

Computing

# OpenGL N-Body Simulator

**A2 Coursework**

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March 2016

Northgate Sixth Form

Typeset with L<sup>A</sup>T<sub>E</sub>X

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# 1. Analysis

## 1.1. Background

Mr Snowden is a Physics Teacher at Northgate High School, teaching the subject at levels from KS3 to A-Level, there are also several other teachers in the department. During lessons, there are some times where it can be useful to the students to be able to see a model of the subject that they are learning about, while the majority of subjects that are taught have a range of models available, both real and software models. However one of the subjects where models are quite limited is in orbital mechanics and circular motion, with most being just simple animations and not a fully interactive or real-time simulation. The main disadvantage of this is that the students are not able to easily see how different modifications to a system will affect the outcome, something that can help to improve the understanding of a particular topic in some scenarios.

### 1.1.1. Project Objective

Mr Snowden requires a teaching tool that can provide a graphical sandbox to create and simulate various scenarios to fortify and replace the currently used tool-set of simple animations and power-point presentations that are used in teaching. This would allow the teacher in a physics lesson to quickly show students how a situation would develop, making use of SI units in order to effectively show students how different parameters (Mass, Size and Velocity) would affect a situation.

### 1.1.2. Prospective Users

While Mr Snowden is intended to be the primary user, other teachers could also have access to the tool as it could be put on the shared network drive, making the tool easy to access and used as a teaching aid in lessons, the tool will also be accessed by students, allowing the tool to be used as a revision resource for individual students on the school computers or at home. Because of this special attention should be paid to make sure that the program is as compatible as possible. Because the users of the program are teachers and students (most likely A-Level) a good amount of technical knowledge in the field can be presumed, the tool will be used as a teaching aide, so it is not a requirement to explain the Physics in the program itself. It will be clear to anybody that studies Physics to a basic level what all the adjustment factors are.

## 1.2. Current System

The current system that is in place for teaching of orbital mechanics and in particular, newtons law of gravitation is mainly through the use of straight mathematical teaching, using the equations. Teachers will often write notes and examples for students to work through, past exam questions are also a popular choice.

Currently, simulation applets such as those found on the phET website are quite commonly used, these are mostly predefined animations with very few of them actually being true simulations. Even the ones that are actual simulations are extremely limited and not particularly visually appealing. It is these tools that this project will aim to replace.

The main disadvantage with the phET tools is that their simulation is often not particularly accurate, one tool has a maximum of 4 bodies that it is able to simulate.

By having a application that has a very basic visual style, it will be a lot easier for a teacher to pause the simulation and annotate the board with extra information for students, such as going through a calculation in the current simulation state to prove the maths.

## 1.3. Requirements and Processes

The interface must be intuitive and self-explanatory in order to ensure that it is easy for teachers to understand, meaning that keyboard short-cuts should be ones that are commonly used in software (Such as *Ctrl* – *S* for save) and any buttons should be clearly labelled. It must also be able to run well on the school computers, most of which are running on low end hardware (Dual-Core HT Intel i3 / Quad-Core Intel i5, 2-4GB RAM) using a 32-Bit Windows operating system, as this is what all computers in the school are running for internal compatibility reasons. This applies a limit of 4GB to the total system RAM but a maximum per program usage of 2GB, meaning that the program will need to handle memory efficiently and potentially have a limit on the number of bodies (In the range of 1000s) to avoid using too much memory and causing the system to crash, a sensible limit would be 100MB. The number of bodies that this would support can be estimated however actual memory usage will likely be higher due to graphics library overhead, meaning that memory profiling should be used.

The program must not require installation on the system or any secondary redistributes to be installed containing dynamic libraries.

What is not needed is a 3D simulation system, as only a simplistic 2D model needs to be explained to students. The use of 3D would require an extremely complex user interface and processing in order to implement in a usable fashion.

Initially, the system will not be handling massive numbers of bodies, at most it will be handling around 1000 bodies at maximum, however no limits should be placed on the number of bodies it is possible to use, as performance will vary depending on the particular hardware the program is run on. Practically, most demonstrations will only make use of two or three bodies.

### **1.3.1. Graphics Processing**

While the resources of these systems are limited, keeping the program to being 2D should help to improve performance as there will be less demand on the integrated GPU (Graphics Processing Unit), meaning that it requires less system resources to display the simulation. This means that an external graphics library would need to be used in order to interface with this hardware and still function across different platforms. (The two main GPU Vendors, AMD and Nvidia, both have different interfaces for using GPUs for compute applications)

Having a 2D model also means that far fewer calculations need to be made for the simulation, making it faster, but it also reduces the load GPU in terms of rendering, As 3D would require shading and lighting to make depth perceivable. Input and camera movement would also be far more complex.

### **1.3.2. Calculations and Trigonometry**

The application must be able to simulate the paths of at least three bodies, as this will encounter the n-body problem ( $n = \text{Any positive integer}$ ), which in essence means that any interaction between two bodies can simply be predicted, however there is no direct solution for three or more bodies, and must be simulated through iterative process between each body to every other body. The main calculation that will be used between bodies is:

$$F = \frac{Gm_1m_2}{d^2}$$

This calculation produces a result for the gravitational force between objects, taking into account the mass of the two planets and the distance between them, based on a universal gravitational constant. ( $6.67408 \times 10^{-11} m^3 kg^{-1} s^{-2}$ ) While this equation only works between two bodies, the equation can be repeated in an iterative fashion in order to calculate the forces between all bodies in a system, the force can then be resolved to a single dimension for X and Y respectively, and then using a value for mass, acceleration can be calculated. By multiplying the acceleration by the change in time ( $\Delta t$ ) of the simulation you can propagate a velocity ( $v$ ) and change the position ( $p$ ) of a body.

$$F_y = F \sin \theta \quad F_x = F \cos \theta$$

The main issue that could crop up is the computational cost of the trigonometric functions, while the instruction set used by processors in nearly all modern desktop computers (x86) has specific instructions for these functions. The issue is that these still take longer than simple mathematical operators, which generally only take single instruction cycles. The difficulty is however that it will be heavily dependant both on the particular CPU running the code, but also the compiler used as well as the implementation used by the programming languages libraries.

There is also the issue that signs of calculated components to be flipped depending on which quadrant the simulated particle is in, adding further computational complexity. Trigonometric functions are also required to calculate the angle that the force is acting relative to the global X axis.

Because of these difficulties, it is worth investigating alternative methods for the calculation of the forces, as there could be far simpler methods that reduce the required complexity of code.

### 1.3.3. Performance and Multithreading

*Since the Mid-2000s, CPU Clock speeds have all but stopped increasing in the consumer market, with most coming in at the 2GHz to 3GHz range. Some high end desktop processors are able to be effectively over-clocked, with processors like the AMD FX-9590 capable of 5GHz as long as adequate cooling is provided, the current world record is 8.794GHz, achieved by an AMD FX-8350, however this with the use of Liquid Nitrogen to keep it cool.*



*While clock speeds are effectively limited by the stability that can be achieved in the material being used (Silicon) and thermal properties, the focus has now switched to increasing the core count of CPUs, as well as improving their power efficiency and increasing their transistor count by shrinking the size of the transistors on die. (Consumer Intel CPUs are currently at 14nm, with 10nm on road map for 2017 and 5nm by 2021)*

*Most Modern CPUs can be found with a core count of 2 or 4, with technologies such as hyper-threading allowing for multiple threads to be executed per core, effectively increasing the number of 'logical cores' by a factor of 2. High end desktop and workstation CPUs can be found with 6-12 physical cores, allowing for 12-24 consecutive threads. (Server CPUs are now coming with upwards of 18 physical cores, giving them 36 threads. These systems can also support multiple CPUs on the same motherboard, up to 8, allowing for effectively 288 consecutive threads on a single server.)*

Mutli-threading leads to is the potential to speed up the execution of the program by computing multiple parts of it at the same time. All of the computers in the school are new enough to have at a minimum dual core Intel i3 with hyper-threading, giving them a total of 4 'logical cores', this means that there is definitely some potential to improve the execution speed of the program.

This opens up new challenges in the form of ensuring that the programming is '*thread safe*', this means that the program is not going to '*step on its own toes*', for example modifying data while another thread is reading it causing lock-ups, or race conditions where one thread completes a task before another and the program fails. When done properly the improvements to performance can be quite impressive.

#### **1.3.4. Saving of Scenarios**

The system will also need the ability to save the current scenario to a file, most likely a plain comma separated value file with a different file extension such as .sav, this will allow a teacher to set up a scenario inside the program outside of lesson time and save it for a future lesson, at which point it can be loaded into the program and used as a quick demo. These files could also be provided to students for use in their revision. Data for every body would need to be stored, namely Mass, Velocity and Position, constants could also be stored which would enable them to be changed in particular scenarios.

### 1.3.5. Scale

It is unlikely that true scale would translate particularly well to a program like this without visual aids added to the program, as distances are extremely massive, and planets are extremely small in comparison to the sun, let alone the distances involved, for example, Earth has a diameter of only 12700 km, however it orbits the sun at a height of 150 Million km. At a zoom where both the sun and earth will be visible, the earth will be too small to make out, even on extremely high resolution monitors.

## 1.4. Data Management

### 1.4.1. Data Sources and Destinations

Table 1: Data Sources and Destinations

| Data Name               | Source                 | Destination                  |
|-------------------------|------------------------|------------------------------|
| Mouse Coord / Key Press | Mouse / Keyboard       | Input Handling               |
| Saved Scenarios         | File System            | Render Scenario              |
| Object Attributes       | User Input / Scenarios | Render / Sim Scenario / File |

### 1.4.2. Data Volumes

Direct user input will be relatively low volume, taken in using keyboard short-cuts and mouse control, the user operates the program in order to place down objects, adjusting the initial size, mass and velocity of bodies in order to set-up the scenario, the user can also stop and start the program and change the speed of time within the simulation as well as other constants.

The loading and saving of scenarios will be sending and receiving similar data to what is provided by user inputs, however the data for an entire scenario will be transferred in one go, this includes the mass and size of bodies, as well as the most recent resolved (X, Y), Force, Velocity and Position of each simulated body, various constants that are set should also be stored, such as the gravitational constant, which can be changed by the user.

During the running of the simulation, the program will aim to keep the frame-rate (screen update) at 60 frames per second, (Ensures very smooth animation and respon-

siveness), this means that the data volumes during the running of the simulation are very high, as each frame could be a simulation update, this includes updating the resolved (X, Y) forces, velocity, acceleration and position of each body, according to how much simulation time passed per each frame.

### 1.4.3. Data Dictionary

The main flow of data around the program will be body data, as there will need to be a container of some data type that will contain a large number of these bodies. Because this is a simulation, I want to make use of the common double data type, this is the IEEE Standard, 64 bit variable providing anywhere from 15-17 digits of precision depending on the specific hardware and software implementation used.

While a float variable is smaller and only 32-Bits long, it is generally considered that there will be minimal computational benefit from using float instead of double on CPU bound simulations. In the majority of language mathematical functions take in double variables. While these can still take float variables, the operation will still performed as a double variable.

