# Option-implied Probability of Default

## A market-implied measure for financial distress

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17-734-377

Workshop in Econometrics II (8299)
Spring Semester 2023

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**ABSTRACT**[1]

In this paper the so-called option implied probability of default is applied to several banks traded in the US market, for the sample period of 1. January 2022 to 07. June 2023, therefore including the US banking crisis in March 2023. A special focus is set on Credit Suisse, which was announced to be taken over by UBS on the 19th of March 2023. The turmoil regarding Credit Suisse is reflected in the option implied probability of default, however, in this application the measure did not give an early warning in this application.

[1]Many thanks to Johannes Vilsmeier and Ilknur Zer who provided me with very useful code used in their studies.

# 1 Introduction and Literature Review

In March 2023, severe stress in the US banking sector was observable, whereby several banks ceased their operations. Loss of confidence in banks, substantial falls in bank stock prices and a bank run of unprecedented proportions characterised this turmoil (Board of Governors of the Federal Reserve System 2023). Also in March 2023, Switzerland's Federal Council announced a package of measures to prevent the globally active and systematically important Swiss bank Credit Suisse from failure and thus averted the onset of a financial crisis with substantial damage to the Swiss financial sector and the national and international economy as a whole (Swiss Federal Department of Finance 2023). These latest incidents in the financial sector show once again the importance of supervising the financial system to sustain financial and general economic stability.

To access the risk within the financial sector, a distinction can be drawn between micro- and macroprudential measures. The former focus on the idiosyncratic risk, i.e. the firm specific risk, while the latter tries to capture the risk of the entire financial sector. Although microprudential measures and its corresponding policy advises and regulations try to ensure stability of individual financial firms, the macroprudential perspective is necessary for example to incorporate interactions among individual financial institutions or feedback loops of the financial sector with the real economy (European Central Bank 2014).

This paper examines a framework to derive the probability of default for an individual firm implied by the market prices of corresponding equity options which was firstly suggested by Capuano (2008) and updated by Vilsmeier (2014). By determining the probability of default of an individual firm, the so-called option implied probability of default can be classified as a microprudential measure. However, Matros and Vilsmeier (2012) propose further an indicator based on the option implied probability which also signals the degree of distress in the financial sector as a whole, extending this concept with a macroprudential component.

***here include some other similar papers***

Capuano (2008) showed for his suggested method and Matros and Vilsmeier (2012) for the updated method that the option implied probability of default is capable to signal the occurrence of adverse shocks to specific financial institution and in Matros and Vilsmeier (2012) also to the financial sector as a whole for the US banking sector. This paper tries to contribute to the existing literature by testing this framework on the latest turmoil in the US banking sector, focusing on Credit Suisse as the only bank that failed during the Banking Crisis in 2023 for which the necessary data could be obtained.

While the option implied probability of default reacts in this application on given events as expected, the results cannot confirm the predictive power of this measure compared to Credit Default Swaps, as was found in Capuano (2008) or Matros and Vilsmeier (2012). However, this application should demonstrate the attractiveness of this measure in terms of the data required and the potential frequency with which estimates can be obtained.

This paper is structured as follows. First, the statistical framework of the method suggested by Capuano (2008) is described. Next, the modifications Vilsmeier (2014) proposed to this method are shown. In section 3, the data sources, the data itself as well as the empirical implementation is discussed. Section 4 shows and discusses the results of the empirical application.

# 2 The Statistical Framework

## 2.1 Methodology proposed by Capuano (2008)

The methodology of measuring the probability of default for a firm implied by observed option prices was first suggested by Capuano (2008). To give intuition about the mechanism behind the suggested methodology, Capuano (2008) remarks can be followed, for which two components are first introduced. The first component assumes the balance sheet structure of a firm according to Merton (1974). Consequently, to finance its assets $V$, a firm either uses its equity $E$ or takes up debt $D$. From the perspective of the equity holders of a firm, their payoff is equivalent to the remainder of the value of assets after the repayment of outstanding debts, since equity is a junior claim on the value of assets. Mathematically, the payoff of equity holders can therefore be defined as

$$E = \max(V - D; 0) \tag{1}$$

The second component that is used for the intuition behind the proposed methodology are equity options, or more precisely call options on the stock of a firm. A call option grants the right to the holder of the option to buy the underlying asset, here equity stocks, within a given time period[2] at a predetermined price, the strike price $K$. The payoff function of a call option at expiration date $T$ is hence given as

$$C_T^K = \max(E_T - K; 0) \tag{2}$$

whereby $E_T$ denotes the price of the stock at expiration $T$. Putting the definition of equity according to equation (1) into the payoff function of a call option according to equation (2), it can be seen that a call option written on a stock can be understood as a call option on a call option (Hull, Nelken, and White 2004):

$$C_T^K = \max(E_T - K; 0) = \max(\max(V_T - D; 0) - K; 0) = \max(V_T - D - K; 0) \tag{3}$$

Given the balance sheet structure, a firm defaults as soon as the debts exceed the value of equity. Put differently, a firm defaults if asset value $V$ falls below the value of debt $D$. Hence, the default domain of asset value can be represented as

$$PoD(D) = \int_0^D f(V_T) dV_T \tag{4}$$

whereby $f(V_T)$ is the probability density function of asset value (Capuano 2008). To solve the integral given in equation (4), both the value of $D$ and the probability density function of $V_T$ need to be determined. The resulting value corresponds to the marked implied probability of default.

To recover the probability distribution of the asset value, Capuano (2008) uses the principle of minimum cross-entropy based on the cross-entropy functional introduced by Kullback and Leibler (1951). Using this principle, the probability distribution of a random variable can be recovered using only what can be observed

[2] American options can be exercised at any date before or at expiration. European options can only be exercised at expiration. In this analysis, we use american style options (due to data availability).

and without any additional assumptions (Capuano 2008). Given that the true distribution is reflected in the observed data, this principle results in an estimated distribution which is the closest to the true distribution (Jaynes 1957). Only the prior distribution ($f^0(V_T)$ in equation (5)) needs to be determined, for which the distribution with maximal entropy on the defined domain, i.e. the the distribution which provides the most uncertainty regarding future outcomes should be chosen (Vilsmeier 2014). In the empirical implementation, the estimations are made on a closed interval, which is why a uniform distribution should be used as the prior distribution (Vilsmeier 2014).

The optimisation problem given from the principle of minimum cross-entropy in Capuano (2008) is as follows:

$$\min_{D}\left\{\min_{f(V_T)}\int_{V_T=0}^{\infty} f(V_T)\log\left[\frac{f(V_T)}{f^0(V_T)}\right]dV_T\right\} \tag{5}$$

whereby $f^0(V_T)$ corresponds to a prior probability density function of asset value, $f(V_T)$ the posterior probability density function and

$$\int_0^{\infty} f(V_T)\log\left[\frac{f(V_T)}{f^0(V_T)}\right]dV_T = CE[f(V_T), f^0(V_T)] \tag{6}$$

the cross-entropy function between the posterior and prior density function as defined in Kullback and Leibler (1951).

