# Bayesian calibration applied to DANAERO data

WINDTRUE: WP2

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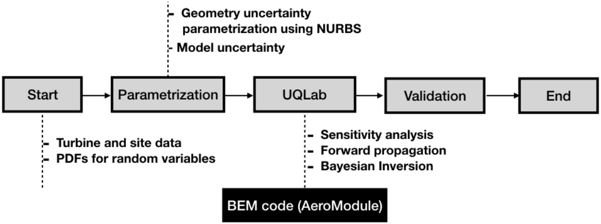
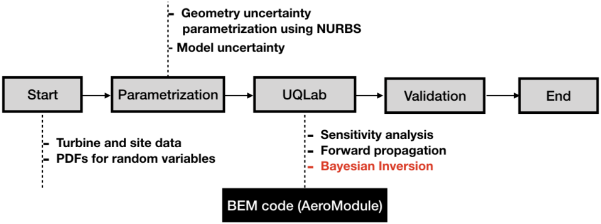
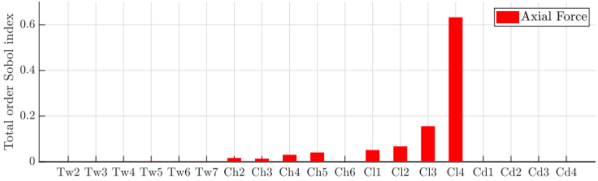
# WINDTRUE workflow



# WINDTRUE workflow



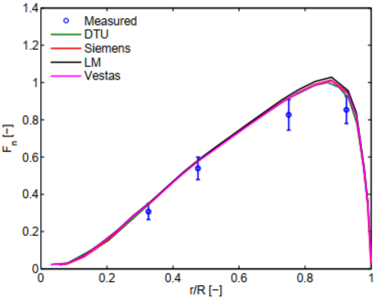
# Results from WP1




**Kumar et al. (2020)**

The results highlight amongst others the importance of the lift coefficient, especially for the axial force prediction.

# DANAERO test case

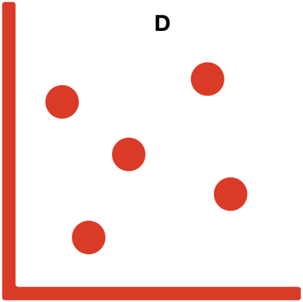



- D: Comparison of measurements and BEM computations of axial force in non-sheared inflow
- Forward model:

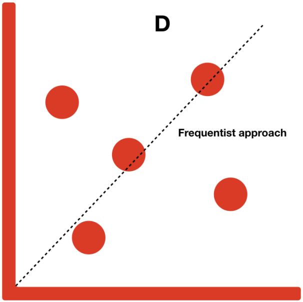
$$F_n^* = 0.5\rho W^2 c(C_L sin\beta + C_D cos\beta)$$

- $\theta = [C_{L1},\ C_{D1},\ C_{L2},\ C_{D2},\ C_{L3},\ C_{D3},\ C_{L4},\ C_{D4}\ ]$

# Inferential statistics

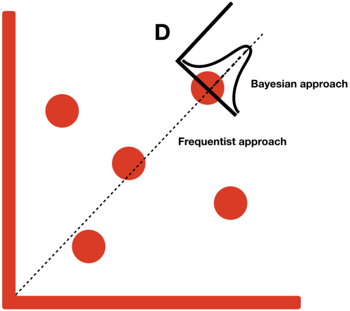


# Inferential statistics




- Frequentist inference: $\mathbb{E}(\theta)$

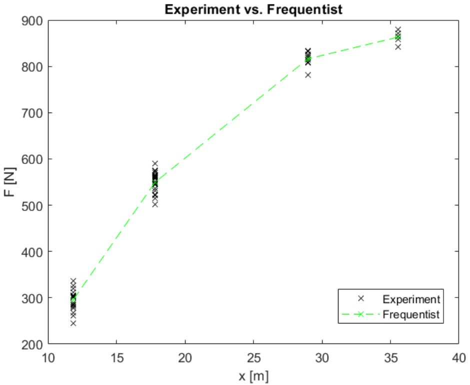
# Inferential statistics




- Frequentist inference: $\mathbb{E}(\theta)$
- Bayesian inference:

$$P(\theta|\mathbf{D}) = \frac{P(\mathbf{D}|\theta) \times P(\theta)}{P(\mathbf{D})}$$

