

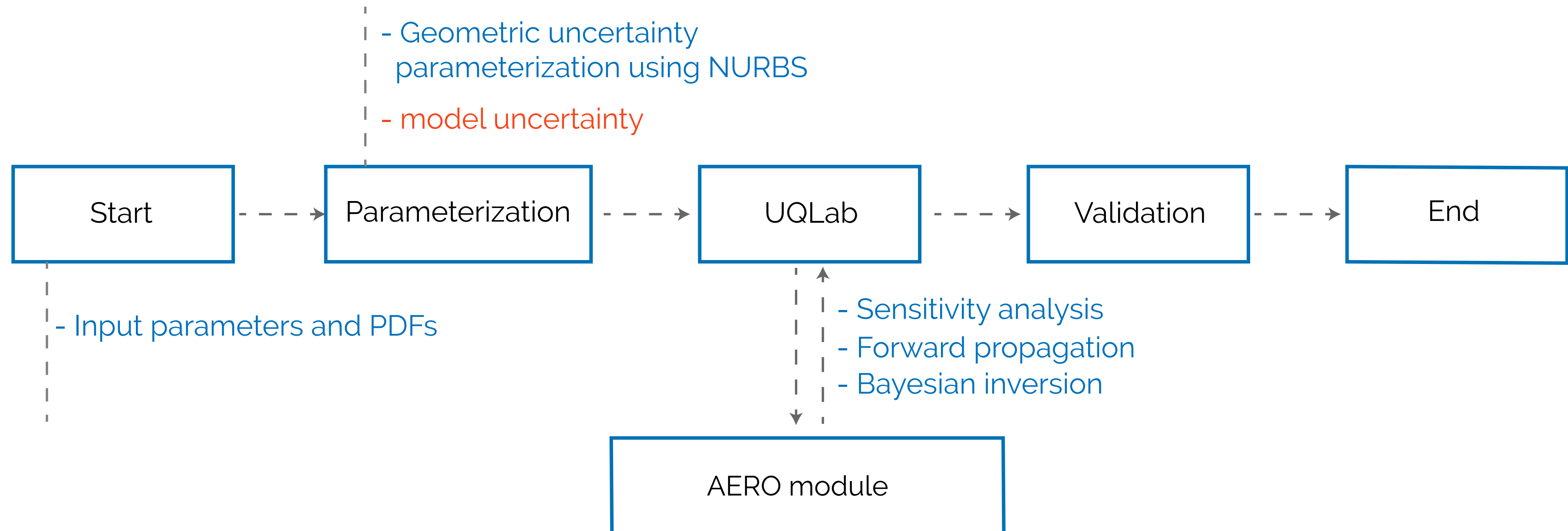
# WindTrue: Sensitivity analysis applied to DANAERO wind turbine

Prashant Kumar  
Benjamin Sanderse

The logo for the Centrum voor Wiskunde en Informatica (CWI) is located in the bottom right corner. It consists of a red parallelogram with the white text "CWI" inside.

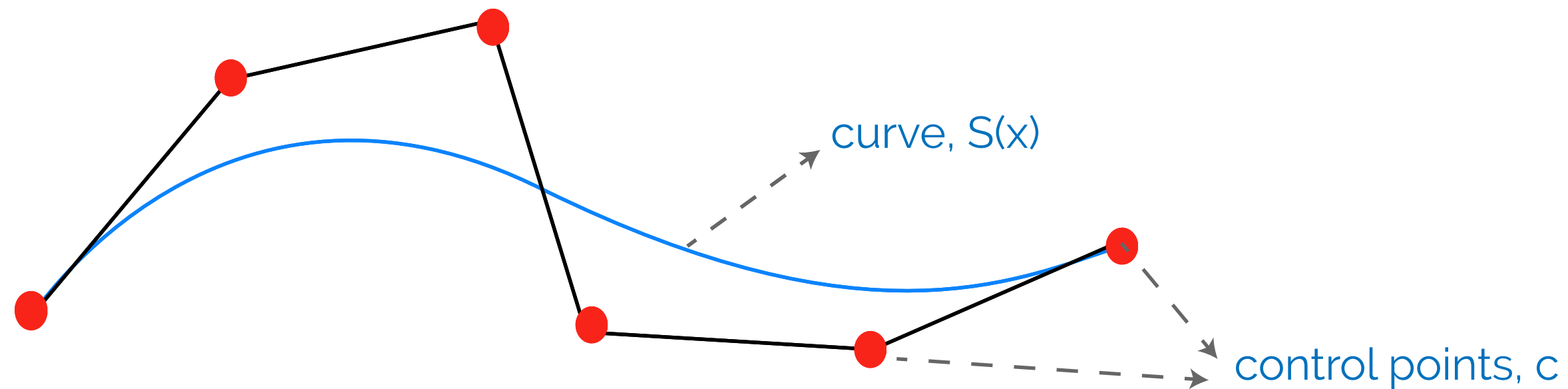
CWI

# Workflow



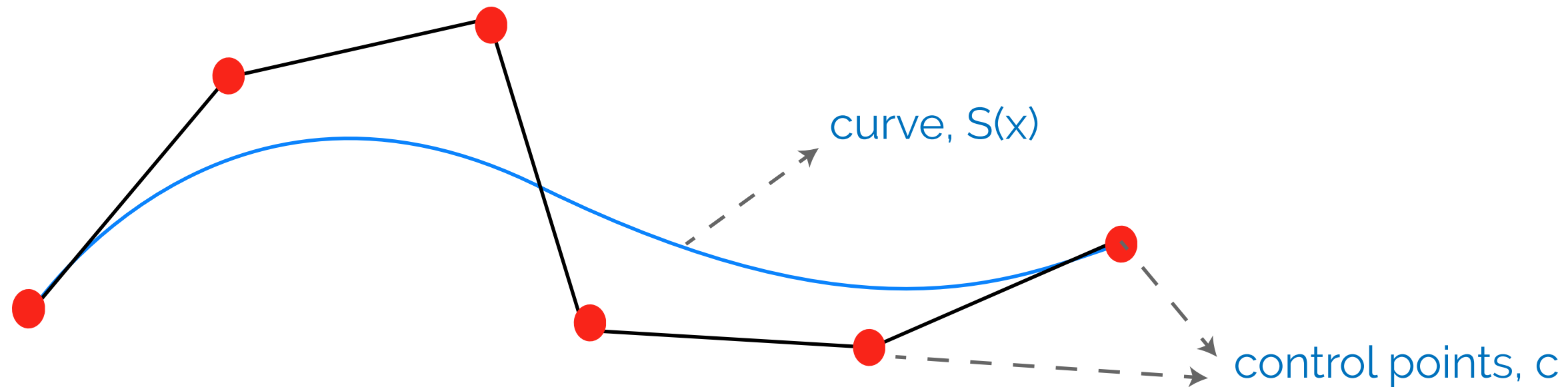
# NURBS based parametrization

# Non-Uniform Rational Basis Spline (NURBS)



$$S(x) = \sum_{i=0}^{N-1} c_i B_{i,p}(x)$$

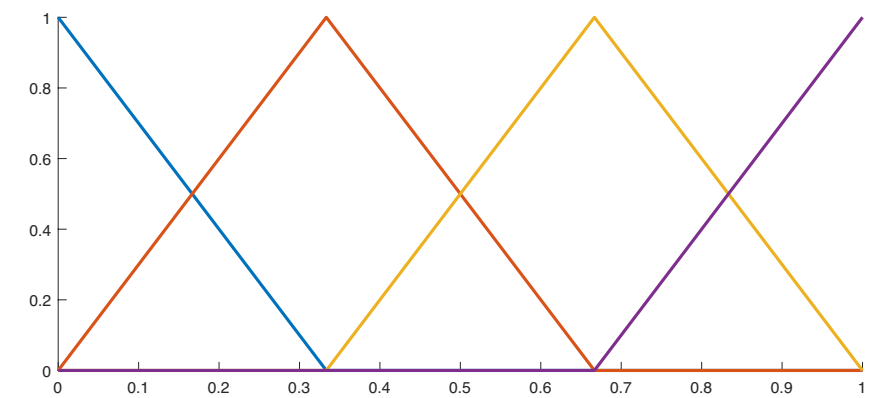
# Non-Uniform Rational Basis Spline (NURBS)