Capuano (2008) introduces following three constraints for the optimisation problem that are all based only on observable information:

Condition 1 states, based on Mertons balance sheet structure, that the value of equity is equivalent to a call options value written on the value of assets. The stock price observed today ($E_0$) must therefore be equivalent to the present value of the stock price at expiration of the corresponding option contract, discounted continuously with the risk free rate $r$:

$$E_0 = e^{-rT}\int_{V_T=0}^{\infty} \max(V_T - D; 0) f(V_T)dV_T = e^{-rT}\int_{V_T=D}^{\infty}(V_T - D)f(V_T)dV_T \tag{7}$$

Condition 2 states that the posterior probability density function needs to be able to price the observable option prices. Therefore the observed price of call options $i$ must be equivalent to the present value of the call options payoff at expiration:

$$C_0^i = e^{-rT}\int_{V_T=0}^{\infty} \max(V_T - D - K_i; 0) f(V_T)dV_T = e^{-rT}\int_{V_T=D+K_i}^{\infty} f(V_T)dV_T \tag{8}$$

The last condition corresponds to the additivity constraint, which ensures that the posterior probability density function integrates to 1:

$$\int_{V_T=0}^{\infty} f(V_T)dV_T = 1 \tag{9}$$

Capuano (2008) solves the optimisation problem stated in equation (5) sequentially in two steps. First, the problem is solved for the optimal $f(V_T)$ as a function of the free parameter $D$, for what following Lagrangian is proposed:

$$\mathcal{L} = \int_{V_T=0}^{\infty} f(V_T)\log\left[\frac{f(V_T)}{f^0(V_T)}\right]dV_T + \lambda_0\left[1 - \int_{V_T=0}^{\infty} f(V_T)dV_T\right] + \lambda_1\left[E_0 - e^{-rT}\int_{V_T=D}^{\infty}(V_T - D)f(V_T)dV_T\right]$$
$$+ \sum_{i=1}^{n}\lambda_{2,i}\left[C_0^i - e^{-rT}\int_{V_T=D+K_i}^{\infty}(V_T - D - K_i)f(V_T)dV_T\right] \tag{10}$$

After taking the first order derivatives and some rearranging, following system of nonlinear equations is derived, which is solved numerically:

$$\frac{1}{\mu(\lambda)}\frac{\partial\mu(\lambda)}{\partial\lambda_1} = E_0 \tag{11}$$

$$\frac{1}{\mu(\lambda)}\frac{\partial\mu(\lambda)}{\partial\lambda_{2,i}} = C_0^i \qquad \text{for } i = 1, 2, ..., n \tag{12}$$

whereby

$$\mu(\lambda) = \int_{V_T=0}^{\infty} f^0(V_T)\exp\left[\lambda_1 e^{-rT}\mathbf{1}_{V_T>D}(V_T - D) + \sum_{i=1}^{n}\lambda_{2,i}e^{-rT}\mathbf{1}_{V_T>D+K_i}(V_T - D - K_i)\right]dV_T \tag{13}$$

Solving this system yields the optimal probability density function $f^*(V_T, D)$ dependent on $D$.

To derive the corresponding optimal $D$, Capuano (2008) substitutes $f^*(V_T, D)$ into the Lagrangian defined in equation (10), resulting to following problem

$$\lim_{\Delta\to 0}\frac{\mathcal{L}(f^*(V_T, D+\Delta)) - \mathcal{L}(f^*V_T, D)}{D+\Delta} = 0 \tag{14}$$

which is again solved numerically. After the probability density function and the threshold for firm default is obtained, the option implied probability can be obtained by calculating equation (4).

## 2.2 Updated methodology by Vilsmeier (2014)

Vilsmeier (2014) mentions some problems in the methodology proposed in Capuano (2008). The first problem concerns the numerical solving of equations (11) and (12), which leads to unstable solutions and sometimes no solutions at all. Vilsmeier (2014) applies an approach introduced by Alhassid, Agmon, and Levine (1978) to overcome this problem and implements a robust and computationally efficient algorithm to obtain the optimal set of $\lambda$ given equation (12) (equation (11) is not relevant for the method proposed by Vilsmeier (2014)). Vilsmeier (2014) furthermore solved the integrals contained in the objective function analytically, which also further stabilises the method. Additionally Capuano (2008) uses accounting data to determine the domain bounds for the risk neutral densities which Vilsmeier (2014) shows is unnecessary from a statistical point of view and can even severely bias the results when the domain derived from accounting data is too short. Vilsmeier (2014) further shows that by numerically solving for the optimal $D$ as in Capuano (2008) can lead to arbitrary results.

In contrast to the structural approach in section 2.1, Vilsmeier (2014) describes the methodology through a purely statistical lens, starting with risk neutral densities, which describe the expectations of the stock value $S$ at time of maturity $T$ implied by observed option prices for different strike prices $K$. To obtain an option implied probability of default, traditional approaches for estimating risk neutral densities have to be adjusted, such that a mass point in the risk neutral density can be estimated which will indicate the probability that stock value $S$ will be zero at the time of maturity of the corresponding option (assuming that stock value of zero implies a firm's default). Since a continuous estimation framework is used to determine the probability of default (and therefore a potential jump in the risk neutral density at a stock price of zero cannot be estimated) Vilsmeier (2014) extends the domain of possible stock prices $S_T$ by shifting its domain upwards by some constant $D$. All realisations within the additional interval $[0, D]$ imply a stock price of zero and therefore a default of the firm. Now, the probability density function of interest is $f(V_T)$ with $V_T = S_T + D$. Using this new domain, the payoff function for a call-option at time $T$ can be described as

$$C_T^{K_i} = \max(V_T - D - K_i; 0) \tag{15}$$

equally as in the structural approach shown in equation (3). When estimating the risk neutral density, the method proposed by Vilsmeier (2014) assigns density to the interval with payoff of zero $[0, D]$ such that an optimal fit to observed option prices is achieved. Analogously to equation (4), the option implied probability of default is given by

$$PoD(D) = \int_0^D f(V_T) dV_T \tag{16}$$

Similarly as in section 2.1, both $f(V_T)$ and $D$ need to be determined for obtaining the option implied probability of default. Vilsmeier (2014) equally minimises the cross entroy function by Kullback and Leibler (1951) stated in equation (6) for a given prior distribution $f^0(V_T)$ under the moment constraints given by equation (8) and equation (9), however Vilsmeier (2014) does not take equation (7) into account for the optimisation. Note that Vilsmeier (2014) also includes the current stock price to the considered options with a strike price of zero. The resulting Lagrangian is given by

$$\begin{aligned} \mathcal{L} = & \int_{V_T=0}^{\infty} f(V_T) \log\left[\frac{f(V_T)}{f^0(V_T)}\right] dV_T + \lambda_0 \left[1 - \int_{V_T=0}^{\infty} f(V_T) dV_T\right] \\ & + \sum_{i=1}^{n} \lambda_i \left[C_0^i - e^{-rT} \int_{V_T=D+K_i}^{\infty} (V_T - D - K_i) f(V_T) dV_T\right] \end{aligned} \tag{17}$$

As described in Section 2.1, a non-linear system of equations is obtained from the Lagrangian, which in Capuano (2008) is solved numerically. Vilsmeier (2014) states, that the process of finding roots is unstable and converges only for a small number of constraints and the initial values of $\lambda$ are chosen near to the final solution. Using an approach introduced by Alhassid, Agmon, and Levine (1978), Vilsmeier (2014) proposes a robust and computationally efficient algorithm which he used to calculate the optimal set of $\lambda$. The new objective function proposed by Vilsmeier (2014) is given by[3]

$$F = -\lambda_0^{Tr''} = \log\left\{\int_{V_T=0}^{\infty} f^0(V_T)\exp\left[\sum_{i=1}^{B}\lambda_i^{Tr}(e^{-rT}\mathbf{1}_{V_T>D+K_i}(V_T-D-K_i)-C_0^{K_i})\right]dV_T\right\} \tag{18}$$