# Frequentist inference



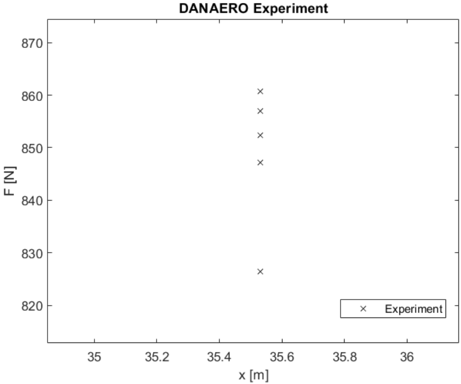

- Frequentist inference (Ordinary least-squares regression):

$$\theta_{OLS} = (X^T X)^{-1} X^T D,$$

$$X = X_{ij} \text{ is the design matrix}$$

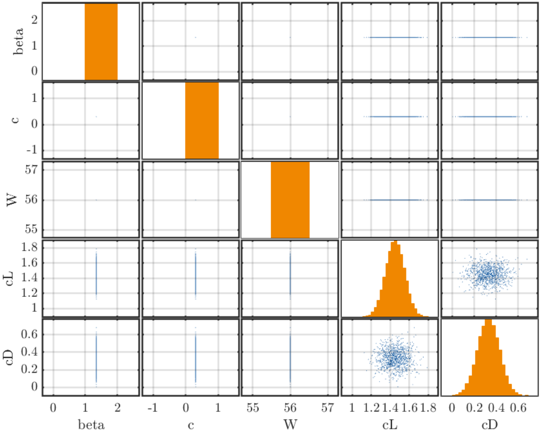
CWI

# Bayesian inference

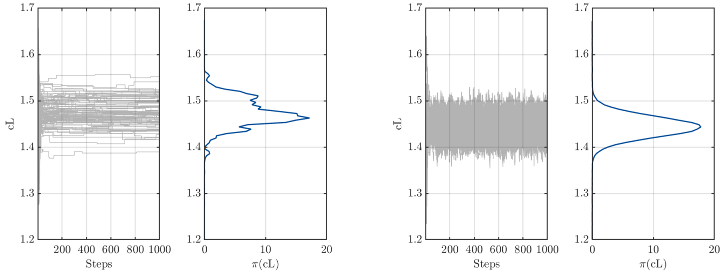



- Radial location: 35.53 m
- Number of data points (N) = 5 (Remember N>$\theta$)

# Bayesian inference - Prior distribution

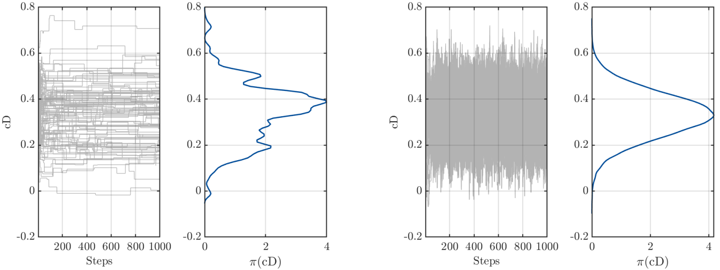


# Bayesian inference - MCMC algorithms



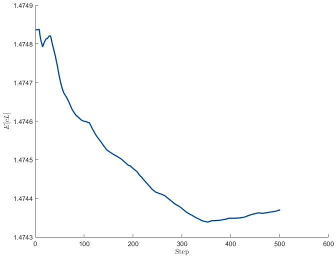

Figures showing trace plots for $C_L$ using Metropolis-Hastings (left) and Affine-invariant ensemble sampling (right) algorithms.

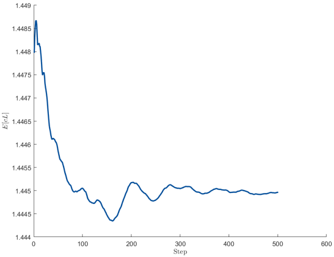
# Bayesian inference - MCMC algorithms




Figures showing trace plots for $C_D$ using Metropolis-Hastings (left) and Affine-invariant ensemble sampling (right) algorithms.

