

Bayesian calibration applied to DANAERO data

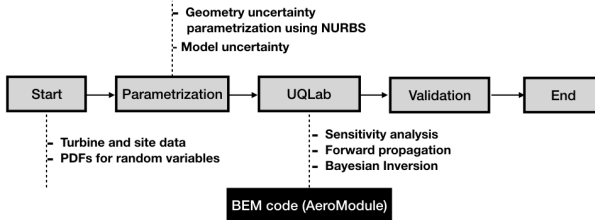
WINDTRUE: WP2

Vinit Dighe & Benjamin Sanderse

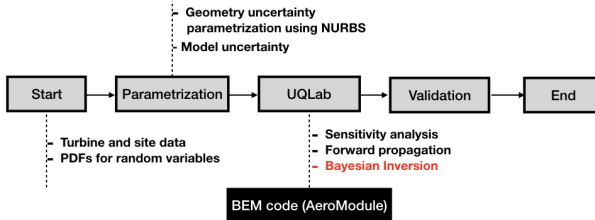
March 16, 2020

CWI

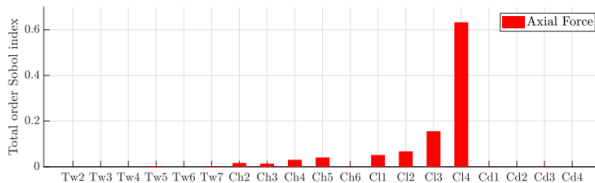
WINDTRUE workflow



WINDTRUE workflow



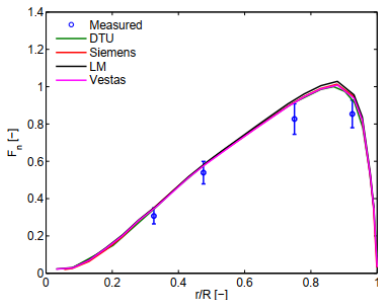
Results from WP1



Kumar et al. (2020)

The results highlight amongst others the importance of the lift coefficient, especially for the axial force prediction.

DANAERO test case

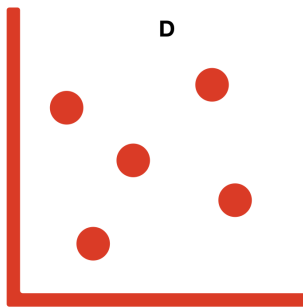


- D: Comparison of measurements and BEM computations of axial force in non-sheared inflow
- Forward model:

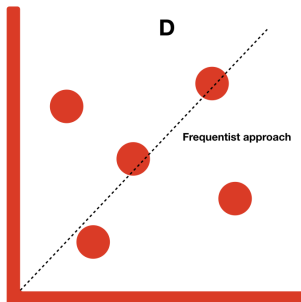
$$F_n^* = 0.5\rho W^2 c(C_L \sin\beta + C_D \cos\beta)$$

- $\theta = [C_{L1}, C_{D1}, C_{L2}, C_{D2}, C_{L3}, C_{D3}, C_{L4}, C_{D4}]$

Inferential statistics

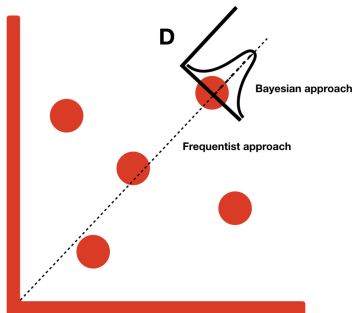


Inferential statistics



- Frequentist inference: $\mathbb{E}(\theta)$

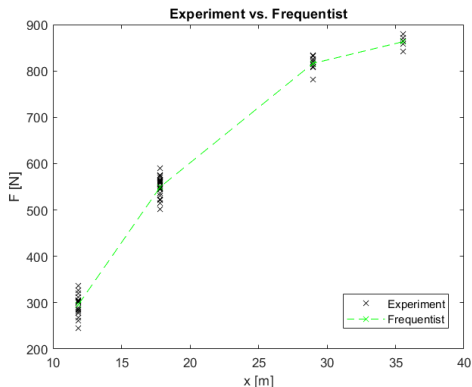
Inferential statistics



- Frequentist inference: $\mathbb{E}(\theta)$
- Bayesian inference:

$$P(\theta|\mathbf{D}) = \frac{P(\mathbf{D}|\theta) \times P(\theta)}{P(\mathbf{D})}$$

Frequentist inference

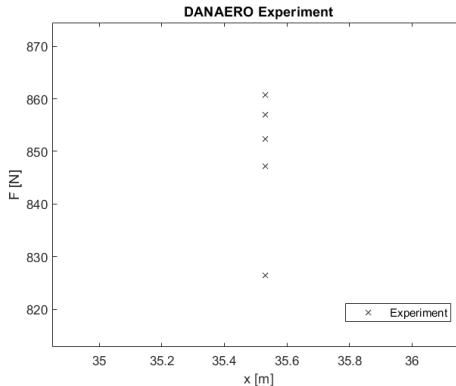


- Frequentist inference (Ordinary least-squares regression):