Furthermore, Vilsmeier (2014) assumes a finite domain for $V_T$ ranging from the lower bound $V_{\min} \in [0; D]$ to the upper bound $V_{\max}$ whereby $V_{\min} < D < V_{\max}$, as well as a uniform prior distribution $f^0(V_T) = \frac{1}{V_{\max}-V_{\min}}$. With this assumptions, the integration in equation (18) can be solved analytically, which is why no numerical quadrature methods are necessary to obtain the $\lambda$'s. After rewriting the objective function without the indicator function and after solving the integrals analytically, the objective function is given by[4]

$$\begin{aligned}
F = \; & \log\left(\frac{1}{V_{\max}-V_{\min}}\right) + \log\Bigg\{\exp\left(-\sum_{i=1}^{B} w_i\lambda_i C_0^{K_i}\right)(D-V_{\min}) \\
& - \sum_{i=1}^{B-1}\Bigg[\frac{\exp\left(\sum_{j=1}^{i} w_j\lambda_j(e^{-rT}(K_i-K_j)-C_0^{K_j}) - \sum_{k=i+1}^{B} w_k\lambda_k C_0^{K_k}\right)}{e^{-rT}(\sum_{j=1}^{i} w_j\lambda_j)} \\
& - \frac{\exp\left(\sum_{j=1}^{i} w_j\lambda_j(e^{-rT}(K_{i+1}-K_j)-C_0^{K_j} - \sum_{k=i+1}^{B} w_k\lambda_k C_0^{K_k})\right)}{e^{-rT}(\sum_{j=1}^{i} w_j\lambda_j)}\Bigg\}\Bigg] \\
& - \left[\frac{\exp\left(\sum_{j=1}^{B} w_j\lambda_j(e^{-rT}(K_B-K_j)-C_0^{K_j})\right) - \exp\left(\sum_{j=1}^{B} w_j\lambda_j(e^{-rT}(V_{\max}-D-K_j)-C_0^{K_j})\right)}{e^{-rT}(\sum_{j=1}^{B} w_j\lambda_j)}\right]\Bigg\}
\end{aligned} \tag{19}$$

Minimizing the objective function stated in equation (19) results in the estimated optimal set of $\lambda$ given the default barrier $D$.

Note that following Matros and Vilsmeier (2012) weights $w$ that are pre-multiplied to the Lagrange multiplier are included in the objective function, such that more liquid option contracts have to be met more precisely by the estimated posterior distribution. Capuano (2008) weighted the different contracts by trading volume. Matros and Vilsmeier (2012) argue for weighting by open interest[5] instead of trading volume, since trading volume of 0 does not contain no information about expectations. Rather, if there was high trading in the past, which is captured in open interest, and no trading today, the expectations simply did not change. Unfortunately, in the data source used in this project, no information on open interest is available, which is why the sample was weighted by trading volume. In the studies of Zer (2015) the option implied probability of default was calculated with both trading volume and open interest, whereby the results were qualitatively similar.

As mentioned the estimation of the risk neutral density is allowed for a mass point at a future stock price of zero. As stated in section 2.1, the prior distribution is chosen to be uniform, with the previously described domain bounds for $V_T$ therefore defined as $f^0(V_T) = \frac{1}{(V_{\max}-V_{\min})}$. Note that the payoff function in equation (15) for given $K$ both for $V_T > D$ and $V_T \leq D$ are the same for arbitrary intervals $[D, V_{\max}]$ respectively for arbitrary intervals $[V_{\min}, D]$ with constant length. Hence, the domain bounds influence the estimates not through their actual values, but solely through the domain length $V_{\max} - V_{\min}$ (Vilsmeier 2014). As a result,

[3]For further details about the derivation of the proposed new objective function, see Vilsmeier (2014) Chapter 3.2
[4]This function is implemented as `objectiveFunction` within the `optipod` function
[5]number of outstanding options

Vilsmeier (2014) states that book value based domain bounds as proposed in Capuano (2008) lack practical significance and can possibly harm the estimation if the domain implied by book value is too short, whereby not enough possible payoffs to price the option contract would be available. When choosing the domain too long, i.e. longer than the "true" domain of the density that priced the observed prices, a density of practically zero will be assigned to the additional domain which exceeds the true upper bound, yielding no harm in the estimation process (Vilsmeier 2014).

The last model parameter to be determined is the threshold $D$ which will define the part of the domain in which the firm defaults. If $V_T \leq D$ the restriction given in equation (8) are zero while the additivity constraint as in equation (9) assigns constant density (Vilsmeier 2014):

$$f^*(V_T) = \frac{1}{V_{\max} - V_{\min}} \frac{\exp(-\sum_{i=1}^{B} C_0^{K_i} \lambda_i)}{\exp F} \quad \forall V_T \in [V_{\min}, D]$$

Since the probability of default is given by the integral over $[V_{\min}, D]$, the probability of default can be defined as

$$PoD(\lambda, D) = \frac{1}{V_{\max} - V_{\min}} \frac{\exp(-\sum_{i=1}^{B} C_0^{K_i} \lambda_i)}{\exp F(\lambda, D)} (D - V_{\min}) \tag{20}$$

Equation (20) reveals that the density assigned to $[V_{\min}, D]$ depends on the parameters $\lambda_i$ that define the shape of the posterior density, which implies that the probability of default and the shape of risk neutral densities for the domain $[D, V_{\max}]$ interact. Given $D$, the combination of the probability of default and the risk neutral density that fits the observed market prices best is optimal (Vilsmeier 2014).

To determine the optimal $D$, Vilsmeier (2014) suggests a procedure based on the evolution of the PoD-function as defined in equation (20) and of the Lagrange multipliers $\lambda$ when estimating the optimal density for different levels of $D$. The characteristics of the evolution of these functions is not yet conclusively determined. The studies in Vilsmeier (2014) indicate, however, that both the value of the probability of default and the sum of estimated $\lambda$'s flattens with increasing $D$, whereby the PoD-function is roughly concave, while the $\lambda$'s exhibit strong fluctuations.

Since there is no exact decision rule for determining the optimal $D$, Vilsmeier (2014) proposes following ad hoc procedure. First, a lower and upper bound for $D$ is set. Concretely, backed by numerical experiments and empirical results, setting $V_{\min} = 0$ and $D_{\max} = 20$, all possible values of $D$ are given by $\mathcal{D} = \{D | D \text{ is an integer, and } 0 \leq D \leq 20\}$. Second, the probability of default is estimated for each $D$ in $\mathcal{D}$. Third, the average estimated probability of default over all considered $D$'s is calculated. Lastly, the $D$ and hence the length of the domain $[V_{\min}, D]$ is determined by choosing the $D$ that provides the estimated probability of default closest to the average probability of default.

After the optimal $D$ is determined, the option implied probability of default can be calculated using the $\lambda$'s obtained in equation (19) to define the shape of the posterior distribution.

# 3 Application of the Method

## 3.1 Data Sources

For this project data for 50 holding companies with reported total assets greater than 10 billion US dollars according to the Federal Financial Institutions Examination Council[6] obtained via the polygon.io API.[7] Polygon is a leading provider of low latency market data, which obtains its options data directly from the Options Price Reporting Authority (OPRA), therefore allowing them to publish reliable and accurate data. All option contracts with an expiration date in 2020 or newer were requested from 07. January 2019 up until 07. June 2023. To extend the observation period for Credit Suisse, the major bank of interest in this study, additional option data since 2005 was collected from OptionMetrics via Wharton Research Data Services. OptionMetrics is an established data provider of historical option data for use in empirical research and econometric studies.