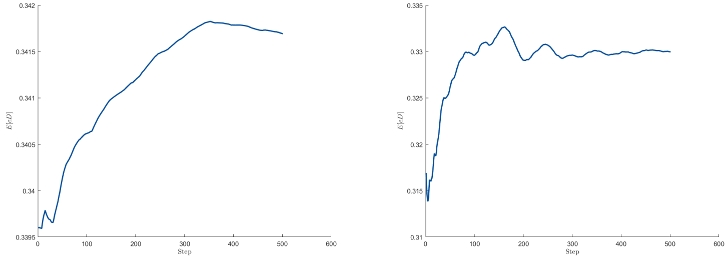
# Bayesian inference - Assessing convergence




**Figures showing convergence plots for $C_L$ using Metropolis-Hastings (left) and Affine-invariant ensemble sampling (right) algorithms.**

# Bayesian inference - Assessing convergence




**Figures showing convergence plots for $C_D$ using Metropolis-Hastings (left) and Affine-invariant ensemble sampling (right) algorithms.**

# Bayesian inference - Posterior distribution

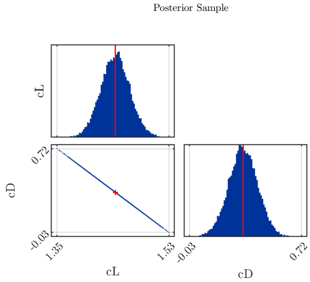
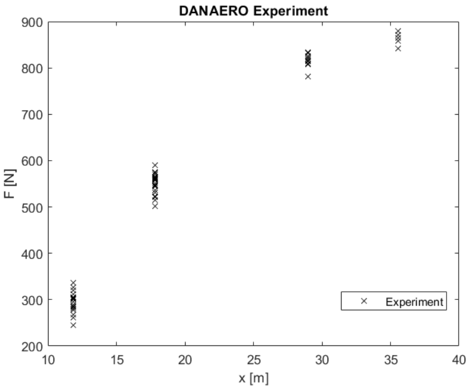



**Figure showing posterior distribution obtained using Affine-invariant ensemble sampling algorithm.**

| | $C_{L4}$ | $C_{D4}$ |
|---|---|---|
| $\mathbb{E}(\theta)$ | 1.4821 | 0.3421 |
| MAP | 1.4828 | 0.3391 |

# Frequentist vs. Bayesian approach



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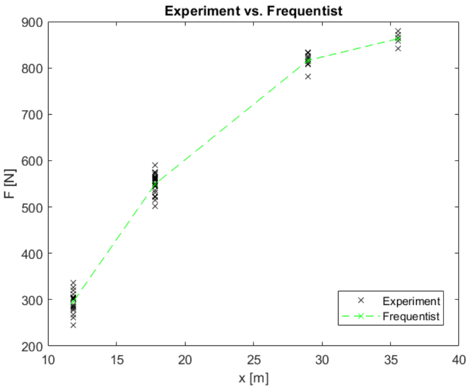

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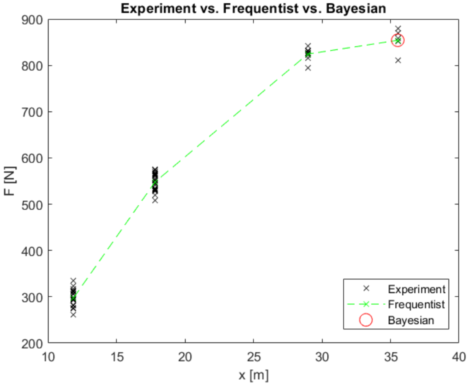

## Concluding remarks

- Bayesian calibration framework using UQLab
  - 4 different MCMC algorithms
  - Gelman-Rubin diagnostics: Assess convergence
  - Trace plots: Evolution of Markov chain
  - Acceptance rate: Quantitative indication of accepted samples
- DANAERO test case

# Next steps

- Experimental data (D)
- Choose relevant calibration parameters ($\theta$)
- BEM code for the forward model
- Choosing prior distribution for Bayesian calibration (Good engineering estimate)