

# WindTrue: Sensitivity analysis applied to DANAERO wind turbine



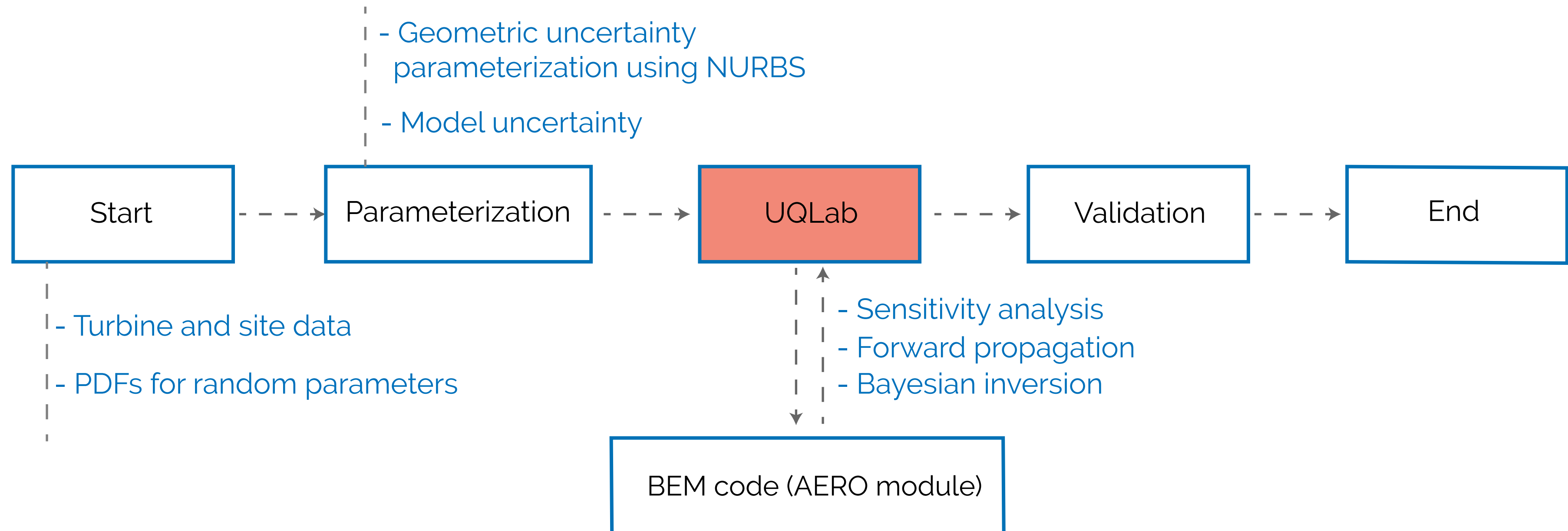
Prashant Kumar



Benjamin Sanderse

CWI

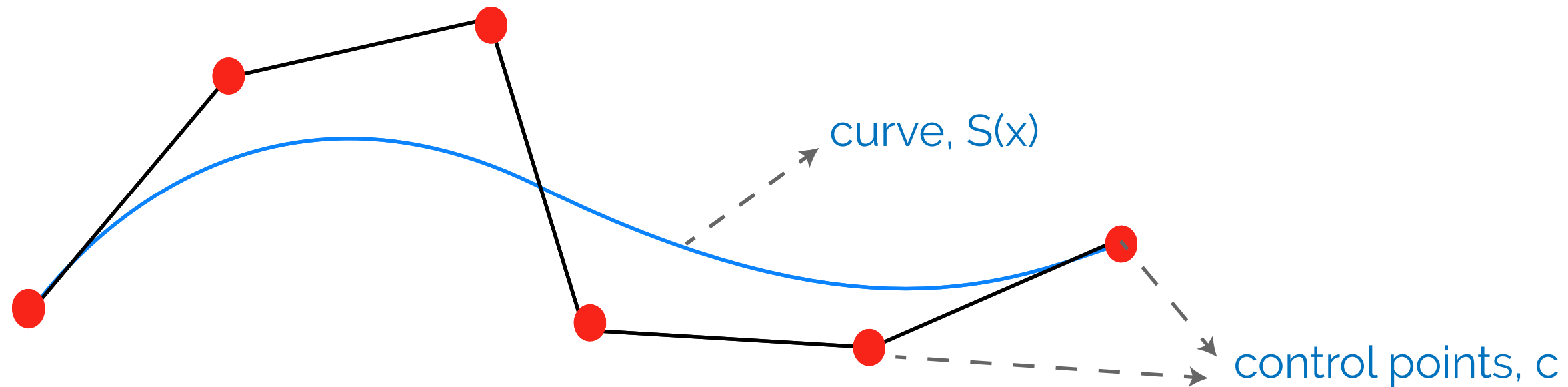
# Workflow



# NURBS based parametrization

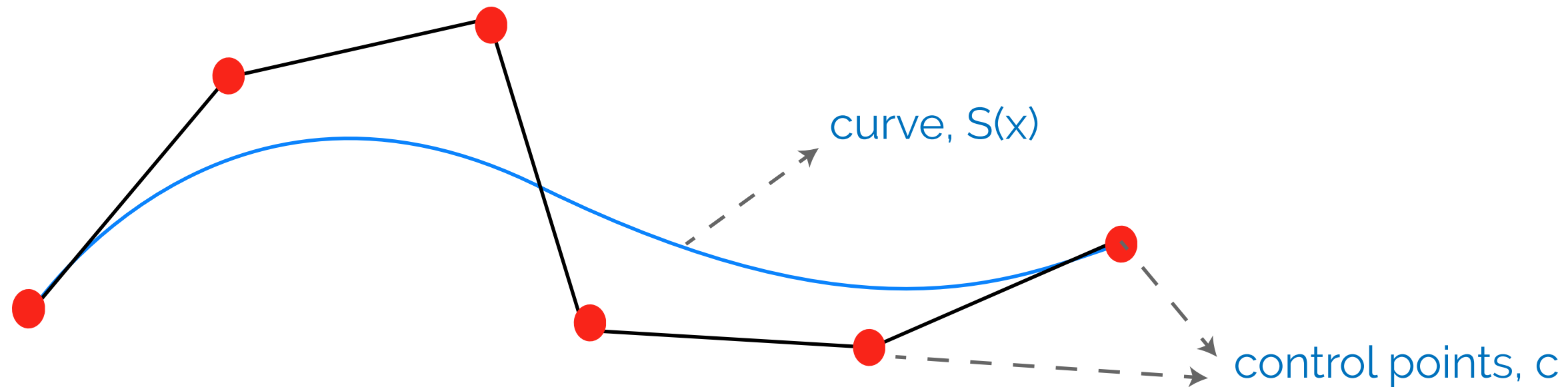
Obtain perturbed chord/twist/thickness curves from  
given reference curves

# Non-Uniform Rational Basis Spline (NURBS)



$$S(x) = \sum_{i=0}^{N-1} c_i B_{i,p}(x)$$

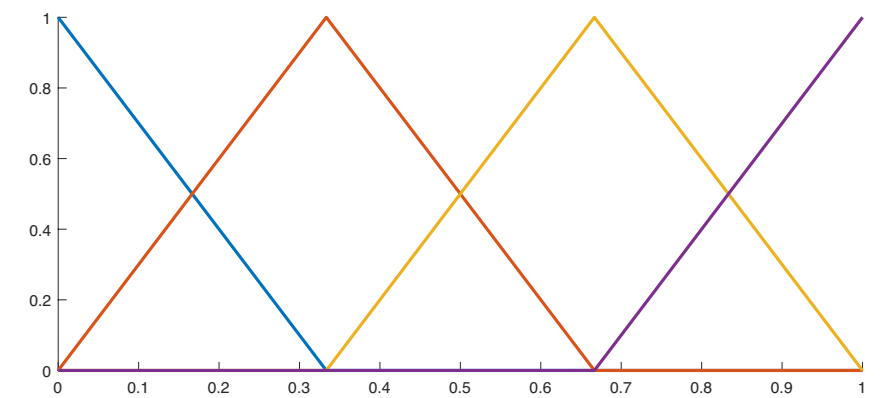
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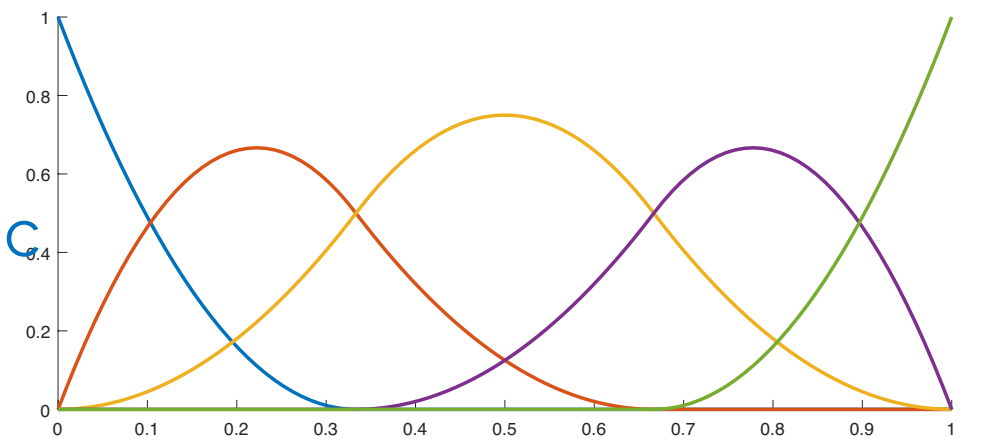
$$S(x) = \sum_{i=0}^{N-1} c_i B_{i,p}(x)$$

