A Global Sensitivity Analysis of Wind Turbine Aeroelastic Models

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Abstract

Keywords: Wind energy, global sensitivity analysis, UQ, aeroelastic models

1. Introduction

2. Aeroelastic models

2.1. BEM models

2.2. Input parameters of aeroelastic models

3. Global sensitivity analysis

3.1. Sobol' indices

3.1.1. PCE Ordinary Least Square (PCE_OLS)

3.1.2. PCE Least Angle Regression (PCE_LAR)

4. Parametrization of uncertain inputs in the BEM model

4.1. Geometric uncertainty

We use Non-Uniform Rational Basis Splines (NURBS) [**?**] to perturb the geometrical parameters of the turbine blade, such as the reference chord and twist curves. The main advantage of using NURBS is that it provides great flexibility to approximate a large variety of curves using a very limited number of control points. Further, the set of control points and knots can be directly manipulated to control the smoothness and curvature.

4.1.1. NURBS based perturbation

The value of NURBS curve at each location x is computed using a weighted sum of N basis functions (or B-splines):

$$S(x) = \sum_{i=0}^{N-1} c_i B_{i,p}(x), \tag{4.1}$$

where $S(x)$ is the value of the curve at location x, c_i is the weight of control point i and $B_{i,p}(x)$ is the value of B-spline corresponding to the i-th control point at x. The subscript p denote the polynomial degree of the NURBS curve. The total number of control points $N = m + p + 1$ where m is the number of knots and the degree of NURBS curve. The above definition can be easily extended to higher dimensions.

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The B-splines are recursive in polynomial degree, for example, we can derive quadratic B-spline ($p = 2$) using the linear B-spline ($p = 1$); cubic ($p = 3$) from quadratic B-splines and so on. Given m knot locations $t_0, t_1, ..., t_{m-1}$, the B-spline of degree 0 is defined as:

$$B_{i,0}(x) := \begin{cases} 1 & t_i \leq x < t_{i+1}, \\ 0 & \text{elsewhere.} \end{cases} \tag{4.2}$$

Higher order B-splines can then be derived using the recurrence relation [**?**]:

$$B_{i,p}(x) := \frac{x - t_i}{t_{i+p} - t_i} B_{i,p-1}(x) + \frac{t_{i+p+1} - x}{t_{i+p+1} - t_{i+1}} B_{i+1,p-1}(x). \tag{4.3}$$

These B-splines can be constructed efficiently using the De Boor's algorithm [**?**]. In Fig. **??**, we show the linear, quadratic and cubic spline for $x \in [0, 1]$. At the interval boundaries these B-splines go smoothly to zero.

Next, we describe steps to generate perturbed samples of chord from a given reference chord, $S_{ref}(x)$, using NURBS based parametrization. The first step is to approximate the given reference chord using a NURBS curve with a fixed degree p. For this, we sample $S_{ref}(x)$ at N locations $\{x_i\}_{i=0}^{N}$ and

perform inversion to compute $\{c_i\}_{i=0}^{N}$ by solving the $N \times N$ linear system:

$$\mathbf{Bc} = \mathbf{S}, \tag{4.4}$$

where $\mathbf{B} \in \mathbb{R}^{N \times N}$ with elements $\mathbf{B}_{ij} = B()$

with a *limited* number of control points. The number of control points are decided based on an error threshold criteria between the approximated and the reference chord.

Remark 4.1. *How to chose x*

4.2. Model uncertainty

4.3. Wind velocity

5. Description of test case

6. Sensitivity analysis workflow

7. Numerical experiment

8. Interpretation of global sensitivity analysis result

9. Conclusions

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