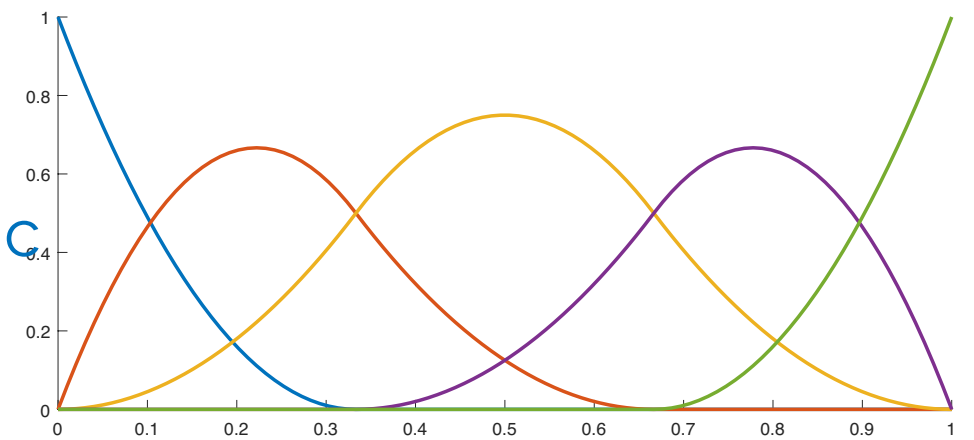
$$S(x) = \sum_{i=0}^{N-1} c_i B_{i,p}(x)$$

Basis functions,  $B_{i,p}(x)$

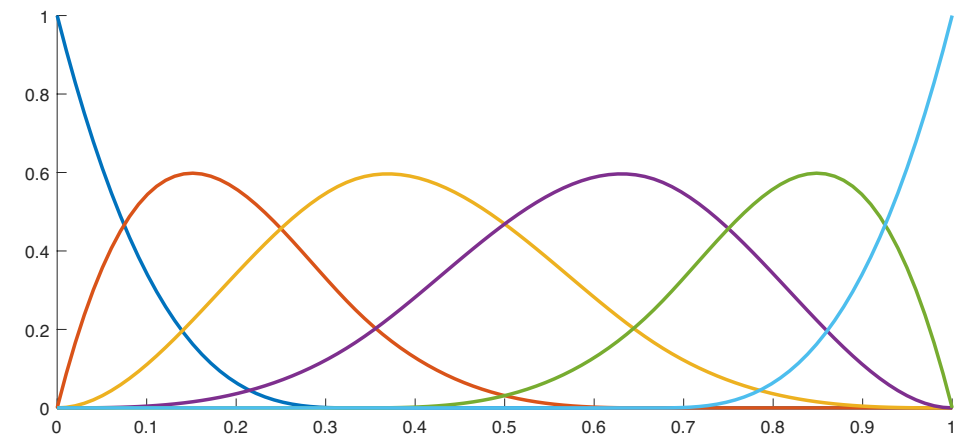
linear



quadratic



cubic



$x$

# Why NURBS?

---

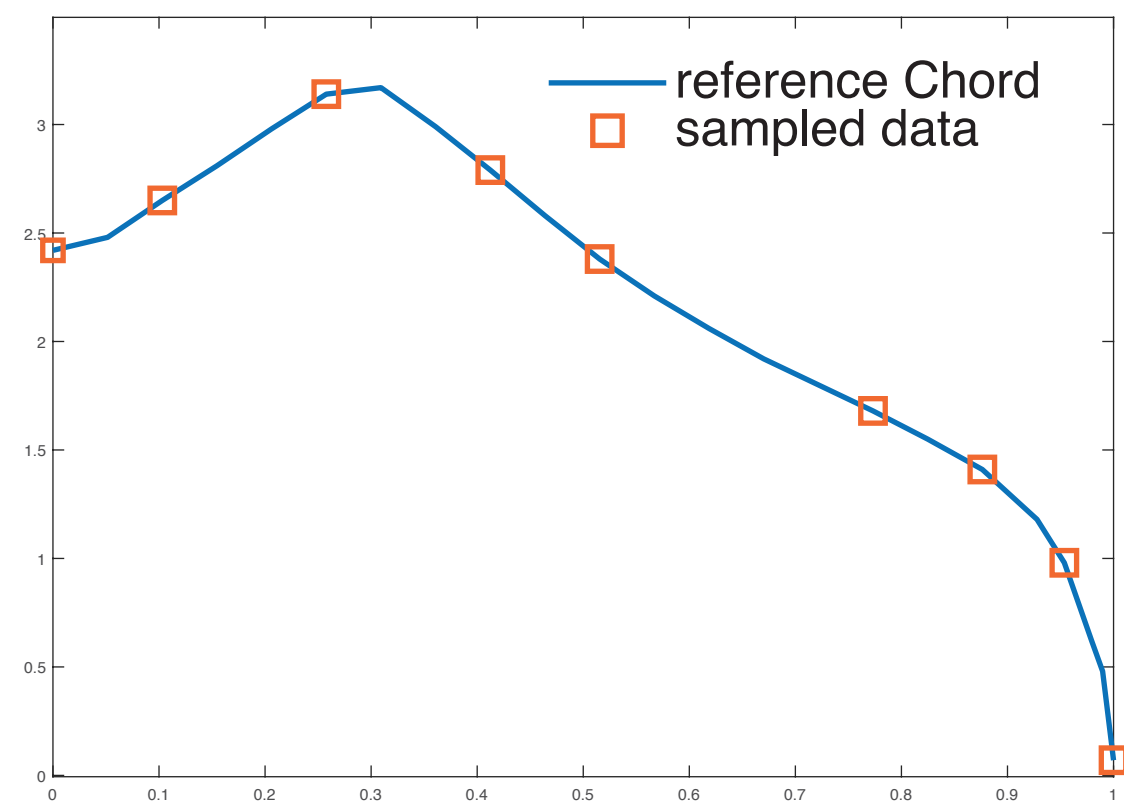
- > Represent complex shapes with very few points
- > Flexibility to design a large variety of shapes
- > Easy to obtain high-order polynomials