Basis functions,  $B_{i,p}(x)$

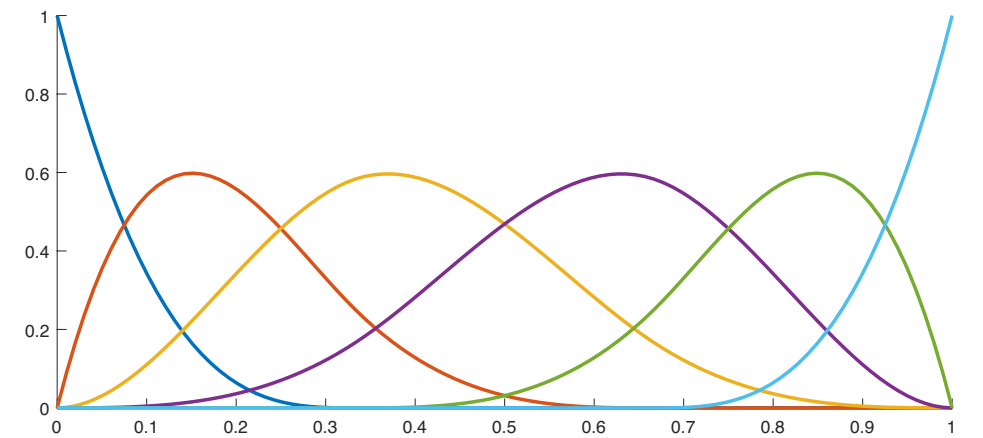
linear



quadratic



cubic



$x$

# Why NURBS?

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- > Represent complex shapes with very few points
- > Flexibility to design a large variety of shapes
- > Easy to obtain high-order polynomials

# Parameterization using NURBS

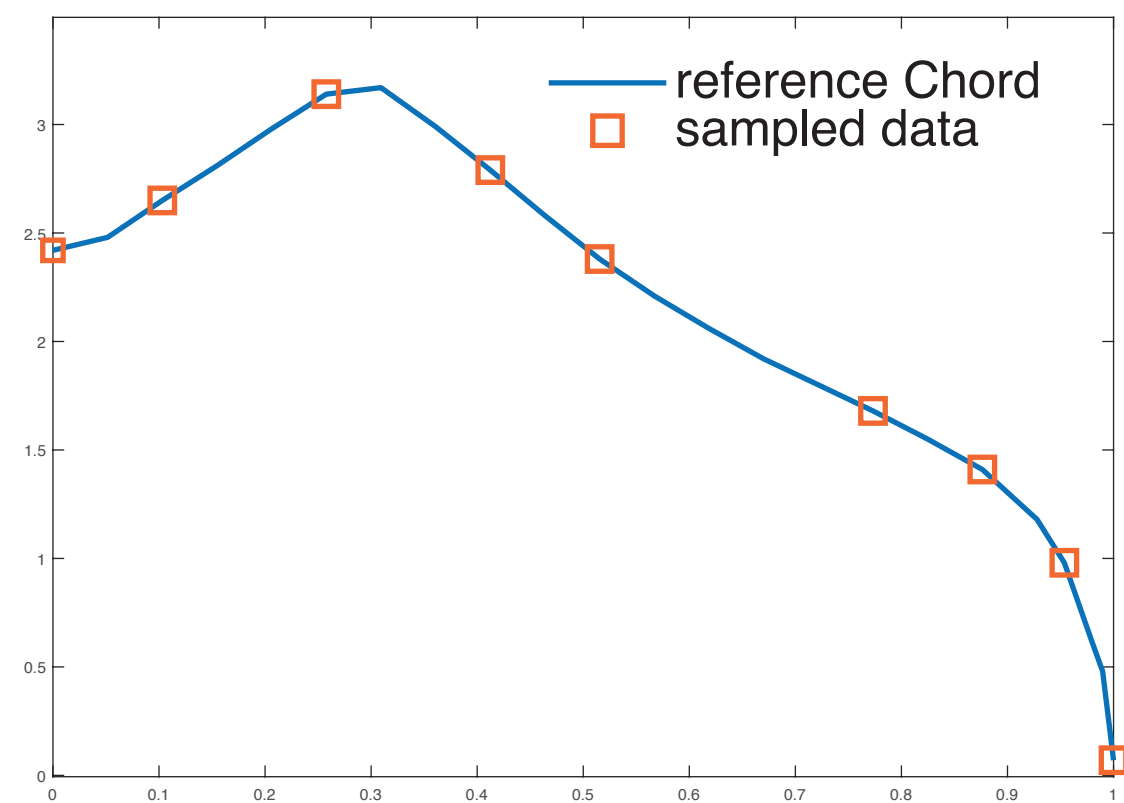
**Goal:** Obtain perturbed chord curves from a given reference chord

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**Step 1:** Sample locations from the reference curve

