A Lean Verification of the Empty Hexagon Theorem

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Abstract

A recent breakthrough in computer assisted mathematics proved that every set of 30 points contains an empty hexagon, thus closing a line of research opened by Erdős and Szekeres in 1935. The proof combines geometric insights with automated resoning techniques, resulting in a sophisticated propositional encoding of the problem with $O(n^4)$ many clauses (where n ends up being 30) that is then solved using a SAT solver and parallel computation. This paper presents a formalization of the proof in the Lean theorem prover, thus certifying its correctness. To achieve this we have formalized several ideas of both discrete computational geometry and SAT-encodings that have been successfully applied in different Erdős-Szekeres type problems. We hope this work sets a new standard for verification when extensive computation is used for discrete geometry problems, and increases the trust of the community in the results obtained by these methods.

1 Introduction

Here goes the intro...