# Parameterization using NURBS

**Goal:** Obtain perturbed chord/twist from a given reference curves

---

**Step 1:** Sample locations from the reference curve



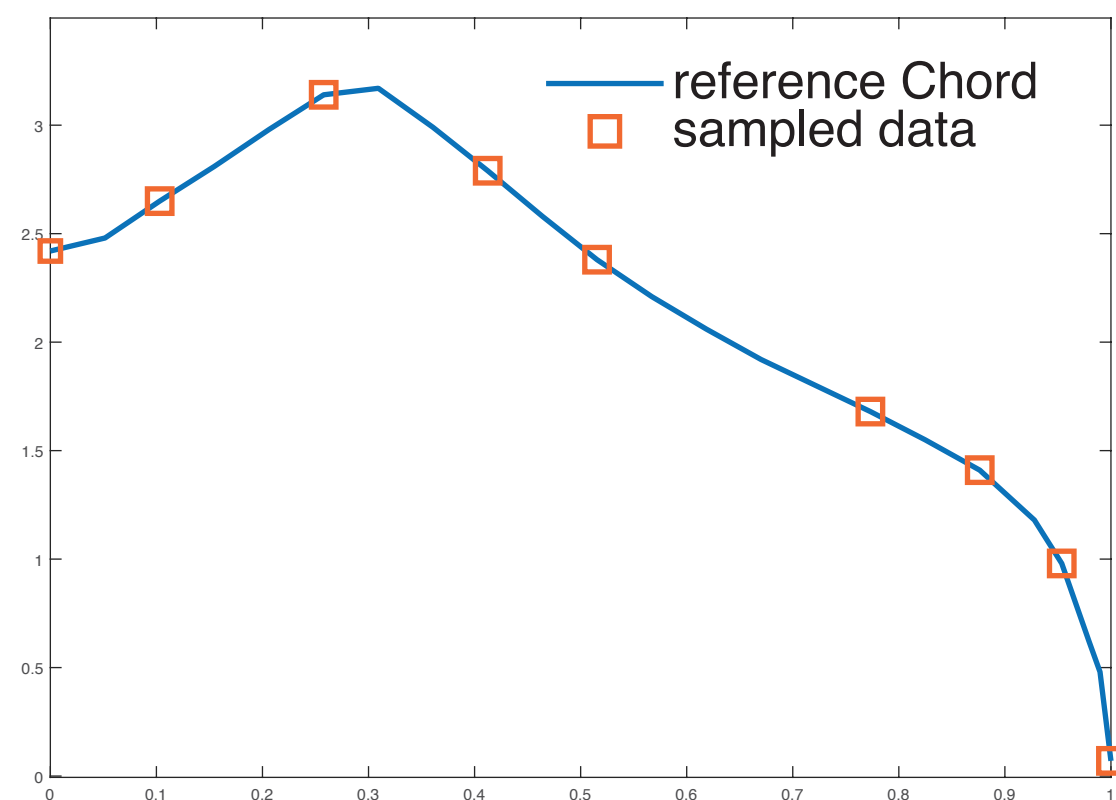
# Parameterization using NURBS

**Goal:** Obtain perturbed chord/twist from a given reference curves

**Step 1:** Sample locations from the reference curve

**Step 2:** Compute control points at sampled location via inversion

$$\begin{array}{ccc} S(x) = \sum_{i=0}^{N-1} c_i B_{i,p}(x) & \implies & \mathbf{B}\mathbf{c} = \mathbf{S} \\ \downarrow & & \downarrow \\ \text{known} & & \text{known} \end{array}$$



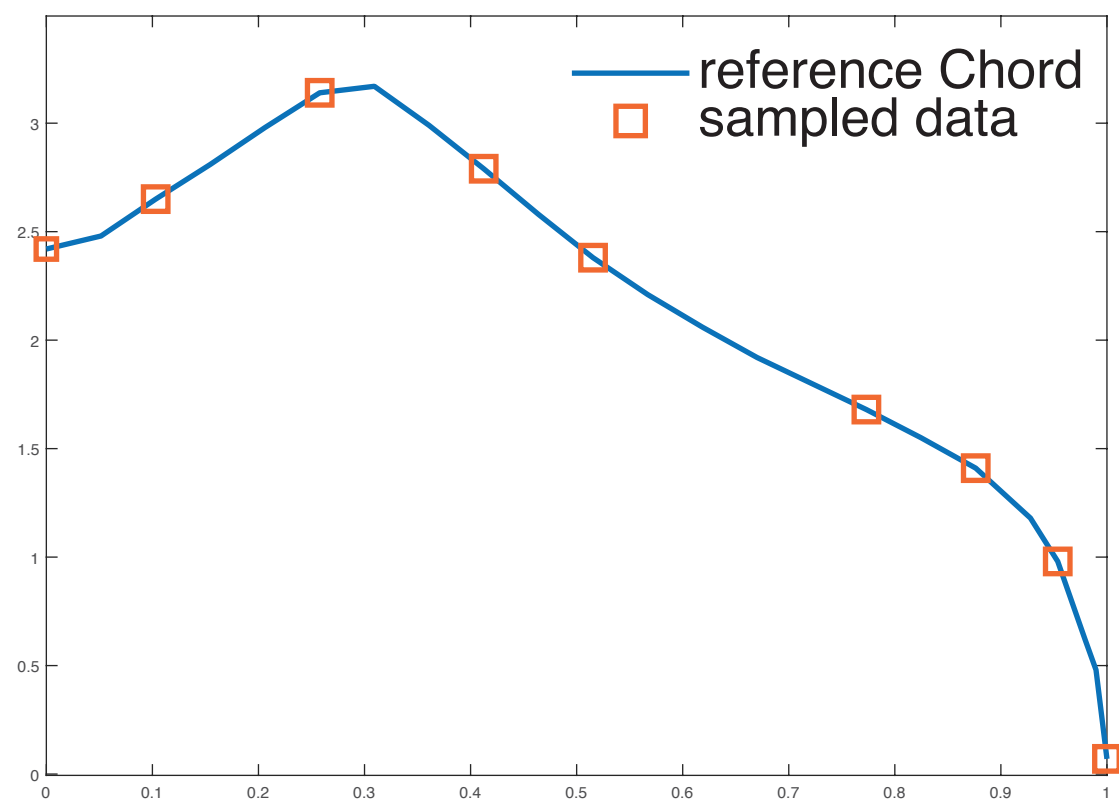
----->



# Parameterization using NURBS

**Goal:** Obtain perturbed chord/twist from a given reference curves

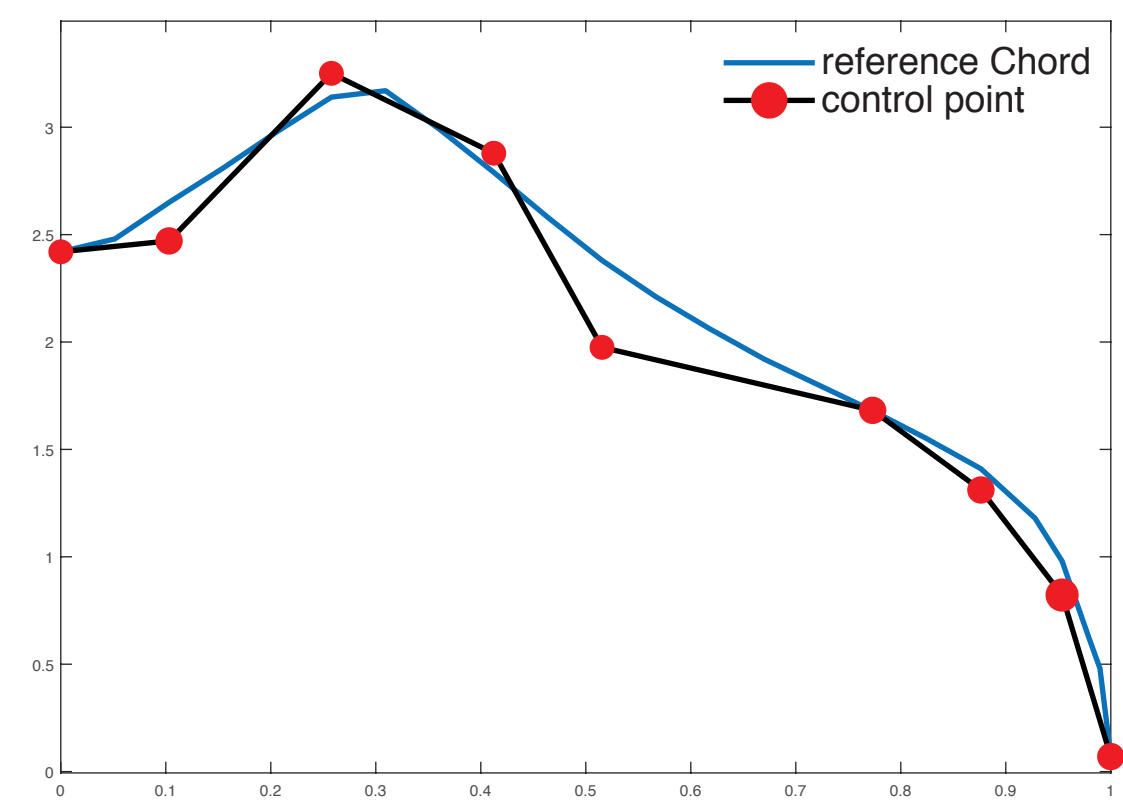
**Step 1:** Sample locations from the reference curve