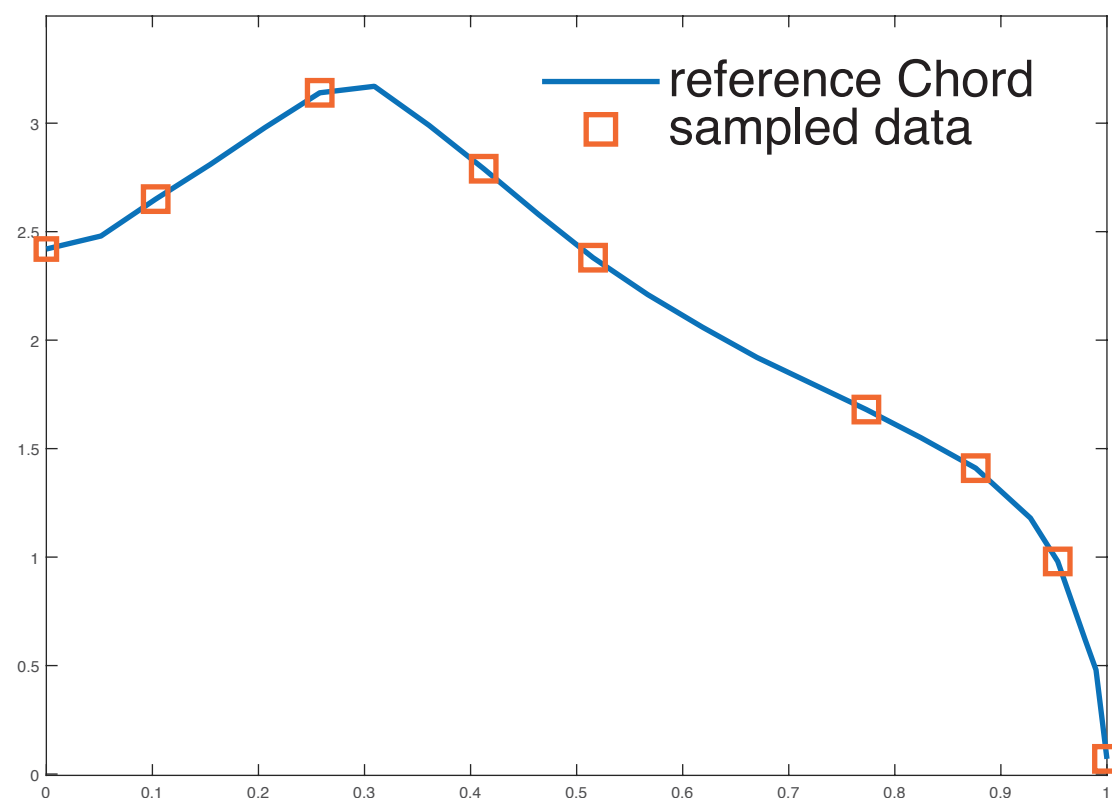




# Parameterization using NURBS

**Goal:** Obtain perturbed chord curves from a given reference chord

**Step 1:** Sample locations from the reference curve



**Step 2:** Compute control points at sampled location via inversion

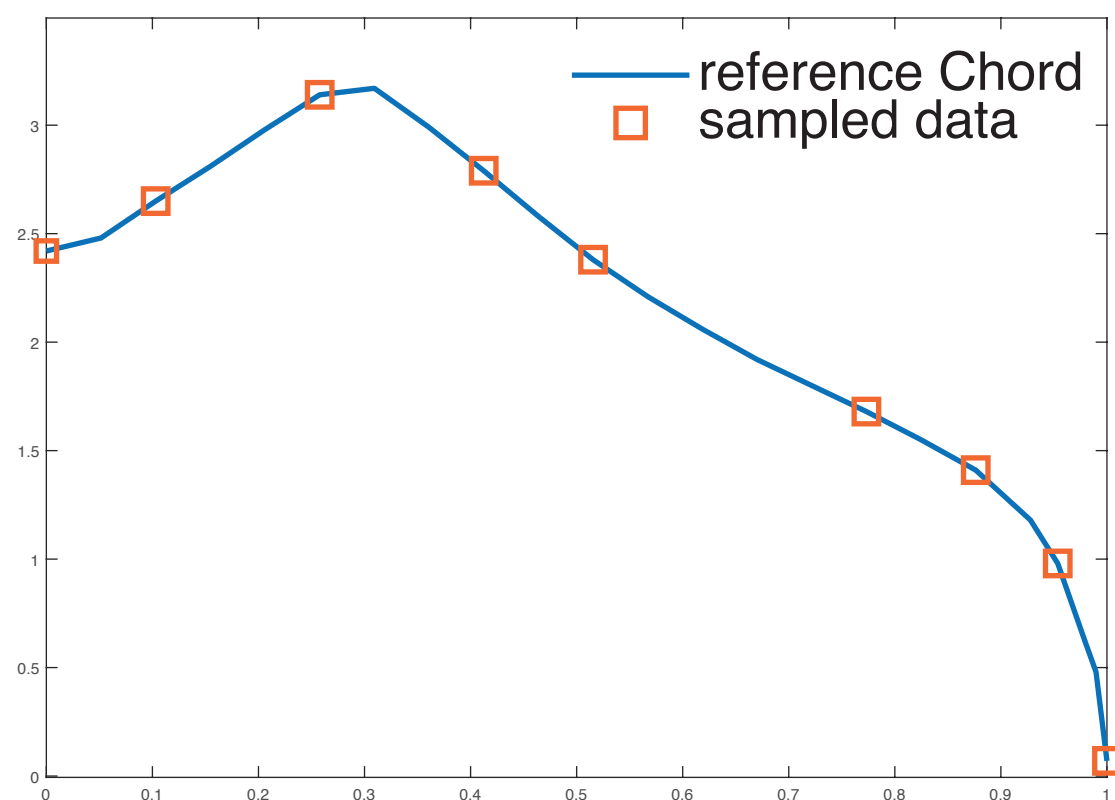
$$\underset{\substack{\downarrow \\ \text{known}}}{S(x)} = \sum_{i=0}^{N-1} c_i \underset{\substack{\downarrow \\ \text{known}}}{B_{i,p}(x)} \implies \mathbf{B}\mathbf{c} = \mathbf{S}$$



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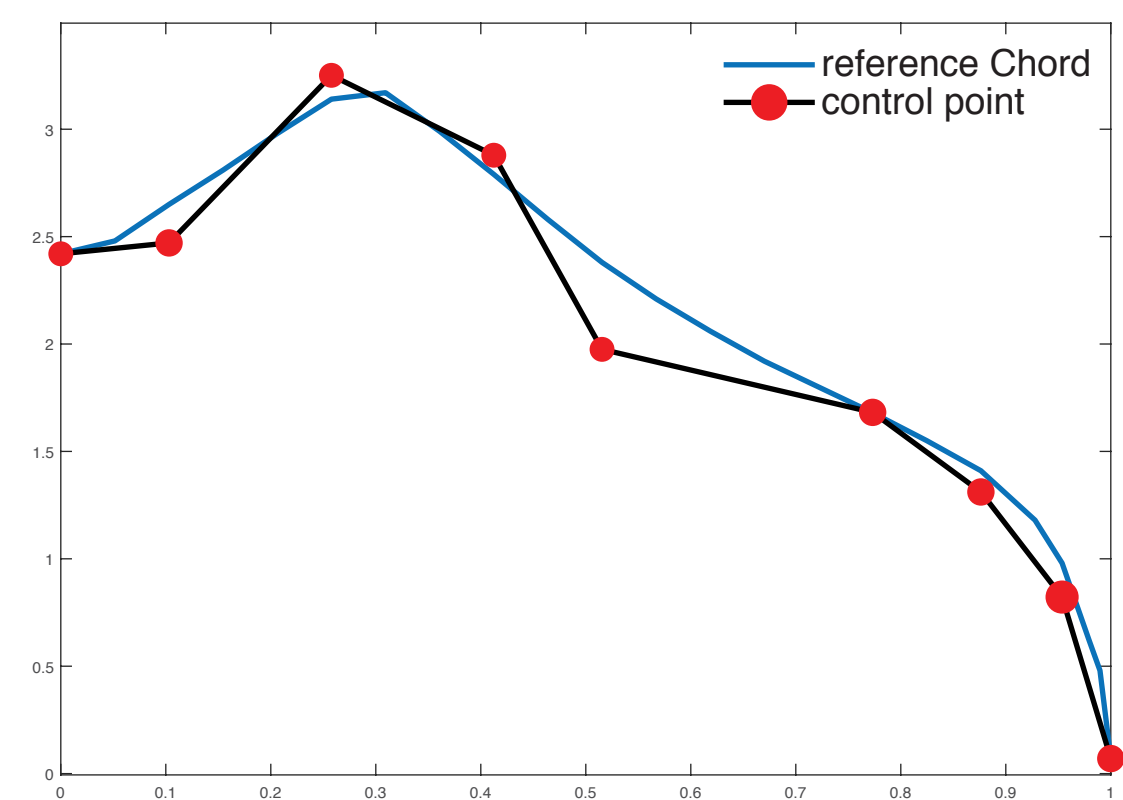


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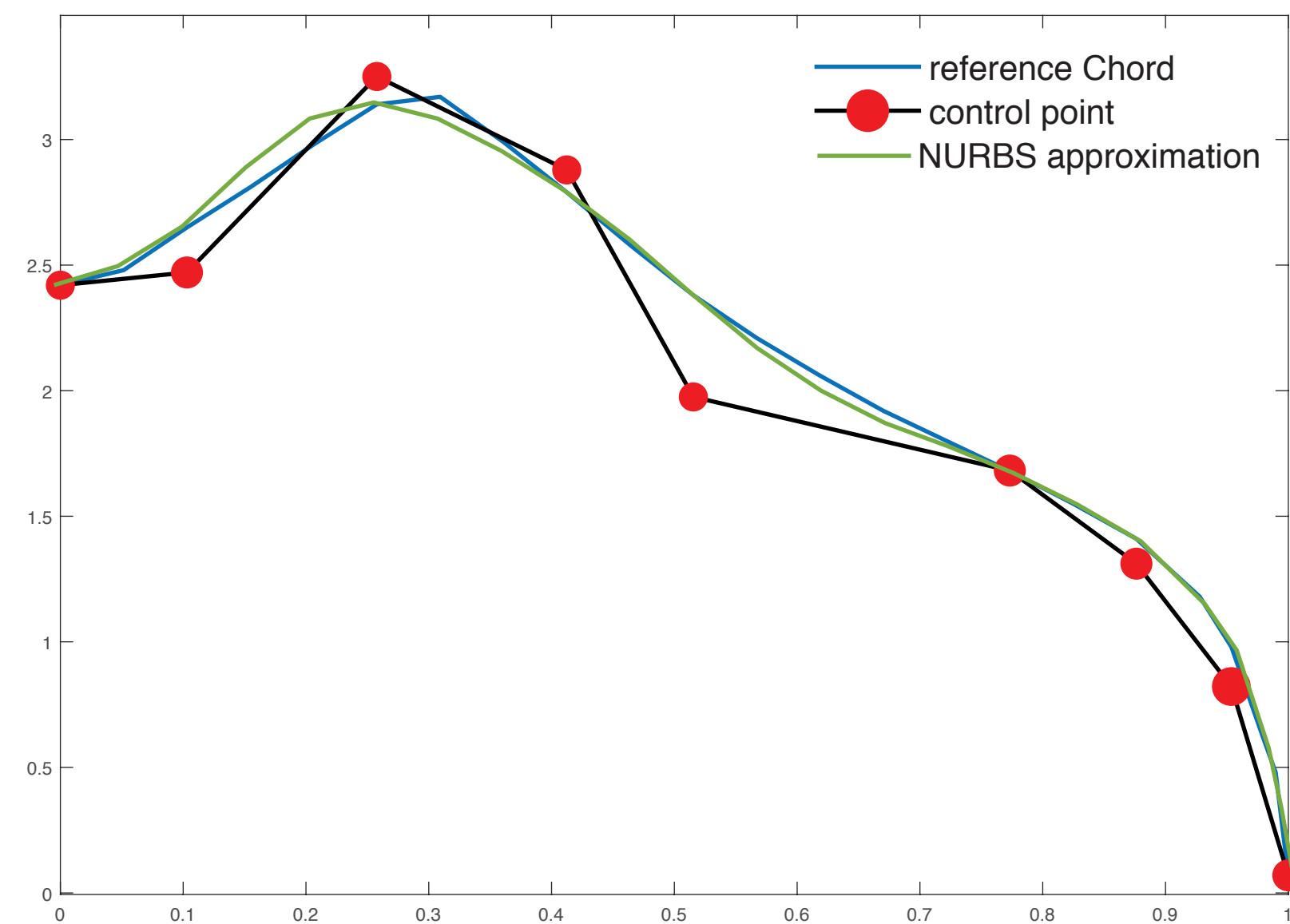
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known                  known