The double variable will also be useful for providing extra precision when it comes to the simulation variables, allowing the simulation to be potentially more accurate, however there may be situations where float can be beneficial to the memory footprint of the program.

It is also worth noting that if the simulation was being written to run on a GPU, the variables would need to be stored as floats unless a GPU with a large amount of double-precision units was being used as the target, as most GPUs will only have a small number of double-precision cores, often resulting in their performance being lower than what could be achieved using a CPU.

The variables written in the table below are likely to change as this project develops, particularly once the specific implementation is picked, things like program flow are more likely to develop during the programming stage.

Table 2: Data Dictionary

| Data Name               | Data Type                            | Description   |
|-------------------------|--------------------------------------|---|
| Mass of Body            | Double                               | The mass of a particular body, every body will have mass. Defined as an objects reluctance to move, allows for calculation of forces, acceleration and potentially collisions.  |
| Radius of Body          | Double                               | The graphical size of an object, could be used for calculation of collisions.   |
| Force between Bodies    | Double                               | Direct calculation for force between two bodies. calculated for every body to every other body. (Vector)  |
| Angle of Force          | Double                               | This will contain the angle of the force acting on the body relative to global Y axis.  |
| Force X/Y               | Double                               | Force between bodies resolved to global X/Y axis.   |
| Acceleration X/Y        | Double                               | Acceleration can be calculated using the mass of the body and the X/Y force.  |
| Initial Velocity        | Double                               | An initial velocity that is specified by the user when a body is placed. (Vector)   |
| Angle of Velocity       | Double                               | This contains the angle of the initial velocity of a body, relative to the positive global Y axis.  |
| Velocity X/Y            | Double                               | The initial velocity will be resolved to global X/Y axis, this value is also modified when the change in velocity is calculated using per frame time and Acceleration X/Y.  |
| Position X/Y            | Double                               | Position is first populated with the position that the user places the body at, it is then modified depending on the X/Y Velocity, updating its position, this is also used when resolving forces and velocity, it is also used to calculate the distance between two bodies to calculate the vector force.   |
| Distance between Bodies | Double                               | The distance between the two bodies in current calculation, calculated using the positions of the two objects being simulated.  |
| Body                    | Complex Structure ( $\approx 120B$ ) | This will define a particular body, containing all the variables that relate to a particular body, stored in some kind of structure / object.   |
| Gravitational Constant  | Double                               | This defines the gravitational constant which is used to calculate the force that gravity will exert on two bodies, real value is $6.67408 \times 10^{-11} m^3 kg^{-1} s^{-2}$ However this value will not be particularly useful and should be something larger to allow for smaller scale simulations. The user will be able to modify this themselves. |
| Time Multiplier         | Double                               | This value will be used when calculating the simulation time per displayed frame to allow the simulation to be run faster or slower than real time  |

Table 3: Data Dictionary Cont.

| Data Name                 | Data Type         | Description   |
|---------------------------|-------------------|---|
| Paused                    | Boolean           | This variable will be a simple TRUE/FALSE describing if the simulation is paused, time will not advance when this is true.  |
| Scenario                  | Complex Structure | Size is dependant on the number of bodies present in the simulation, Also contains constant variables that affect simulation, namely time multiplier and the gravitational constant, as well as if the simulation is paused, it may be more efficient and safer to have separate scenarios for simulation and rendering, copying the simulation scenario over to render when a frame has finished simulation. |
| Pi $\pi$                  | Constant          | Pi is the ratio of a circle's circumference to its diameter, it is defined as 3.14159265358979323846 as a constant in the C++ cmath library. While Pi is not entirely necessary for the calculations used in the simulation, it may be useful for writing other functions, particularly when converting between Radians and Degrees.  |
| $F = \frac{Gm_1m_2}{d^2}$ | Formula           | This equation can be used for calculating the force between two bodies due to gravity, F is a vector Force, G is the Gravitational Constant, m is the mass of a body, d is the distance between the bodies.   |
| $F_y = F \sin \theta$     | Formula           | This equation allows the vector Force or Velocity to be resolved to a single axis/dimension. This resolves the y axis. This will be the main computational cost of the program.   |
| $F_x = F \cos \theta$     | Formula           | This equation allows the vector Force or Velocity to be resolved to a single axis/dimension. This resolves the x axis. This will be the main computational cost of the program  |
| $F = ma$                  | Formula           | This equation is one of the first principles of physics, Newton's Second Law, F is force, m is Mass, a is Acceleration.   |
| $p = mv$                  | Formula           | This equation describes how momentum is related to mass and velocity, this equation can be performed on both the x and y axis in the event of a collision between bodies to calculate its final trajectory, momentum is conserved through collisions.   |

#### 1.4.4. Data Flow Diagrams

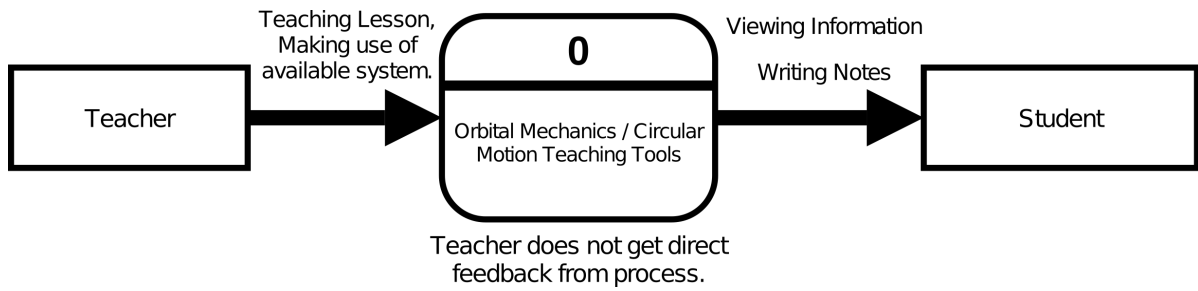


Figure 1: Current System, Level 0

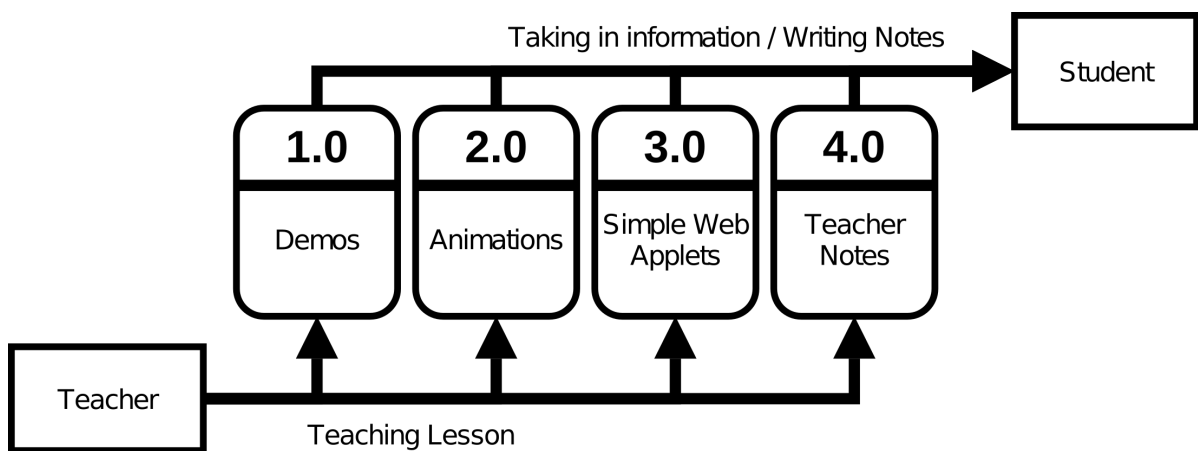


Figure 2: Current System, Level 1

The program can be used by either teachers or students, teachers can also provide preset scenarios for pupils to make use of in their study.