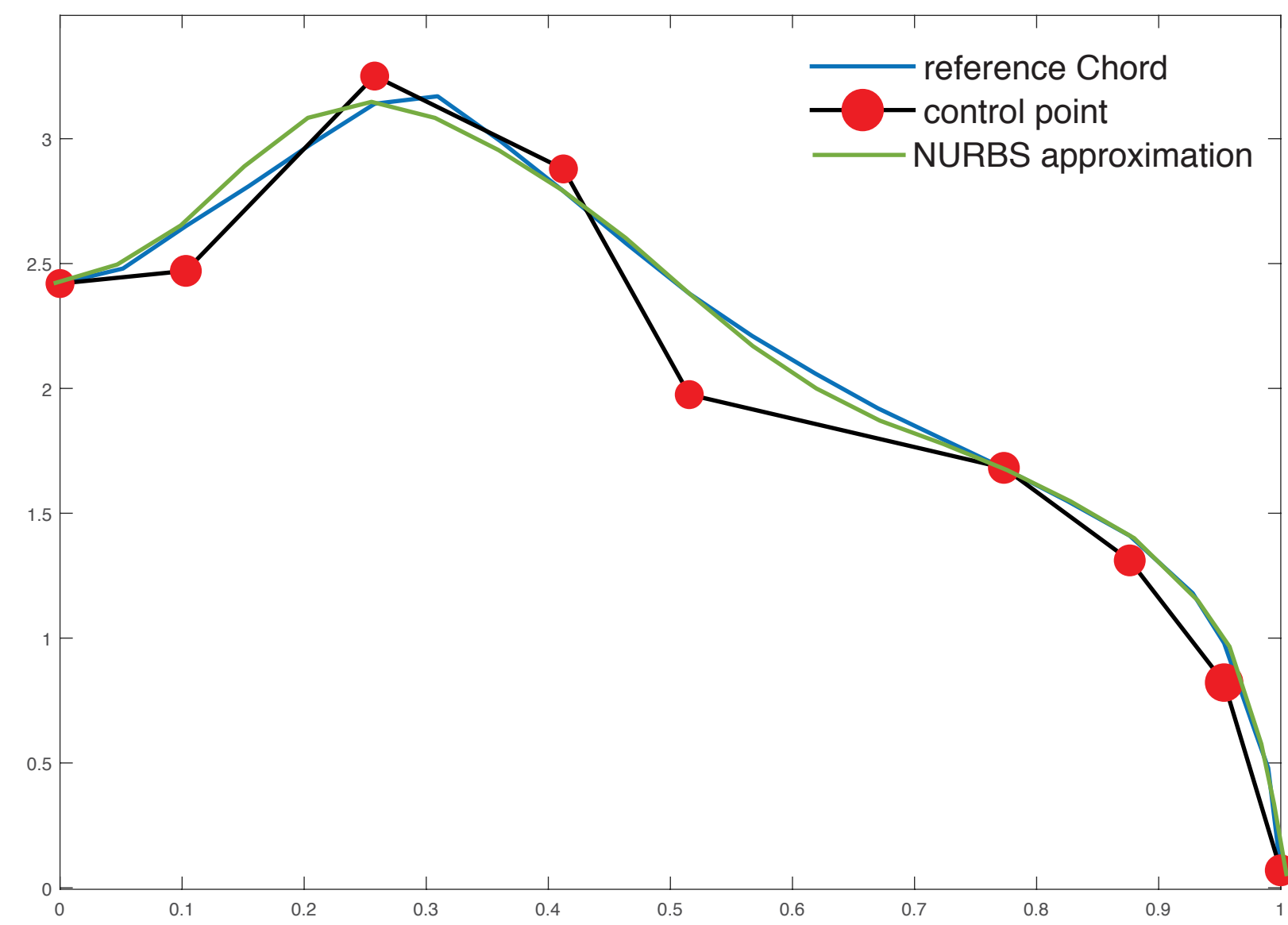
**Step 2:** Compute control points at sampled location via inversion

$$S(x) = \sum_{i=0}^{N-1} c_i B_{i,p}(x) \implies \mathbf{B}\mathbf{c} = \mathbf{S}$$