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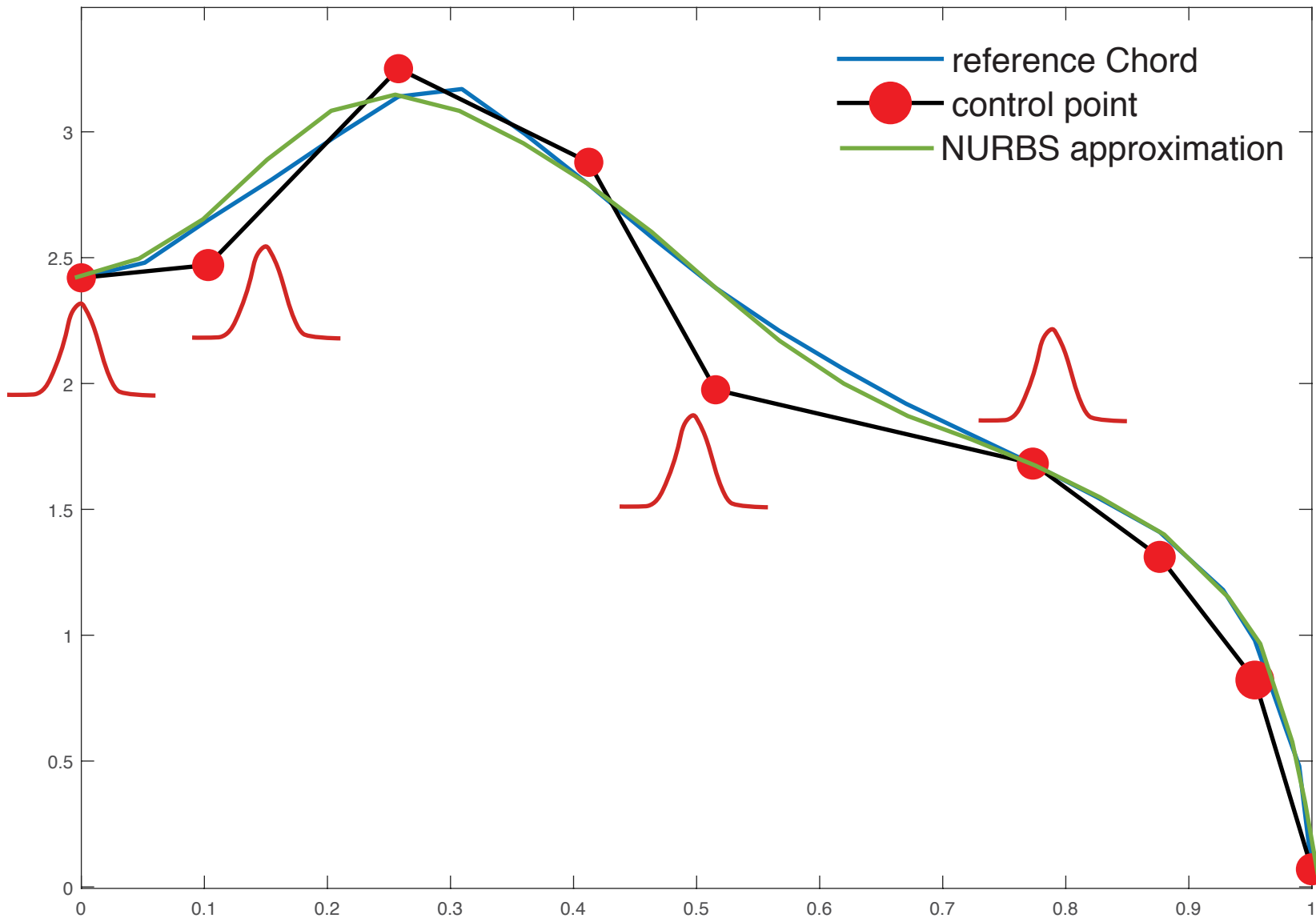