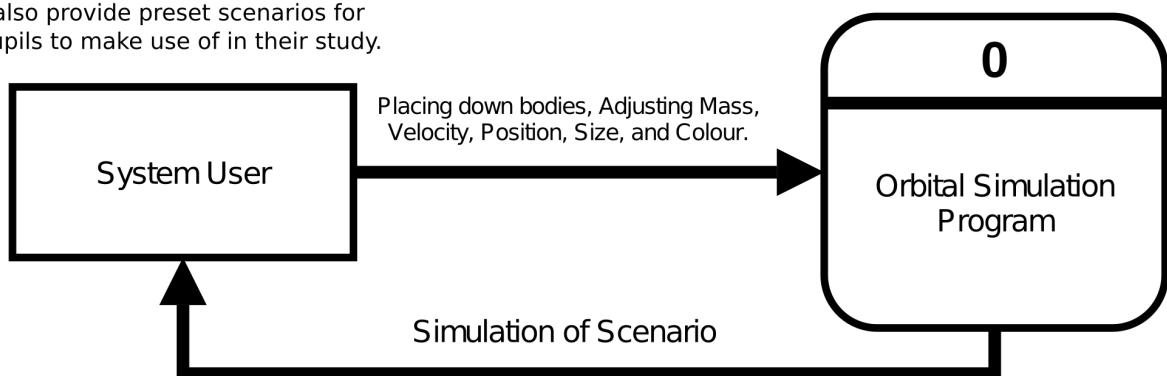


Figure 3: New System, Level 0

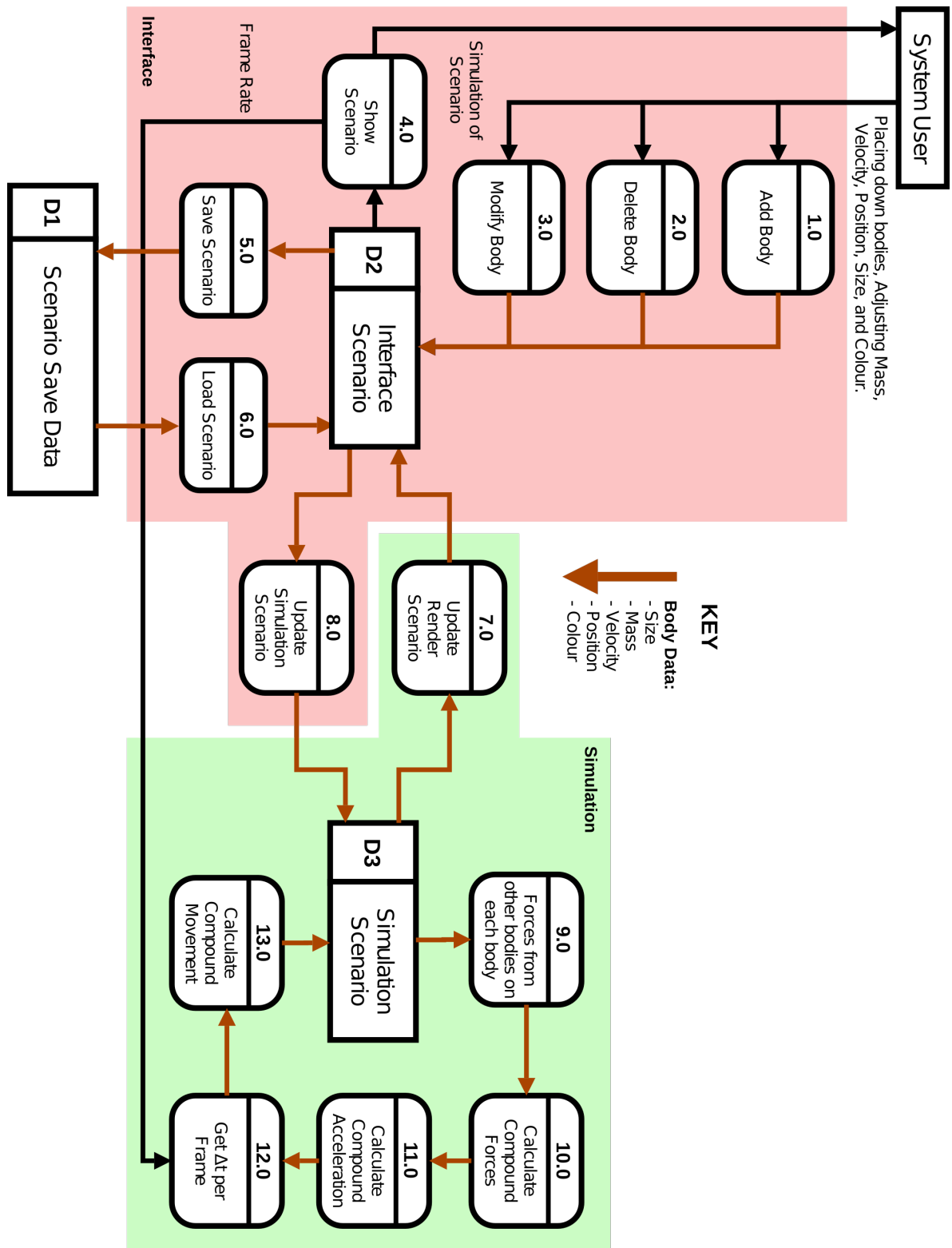


Figure 4: New System, Level 1



## 1.5. Object Orientation

Because of object based nature of the program, object orientation may be used to satisfy the need to support a fully dynamic system that allows any number of bodies to be added or deleted at any time during the runtime of the program.

The benefit of object orientation is that objects can be defined as a structure where access to certain elements is controlled, requiring methods to change attributes allowing validation to be done on objects themselves, objects that are similar to each other can inherit a common base to minimise repetition of code.

The basic structure of this program would require local classes for the render (main) thread and for the simulation, these will need to be similar but will require their own specific sets of functions, meaning that inheritance can be used to create a base class and then add specific functions to each individual class.

A class is also required for the shared area, this is a separate entity containing two body stores, one for each local thread to send data to for the other thread to pull from.

The body object itself will contain all of the attributes required for the bodies to 'exist' in the scenario, such as mass, radius (etc), these bodies will be contained within the local classes inside container objects (array/vector)

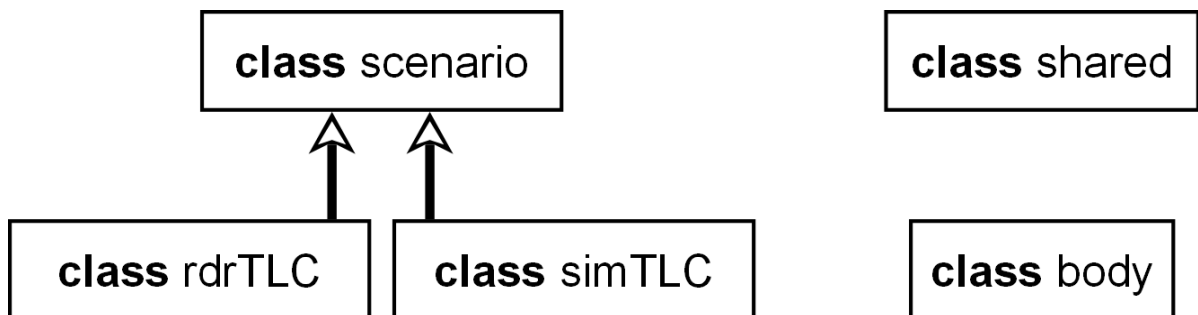


Figure 5: Basic Inheritance Diagram

## 1.6. Limitations

When it comes to limitations, the main one to consider will be the dimensions of the simulation, while it would be relatively straightforward to program a 3D simulation, it would increase the computational requirement for the program as well as make the rendering far more complex, requiring some form of lighting and shading calculations to make the display useful in any way.

The disadvantage of having a more complicated visual display is that it will be much easier to miss displayed information on the screen, particularly from the view of students. This is because the teacher will likely be projecting the application onto the whiteboard, due to the nature of the projectors they don't produce the best quality image and often struggle with contrast, having complex shading would just serve to make matters worse.

The system being used is as mentioned previously something of a brute force method, while more efficient algorithms exist, such as Barnes-Hut I do not plan on implementing these, as they are more suited to extremely large simulations with hundreds of thousands to hundreds of billions of bodies over galactic scales. Often these are implemented on supercomputer clusters.

## 1.7. Objectives

### 1.7.1. System

1. Target performance is 60 frames per second with 500 bodies on screen at minimum.
2. The graphics used in the program should be rudimentary 2D vector shapes and lines of varying colours in order to effectively convey the information without becoming too crowded.
3. The program must be compiled to run under 32-bit Windows. it should also not require any external dependencies to be installed in order to run and should be able to be run from any folder on any computer.
4. The size of the scenario should be limited so that objects the precision on the position of the far bodies is not too large, which would potentially affect the simulation. A large limit that makes full use of the precision of the double floating point used in the simulation would be preferred. (15 Digits)

### 1.7.2. Processing

1. The application will use the gravitational force equation to calculate the force between two bodies and newtons second law to convert to acceleration.
2. Trigonometric functions are the currently known way to resolve vector forces to a global X and Y axis. Because of the computational cost these are likely to have an alternative should be investigated.
3. Collisions can be simulated using basic conservation of momentum equation, in order to calculate the resultant direction of travel.
4. The simulation should be developed with memory usage in mind and tailor algorithms to be memory efficient, the absolute maximum the program can use is 100MB.
5. The system should make use of dedicated hardware features such as the GPU to accelerate rendering, this will require external libraries for ease of porting.
6. The system must be able to remain responsive even with a large number of bodies on screen, meaning there must be a degree of separation between the execution of the simulation and rendering.
7. The system will only simulate a 2D scenario as 3D simulation would increase the computational requirement and require a much more complex user interface.

### 1.7.3. User

1. The interface to the program should be basic and self explanatory, making use of the mouse for the placement (position) of bodies and definition of size, mass and velocity, the keyboard can be used for more precise entry of values.
2. It should be possible for the the user to save scenarios that have been set-up so that they can be accessed at a later date or shared with others. The data can be saved to a single file that can be loaded into the program, allowing teachers to set up a scenario outside lesson time to use as a demo.
3. The output of the program should be able to show up well on a projector, meaning that bodies must contrast highly with the background as well as with each other, it may be useful to allow the user to choose the colour of bodies.
4. The user should be able to change any of the values for bodies, position, mass, velocity, should all be possible to change on the fly even when the simulation is running.
5. The user should also be able to change variables such as the gravitational constant

and time acceleration to suit smaller scale simulations which will be more visible and understandable to students.

## **1.8. User Dialogue**

Email correspondence with the user can be found in Appendix A.

## 1.9. Potential Solutions

### 1.9.1. C++, OpenGL, GLFW

C/C++ using the GLFW OpenGL library, this library provides a window management library, handling operating system specific tasks. OpenGL is mainly geared towards 3D rendering, however it is still possible to create a simple 2D scene in the software. The C++ standard library also contains support for multi-threaded applications.

#### Advantages

- I already have an understanding of the C Programming Language, the syntax used by C++ is near identical to C and has a more modern feature set as well as some extra features, such as support for object oriented programming.
- OpenGL makes direct use of the GPU for rendering of scenes, meaning that high performance will be easier to achieve than with a software based renderer (Using the CPU).
- The GLFW library contains support for multithreading, this can potentially be used to gain large performance boosts particularly for simulation. (Multithreading is also defined in C++ the standard library. (C++2011))
- GLFW takes a lot of the tedium out of programming for OpenGL, mainly in the creation and display of a window that fits in with the current operating system. It also contains support for keyboard and mouse inputs to the program. Traditionally, setting up a window for an OpenGL context would require a large amount of operating specific programming that does not warrant itself for small projects.
- GLFW is also a multi-platform library, making it significantly easier to port the program to other platforms should it be necessary.
- GLFW is an extremely minimal library, simply providing the utility to create OpenGL contexts to draw to and get the input from it. By including the GLFW header you also include the OpenGL headers as well, which are generally found in any standard development package.

#### Disadvantages

- On Windows systems, and external .dll (Dynamic Linked Library) will be required in the program directory in order for the program to run.
- The library may contain excess bulk that will increase the size of the program when compiled, even if the entire library isn't being used.

- There will be a large increase in the memory usage of the program due to the use of the library and set-up that is required, particularly for OpenGL, this will be in the range of 200MB.
- As the library is not written by me, there may be bugs in the code, however the library is open source and maintained by a relatively large group of people, meaning this would be quickly fixed.
- C++ is a complicated language and is known for having longer compile times than a language like C.
- GLFW has no inbuilt support for GUI, there are extra libraries available that could be used in order to create basic GUIs, such as AntTweakBar.
- The issue with both GLFW and AntTweakBar is that they are written with C style interfaces, this can make integrating into a C++ program in a clean way somewhat more difficult.
- Hardware will support a particular version of OpenGL and below, for example a GPU that supports OpenGL 3.0 will not support programs written for OpenGL 4.0, and most computers in the school will have a processor with new enough integrated graphics to support GLFW, which requires an OpenGL version above 3.2.
- OpenGL has some added complexity in terms of its use of matrix mathematics, something that can be something difficult to follow at times.

### 1.9.2. Pascal, Graph, Forms

Pascal contains an inbuilt unit known as graph, this allows the program to create a window and provide basic graphical output to the user.

#### Advantages

- Pascal is a language that I am familiar with and would be relatively understandable to write the program using Pascal, the more modern Free Pascal Compiler also supports Object Pascal, which is an object orientated subset of Pascal.
- The graph library is self-contained and part of the default pascal library included with the compiler, meaning that no libraries for external sources would be necessary for display output.
- I have a decent understanding of writing programs in Pascal from my studies for my AS qualifications.
- Lazarus, one of the main IDEs for Pascal has inbuilt support for Forms, these allow the developer to produce a GUI application, a small form could be created alongside the graph window for the modification of properties of bodies.
- Pascal does support multi-threading, which could be used to improve the speed of computation.
- Pascal can be compiled for any platform without the need to change code, assuming external libraries are not used.