known                      known

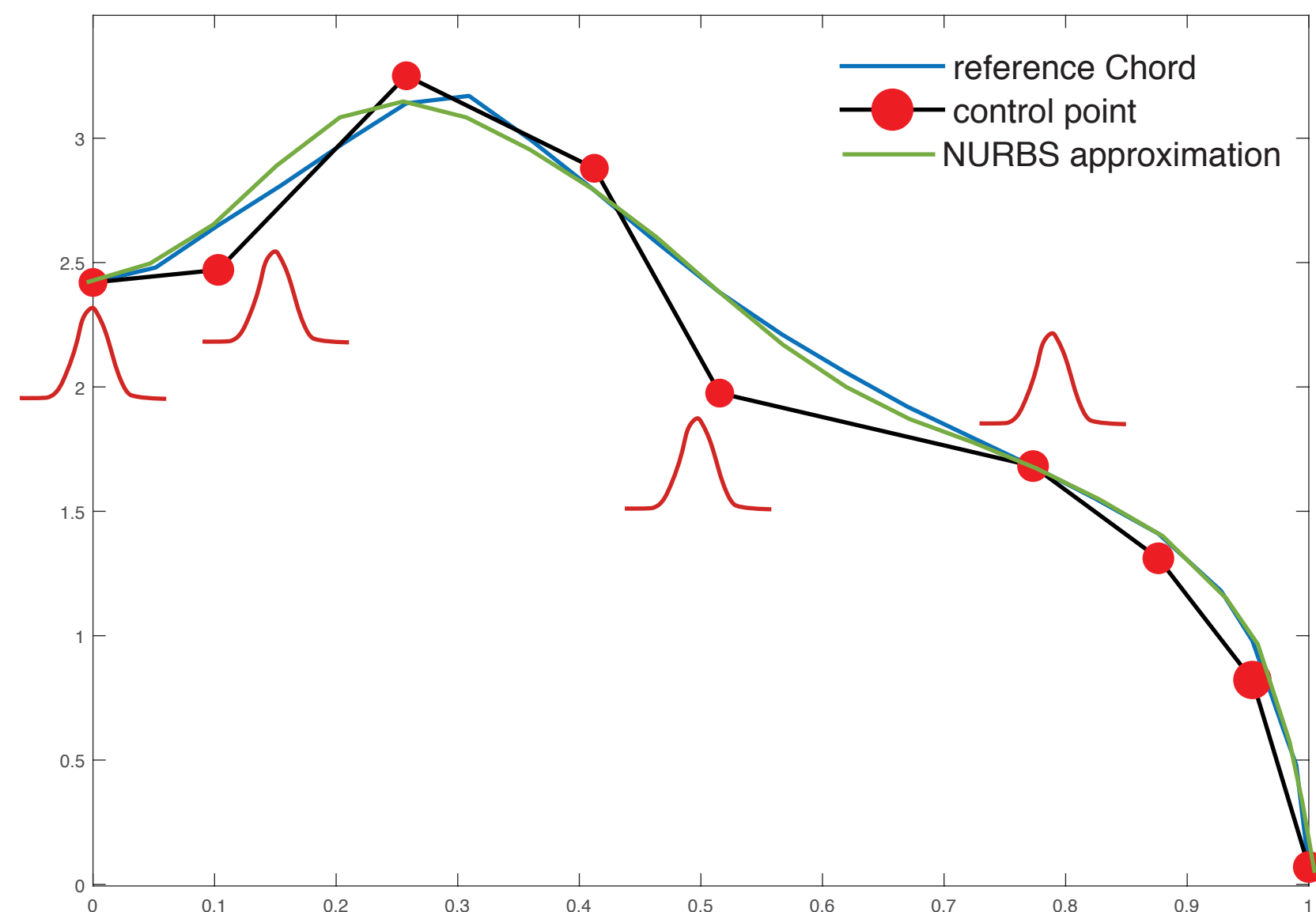


# Parameterization using NURBS



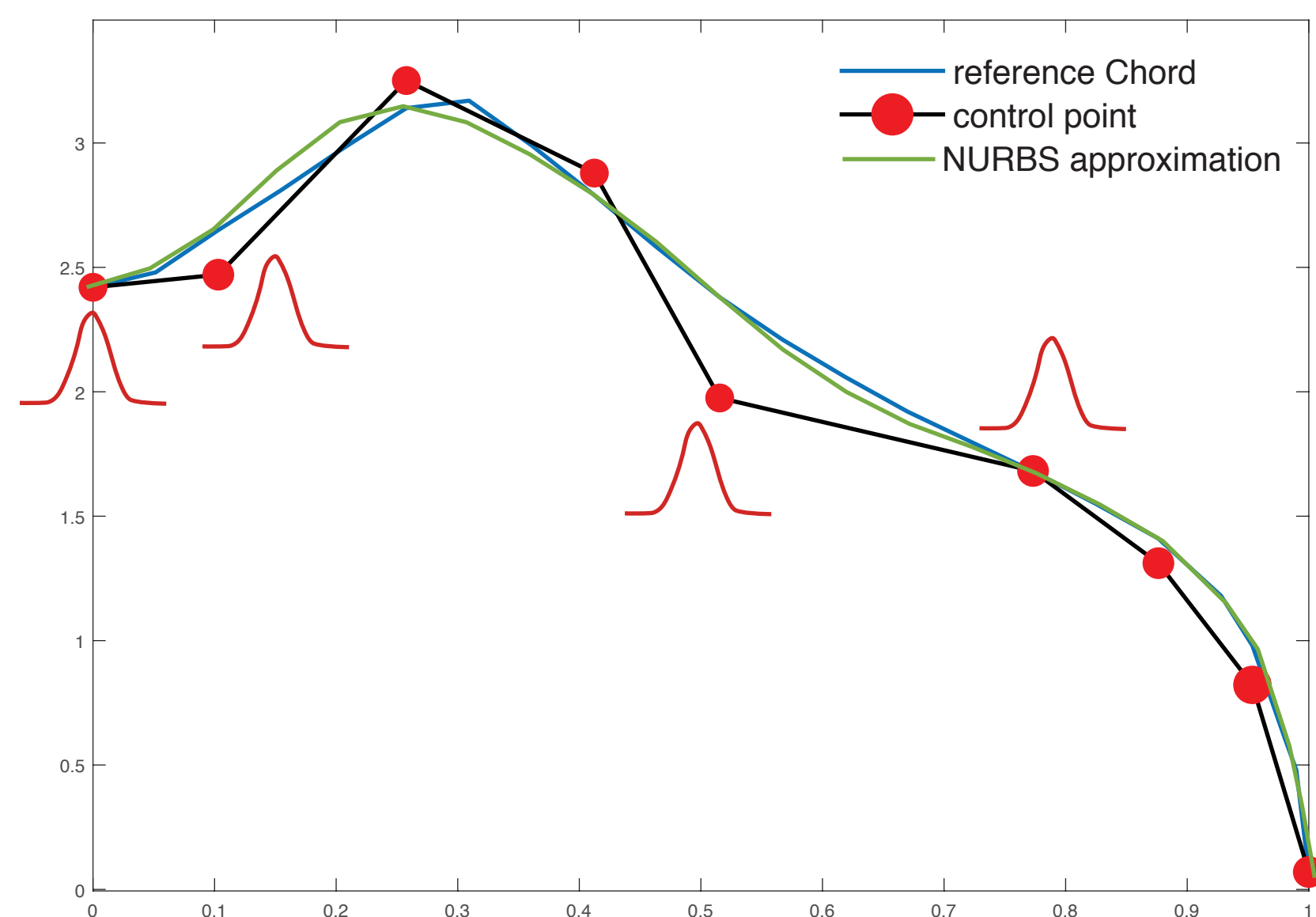
# Parameterization using NURBS

**Step 3:** Perturb control point values using some PDF

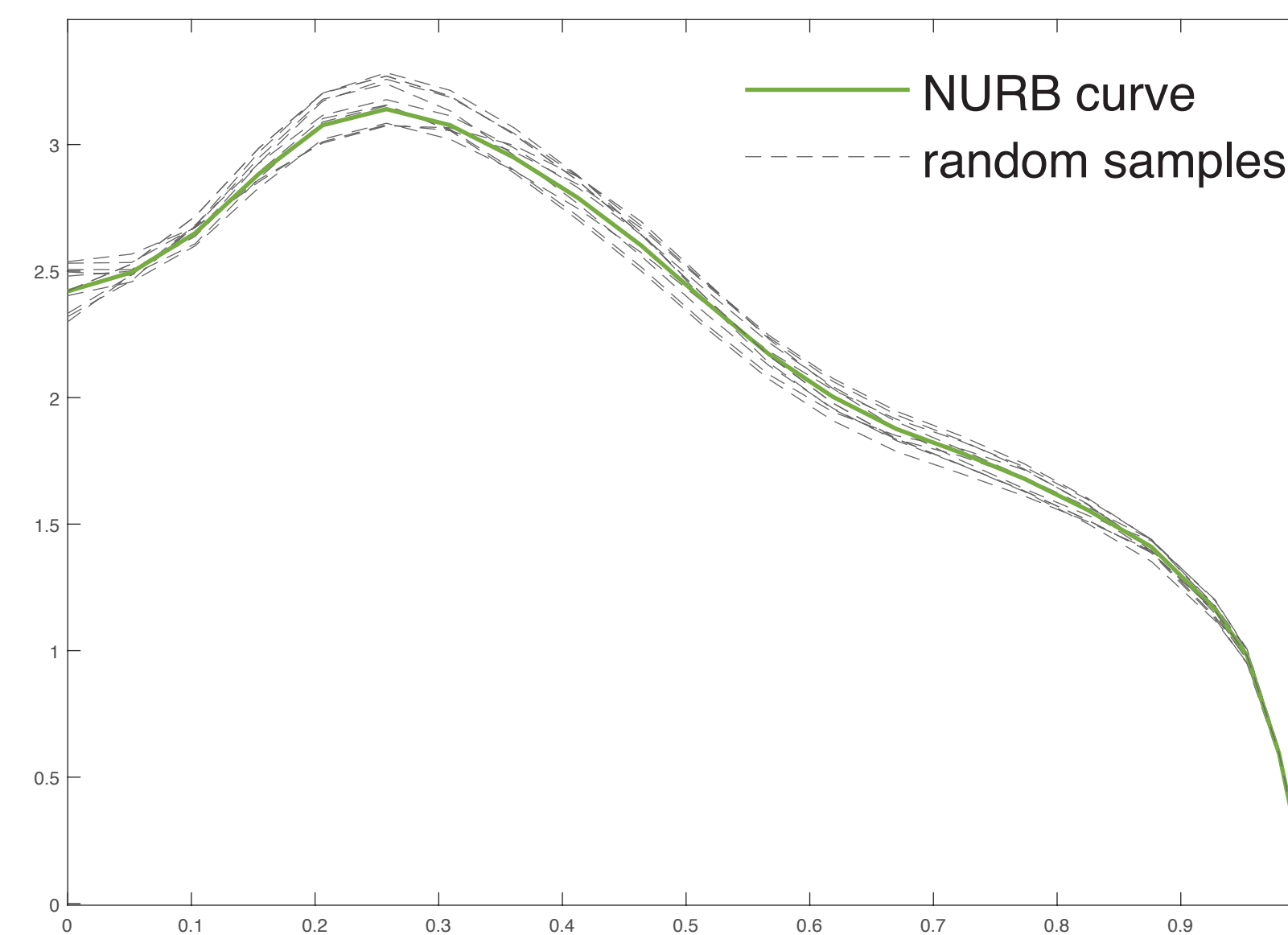


# Parameterization using NURBS

**Step 3:** Perturb control point values using some PDF



**Step 4:** Sample perturbed curves





# Global sensitivity analysis

# Global sensitivity analysis

- > Goal is to rank the uncertain parameters in the order of importance
- > Global approaches cover the uncertainty spaces more exhaustively
- > Better able to capture uncertainty in the model output