# Parameterization using NURBS



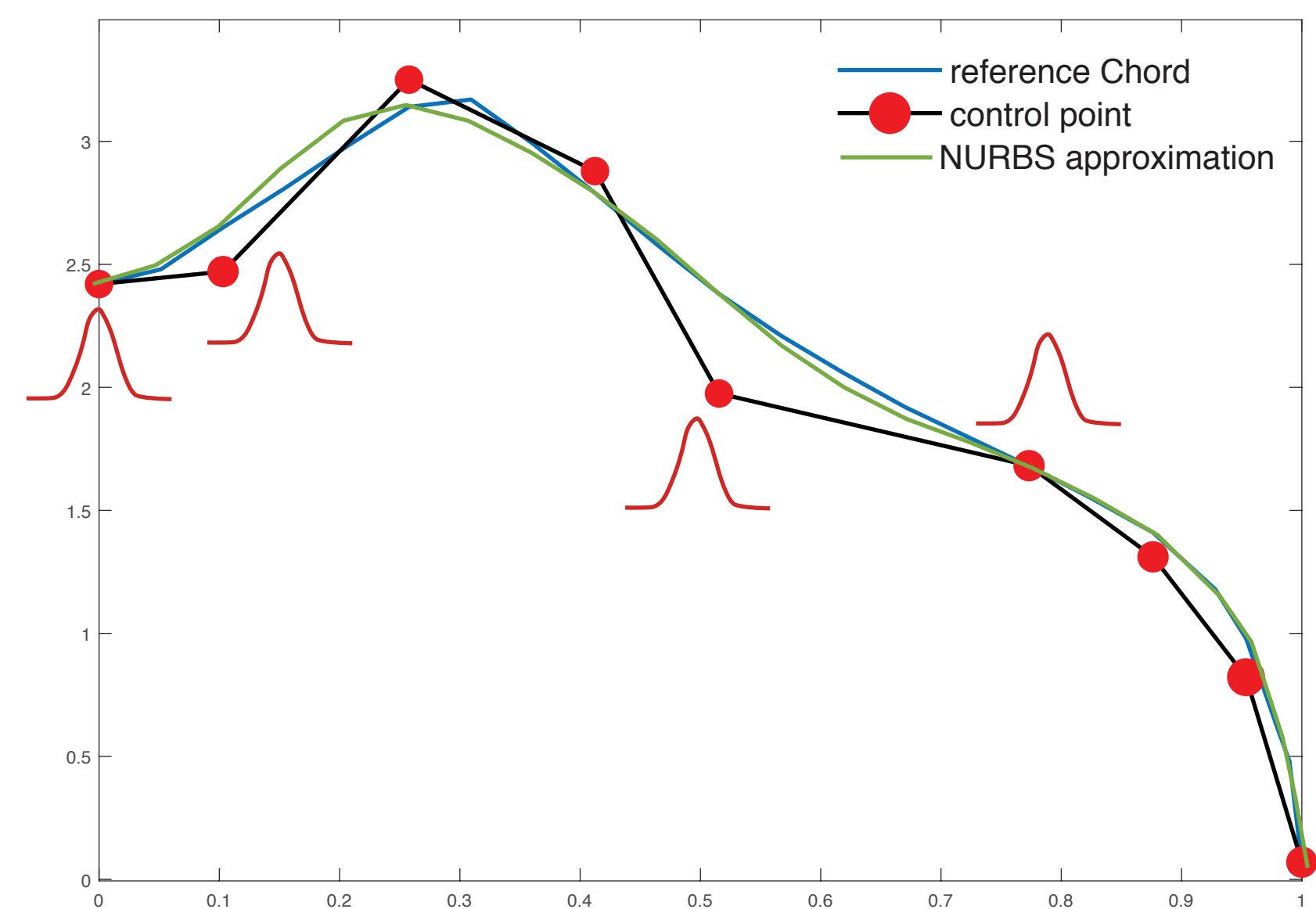
# Parameterization using NURBS

**Step 3:** Perturb control point values using some PDFs

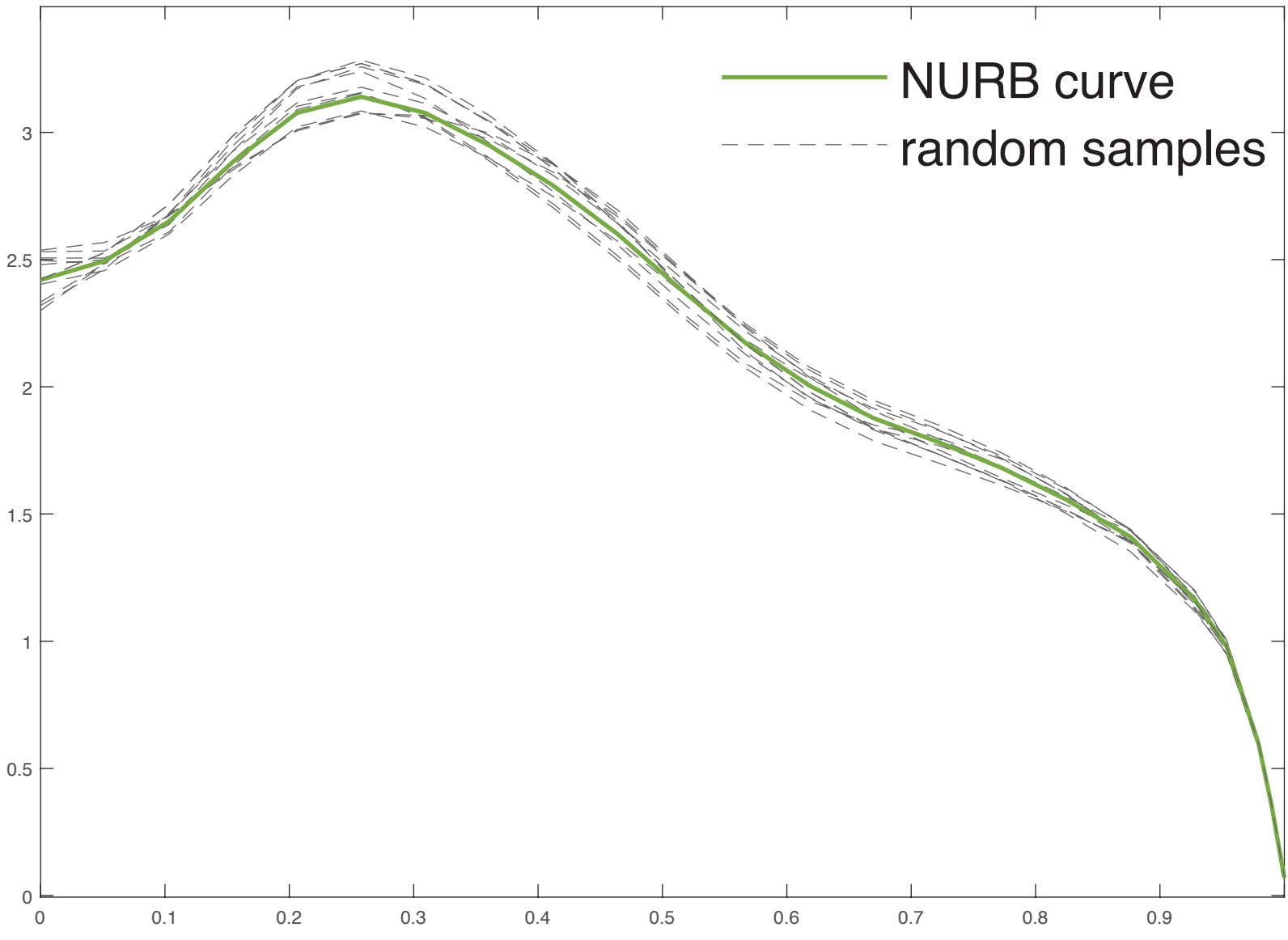


# Parameterization using NURBS

**Step 3:** Perturb control point values using some PDFs



**Step 4:** Sample perturbed curves



# Global sensitivity analysis

# Global sensitivity analysis

- > Goal is to rank the uncertain parameters in the order of importance
- > Global approaches cover the uncertainty spaces more exhaustively than local approaches
- > Better able to capture uncertainty in the model output

# Sobol sensitivity indices

**Main idea:** Decompose the variance of model output in terms of contribution from individual input parameters and their combinations.

$$V(y) = \sum_i V_i + \sum_{i,j} V_{i,j} + \text{higher order terms}$$

First order indices

$$S_1 = \frac{V_1}{V}, S_2 = \frac{V_2}{V}, \dots$$

Second order indices

$$S_{1,2} = \frac{V_{1,2}}{V}, S_{1,3} = \frac{V_{1,3}}{V}, \dots$$



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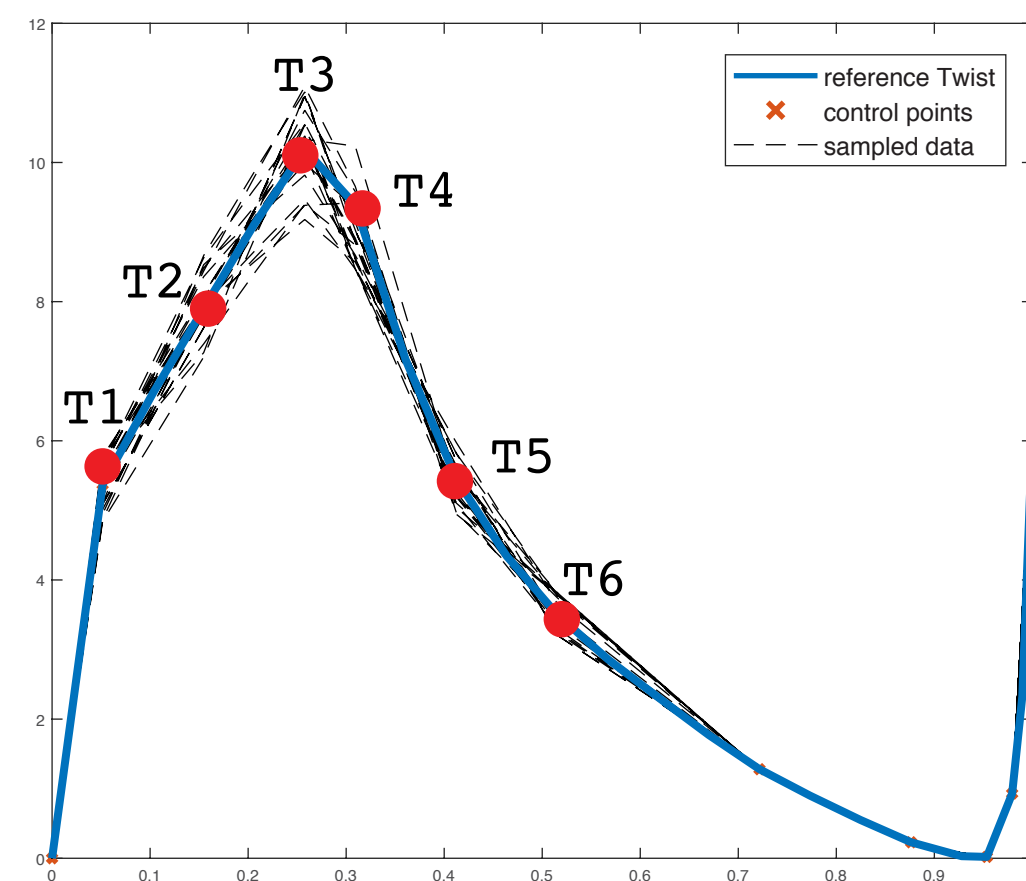
Second order indices