#### Disadvantages

- Pascal is much less supported at present than languages such as C or C++, while the compiler still gets updates, it has much less of an active community, the most likely place where this would affect the program is in the optimisations that have been applied to the compiler and how it translates code.
- Pascal has been shown to have lower performance than C in certain examples, which could be a problem as the application will be quite demanding of performance.
- Pascal has no native way to obtain mouse input from the graph screen output, meaning that the program would either be limited to just keyboard inputs or an external library of some kind would likely be required in order to get mouse input.
- Pascal as a language is extremely verbose, using full words such as begin and end as opposed to braces, { and } as delimiters, this can lead to making the code

extremely difficult to read when there is a large number of these on the screen.

- Pascal also is not case-sensitive, which can lead to some confusion when a particular case styling is not kept throughout the code.
- Pascal has a large amount of functions included in the standard library, without needing to define any extra uses, leading to potentially larger binaries as these are linked in even if they are not used.
- The graph library is also not particularly efficient, and relies on CPU software bound rendering to draw to the display, meaning that it is unlikely that the target update rate could be achieved.



### 1.9.3. C++, OpenGL, SFML

SFML (Simple and Fast Multimedia Library) and C/C++ (Also has bindings for .NET, Java, Ruby, Python, Pascal and many other languages, this library is geared towards the development of 2D applications, it is a more modern alternative to SDL (Simple DirectMedia Layer) built in C++ from the ground up with object orientation in mind. SFML can be used a lightweight window manager for an OpenGL application or as a full multimedia library, making use of all of the features that it provides.

#### Advantages

- SFML is relatively modern and is built around the idea of object orientation.
- SFML provides a window management wrapper for the creation of OpenGL applications, similar to GLFW, however it provides many more features, such as a layer of abstraction on top of the OpenGL, Networking (TCP / UDP) and Audio (OpenAL) features.
- It is possible to choose which parts of the library that you want to use, split into the following:
  - Systems
  - Window - Depends on System
  - Graphics - Depends on System, Window
  - Audio - Depends on System
  - Networking - Depends on System
- SFML will work on any platform that has an OpenGL version above 3.0.
- The library is cross platform and can be compiled to run on Windows, Linux and OSX.
- Similar to GLFW, SFML removes the need to produce complex code that allows the basic set-up of an OpenGL context that fits in with the surrounding operating system.
- Multi-threading support is also provided, however there seem to be fewer features for controlling the flow of said runtime.

#### Disadvantages

- The Audio and Networking components of SFML are not required for this project, while it is not necessary to include them, it removes most of what sets SFML apart from GLFW, the only functionally different part would be graphics, which

contains pre-defined functions for loading images and drawing shapes, which is also unnecessary as this is relatively straightforward to to using plain OpenGL, at this stage SFML is functionally identical to GLFW.

- Because of its relative newness (First released in 2007), SFML does not have nearly as much support available compared to GLFW, which was originally released in 2001.
- This library will also share many of the same disadvantages as GLFW due to its similarities.

#### **1.9.4. Conclusion**

Based on this analysis, I feel that Solution 1 would be best choice, namely because the minimal nature of the GLFW library, providing only the bare essentials for setting up an OpenGL context and system utilities such as multi-threading, it also can be programmed using C/C++, a language that I am familiar with and would be comfortable with writing a large scale program with, There are also tools that I can use that integrate well into the language to allow me to debug my code and step through it on a line by line basis.

While it would still be valid to use SFML and simply make use of the System and Window components and use it in a similar way to GLFW. It makes more sense to use the library that is more focused on the features that I plan to use.

## 1.10. Management and Tools

### 1.10.1. Linux and Development Environment

When it comes to the management and writing of the code, I will be using a bare-bones text editor with auto-complete plug-ins to make the experience somewhat smoother. A custom makefile will be used in order to create a custom build system which will make building an application with multiple source and header files much easier.

I plan on using the Linux operating system for the main development of the program, the main reason for this is that there are several tools that integrate very well into the Linux environment. As well as this, build tools and compilers are generally far better supported and more up to date on Linux.

In order to improve the organisation of code, different files will be split into separate directories source files will be found in *src/*, header files in *include/*, compiled object (.o) files will be stored in *bin/*. Having .o files in a separate directory means that the files can be kept after the program is linked, if any change is made to a particular source file only that file needs to be recompiled and the older files can just be linked together to produce a new executable.

Because I will be developing the software using Linux, I will need to work out an alternative build system to build an executable binary for Windows. It is possible to do this from within the Linux Environment. (e.g: CMake)

### 1.10.2. Git - Version Control

I will make use of tools such as *Git* for the management and version control of the source code, this allows me to make changes to my code and upload it to a remote server. The code can be 'pulled' on other computers, changes can be 'staged' and 'committed', the resulting 'commits' can then be 'pushed' to the remote server. Git provides options to split changes to code into separate branches for making experimental changes to the code without making changes to the 'master' code.

Another feature of Git is the ability to automatically merge code, if a multiple changes are made to code on different computers (Such as forgetting to pull changes from remote.) the modified code can be staged to a commit and the previous remote commit can be

pulled from the server. Git will then attempt to automatically merge the code together in a way that preserves its function. (In the case that it cannot, it presents differences to you and asks you to merge the code manually.)

It is possible to completely revert code back to any previously commit, all of these features make Git an invaluable tool for the maintenance of source code, even for small project. (It makes working on collaborative projects far easier also.) The fact that code is stored on an external server is an added bonus as it provides an off site backup and makes it possible to access code anywhere with an internet connection, but still make changes and commits to a local copy without a connection.

### **1.10.3. GDB - Debugging**

The GNU Debugger is an invaluable tool for the 'real-time' debugging of applications, assuming that the application has debugging symbols compiled in it is possible to run through code line by line stepping through code.

The tool allows individual variables to be looked at, allowing you to check that the program is running as intended. It will also return more information in the event of exceptions.

The main disadvantage to this tool is that even when the program is set to continuously run, it will run at a much lower speed than the program running outside the tool would run at, making this not a tool for profiling the performance of the application.

### **1.10.4. Valgrind - Profiling and Memory Analysis**

Valgrind is a tool which can be used for profiling certain aspects of the programs runtime, mainly looking at the programs use of memory.

The tool will print errors to a terminal in the case that it detects programming that could potentially be causing memory leaks to occur. After the program finishes or is exited the tool will print the total memory usage at the end of the program, it will also print out its analysis of potentially lost memory.

### **Included Tools**

- Memcheck - Memory error detector <sup>1</sup>

- Illegal Access
- Undefined Variables
- Incorrect Memory Management
- Overlapping Allocation
- Negative Memory Sizes
- Memory Leaks
- Cachegrind - Cache Interaction Simulator
- Callgrind - Call Profiling Tool <sup>1</sup>
  - Function Calls
  - Function 'Cost'
  - Includes Cache Simulator - Cachegrind
  - Other Events
- Helgrind - Multi-thread Synchronisation <sup>1</sup>
- DRD - Multi-threading Errors <sup>1</sup>
- Massif - Heap Profiler <sup>1</sup>
  - Memory Usage
  - Stack Usage
  - Long Term Leaks
- DHAT - Heap Allocation <sup>2</sup>
- SGCheck - Stack and Global Array Overrun
- BBV - Basic Block Vector <sup>2</sup>

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<sup>1</sup>Will Use

<sup>2</sup>May Use

## 2. Design

### 2.1. Simulation Efficiency

It is also worth noting that the method that I am using to simulate the orbits is generally known as being a brute force method, where every body in the simulation has all of its forces simulated based on every other body in the simulation, while this enables the most accurate method of simulation, it is not particularly efficient, and only really works well for small numbers of bodies, meaning that it would suit the purposes of this program.

There are some forces that will be calculated to be so insignificant that it will not majorly affect the outcome of the simulation. For example, the orbit of two bodies will not be massively affected by a third body that is extremely far away.

This principle is applied in the Barnes-Hut Algorithm, in which space is divided up into a quadtree. (octree for 3D simulation.) In this method, only particles/bodies that are close to the particle being simulated will be individually simulated. Particles that are further away will be simulated as one large particle based on its center of mass. This mainly increases the effectiveness if there are a large number of particles in a particular area, as this greatly reduces the number of interactions that need to be calculated.

While this method would have some benefits, the method is mainly designed for simulations with thousands of bodies or particles, something that is outside the scope for this project, it also makes use of a relatively complex algorithm that could be difficult to replicate in the time that I have available.

This may be something that is worth investigating at a later date should there be time, particularly if there is a need to simulate multiple thousands of bodies.

It may be possible to save some computational time by computing the values force forces inside a matrix. This would describe every particle to particle permutation that is possible and thus the forces between them in a undirected, weighted graph. While it won't really be used as a graph, it is a valid abstraction for the computation, removing duplication of a calculation that would decrease calculation performance.

## 2.2. Simulation Prototyping

The main thing that I wanted to check in the design of the project is that the mechanics that I plan to use will work correctly and as intended, due to the sheer volume of data involved, the easiest way to deal with this is to write a small program that runs through a simple scenario.

I started by writing a program that ran through a two body problem, using the intended algorithm and a fixed one second time step, I adjusted the gravitational constant to a more manageable number (1) that would work for small values of mass and velocity.

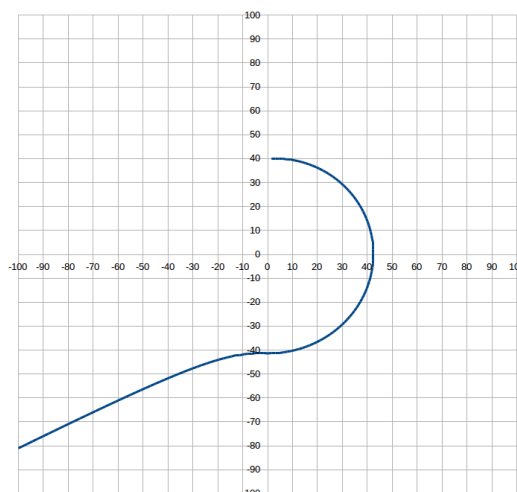


Figure 6: Inverted Negative Quadrant

After the program was running, I set up a quick terminal output of the position of the body, these values were then pasted directly into a spreadsheet application and displayed on a graph, much to my dismay, it did not quite work correctly, the expected result of this should be a near perfect circle that loops back on itself, the system is theoretically closed and should remain constant.

However this output clearly shows that the object completes half an orbit, and then suddenly decides to veer off in completely the wrong direction, this results in the distance to the body becoming greater and the force diminishing.

When looking at the other values, particularly the angle of the orbiting body to the barycentre (0, 0, fixed mass), the angle flips its sign, from -1.5 Radians to 1.5 Radians, this causes the forces to also flip their sign and the forces then act in the opposite direction, causing acceleration away from the central body into infinity.

The fix for this was to check when the orbit crosses the X axis and flip the sign of the resultant forces when it does so that the system remains in orbit, this works in preventing this problem, however it manages to highlight a new issue.