Additionally to the options data, stock price data and data for the risk free rate for discounting was collected. For the period from mid 2021 to 2023, the stock data was retrieved via the polygon.io API. For Credit Suisse, the additional stock data since 2005 was obtained from Yahoo Finance. The risk free interest rate was downloaded from Kenneth R. Frenchs website.[8]

The data for one options chain, which consists of all option contracts with the same expiration date measured on the same date, can be represented as follows:

Table 1: Example of Options Chain

| Date | Expiration Date | Price | Strike Price | Risk Free Rate | Time to Maturity | Weight |
|---|---|---|---|---|---|---|
| 05.04.2022 | 13.05.2022 | 133.34 | 0 | 0.001 | 38 | 1.00 |
| 05.04.2022 | 13.05.2022 | 4.21 | 135 | 0.001 | 38 | 0.06 |
| 05.04.2022 | 13.05.2022 | 2.24 | 140 | 0.001 | 38 | 0.42 |
| 05.04.2022 | 13.05.2022 | 1.15 | 145 | 0.001 | 38 | 0.16 |
| 05.04.2022 | 13.05.2022 | 0.57 | 150 | 0.001 | 38 | 0.02 |
| 05.04.2022 | 13.05.2022 | 0.15 | 160 | 0.001 | 38 | 0.34 |

Each row corresponds to a specific option contract, which grants the holder of the option the right to buy the underlying stock at the corresponding strike price within the time to maturity.[9] Note that the first row corresponds to the current stock price which is included as an option contract with strike price 0 and a weight of 1, as mentioned in Section 2.2. Using this options chain, the option implied probability of default as of the 5th of April 2022 can be calculated. When more than one option chain was traded on one day, i.e. when more than one expiration date were traded, the probability of default was calculated for each option chain. To get the daily estimate of the probability of default, the average probability of default over all option chains traded on this day was calculated.

To classify the predictive power of the option implied probability of default, data for Credit Default Swaps for Credit Suisse were obtained from investing.com. Credit Default Swaps are financial derivatives, with

[6] https://www.ffiec.gov/npw/Institution/TopHoldings

[7] All necessary functions to get the data are available on `GitHub`. Most of the data can be obtained with a free plan, however the API requests are limited to 5 calls per minute.

[8] https://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html, function for auto-download available in `optipod`. For days which were not yet included in this data source, a constant risk free rate since the last observed value was assumed.

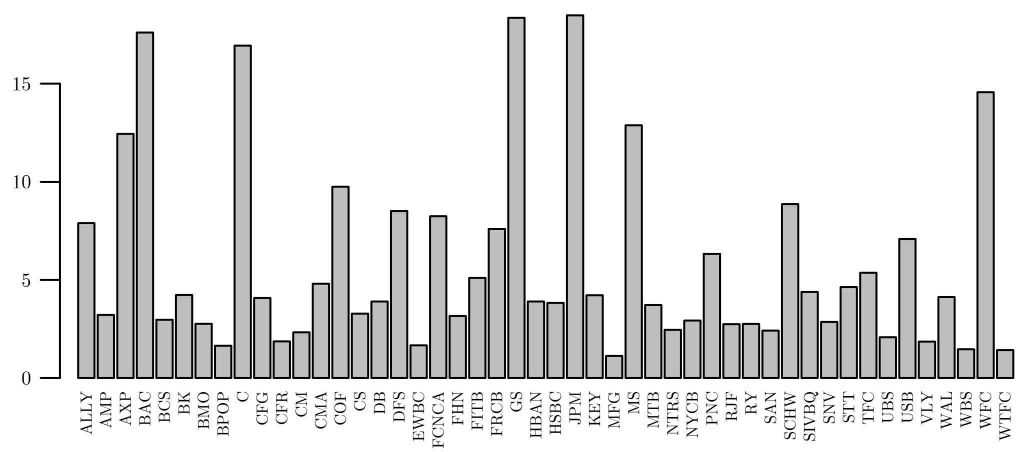
[9] As the options under consideration have American exercise style.

which the buyer of the derivative insures against the possibility of default on a bond issued by a given entity, therefore making it possible to trade risk associated with the underlying entity (Longstaff, Mithal, and Neis 2005). Therefore, higher default risk should be represented by higher prices for Credit Default Swaps.

### 3.2 Exploratory Data Analysis

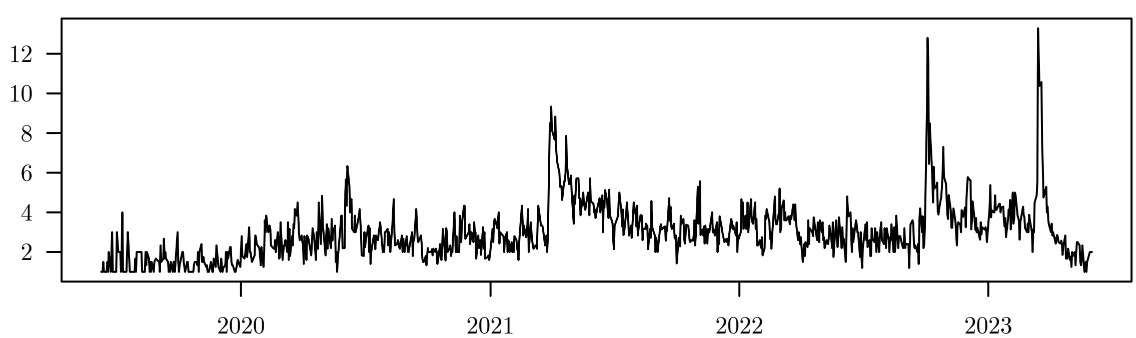
As can be seen in equation (8), the number of conditions for the optimisation problem stated in equation (19) is dependent on how many options with different strike prices were traded and therefore priced on a specific day for a given expiration date. As the number increases, more conditions for the optimisation problem are available, possibly leading to a more accurate estimation of the posterior distribution. As can be seen in Figure 1, the number of traded options with different strike prices varies strongly across different banks.

Figure 1: Average number of traded strike prices per option and day



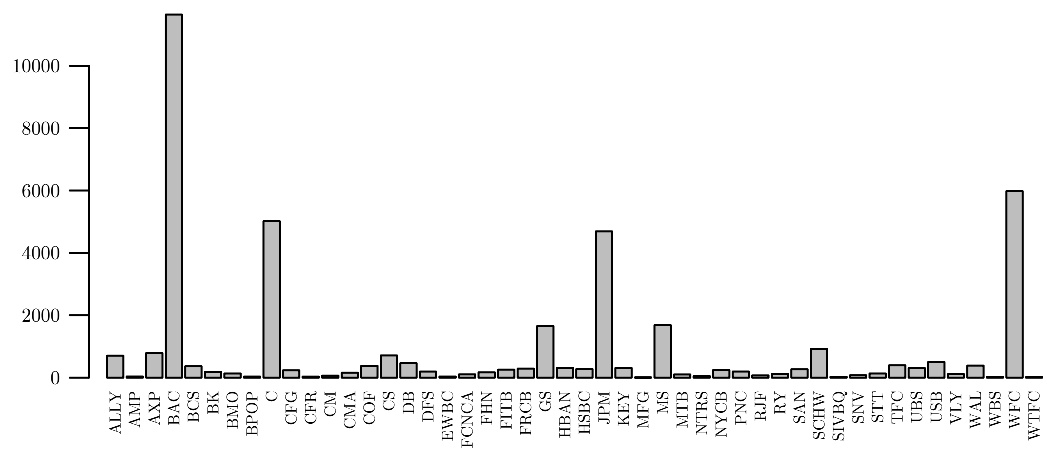

Additionally, when examining the number of options with different strike prices traded for a specific bank over time, it can be seen that this number also varies strongly over time. Figure 2 shows the dynamic of the number of traded options with different strike prices over time for Credit Suisse.