$$S_{1,2} = \frac{V_{1,2}}{V}, S_{1,3} = \frac{V_{1,3}}{V}, \dots$$

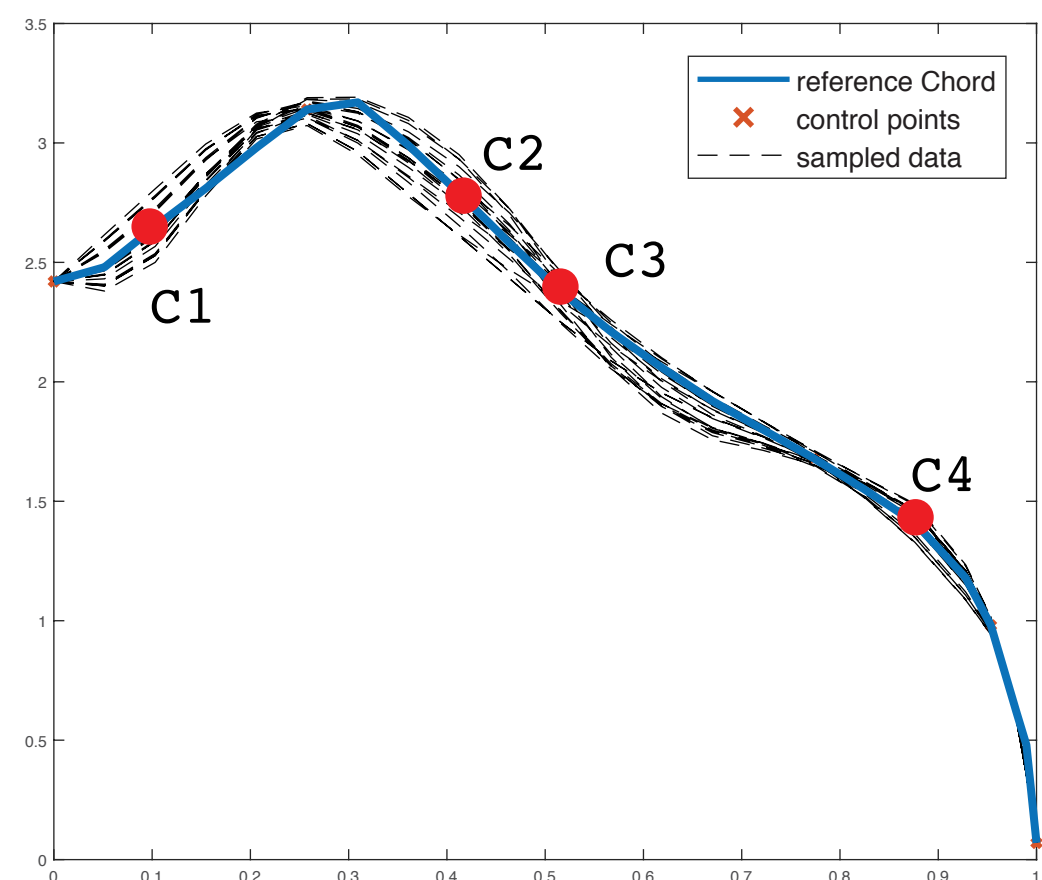
We use **adaptive Polynomial Chaos Expansion** (Least angle regression) to compute variances