As the body orbits, its height increases dramatically, and mainly on the negative X side of the graph, it is pretty clear that energy is not being conserved in this closed system and the orbiting body is gaining energy allowing it to reach a higher and higher orbit.

Upon further reading, it appears that this is simply a flaw of the system that I have elected to use, known generally as Euler's Method, while this method is relatively simple to implement, it comes with significant disadvantages, the main one being is that a much smaller time base is required when bodies become extremely close together, meaning that a fixed time base will not be an option. The method does not maintain conservation of energy and orbits will become higher over time due to error accumulated in the equations, the sheer amount of calculations that need to be done only worsen this effect, this is more obvious when looking at a highly eccentric orbit. while I understood that this would be the case, I did not consider that it would be quite this severe.

At a mathematical level, the algorithm is performing a method of numerical integration through the equation  $s = ut + \frac{1}{2}at^2$  which in essence will find the area under a velocity / time graph which equates to the distance travelled. This method performs this equation at every single time step, during the time the acceleration is constant and it is performed over the provided time step. This results in an issue when the orbiting body is close to the central

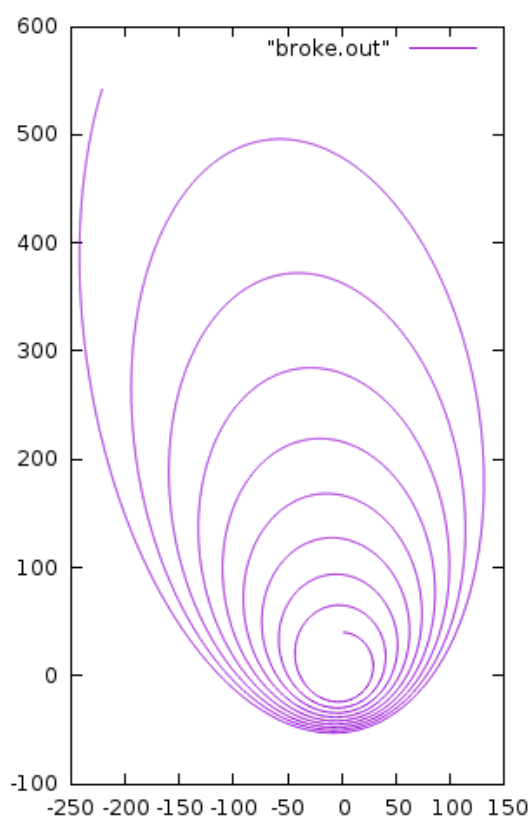


Figure 7: Extreme KE Increase

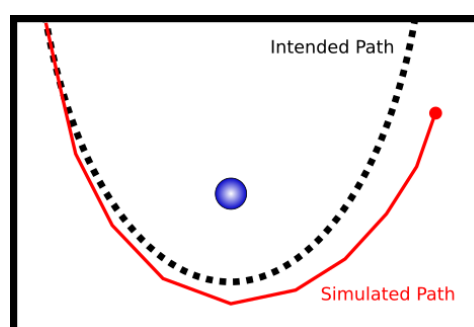


Figure 8: Integration Deviation



body, as the distance travelled over the fixed time step becomes too large to correctly map to the intended orbit path, which causes the accumulation of error and instability over time. This is widely known as Euler Integration, Acceleration, Velocity and Position ( $a \rightarrow v \rightarrow r$ ) are all updated sequentially in each time step.

### 2.2.1. Leapfrog Intergration

After some research into this problem, it was clear that this is not a problem that can be solved while still retaining the simple sequential  $a \rightarrow v \rightarrow r$  calculation flow and would require some investigation into alternate methods. One of the numerical integration methods that is commonly used in gravitational particle simulations is Leapfrog integration, this method involves updating the velocity half time step out of phase with the acceleration and position, hence the name leapfrog.

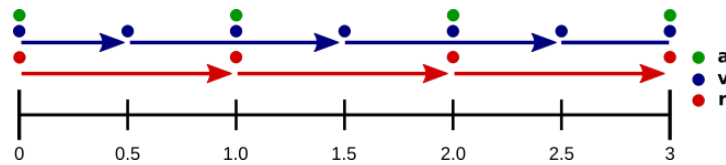


Figure 9: Leapfrog Flow

Because it is somewhat difficult to step for half an iteration as integer loop counters are generally preferred for performance reasons, there are versions of these equations that are synchronised to update simultaneously. This effectively then becomes a calculation flow of  $\frac{1}{2}v \rightarrow r \rightarrow a \rightarrow \frac{1}{2}v$ .

$$x_{i+1} = x_i + v_i \Delta t + \frac{1}{2} a_i \Delta t^2$$

$$v_{i+1} = v_i + \frac{1}{2} (a_i + a_{i+1}) \Delta t$$

Figure 10: Time-step Synchronised Leapfrog Formula

### 2.2.2. Vector Equations

These equations will apply to the the components of the vectors, the acceleration of the body is calculated using the universal gravitation calculation and the resulting force can be divided by the mass of the body in order to get the acceleration. This can then

be broken into x and y components and calculations can be performed on individual axis. The issue with this is that this requires the use of trigonometric functions both to find angles and to work out the ratios for components, these can come a quite high computational cost, while in low volumes this should not be an issue, with a larger number of bodies it could cause significant slow down.

Another issue that these functions can cause is when it comes to moving through a coordinate system, when an object goes negative on the x axis the sign on the angle becomes negative which results in the forces flipping, while this can be corrected it is somewhat messy and could cause issues further down the line.

A better way to compute the components is to directly calculate the forces as components by using a vector equation on both axis, this means that the trigonometric functions will not be necessary for the main simulation loop of the program, greatly improving the performance of said simulation.

In this form the equation for force becomes:

$$F_{xy} = \frac{Gm_1m_2}{r^3} \cdot \hat{r}_{xy}$$

The proof for this formula can be found in the document appendix.

The benefit of using this equation over the trigonometric equation is found in small benefits to performance, when it comes to the large amount of calculations that are required per second, but also when it comes to the complexity of the code, resulting in a program that is far easier to maintain and understand in the long run.

There will still be some cases where the trigonometric functions are required, for example it may be necessary when setting up bodies and the direction of their initial velocity, as it would be more understandable to the user to define starting conditions in terms of angle and magnitude. The main design points to focus on going forward are:

- OpenGL Rendered Output and Fully Interactive Interface
- Separate Threads for Simulation and Graphics/Interface
- The user Set-up of Scenario
- The user is able to save and load scenarios to and from the file-system.
- The user is able to modify variables in the simulation

- Mass of Bodies
- Position of Bodies
- Velocity of Bodies
- Radius of Bodies
- Gravitational Constant
- Simulation Time Step
- Iterations Per Frame
- It will be necessary to synchronise the threads in a way that limits the speed that the simulation can run at to the update speed of the renderer, which will likely be the bottleneck at low body counts.
- Target frame-rate for the display should be 60FPS to maintain a smooth interactive experience.

## 2.3. Threading

The threaded nature of this program lends itself extremely well to modular design, as new threads are assigned a thread from which they start, effectively becoming their own 'main' function.

The main difficulty when working with multiple threads is ensuring that they are well synchronised as to prevent race conditions, these occur when one thread completes a task before another which could lead to incorrect program execution. A similar issue is when working with shared data, multiple threads writing to the same data will cause exceptions in the program; reading data while it is being written may not throw an exception, but it will cause errors.

The main way that this will be dealt with this is to implement locks, these are variables that are written before a thread accesses certain data, another thread attempting to write to the data must first check the lock variable to ensure that data is free to write. Once a thread is done, the lock variable is cleared and another thread is safe to write to the shared data.

The benefits of using threading means that while one part of the program is managing a time consuming task such as rendering the display, the simulation can still be running on another thread, thus significantly improving the performance of the program by ensuring that the program is making the best possible use of time.

When it comes to the actual scenario containing body information, each thread will have its own local copy of the scenario, at the beginning of a simulation or graphics loop copy their current data to respective shared access stores, threads also have their own status 'registers' in shared memory area and can change bits in these variables to denote their current status throughout their execution path. In the case of the graphics and interface thread, the thread will turn on a certain bit in its register when it has received user input and a change has been made, when the simulation thread gets to this point it will read this bit and copy the changes to its local storage. This algorithm is best explained using a state transition diagram due to its branched complexity. Another method of locking the variables that may end up somewhat cleaner is to make use of Mutual Exclusion Objects; these are a feature of C++ which would produce a similar result to the prior explanation, they are however implemented in the compiler itself.

In short, thread safety will be ensured through the use of Thread-Local Storage (TLS) and locks on any shared data to prevent concurrent writes/reads. Thread registers will also be found in shared data describing the current state of a thread and its current task, as well as flags denoting new available data or requests for new data. The system will make use of a basic request > acknowledge system to ensure that it is correctly synchronised at all times. When the status registers are included. This means that the thread also has the ability to be re-entrant, allowing it to resume should the thread be interrupted.

In order for this re-entrancy to work correctly, the graphics processing and handling must be done on the initial thread, as the OpenGL wrapper library (GLFW) states that the OpenGL code is not thread-safe. (i.e. not re-entrant) The simulation component can then be handled in a secondary thread and can provide new simulation data to render into the shared data area, as well as setting register bits to denote that new data is available, when the render thread sees that new data is available, it will set its own bits to acknowledge that it has taken the new data, on the next loop, the simulation thread will clear its new data bit if the render thread has acknowledged and will eventually repeat the process, If for some reason the rendering is taking longer than usual and the new data has not been taken by the next loop the simulation thread will sleep until the new data has been taken, meaning that there is a small 1 deep buffer for the scenario.

It may be beneficial to implement a larger buffer for the simulation so that it can build up a backlog of new data frames to improve performance, however this is not a

priority as if user input is given the backlog would need to be discarded, meaning that this would only provide a performance gain should no changes be made by the user, and only if the simulation is running much faster than the renderer.

Another point for consideration is the actual parallelisation of the simulation, while currently the simulation is broken up into its own thread the carries out computations for all bodies, it would be possible to split the simulation itself onto multiple threads to improve the simulation speed further by running computations for each body on an individual thread, this would result in some duplication of calculations but the sign does not need to be reversed later on.

On a similar note, this same idea of parallelisation can be expanded much further, while an average consumer CPU may have the ability to run 4 concurrent threads, with high end servers supporting multiple processors with capability of upwards of 36 threads each. This still somewhat pales in comparison to GPUs which can have hundreds or even thousands of compute cores. These are built for crunching numbers; the cores are far simpler but can carry out similar tasks to the main processor. The difficulty comes in accessing them because there are several different manufacturers of graphics processors with vastly different software interfaces.

OpenCL is an API that allows easier access to the graphics processor and leverage the computing power that is available in them. When code is written in a particular and correct way and the memory of the graphics card is managed correctly, performance gains can be huge. The disadvantage to this system is that it tends to require dedicated graphics hardware as opposed to integrated graphics, as integrated graphics processors will have a far lower amount of cores; it would also likely dramatically affect the performance of the rendering of the scene.