Figure 2: Dynamics of average number of traded strikes per option and date (Credit Suisse)



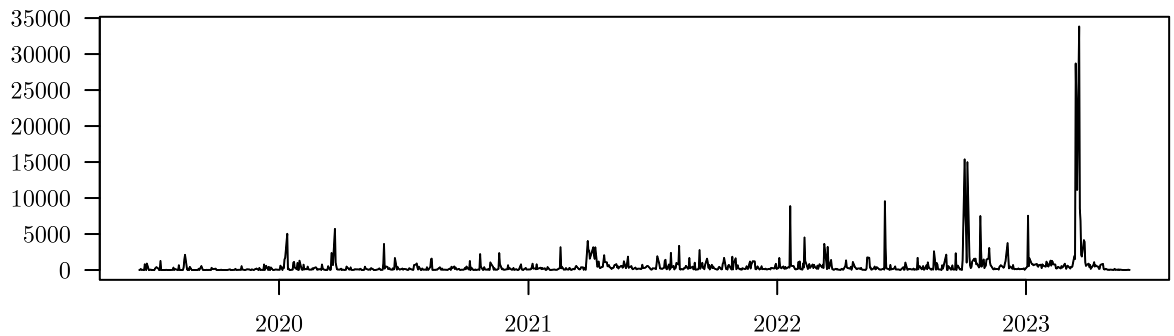

Furthermore, the trading volume, i.e. the number of traded options on a specific day, varies tremendously across different banks as visualised in Figure 3.

Figure 3: Average Trading Volume per Bank




Again looking at the dynamics for Credit Suisse, Figure 4 shows that trading volume can vary strongly over time.

Figure 4: Dynamics of Trading Volume for Credit Suisse over time




In general, ceteris paribus it can be assumed that with more strike prices (leading to more constraints in the optimisation problem) and with higher trading volume (information about the expectations of more investors revealed), the posterior distribution can be estimated more accurately. The underlying argument is, that the principle of minimum cross-entropy can only recover the distribution closest to the true distribution if the true distribution is reflected in the observable data, as mentioned in Capuano (2008).

## 3.3 Empirical Implementation

With more strike prices and therefore more constraints that need to be satisfied, the computation time for estimating the probability of default increases quite drastically. For estimating the probability of default for Credit Suisse between January 2022 to June 2023, 9’838 option contracts are available for estimating 2’115 probabilities of default in total. Therefore, on average 4.65 conditions are considered for the optimisation problem. The estimation process for Credit Suisse took about 40 minutes. For the same period of time 78’541 are available for estimating 2’115 probabilities of default in total (21.03 on average) for JPMorgan Chase & Co. (JPM). The estimation process for JPM took about 36 hours. Hence, due to this computational limitations, the probability of default could not be estimated for all retrieved options. The sample was reduced on Credit Suisse, 3 banks similar to Credit Suisse in terms of total asset value according to the

Federal Financial Institutions Examination Council[10], JP Morgan Chase & Co. as the largest bank as a benchmark and UBS as the other Swiss Bank traded in the US market. Additionally, the observation period was limited to 2022 onwards. The procedure for estimating the option implied probability of default, as theoretically described in Section 2.2, can be summarised and represented in Pseudo-Code as follows:

Algorithm 1: Estimation of Option implied Probability of Default

```
procedure ESTIMATE IPOD

    for every D in 𝒟 do
        solve objective function defined in equation (19)
        with returned λ's, determine posterior distribution
        PoD ← integration of posterior over [0, D]
    end for

    average PoD ← mean over all obtained PoD's
    optimal D (D*) ← D for which min(|PoD(D) − average PoD|)

    return estimated PoD with D*

end procedure
```

This algorithm is implemented in `R` and is available as the function `optipod` within the `GitHub`-Repository[11] which was created for this project. Note that the code underlying this function was provided by Johannes Vilsmeier. Since the code was refactored in the process of this project, possible errors might were included in the original code.

For about 1.89% of considered option chains, the estimation process returned no result. The reason for the failed estimates still needs to be clarified. Additionally, for some option chains the estimated result explodes and reaches values in the trillions, despite that the expected results should be bounded between 0 and 1. Note that this only happened to the four banks Credit Suisse, Silicon Valley Bank, First Republic and Mizuho Financial Group, whereby the first three went bankrupt in 2023. For Credit Suisse, this explosion happened at the beginning of the Covid-19 pandemic in March, April and May 2020 as well as in every month since June 2022. For Mizuho Financial Group, during the whole sample since January 2022 exploded results can be observed.

The reason for this problem might be that the considered domain $[D, V_{\max}]$ might have been too short. Following the code which was provided by Johannes Vilsmeier, $V_{\min}$ was chosen to be 0 and $V_{\max}$ was chosen to be five times the current stock price. Vilsmeier (2014) states that $V_{\max}$ should be large but can be arbitrarily chosen since it does not significantly influence the estimates. With the current approach to determine $V_{\max}$ using a multiple of the current stock price, the set value of $V_{\max}$ is obviously dependent on the development of the stock price. The stock price for example for Credit Suisse, as visualised in Figure 5, varied strongly within the past 18 years.

[10] https://www.ffiec.gov/npw/Institution/TopHoldings
[11] https://github.com/bt-koch/optipod

Figure 5: Credit Suisse Stock Price

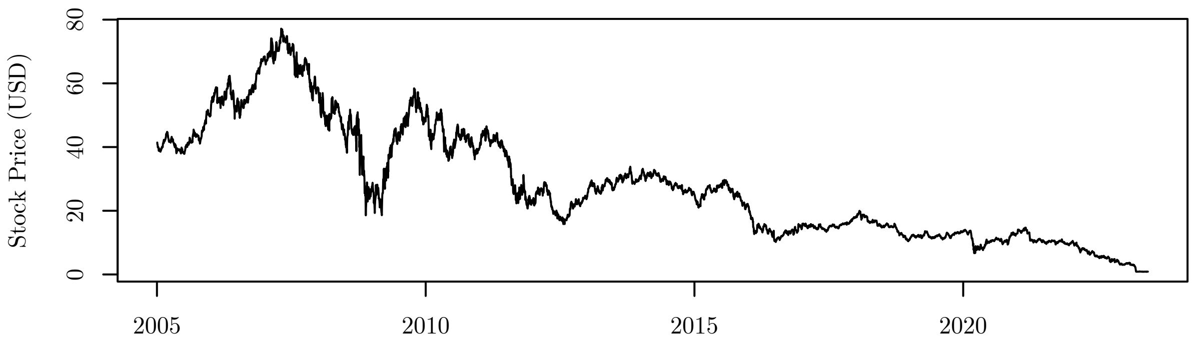
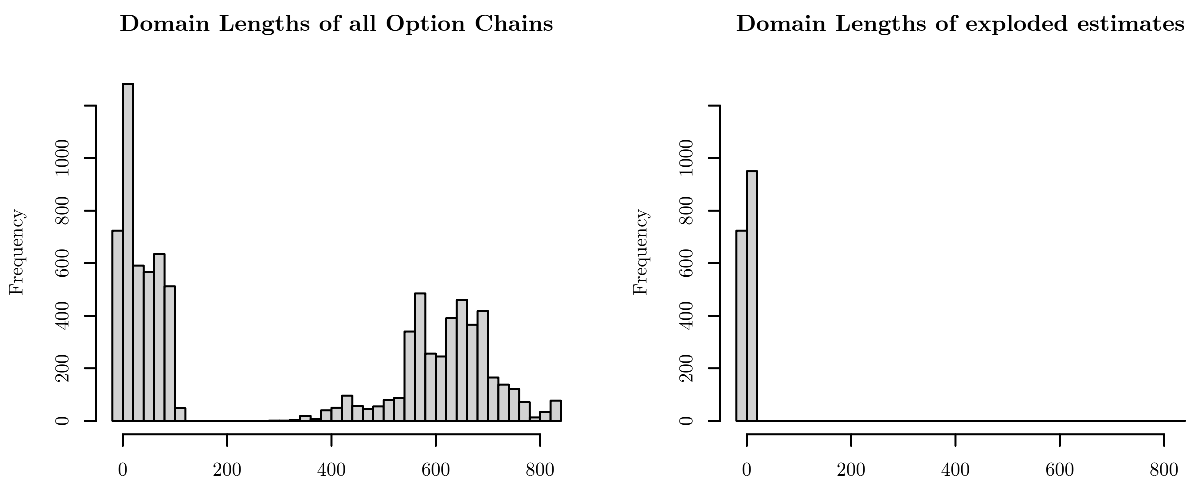



