and then (subsequently)

2. $\ln P_i$ and $\left(\ln P_i - \ln P_j\right)$

Supply-Side TFP Shocks (continued)

This sequence of substitutions gives

$$\ln C_i = \tilde{\mu}_i + \ln A_i + \sum_{j=1}^{N} \gamma_{ij} \ln C_j$$

where the industry-specific intercept equals

$$\begin{aligned}\tilde{\mu}_i \stackrel{def}{\equiv} & \ln \alpha_i + \beta_i \ln \beta_i \\ & + \sum_{j=1}^{N} \gamma_{ij} \left(\ln \gamma_{ij} - \ln \alpha_j \right) + \beta_i \ln \left(\frac{\bar{L}}{1+\delta} \right)\end{aligned}$$

Supply-Side TFP Shocks (continued)

In matrix form...

$$\ln \mathbf{C} = \tilde{\mu} + \ln \mathbf{A} + \Gamma \ln \mathbf{C}$$

...so that, for $\mu = (I - \Gamma)^{-1} \tilde{\mu}$,

$$\ln \mathbf{C} = \mu + (I - \Gamma)^{-1} \ln \mathbf{A}$$

Taking the total derivative we get

$$\mathrm{d} \ln \mathbf{C} = (I - \Gamma)^{-1} \mathrm{d} \ln \mathbf{A}$$

Supply-Side TFP Shocks (continued)

Note from (2) and (5) and (6) we have (w/ $G_i = 0$)

$$
\begin{aligned}
Y_i &= C_i + \sum_{j=1}^{N} X_{ji} \\
\frac{Y_i}{C_i} &= 1 + \sum_{j=1}^{N} \gamma_{ji} \frac{P_j Y_j}{P_i C_i} \\
&= 1 + \sum_{j=1}^{N} \gamma_{ji} \frac{\alpha_j Y_j}{\alpha_i C_j}
\end{aligned}
$$

...which implies that sector-specific consumption and output move in lock-step so that

$$
\boxed{\mathrm{d} \ln \mathbf{Y} = (I - \Gamma)^{-1} \mathrm{d} \ln \mathbf{A}}
$$

Supply-Side TFP Shocks (continued)

$$\boxed{\mathrm{d}\ln \mathbf{Y} = (\boldsymbol{I} - \Gamma)^{-1}\, \mathrm{d}\ln \mathbf{A}}$$

$\ln Y_i$ varies with productivity growth in other sectors according to i^{th} row of Leonteif inverse.

Elements of $\iota' (I - \Gamma)^{-1}$ indicate which sectors the aggregate economy is most vulnerable to TFP shocks in.

Demand Side Shocks

Acemoglu et al. (2015) use changes in government spending to simulate demand side shocks to specific sectors.

Main finding is that demand shocks move *upstream.*

For simplicity assume that $\mathrm{d}\ln\mathbf{A} = 0$.

Demand Side Shocks (continued)

Begin with the market clearing condition (2). Multiplying both sides by P_i yields

$$P_i Y_i = P_i C_i + \sum_{j=1}^{N} P_i X_{ji} + P_i G_i. \tag{11}$$

Next, from the labor supply condition (8), we have

$$P_i C_i = \frac{\alpha_i}{1+\delta}\left[\bar{L} - \sum_j P_j G_j\right] \Rightarrow \mathrm{d}\left(P_i C_i\right) = -\frac{\alpha_i}{1+\delta}\sum_j \mathrm{d}\left(P_j G_j\right). \tag{12}$$

Demand Side Shocks (continued)

Combining equations (11) and (12) and using the implication of the firm's FOC that

$$X_{ji} = \frac{\gamma_{ji} P_j Y_j}{P_i}$$

we can derive, for $i = 1, \ldots, N$

$$\mathrm{d}\left(P_i Y_i\right) = \sum_j \gamma_{ji} \mathrm{d}\left(P_j Y_j\right) + \mathrm{d}\left(P_i G_i\right) - \frac{\alpha_i}{1+\delta} \sum_j \mathrm{d}\left(P_j G_j\right).$$

<u>Demand Side Shocks (continued)</u>

Let $\tilde{Y}_i = P_i Y_i$ and $\tilde{G}_i = P_i G_i$ and stacking things in matrix form yields

$$\mathrm{d}\tilde{\mathbf{Y}} = \Gamma' \mathrm{d}\tilde{\mathbf{Y}} + \left[I - \frac{1}{1+\delta} \iota \alpha' \right] \mathrm{d}\tilde{\mathbf{G}}.$$

Solving for the change in nominal (real) output yields

$$\boxed{\mathrm{d}\tilde{\mathbf{Y}} = \left(I - \Gamma' \right)^{-1} \mathrm{d}\tilde{\mathbf{G}} - \frac{1}{1+\delta} \left(I - \Gamma' \right)^{-1} \iota \alpha' \mathrm{d}\tilde{\mathbf{G}}.}$$

Row sums of $(I - \Gamma')^{-1}$ provide a measure of *buyer* centrality.