The main issue that presents itself with GPU calculation is that in order to get the large improvements to performance you are limited to using single precision floating point arithmetic, this reduces the maximum attainable precision of the simulation as well as limiting the effective size that is possible. There are specialist GPUs that exist which are built with double precision floating point, these are extremely expensive and out of the reach of most people and establishments.

Because of the low amount of bodies that are likely to be used in this simulation, I do not plan to implement either CPU Simulation parallelisation or OpenCL based parallelisation as it is unlikely to be something that most people will be able to leverage and gain an effective performance benefit without a dedicated high end PCI-E graphics card. Along with implementing the Barnes-Hut simplification algorithm it could make for an interesting extension task.

## 2.4. Libraries

Libraries will be a critical part of the programming and code structure for this project, both third party and self-programmed.

The first and most obvious library is GLFW, a minimal wrapper that allows for the creation of OpenGL contexts which can be used to render graphics. It also provides easy access to inputs from keyboard and mouse. The benefit of this library is that it provides an extremely lightweight abstraction layer for doing all of these things across different OS platforms, making it far easier to port code from one platform to another.

The main reason for this is that setting up windows for displaying OpenGL contexts can be somewhat involved when it comes to writing operating system specific code, not something that is particularly necessary in this application. The library also provides full access to keyboard and mouse inputs which are critical to the operation of the program and are another feature that would normally require a large amount of somewhat messy operating system specific code, making porting to different target systems somewhat difficult.

Another library that will be used is AntTweakBar, this library provides a quick and simple to use utility for exposing certain variables and information to the user through a variety of control methods such as buttons and sliders for setting variables to particular values. It also provides a good way of displaying textual information to the user such as a basic help screen.

The C and C++ standard libraries will also be used in this program; these contain functions that allow the use of input and output streams as well as some higher level features such as string handling, threading and vectors, the main two here are threading and vectors. The threading library provides an object orientated implementation for

threads, while vector provides an object orientated implementation of a dynamic collection, or array that can change size during program runtime. The benefit of using the standard template libraries is that they have been optimized over a long period of time by the maintainers of the particular compiler, in this case GCC, which is open source and has thousands of contributors.

The library structure can also be used for my own code in order to split it up, code can be split into separate code files (.c / .cpp) and linked together using header files (.h / .hpp), these files contain function prototypes for functions that reside inside the code file that it describes. The benefit of this is that the function prototypes are not required inside the code file. It also means that header 'guards' can be used in order to prevent the code being duplicated at the link stage. The result of this is much cleaner code that is much easier to reuse in other projects.

When it comes to the compilation and linking of programs, there are generally two methods to this for libraries, static and dynamic, while static linking will link all of the library code into the application file, dynamic relies on libraries being compiled ahead of time and being shipped with the application or as redistributables, which the user can install to their system and any application that makes use of them can in theory make use of the existing library rather than needing its own copy.

The benefit of dynamic linking is that the final size of the application file is much smaller, it also means that libraries do not need to be compiled alongside the application, reducing the time to compile a fresh copy of the target application. The disadvantage is that more files are required to be placed alongside the application or added to a system directory which requires installation. It can also result in lower resource usage if multiple applications use the same libraries.

In the case of this project, static linking is a more attractive option, as while the size of the executable is somewhat larger, other files are not required and the executable can be placed anywhere and run without the need for other external files. When looking at compile time, parts of code that are compiled generally only need to be recompiled if those parts of code change as object files are retained, meaning that compile time reductions are likely to be minimal when going with the dynamic approach.

## 2.5. Structure

This program has two main identifiable components, the simulation and graphics/interface, because these parts are sufficiently different from each other, it makes sense to split the processing into threads, potentially making code more readable but also providing the potential for some performance improvements.

### 2.5.1. Overall System Design

Table 4: Overall System Design

| Input                                       | Storage         | Process                              | Output   |
|---|-----------------|--------------------------------------|--|
| Mouse, Keyboard,<br>Body Data,<br>Constants | Saved Scenarios | Simulation,<br>Drawing,<br>Rendering | GUI, Control,<br>Rendered Output,<br>Real-Time |

Due to the interactive and real-time nature of the program, the main input method will be through the mouse, relying on the user to select bodies in the current screen, then select attribute boxes and use the keyboard to change the value in those boxes. (The user is able to change both individual bodies and the simulation control constants, such as the gravitational constant.

The only data that will be put onto a secondary storage medium is any scenarios that may be saved, these will be stored in a human-readable and editable text file in a format that will be discussed in a later section. The files will contain all the information required to re-produce a scenario.

The main simulation is performed on a separate thread, handed to a shared area, the data will then be taken from the shared area by the main thread and rendered by OpenGL.

The basic layout of the program is an Input -> Simulate -> Render loop, however in order to produce an output that is actually useful a certain amount of management must be in place to control how much simulation time passes in each frame, meaning that the simulation will only calculate a number of iterations per frame.



GLFW and OpenGL are not handled using object orientation, and instead are handled in a more traditional procedural way. Input in GLFW is handled through the use of callback functions (Event-Driven Programming), these must be setup by calling a particular library function and passing it a programmer-defined function that can accept specific attributes that relate to that particular input. (Listed in GLFW Documentation) At the time that that particular input is received that function will be called and the variables passed to it reflect the input given. (Happens at call of `GLFWPollEvents()`; in render loop.)

```
glfwSetCursorPosCallback(window, cursorPosCallback);
```

When `GLFWPollEvents()` is called, the function `cursorPosCallback()` will be called if the mouse moves in the GLFW window and its window coordinates will be passed to the function which can then handle any programmer-defined actions with that data.

The interface will be constructed using input taken from GLFW, these input events will contain a check which passes the control onto the check function for the main interface library, `AntTweakBar`. This ensures that the inputs are available to both components, even though there is a degree of separation between them. Unfortunately it is not possible to have assign callback functions to class-member functions, this is something that would make organisation somewhat easier.

While the rendering and interface will be mostly programmed using callbacks and procedural programming, however the simulation; due to its nature of distinct bodies or objects, is extremely well suited to an object orientated method of programming.



## 2.5.2. Design Data Dictionary

Table 5: Design Data Dictionary

| Data Name            | Data Type                | Description   |
|----------------------|--------------------------|---|
| Mass ( $m$ )         | Double (8 B)             | The mass of an particular body or system in the simulation, used in calculations for forces and collisions.   |
| Radius ( $r$ )       | Double (8 B)             | Measurement of circular objects, specifically half the width, used for sizes of circular bodies, systems or orbits. Used for collision checking and rendering.  |
| Position ( $r$ )     | Double $\times$ 2 (16 B) | The position in world space of a particular body or object, independent of units, up to users interpretation. Used in simulation. (XY)  |
| Velocity ( $v$ )     | Double $\times$ 2 (16 B) | The velocity of any object or system in the simulation, change in unit position per unit time. Used in simulation. (XY)   |
| Acceleration ( $a$ ) | Double $\times$ 2 (16 B) | The instantaneous acceleration acting on a body, change in unit position per unit time per unit time, 2nd derivative or rate of change, calculated using $a = F/m$ . (XY)   |
| Force ( $F$ )        | Double $\times$ 2 (16 B) | The instantaneous force acting on a body, calculated as the relationship between two bodies and summed on individual components (XY), calculated using $\frac{-GMm}{r^3}\hat{r}_{xy}$   |
| Fixed                | Bool (1 B)               | This variable will control if a body is simulated in terms of movement, if it is fixed, its neither its position or velocity will update.   |
| Colour               | Float $\times$ 3 (12 B)  | This is a array of 3 floating point variables that describes the colour RGB of an object, the American spelling may be used for continuity with libraries.  |
| Body                 | Object (93 B)            | This object structure represents a body, it has mass, radius, position, velocity and can be acted on by force which translates to acceleration, size estimate is 93 Bytes.  |
| Vector               | C++ Container            | <i>Vector</i> is a data structure provided by the C++ STL (Standard Template Library), it acts as a dynamically resizing array, items can be deleted or added and the vector will resize, it can be accessed like a normal array. |
| Body Store           | Vector                   | The scenarios contain a body storage vector, effectively array that contains every body object in the current scenario, allowing iterative and management of the bodies.  |
| UGC ( $G$ )          | Double                   | This is a control variable that specifies the <i>Universal Gravitational Constant</i> . It is the constant of proportionality between the product of masses and the inverse proportionality of the square of the distance.        |

Table 6: Design Data Dictionary Cont.

| Data Name                            | Data Type | Description   |
|--------------------------------------|-----------|---|
| IDT ( $\Delta t$ )                   | Double    | This is a control variable that specifies the <i>Iteration Delta Time</i> . This is the graduation of time that the simulation will iterate for. A smaller value results in a slower but more accurate simulation.  |
| IPF                                  | Integer   | This is a control variable that specifies the <i>Iterations Per Frame</i> . This value describes how many simulation iterations are carried out before sending frame data, this allows a speed-up in the display but retains accuracy.  |
| Paused                               | Boolean   | This is a control variable that specifies if the simulation is running or not.  |
| Scenario                             | Object    | The scenario object will contain a body storage vector as well as all of the relevant control variables that enable the simulation, separate scenarios are allocated for simulation and render, enabling Thread-Local Storage.  |
| Pi ( $\pi$ )                         | Constant  | Pi is the ratio of a circle's circumference to its diameter, infinitely recursive, it is defined as 3.14159265358979323846 in the C++ standard library. Not used for any simulation calculations.   |
| $F = \frac{GmM}{r^2}$                | Equation  | Newtons Law of Gravitation, states that particles attract every other particle with a force that is directly proportional to the product of masses but inversely proportional to the square of the distance between them. Meaning this equation gives the force that two particles will exert on each other due to gravity. |
| $F = \frac{GmM}{r^3} \hat{r}_{xy}$   | Equation  | This equation is the vector form of the previous, $\hat{r}_{xy}$ is the component distance for the axis the force is being calculated on. The force that is calculated applies to both bodies.  |
| $F = ma = \frac{\Delta p}{\Delta t}$ | Equation  | This equation describes newtons second law, which states that force is equal to the rate of change of momentum. Rearrange to $a = \frac{F}{m}$ to calculate acceleration due to force.  |
| $p = mv$                             | Equation  | This equation relates mass and velocity and momentum. Momentum will be conserved in collisions as a vector quantity, and thus can be calculated on the X and Y components.  |
| $A = \pi r^2$                        | Equation  | This equation is used for calculating the area of a circle, used for adding together areas of bodies during collisions.   |