Figure 6 shows the number of considered domain lengths as well as the number of exploded estimates per domain length. It can be seen that the optimisation procedure breaks if the domain length is too short. Since computational limits were present in this project because of the reported running time, a detailed analysis of the behaviour of the optimisation procedure and its results for different multiplication factors was not possible.[12]

Figure 6: Histrograms of Domain Lengths



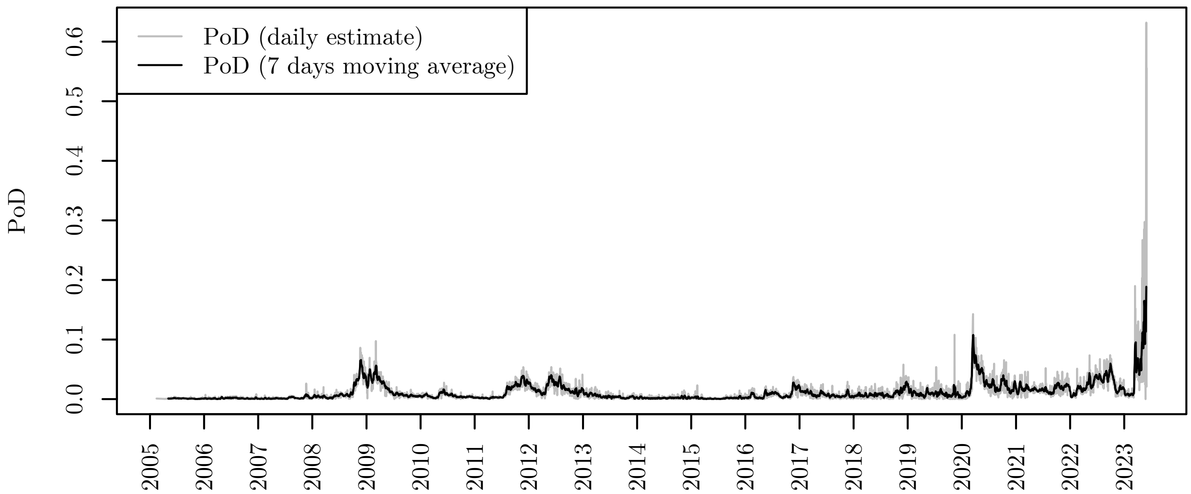

Another issue in the estimation process discussed in Matros and Vilsmeier (2012) is the maturity dependence of the risk neutral density estimates. As described, risk neutral densities reveal information about the expectations regarding the value of an options underlying at the date of expiration. If risk neutral densities are estimated for subsequent days using options with the same date of expiration, risk neutral densities that are estimated closer to the date of expiration will exhibit ceteris paribus less uncertainty regarding the value of the underlying at expiration date. Matros and Vilsmeier (2012) apply a regression based procedure to remove this maturity dependence. Currently, the results presented in this analysis are not corrected for maturity dependence. However, Matros and Vilsmeier (2012) stated that this maturity correction only increase the estimates systematically, but does not change the dynamics of the obtained time series. As will be addressed later, it seems that the dynamics of the estimates is of more interest than the estimated value itself, this correction is omitted for the time being.

[12]Running time increased further with higher domain lengths. Using a multiplication factor of 5, 10 and 30, running time was around 40 minutes, one hour and two hours respectively.

# 4 Results

Figure 7 shows the daily estimates of the option implied probability of default for Credit Suisse since 2005 as well as the corresponding 7 days moving average to make the dynamics more visible. Note that for obtaining this estimates, the upper limit of the domain $[D, V_{\max}]$ was chosen as the current stock price multiplied by 30 to avoid exploded estimates which occured during the Covid-19 Pandemic and the most recent banking crisis when estimating with a multiplication factor of 5, which was used when first estimating and for the other banks considered.

Figure 7: Estimated Probability of default for Credit Suisse (2005-2023)

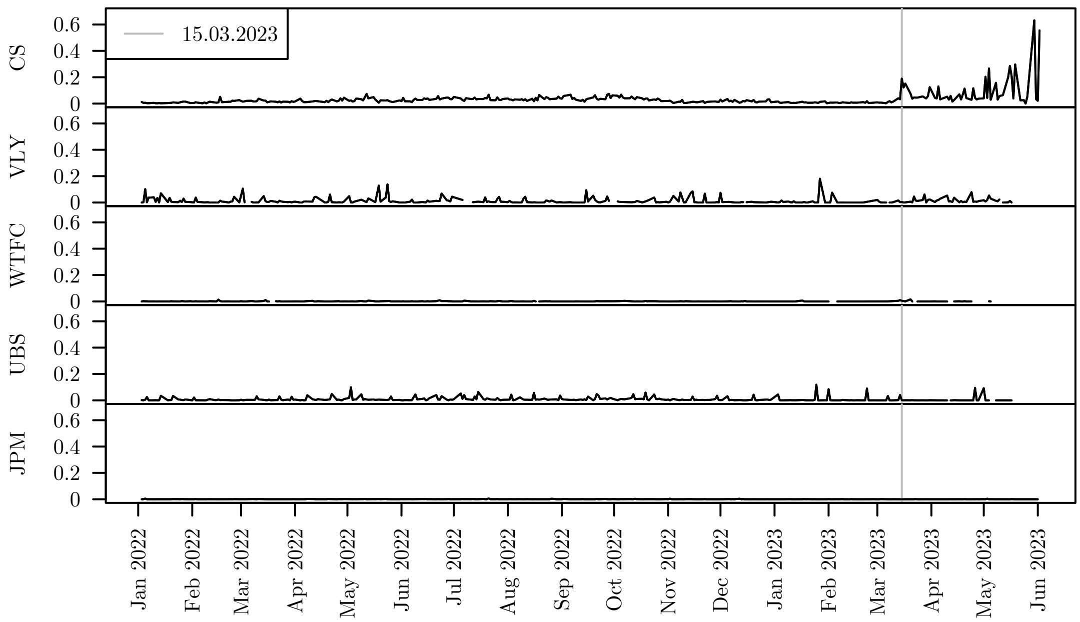



In the dynamics of the obtained estimates, episodes of increased stress in the financial sector, such as the World Financial Crisis 2008/2009 and its aftermath, the Euro Crisis or the Covid-19 Pandemic, are visible and represented as episodes with relatively higher estimates for the probability of default. The most visible dynamic in Figure 7 is the huge increase in the estimates for Credit Suisse since its collapse in March 2023.

In put this huge increase into perspective, Figure 8 compares the estimates of the option implied probability of default for Credit Suisse (CS) with estimates of selected other banks. Valley National Bancorp (VLY) as well as Wintrust Financial Corp (WTFC) were selected because they exhibit a similar value of total assets to Credit Suisse (57.5 mio USD, 52.9 mio USD and 57.4 mio USD accordingly)[13]. Further UBS, as the other Swiss Bank, was included in the sample to observe whether or how market participants' expectations changed for this bank as well. Additionally, since Credit Suisse will take over Credit Suisse, observing the dynamics of the estimates of UBS after the collapse of Credit Suisse are of interest. Finally, JPMorgan (JPM) as the biggest bank according to the value of total assets is included.