# Sobol sensitivity indices

**Main idea:** Decompose the variance of model output in terms of contribution from individual input parameters and their combinations.

$$V(y) = \sum_i V_i + \sum_{i,j} V_{i,j} + \text{higher order terms}$$

First order indices

$$S_1 = \frac{V_1}{V}, S_2 = \frac{V_2}{V}, \dots$$

Second order indices

$$S_{1,2} = \frac{V_{1,2}}{V}, S_{1,3} = \frac{V_{1,3}}{V}, \dots$$

# Polynomial Chaos Expansion (PCE)

To compute individual variances, we use PCE based methods

Three variants of PCE:

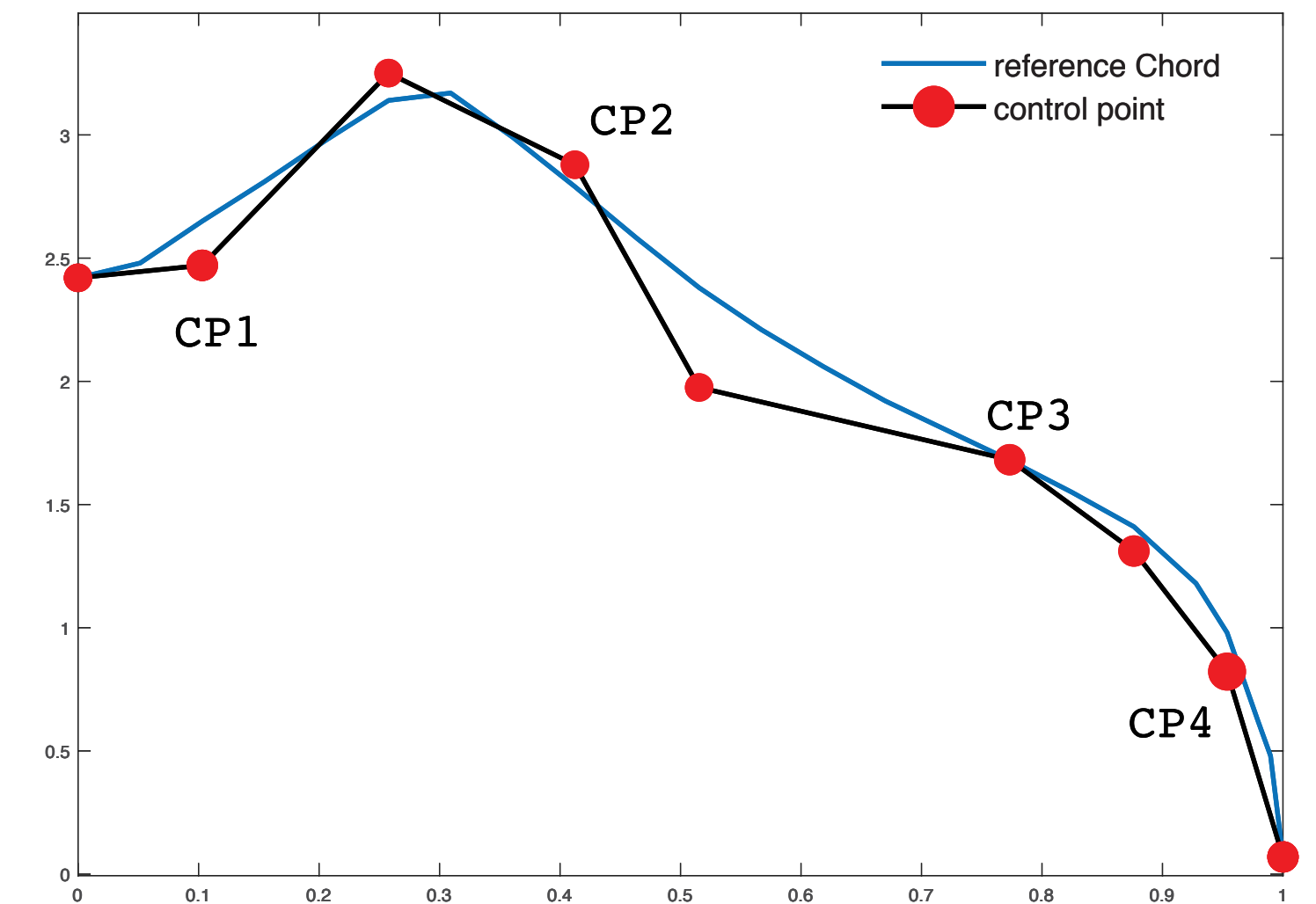
- 1) Quadrature (PCE\_QUAD)  $\Rightarrow$  Deterministic sampling, fixed in degree
- 2) Ordinary Least square (OLS)  $\Rightarrow$  Random sampling, adaptive in degree
- 3) Least Angle Regression (LAR)  $\Rightarrow$  Random sampling, adaptive in degree