Empirical Implementation

Define the projection

$$\mathbb{E}\left[\ln A_{it} \middle| \, \text{ForeignPatent}_{it}\right] = \xi_i + \tilde{\rho}_t + \lambda \text{ForeignPatent}_{it}$$

Foreign patent stocks are an 'exogenous' component of available knowledge and hence TFP (caveats).

Let $\Theta = (I - \Gamma)^{-1} = \left[\theta_{ij}\right]$ and define the regressors

$$\text{Downstream}_{it}^{\text{FP}} = \sum_{j \neq i}^{N} \theta_{ij} \text{ForeignPatent}_{jt}$$

$$\text{Own}_{it}^{\text{FP}} = \theta_{ii} \text{ForeignPatent}_{it}$$

Empirical Implementation (continued)

Using the projection and our definitions we get

$$\begin{aligned}\triangle \ln Y_{it} =& \rho_t \left[\sum_{j=1}^{N} \theta_{ij}\right] + \lambda \triangle \mathsf{Own}_{it}^{\mathrm{FP}} \\ &+ \lambda \triangle \mathsf{Downstream}_{it}^{\mathrm{FP}} + \sum_{j=1}^{N} \theta_{ij}\epsilon_{jt}\end{aligned}$$

for $\rho_t = \tilde{\rho}_t - \tilde{\rho}_{t-1}$ and ϵ_{jt} our projection error.

Empirical Implementation: Three Problems

1. Heterogenous coefficient on aggregate time effect (easy fix);

2. Cross-sector correlation in regression errors (harder, GLS?);

3. ...making static model dynamic for empirics (see paper).

Table 5A
Foreign Patent Shock Analysis

	Δ Log Real Value Added	
	(1)	(2)
Δ Dependent variable $t-1$	0.020	0.020
	(0.025)	(0.025)
Δ Dependent variable $t-2$		0.051**
		(0.023)
Δ Dependent variable $t-3$		0.037*
		(0.021)
Downstream effects $t-1$	0.043***	0.044***
	(0.011)	(0.011)
Upstream effects $t-1$	–0.000	0.000
	(0.005)	(0.005)
Own effects $t-1$	–0.006	–0.007*
	(0.004)	(0.004)
Observations	6,543	5,761
P-value: Downstream = own	0.000	0.000

Empirical Implementation (continued)

Note that their set-up also implies that

$$\begin{aligned}\triangle \ln Y_{it} - \sum_{j=1}^{N} \gamma_{ij} \triangle \ln Y_{jt} =& \triangle \ln A_{it} \\ =& \rho_t + \lambda \triangle \text{ForeignPatent}_{it} + \epsilon_{it}.\end{aligned}$$

This results in a textbook application of OLS analysis...

...it probably treats the model too seriously.

If the model is correct, then any average of $\triangle$ForeignPatent$_{jt}$ over $j \neq i$ should enter with a coefficient of zero.

Demand Shocks

Acemoglu et al (2015) also study the demand side shocks, operationalized as government purchases of goods, interacts with the network structure of production.

Main theoretical finding is that demand side shocks move *upstream*.

Construct upstream shock variables as, for example,

$$\text{Upstream}_{it}^{\text{FP}} = \sum_{j \neq i}^{N} \theta_{ji} \text{ForeignPatent}_{jt}$$

Evidence presented in paper broadly consistent with theory, at least qualitatively.

Wrapping Up

The idea of modeling the economy as a set of interlinked sectors has a long tradition in economics (and in government statistical offices).

Tools from network analysis provide an opportunity to revisit and extend this tradition in important ways.

Acemoglu et al. (2012, 2015) work at the sector level, but the theory is perhaps even more applicable at the *firm* level.

Interesting questions related to technical change, industrial policy (e.g., the effects of mergers on network structure) and other policy questions.