# Sensitivity analysis results

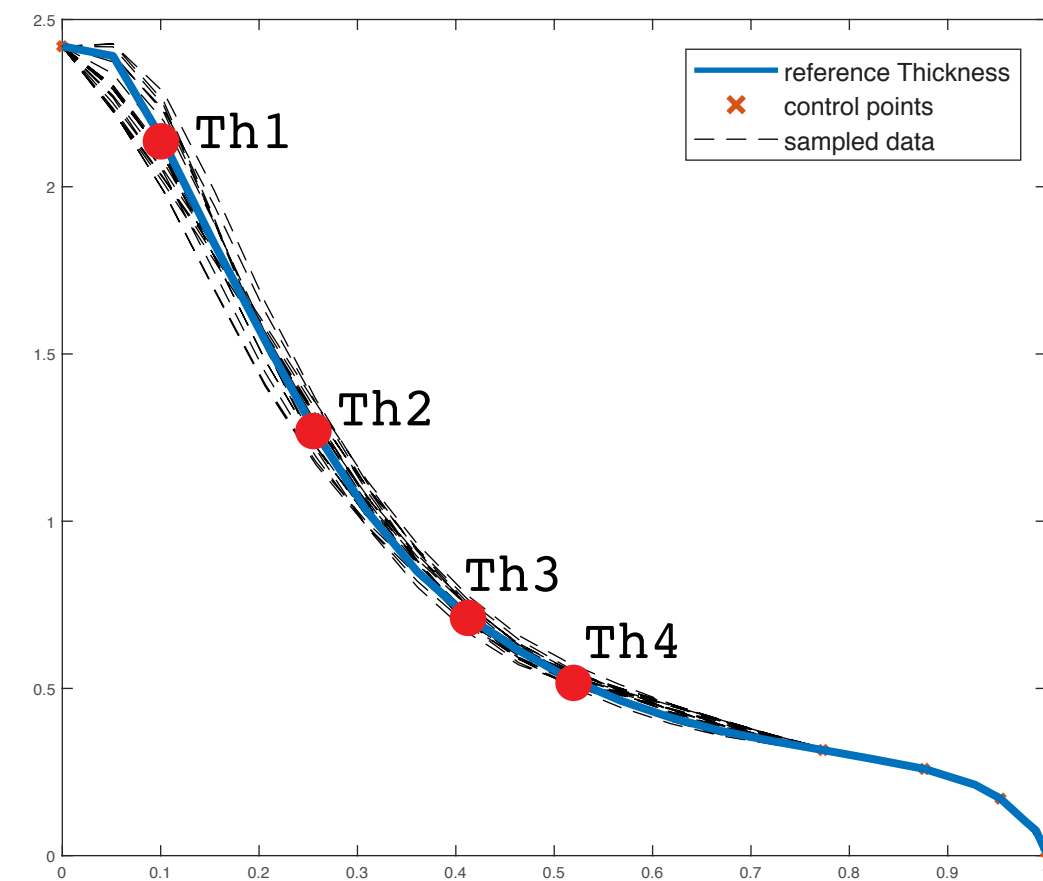
# Geometric Uncertainty



Twist



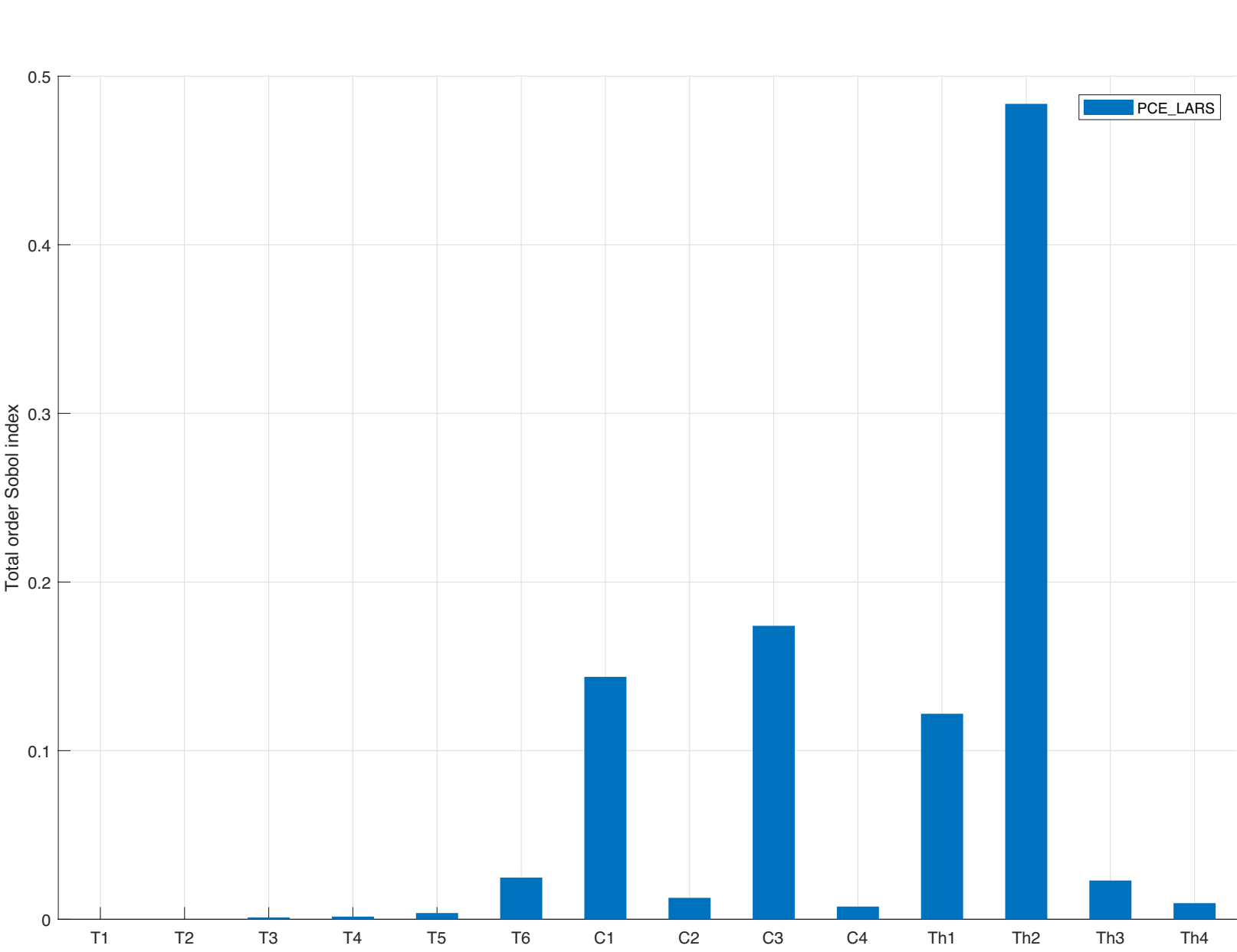
Chord



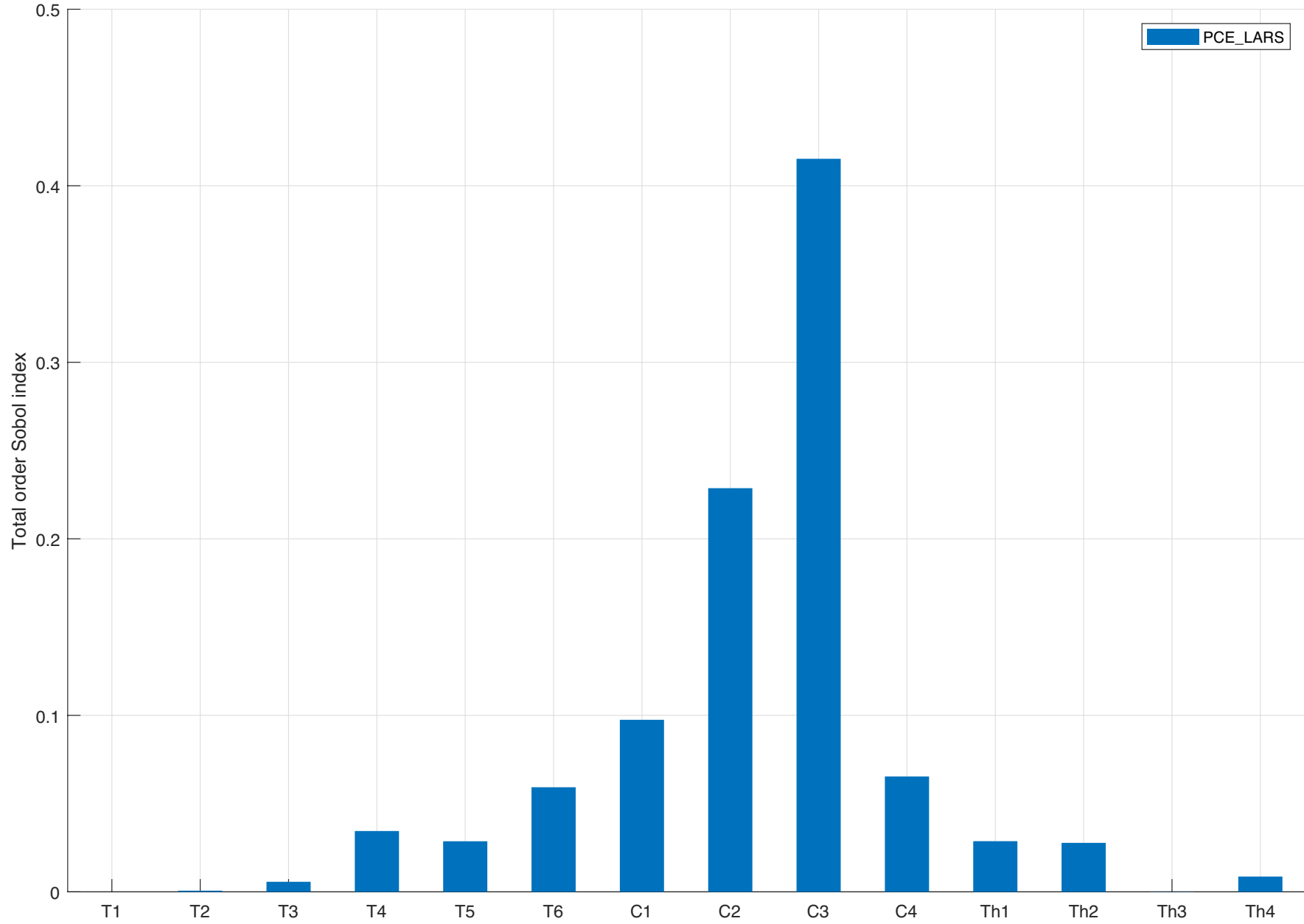
Thickness

10% (uniformly distributed) uncertainty in chosen control point

# Sobol indices



Power



Axial Force

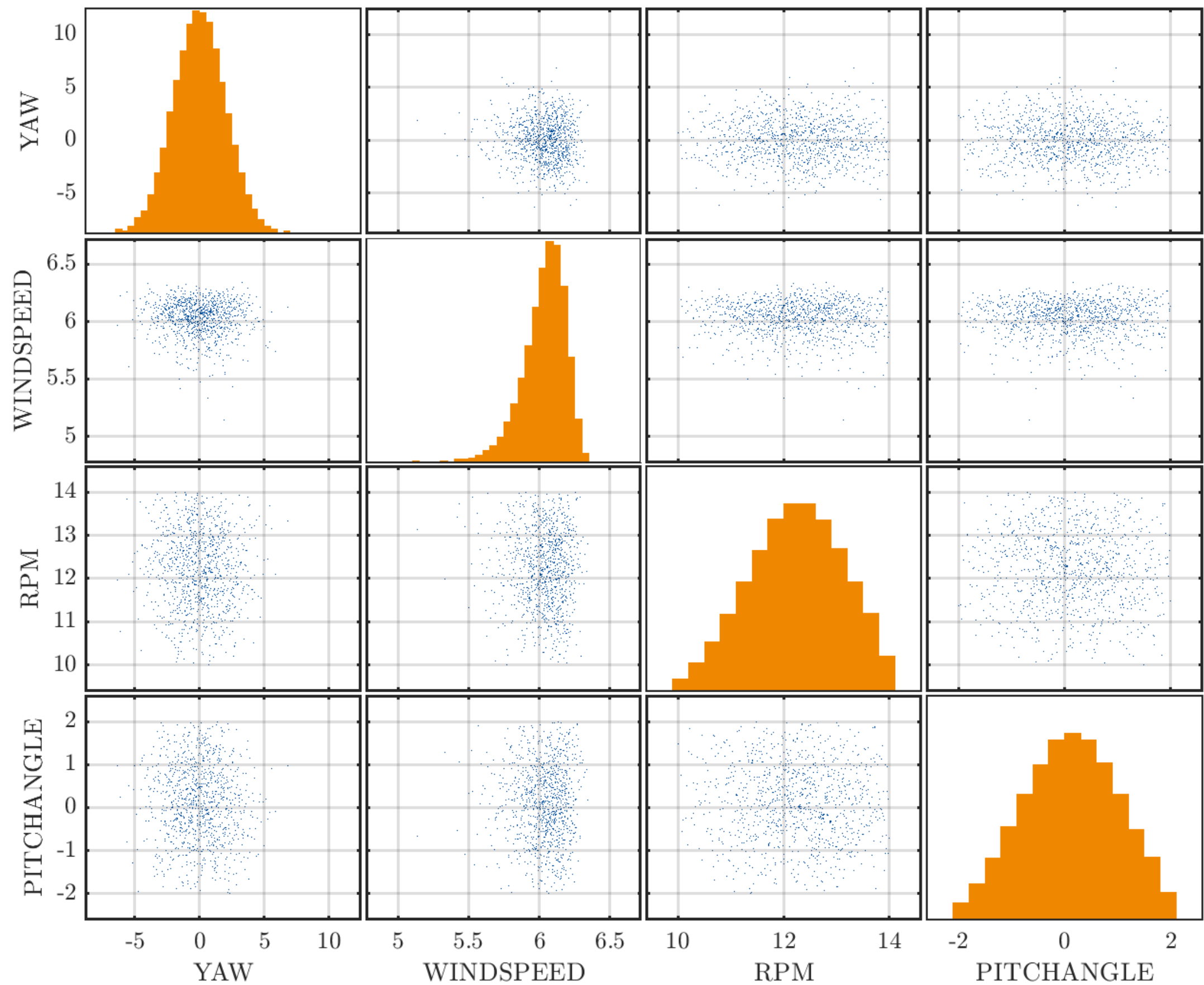
# Operational Uncertainty

**Yaw**  
Truncated Gaussian [mean = 0, std = 2, LB = -10, UB = 10]

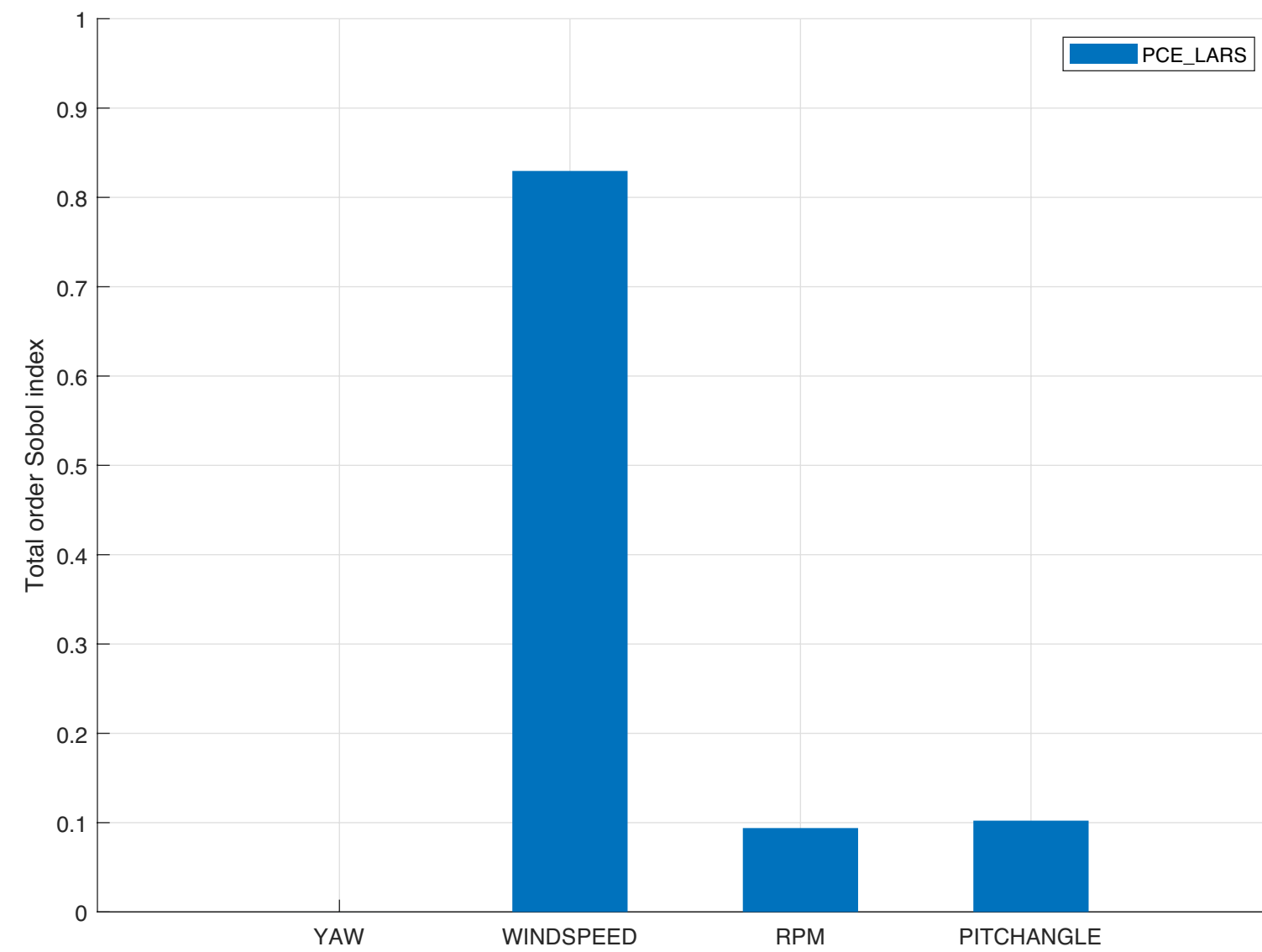
**WindSpeed**  
Weibull distribution [Scale = 6.1, Shape = 50]

**RPM**  
Truncated Gaussian [mean = 12.3, std = 1, LB = 10, UB = 14]