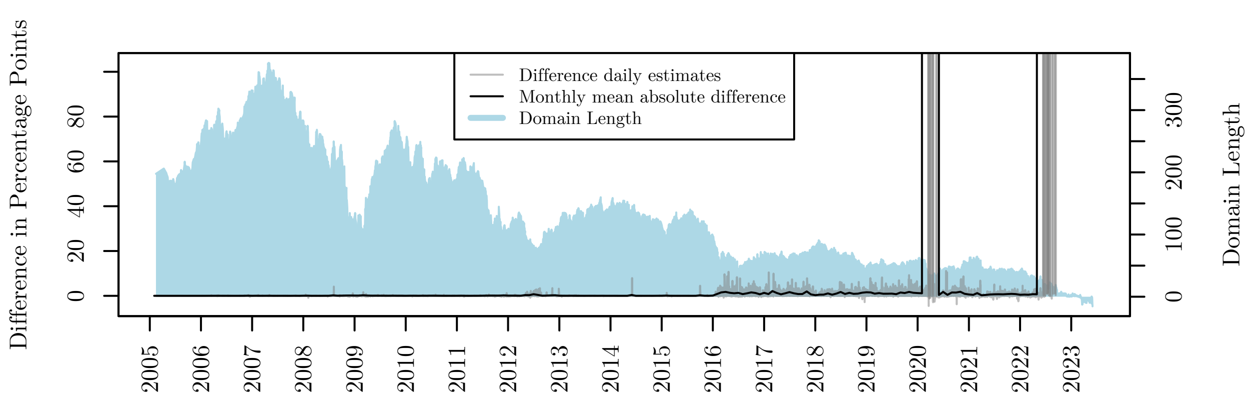
[13] https://www.ffiec.gov/npw/Institution/TopHoldings

Figure 8: Comparison of selected banks (January 2022 - March 2023)




As can be seen in Figure 8, when the Swiss National Bank (SNB) and the Swiss Financial Market Supervisory Authority (FINMA) published their media statement about market uncertrainty and that the SNB will provide liquidity to Credit Suisse if necessary[14] on the 15. of March 2023, the estimates for Credit Suisse increased strongly relatively to its own history, while the estimates of the other banks remained relatively constant. However, when looking at the estimates for Valley National Bancorp, the magnitude of the increase for Credit Suisse is somewhat mitigated. An important note is that for Credit Suisse's estimates the multiplication factor for the current stock price to obtain $V_{\max}$ was set to 30 to avoid breaking the optimisation problem, while for the other banks this multiplication factor was set to 5 as in the application of the method in Vilsmeier (2014). According to Vilsmeier (2014), the domain length for estimating the posterior distribution and therefore the value of $V_{\max}$ can be chosen very large without significantly altering the results, which is why the obtained estimates should theoretically still be comparable, even with the different multiplication factor. Figure 9 displays the difference in the estimates obtained using a multiplication factor of 5 compared to 30:

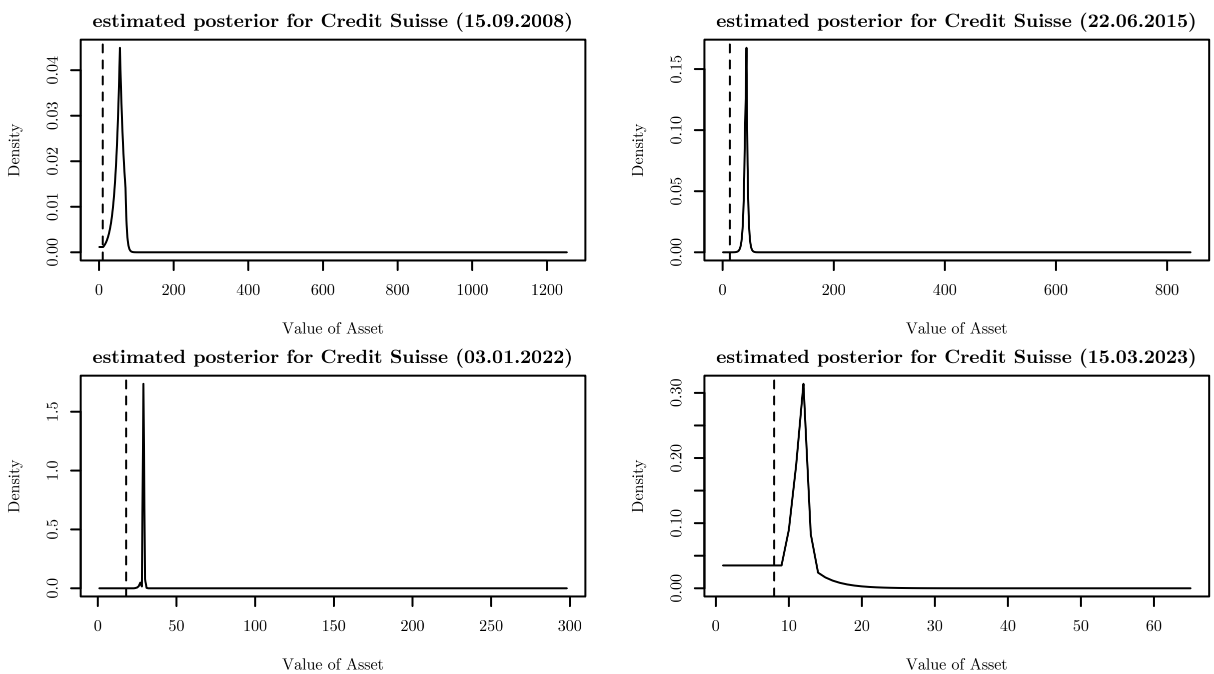
Figure 9: Difference in estimates (multiplication factor 5 vs. 30)




[14]https://www.snb.ch/en/mmr/reference/pre_20230315/source/pre_20230315.en.pdf

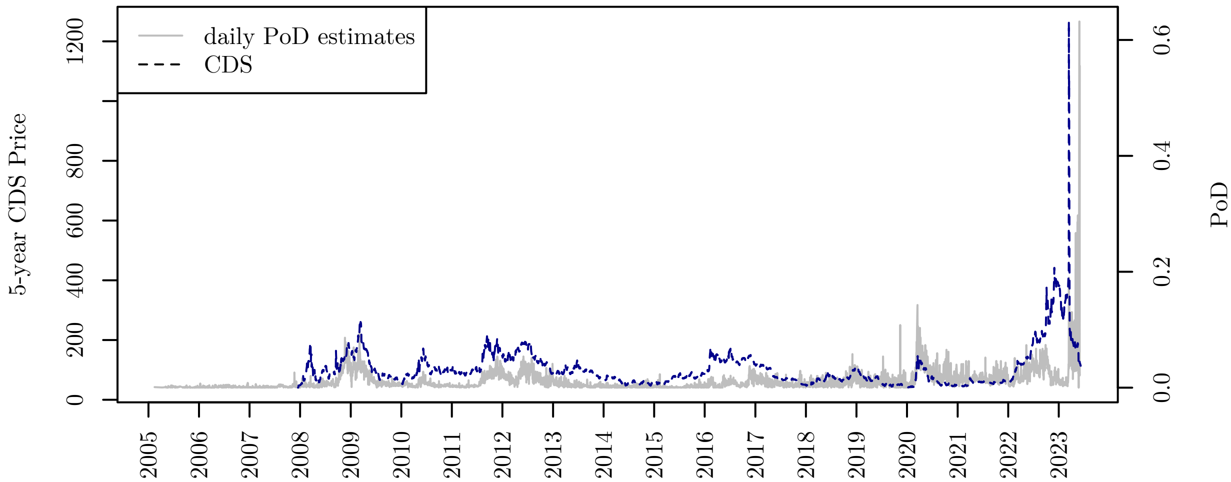
Note that the maximal displayed difference is limited to around 100 percentage points to keep the dynamics of differences as well as the relationship between the differences and the domain length more visible. In case of exploded estimates, differences reach values in the quadrillions percentage points. As visible in Figure 9, the differences between the estimates for larger domain lengths are neglectable and increase as domain length decreases. Figure 9 seems to confirm Vilsmeier (2014), who states that very large domain lengths do not influence the estimates significantly, while domain lengths that are too short can seriously distort the results. As stated in Vilsmeier (2014), the reason why domain lengths can be chosen to be very large is that the density that will be assigned to high asset value will practically be zero. Figure 10, which displays the probability density functions of some obtained posterior distributions confirms this statement by Vilsmeier (2014).