# Sensitivity analysis results

**Output quantity:** Average power output of turbine

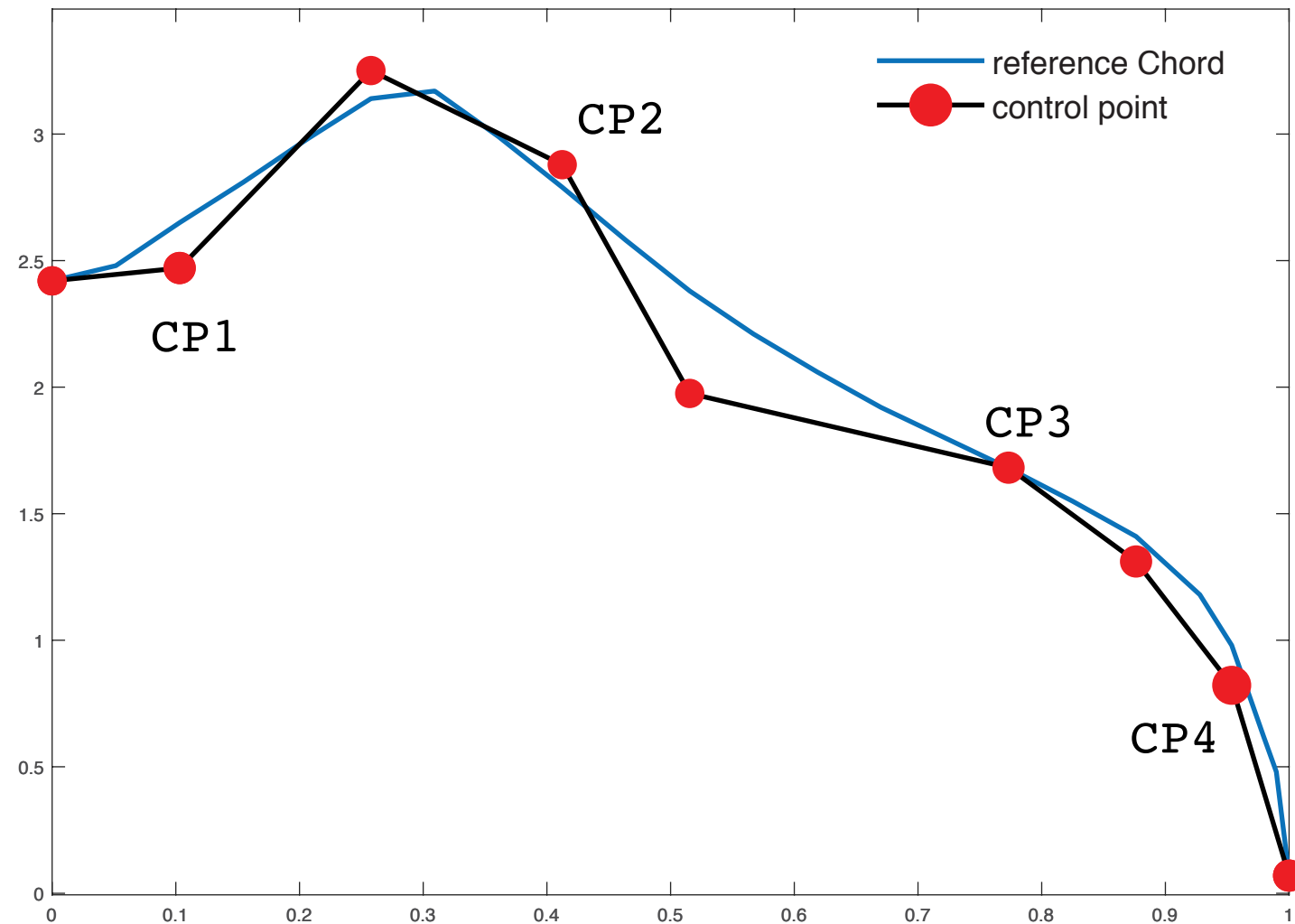
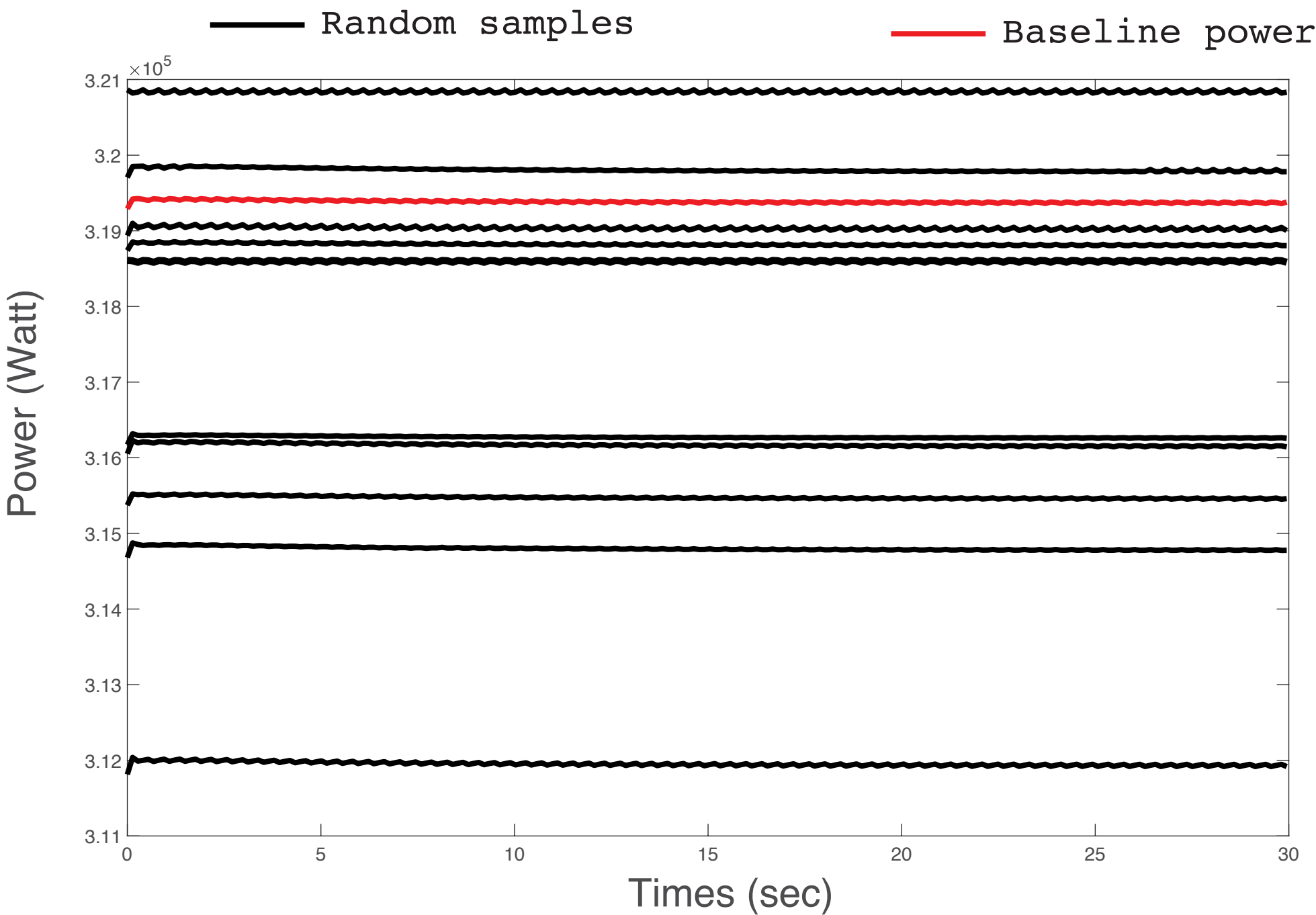
**Uncertainty:** Introduce 10% (uniform) uncertainty in 4 control points for Chord