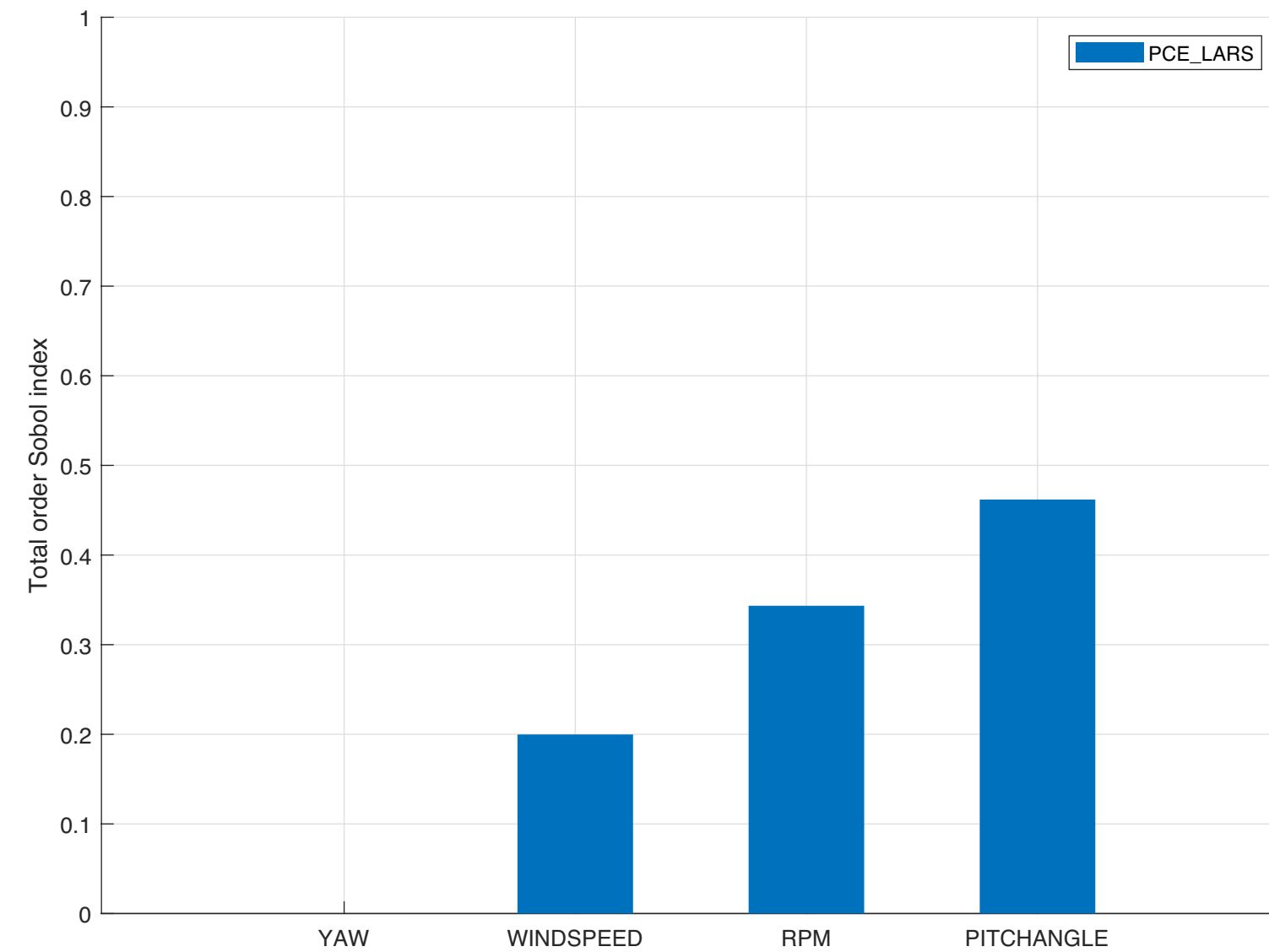
**PitchAngle**  
Truncated Gaussian [mean = 0.15, std = 1, LB = -2, UB = 2]



# Sobol indices



**Power**



**Axial Force**

# Conclusions

- > Global SA is a powerful method to analyze uncertainties in BEM models
- > Chord and Thickness more sensitive compared to Twist
- > Yaw angle is least sensitive parameter compared to WindSpeed, RPM and Pitch

# Next Steps

- > Determine realistic amount of perturbations for uncertain parameters
- > Parameterization for other random inputs, model-form uncertainty
- > Include other BEM codes in the workflow and perform comparisons