2.5.3. Program Flowchart

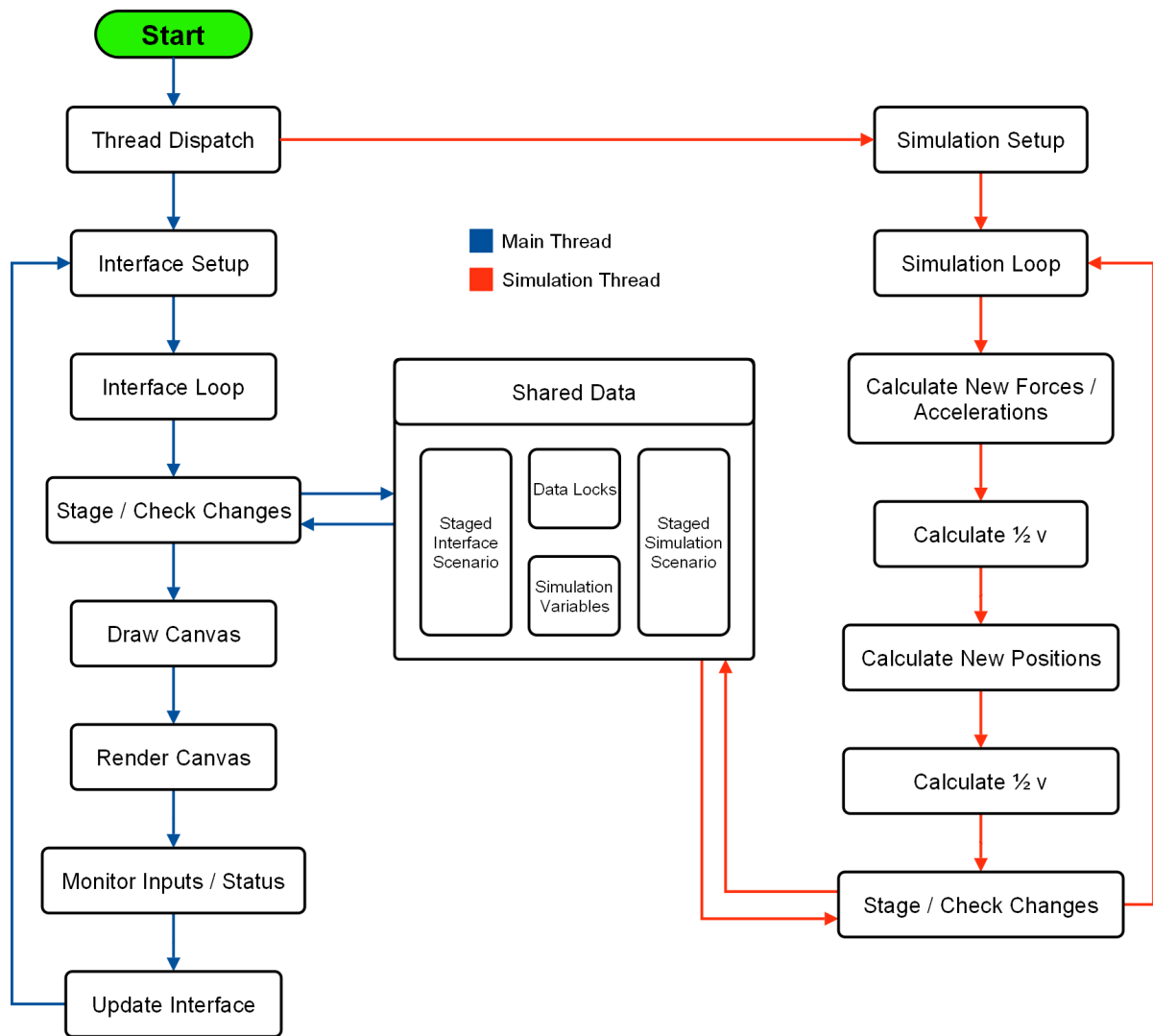


Figure 11: System Flowchart

#### 2.5.4. Data Range and Validation

The main issue with the validation of data is that it does not play particularly nicely with the medium of the simulation, which is Space, in reality, an orbit is massive because the size of the bodies and their velocities are massive, for example, the earth orbits about 149.6 million kilometres from the sun and at a velocity of 29.78 kilometres per second. The (current) furthest known common body in our solar system, Pluto, orbits at a range of 4.4-7.4 billion kilometres and at a velocity of only 4.67 kilometres per second.

The main issue here is the range of distances; from this view it makes it quite difficult to set limitations of the simulation because of the simple fact that space is vast and distances are vast. It is however necessary to take into account that this scale would also not be practical in the simulation as it would be nearly impossible to make out individual bodies inside an accurate solar system model. (The earth has a radius of just 6,371 kilometres.)

There is also the issue that when bodies are too close together and the velocities are too great, the simulation method will breakdown; even with the improved Leapfrog method for integrating the acceleration, meaning that the scale of the simulation cannot be confined to being too small.

In terms of massive scale, something to look out for is the loss of precision when the numbers get too big, because the simulation will make use of the double floating point data type, as with float precision is not constant through a range in numbers as the mantissa is taken up by the most significant bits in a large number. C++ implements a IEEE variant of double-precision floating point, a 64 bit variable, 1 sign bit, an 11 bit exponent and a 52 bit fraction. (IEEE 754) (There are some other differences compared to twos complement floating point.)

These bit depths mean that (depending on particular implementation) the double precision type has 15-17 digits of precision, this would be enough digits to describe the position of Pluto in our solar system, and still have about 5 digits of precision remaining for the decimal, one problem with this is that the force that is provided by other bodies in the solar system could be quite small and result in a cancellation error in calculation that does not take into account the smaller forces, this will increase the error in the simulation. However due to the size and mass of bodies in our solar system all of the

interactions are still likely to be within range of each other and only a few powers of ten apart, meaning that it is still something that is possible to calculate without cancellation the most significant bits.

There will however be cancellation when it comes to the calculation and use of the mass of bodies, for example, the sun has a mass of  $1.989 \times 10^{30}kg$ , this is easily possible to represent however there will be very little precision, only the first 15 most significant powers of ten change will have any effect to the mass (i.e: anything with a mass of less than  $1 \times 10^{15}kg$ ). In the grand scheme of things this is not a problem that is likely to have much of an effect on the simulation, especially at the scale that will be likely to be used.

The force that the sun exerts on Pluto at its highest point results in a change in acceleration of  $2.42 \times 10^{-6}$ , at this height Pluto is orbiting at  $3.71kms^{-1}$ , while this will be in range and not result in cancellation, when looking at the force exerted by the relationship between Earth and Pluto, the change in acceleration is  $7.05 \times 10^{-12}$ , this is only just within the range of the double precision when added to the velocity of Pluto, which is orbiting quite slowly to begin with.

Due to the nature of a n-body system, it is impossible to predict the outcome over a period of time, while a two body system can be described perfectly (in an ideal sense) with equations, an n-body system cannot, and will generally become chaotic over a relatively short length of time. This again makes it something that is difficult to place validations and limitations on, as a lot of possible restrictions could end up crippling the usage of the program in certain scenarios.

It may make a certain degree of sense to look at the maximum distance that can be represented using a double variable yet still maintain a certain number of decimal places for the precision of the calculations (At least 2). This would mean that assuming a base unit of 1 meter, and a precision of 15 digits, the orbit of Pluto can still be represented on a centimetre scale and still have orbits further out.

Worth noting is that there will be a fair amount of error due to the way that decimals are represented in binary. For example, it is generally not possible to exactly represent ...0.01, and it will instead be represented as ...0.00996, the rest of the digits are due to the implementation that is used both by the compiler and the hardware being used.

This will result in an error that can accumulate, adding to the potential for chaos and other miscalculation errors in the simulation.

While this would work as detached simulation engine without a rendering counterpart, the inclusion and requirement of a graphical front-end and display for the results of the simulation places another limitation on the available range. This is because rendering at a hardware level, handled by a GPU is carried out using single precision floating point, with only high-end GPUs having any kind of FP64 capability and these are generally suited to raw compute rather than actual graphical rendering.

When taking this into account, standard float has only 7 digits of precision, meaning that when representing the orbit of Pluto, you only end up with a precision in the range of  $1000km$  which is a considerable distance on a smaller scale, it also makes things like rendering an accurate circle of smaller radius somewhat difficult, especially when you consider that the radius of Pluto is only  $1186km$ . (The circle drawing algorithm discussed later on will end up drawing a square.)

While this would be an issue for rendering full circles at close up distances, it will still be possible to render singular points, *GL\_POINTS* will always render, regardless of how small the body itself it, and is far more tolerant to the loss in precision, because of this, the limit for distance will be set at  $1 \times 10^{15}m$  (Larger bodies will still be viewable as circular objects at the extents)

Other more definite limitations include mass, which cannot be 0 or negative, while hypothetically possible, this would result in complications and odd behaviour in the simulation, something that is best to avoid. It makes little sense to implement any maximum for mass, as on a stellar scale, masses are extremely variable due to composition, type and size of bodies. (A good example of this are black holes, these are relatively small in size but super-massive.)

Maximum radius should also be limited, however it does once again bring up the issue of what makes sense, the sun in our solar system has a radius  $696,000km$ , while the largest known star, VY Canis Majoris reaches a staggering 1 billion km in radius (About 1420 Sol Radii), this actually be near the limit of the size of the simulation, making a body of this size quite pointless, a much more reasonable maximum would be 1 million km , allowing for representation of bodies somewhat larger than our own sun.



Other variables, such as time step, should be limited to being positive (It is however worth noting that the leapfrog integration method allows for time reversal, this requires some modification however.) so that time can only move forward. The number of iterations per frame should be limited to a range of 1 to 10000, while the upper range will likely be only run at target frame rate on extremely high end systems or with low numbers of bodies, this value will be the main control the user has for increasing the speed of the simulation without sacrificing on accuracy.

Gravitational constant should be constrained to being a positive number as a negative number would reverse the effects of gravity, the upper limit of the gravitational constant will be 10, with a lower limit of  $1 \times 10^{-12}$ . Other values involved primarily in calculation, such as velocity and acceleration will not have the limitation set to the speed of light in vacuum, or  $3 \times 10^8 \text{ms}^{-1}$ . This is a law of the universe and cannot be bypassed. In reality it cannot be reached, as at relativistic speeds the mass of the approaching body will increase, this simulation does not cover relativistic effects. (Because these are vector quantities, the same limit exists for the negative range.) If a body reaches or exceeds this velocity it will be deleted.

I am still somewhat hesitant to implement limitations that are too stringent as it may result in a crippled simulation engine, however unlikely that its intended use is to reach those limits.

While all of the data is primarily numerical, notation such as 'E' ( $\times 10^x$ ) is likely to be used to make very large or small numbers easier to type. The numerical validation is by default handled by the AntTweakBar GUI and the data boxes will not accept text as they are defined variable types. While custom validation functions could be added by setting the variable type to string and manually reading and converting it, there are other areas where validation could be manually implemented. (This would only add extra overhead and reduce features.)

#### **2.5.5. Data Security and Integrity**

None of the data handled by the application is considered sensitive from a user sense, however all of the data is considered critical to the correct and accurate operation of the calculation. The main place that integrity is important is when it comes to sharing data between the simulation and the renderer, as both threads have the potential to

be reading and writing to the same data, race conditions come into play and are likely to cause issues, such as reading data that is only partially updated. This area will be managed using a state machine like algorithm to ensure that data is written to and read at the correct times.