$$\theta_{OLS} = (X^T X)^{-1} X^T D,$$

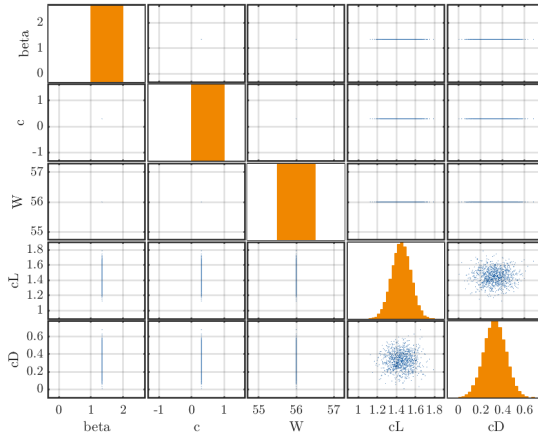
$X = X_{ij}$ is the design matrix

Bayesian inference

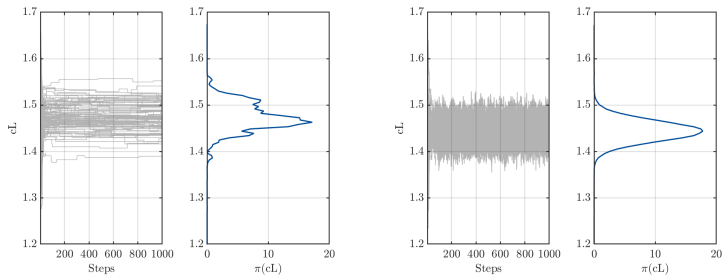


- ▶ Radial location: 35.53 m
- ▶ Number of data points (N) = 5 (Remember $N > \theta$)

Bayesian inference - Prior distribution $\frac{P(\mathbf{D}|\theta) \times P(\theta)}{P(\mathbf{D})}$

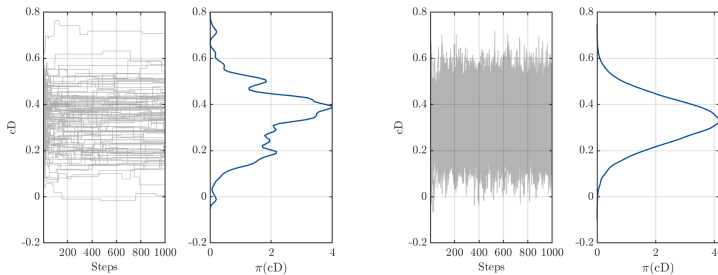


Bayesian inference - Posterior distribution $P(\theta|\mathbf{D})$



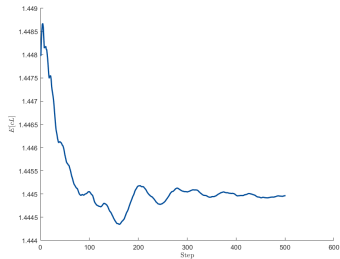
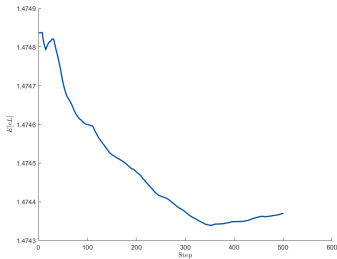
Figures showing trace plots for C_L using Metropolis-Hastings (left) and Affine-invariant ensemble sampling (right) algorithms.

Bayesian inference - Posterior distribution $P(\theta|\mathbf{D})$



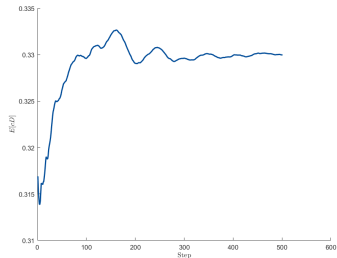
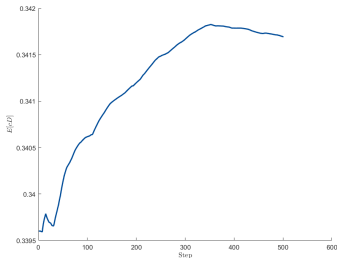
Figures showing trace plots for C_D using Metropolis-Hastings (left) and Affine-invariant ensemble sampling (right) algorithms.

Bayesian inference - Assessing convergence



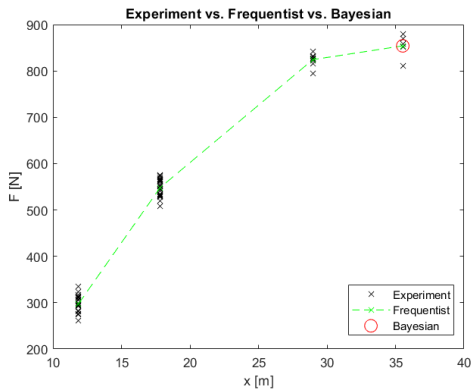
Figures showing convergence plots for C_L using Metropolis-Hastings (left) and Affine-invariant ensemble sampling (right) algorithms.

Bayesian inference - Assessing convergence



Figures showing convergence plots for C_D using Metropolis-Hastings (left) and Affine-invariant ensemble sampling (right) algorithms.

Point estimate for C_{L4} and C_{D4}



	C_{L4}	C_{D4}
Frequentist: $\mathbb{E}(\theta)$	1.4821	0.3421
Bayesian: MAP	1.4828	0.3391

Concluding remarks

- ▶ Bayesian calibration framework using UQLab
 - ▶ 4 different MCMC algorithms
 - ▶ Gelman-Rubin diagnostics: Assess convergence
 - ▶ Trace plots: Evolution of Markov chain
 - ▶ Acceptance rate: Quantitative indication of accepted samples
- ▶ DANAERO test case

Next steps

- ▶ Experimental data (D)
- ▶ Choose relevant calibration parameters (θ)
- ▶ BEM code for the forward model
- ▶ Choosing prior distribution for Bayesian calibration (Good engineering estimate)