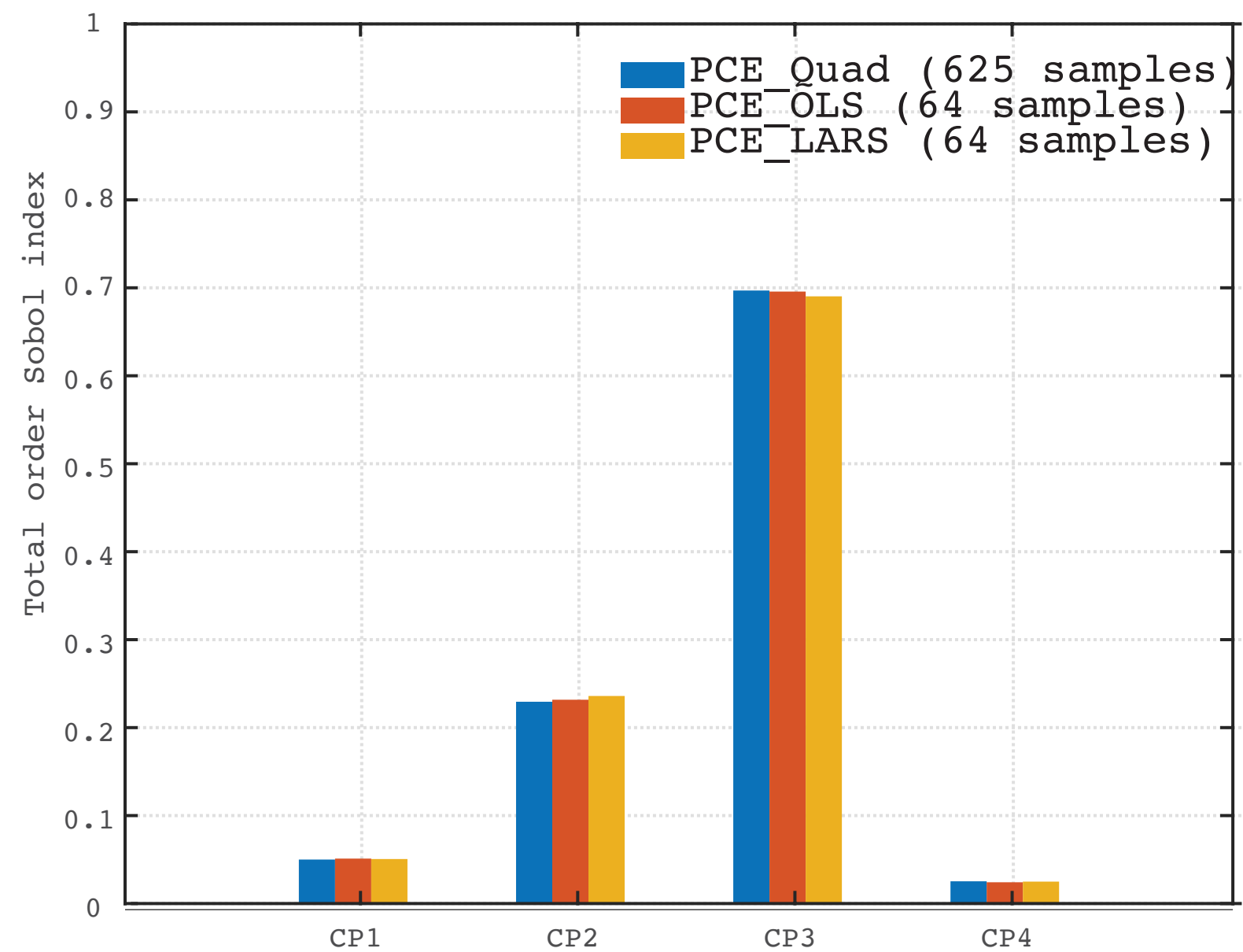
# Sensitivity analysis results

**Output quantity:** Average power output of turbine

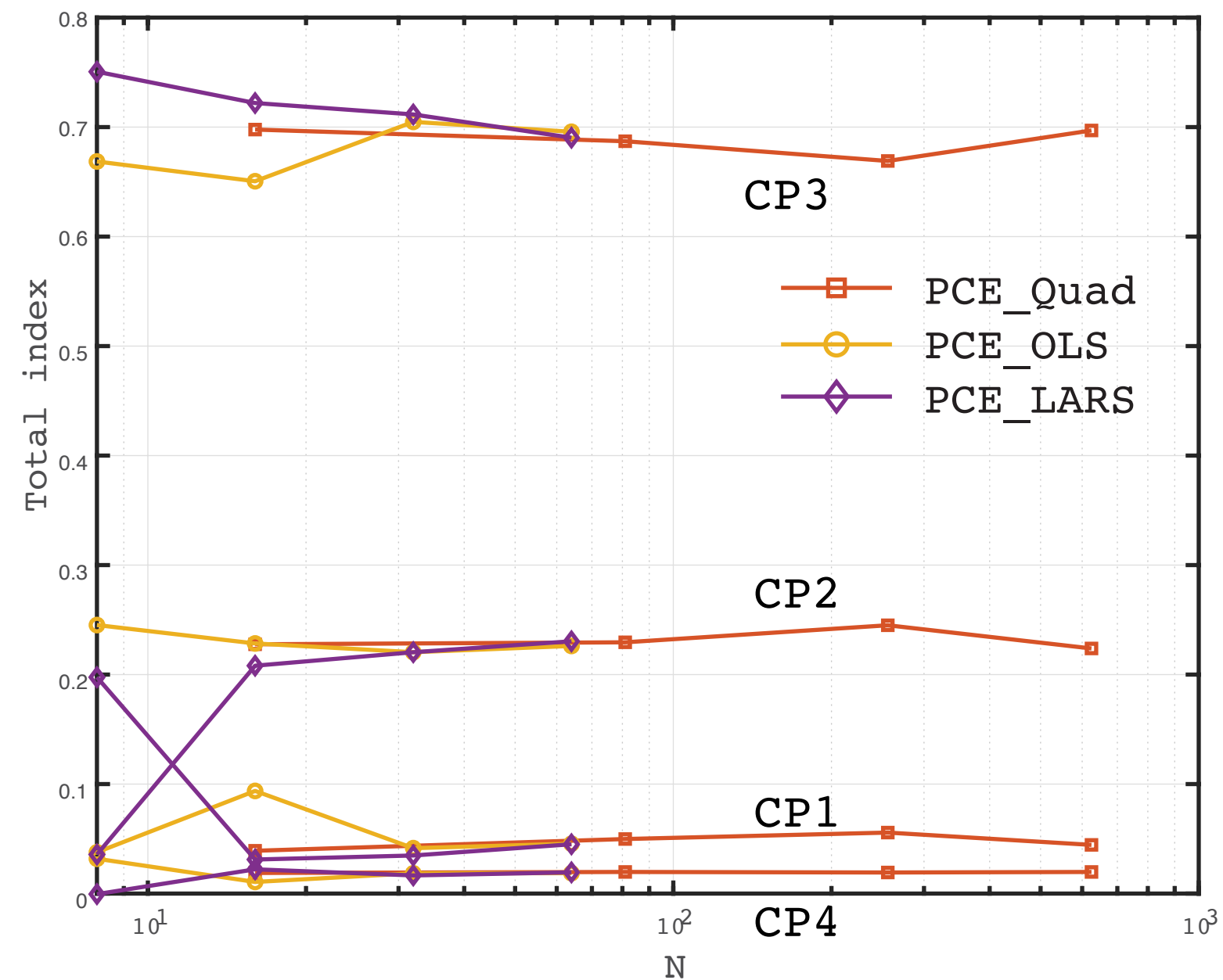
**Uncertainty:** Introduce 10% (uniform) uncertainty in 4 control points for Chord



# Sensitivity analysis results



Comparison of total Sobol indices



Convergence

# Conclusions

- > CP3 is the most important parameter
- > 64 samples using LARS and OLS is able to achieve similar accuracy as PCE\_QUAD with 625 samples
- > LARS and OLS are more suitable for models with a large number of uncertain inputs

# Next steps:

- > Parameterization of other random inputs
- > Determine realistic amount of perturbations for uncertain parameters
- > Include Bladed in the workflow and compare with AERO module results