Figure 10: Estimated posterior distributions for Credit Suisse



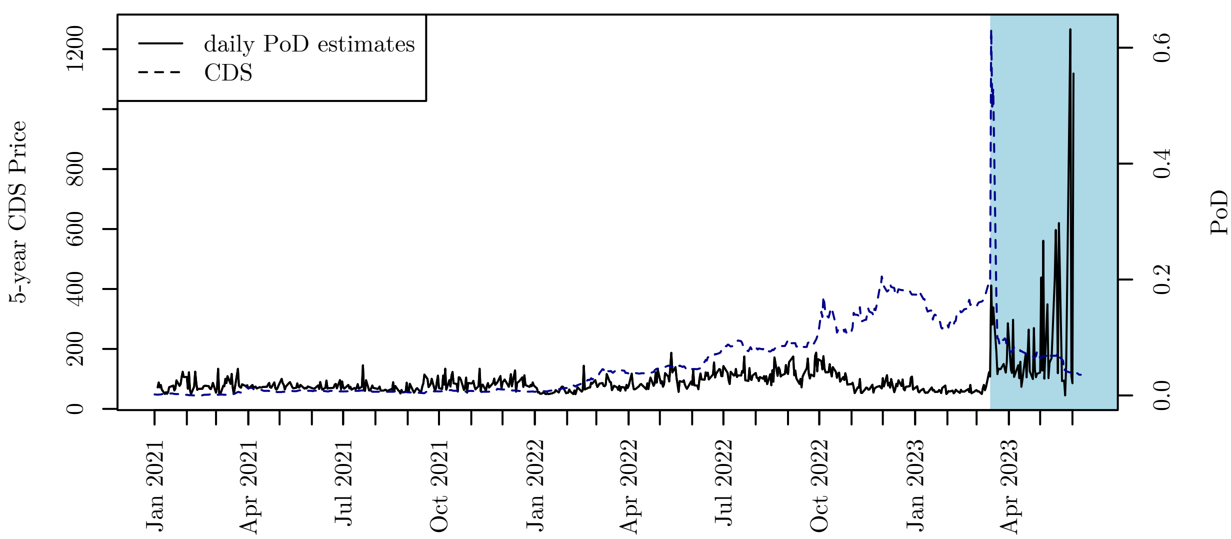

In order to assess the predictive power of the option implied probability of default, Figure 11 and 12 compare the movement of this measure to the movement to Credit Default Swap prices. Figure 11 shows the entire time series since 2005, while Figure 12 focuses on the most recent turmoil of Credit Suisse, whereby the period since the publication of the statement by SNB and FINMA is marked blue. Note that the magnitude of the measures cannot be directly compared using this plot, only the change of the measures relative to their own history can indicate how the measures react to increased or decreased episodes of financial stress. Figure 11 shows that both measures identify the same episodes of increased financial distress for the period since 2005 up until the end of 2021.

Figure 11: Probability of Default and CDS 2005-2023 (Credit Suisse)




Examining the period since 2022 more carefully, Figure 12 shows that Credit Default Swaps seem to indicate a steadily increase of risk for Credit Suisse from the second half of 2022 onwards. Meanwile, the option implied probability of default rather shows the opposite up until the beginning of March 2023, when first a modest increase is visible, which results in a sharp increase on the 15. of March. After a small descent at a high level, the CDS price rises starting in February 2023, again with a huge increase observable on the 15. of March.

Figure 12: Probability of Default and CDS 2022-2023 (Credit Suisse)




In this concrete application, using the data described in Section 3.1 and the method described in Section 3.3, the prices of Credit Default Swaps appear to have more predictive power than the option implied probability. The option implied probability of default estimates obtained here did not indicate the increased risk in advance to officially released information. However, the measure reacted as expected to the official reported default of Credit Suisse.

## 5 Conclusion

The results of this project show, that the measure of option implied probability of default reacts to major events as expected, but the predictive power of this measure as found in Capuano (2008) and Matros and Vilsmeier (2012) compared to Credit Default Swaps could not be confirmed for this specific application on Credit Suisse. However, the results are not intended to prove the validity or invalidity of this measure but might indicate that further research and a more extensive application on the banking crisis could be worthwhile to further assess why this measure works for some applications and does not work in other. For more detailed information about the measure and its properties, please refer to Vilsmeier (2014). Further research could address in particular the determination of the threshold $D$ and $V_{\max}$ which will define the domain bounds considered in the optimisation problem.

A potential problem within this project could be the data used. All available option chains were used for the estimations, whereby the weighting of specific option contracts was made only within the corresponding option chains. Therefore, the estimate which is based on an option chain which includes many different option contracts which were traded frequently is treated equally with an estimate obtained from an option chain which possibly only includes one option in addition to the current stock price. Put differently, estimates based on option chains that contain a lot of information, and therefore could potentially better reflect market participants' true expectations, are treated equally to estimates based on option chains that contain little information, and potentially do not reflect market expectations at all. This could potentially alter the dynamics of the resulting time series or can directly alter the daily estimate of the probability of default, since the daily estimates were obtained by taking the unweighted average of the estimates based on all option chains on the corresponding day. Therefore, it would need to be examined in more detail whether the data set provided to the estimation process would need to be filtered before use. In addition, the method of aggregating estimates that are based on different option chains, but were traded on the same day, needs to be further investigated. The probably better approach would have been to consider all contracts traded on the same day within the same optimisation problem, since the different expiration dates of the contracts would have already been considered by the discounting factor that depends to the time to maturity. Since this approach would require strong modifications of the code for the estimation process, this approach was not implemented within this project. The problem could potentially also be addressed by using a weighted rather than an unweighted average, giving greater weight to option chains that reveal more reliable information.

Matros and Vilsmeier (2012) demonstrate in their studies further possible applications of the option implied probability as an indicator for systemic risk, thus expanding this concept as a purely idiosyncratic measure. Analysing the dynamics of their obtained systematic risk, Matros and Vilsmeier (2012) showed high signalling and predictive power of this measure. Additionally, Matros and Vilsmeier (2012) showed three indicators measuring the risk of a specific bank relative to the prevailing risk in the financial sector, relative to the risk of the most resilient bank within the sector as well as relative to the bank's risk in the past. With their relative risk analysis, Matros and Vilsmeier (2012) could identify the high risk banks before the bankruptcy of Lehman Brother's.

Since only observed market data is used for estimating the measure of the option implied probability of default, this measure could indicate the build up of default risk for a specific firm, or with the extensions from Matros and Vilsmeier (2012) of the financial sector as a whole, even on an intra-day frequency. Therefore, this measure could be a highly attractive indicator for monitoring the banking sector.

# References

The code used for this project can be found on GitHub: `https://github.com/bt-koch/optipod`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