When it comes to the saving of scenarios, these files will be by default outputted to a directory alongside the main executable. The user will be able to choose the name of the saved file, if the file already exists, the user should be prompted to if they want to overwrite the existing data. On an expected exit of the program, the simulation data should be saved to a default storage file that keeps a backup of the last data in the program, this will be opened on start-up without prompt.

Because these saves are files normal text files, they can be moved around by users at will, meaning that if required they can be stored on a distributed storage medium such as the school server, for backup and sharing with other students and teachers. The main application will also just be an executable, requiring no external files to run and no installer, meaning that this could also be stored on a shared drive and copied to a users local machine in order to run it.

It is important that thread shared data is managed effectively so that there are no concurrent reads or writes as this will cause the corruption of data and potential issues further down the line in the event that a variable changes while another thread is reading the data. This can be managed via the use of *Mutex* lock objects, if one of these are locked and another thread attempts to lock the same mutex it will pause until the locking thread unlocks the mutex, thus preventing concurrent read/write access.

#### **2.5.6. Data Record Structure**

The user has the ability to save the current state of the simulation, this will be stored in a file (.sav, text format) inside a sub-folder next to the application executable. The user has the ability to choose the particular name of the file.

The data file must be able to store the following:

- Gravitational Constant
- Simulation Iteration Time-step
- Simulation Iterations per Frame

- Number of Bodies
- Simulation State Data
- Body Storage (For Each)
  - Mass
  - Radius
  - Colour
  - Position (X, Y)
  - Velocity (X, Y)
  - Fixed

The save file will only contain data which does not get produced as a result of the running of the simulation.

Preferably the data files should be in a human readable format, allowing an alternative / advanced method for setting up the scenarios. Based on these requirements the following format will be used:

```

1 // Save File Template
2
3 // Constants
4 // Gravitational Constant
5 UGCONST = VALDOUBLE
6 // Simulation Iteration Timestep
7 SIMITTS = VALDOUBLE
8 // Simulation Iterations per Frame
9 SIMITPF = VALINT
10 // Scenario Data
11 // Number of Bodies
12 NUM_BODY = VALINT
13 // Simulation State
14 STATREG = VALINT
15 // Body Data
16 BID = VALINT {
17     MASS      = VALDOUBLE
18     RADIUS    = VALDOUBLE
19     COLOUR    = VALFLOAT, VALFLOAT, VALFLOAT
20     FIXED     = VALBOOL
21     POSITION   = VALDOUBLE, VALDOUBLE
22     VELOCITY  = VALDOUBLE, VALDOUBLE
23     ORBITALV  = VALBOOL
24 }
```

As with C++, "//" denotes a comment, any line in the save file will be ignored. White-space will also be skipped by the parser. Variables can be written in any order,

as long as multi number variables are kept together.

If data that is read in is deemed to be incorrect, the file will not be loaded and the user will be notified as to the particular problem with the save file, giving the opportunity to manually repair the file.

Here is an example showing the use of the format:

```
1 // Save File Template
2
3 // Constants
4 // Gravitational Constant
5 UGCONST = 1
6 // Simulation Iteration Timestep
7 SIMITTS = 0.01
8 // Simulation Iterations per Frame
9 SIMITPF = 6
10 // Scenario Data
11 // Number of Bodies
12 NUM_BODY = 3
13 // Simulation State
14 // 1 = Default State, Paused
15 STATREG = 1
16 // Body Data
17 B_ID = 1 {
18     MASS      = 10.0
19     RADIUS     = 100.0
20     COLOUR    = 1.0f, 0.8f, 0.1f
21     FIXED     = TRUE
22 }
23 B_ID = 2 {
24     MASS      = 0.1
25     RADIUS     = 1.0
26     COLOUR    = 0.0f, 0.4f, 0.1f
27     POSITION   = 1000.0, 0.0
28     ORBITALV  = TRUE
29 }
30 B_ID = 3 {
31     MASS      = 0.1
32     RADIUS     = 1.0
33     POSITION   = 0.0, 2500.0
34     VELOCITY  = 0.0064, 0.0
35 }
```

As shown in this example, certain values can be omitted and default values will be assumed, such as velocity and position, will both be considered as 0 and fixed will be considered false. If a colour is not present a random colour will be chosen. (These values will be always included in generated save files.)

Also, if velocity is omitted and the ORBITALV flag is set for the body, the parser will automatically calculate the velocity required to place that particular body in a close to circular orbit, speeding up the set-up of a simple scenario, this is a user set only flag and will not appear in generated files.

If either Mass or Radius are not defined the body will be ignored and removed from the scenario.

## 2.6. Algorithms

### 2.6.1. Simulation

**Algorithm 1: Force Calculation Between Two Bodies** This equation can be used to calculate the gravitational force exerted by two masses on x and y components without the need to use the trigonometric functions.

$$\begin{bmatrix} F_x \\ F_y \end{bmatrix} = \frac{Gm_1m_2}{r^3} \begin{bmatrix} r_x \\ r_y \end{bmatrix}$$

Required Parameters:

- Gravitational Constant ( $G$ )
- Mass of Body 1 ( $M_1$ )
- Mass of Body 2 ( $M_2$ )
- Distance Between Bodies ( $r$ ,  $r_x$  and  $r_y$ )

**Algorithm 2: Calculation of all Forces** In order to simulate the interaction of multiple bodies, all relationships between bodies must be calculated, this can be presented as an undirected graph in an abstract sense. This also means that it can be represented as a matrix.

When showing this organisation structure as a matrix, it becomes clear as to how many calculations need to be done in order to calculate all the required relationships.

Because the 'graph' is undirected, the matrix would be mirrored, the same force applies to both bodies in a relationship. In order to reduce wasted memory footprint and increase ease of passing the variable, the x and y forces can be packed into the same matrix. (Will require slightly more complex programming when accessing the forces.)

Sign of the force must be retained, meaning that the way round that the force is calculated must be constant throughout all (x to y). The number of individual calculations that need to be done becomes  $\frac{1}{2}n^2 - n$  for the main force calculation and  $n^2 - n$  for the directional vector, this is still  $O(n^2)$  time complexity per body.

Required Parameters:

- Current Number of Bodies
- Scenario Body Structure

|   | 0        | 1        | 2        | 3        | 4        |
|---|----------|----------|----------|----------|----------|
| 0 | —        | $y_{01}$ | $y_{02}$ | $y_{03}$ | $y_{04}$ |
| 1 | $x_{01}$ | —        | $y_{12}$ | $y_{13}$ | $y_{14}$ |
| 2 | $x_{02}$ | $x_{12}$ | —        | $y_{23}$ | $y_{24}$ |
| 3 | $x_{03}$ | $x_{13}$ | $x_{23}$ | —        | $y_{34}$ |
| 4 | $x_{04}$ | $x_{14}$ | $x_{24}$ | $x_{34}$ | —        |

Figure 12: Example Force Matrix Representation (Body Count = 5)

**Algorithm 3: Summation of Forces on Bodies** The forces on each body must be summed up so the total force on each body can be resolved to two numbers for x and y respectively.

Because the force is only stored in the matrix an implied direction, in certain cases it may be necessary to flip the sign of some of the results to correctly sum the answers, Results will be stored in an individual bodies dataset.

The force is calculated x to y on the matrix, meaning that all forces will initially be calculated with a sign that represents this direction (The sign of the unit vector distance). This function will need to 'traverse' the matrix in a similar way to the previous function, but only add a force if one of the loop axis is equal to the current body ID.

If the body ID is found in the first array access column, the original sign is kept. If it is found in the second, the sign must be flipped to preserve the correct direction of force relative to the body because of the way round that force has been calculated.

Required Parameters

- Current Number of Bodies
- Scenario Body Structure
- Scenario Component Force Matrix

**Algorithm 4: Integration of New Position** Using the second order leapfrog integration method, calculates change in velocity and change in position/acceleration out of step with each other in order to reduce deviation.

Each iteration will be calculated in the order  $\frac{1}{2}v \rightarrow r \rightarrow F \rightarrow a \rightarrow \frac{1}{2}v$ , but initial acceleration should be calculated should any new bodies be added into the simulation as velocity is calculated at the start of each iteration.

In order to calculate the acceleration of each body the force gets calculated for every body and given to the individual body objects, the instantaneous force for that iteration is then used to calculate instantaneous acceleration based on Newton's Second Law or  $a = \frac{F}{m}$  after which all other operations are done in the body object methods instead.

Velocity and Position are both accumulative properties, meaning that they are conserved in the body objects iteration to iteration. Velocity is  $\Delta v = a\Delta t$ , but this will be calculated as  $\Delta v = \frac{1}{2}a\Delta t$  and will be done at two points in the iteration. Position is calculated as  $\Delta r = v\Delta t$ . (Velocity and Acceleration are first and second derivatives of position respectively.)

References: [2]

Required Parameters

- Current Number of Bodies
- Scenario Body Structure

**Algorithm 5: Body Collision Detection and Calculation** The simulation will have a simple body collision detection algorithm implemented that makes use of basic inelastic collisions, no fragments will be created and colliding bodies will simply merge together in mass and circular area.

Detecting if bodies are colliding can be done using the radius property of bodies, if two bodies vector distance is smaller than the sum of their radii, the bodies can be considered to have collided and the result can be calculated. Because this method checks every single body, the time order complexity is  $O(n^2)$ .



If a collision is detected, the momentum for individual bodies can be calculated using the equation  $p = mv$ , momentum will be conserved in an inelastic collision, meaning that the momentum for both bodies can be added together and then divided by the total mass of both bodies combined to get the new velocity for the result. This will need to be done for both X and Y vectors.

Required Parameters

- Current Number of Bodies
- Scenario Body Structure

### 2.6.2. Graphics

**Algorithm 6: Circle Drawing** Due to the fact that OpenGL has no inbuilt support for drawing circle primitives, they must be constructed using vertices and line segments in order to form a polygon (GL\_POLYGON) and fill it in to create a approximated circle.

To do this, the center coordinates will be used in conjunction with the radius to calculate trigonometric factors in order to speed up the calculation of the circle. Traditionally, you would calculate the position of the next point in respect to radius by calling the trigonometric functions for every vertex, however with this method they only need to be called once per object as they will be the same for every vertex.

The benefit of this is that it still allows for different numbers of segments to be used, higher numbers will make a much smoother looking circle, however it will be dependant somewhat on the radius of the drawn circle, as being able to zoom in will make the straight edges a lot more visible, even when anti-aliasing is used. Due to performance constraints, a lower number here would be better, 16 or 32 is sensible.

$$\theta = 2\pi / Segments \quad k_{\tan} = \tan \theta \quad k_{\cos} = \cos \theta$$

$$x_{pos} = r \quad \& \quad y_{pos} = 0$$

$$x_{pos} + (-y_{pos}k_{\tan})k_{rad} \quad \& \quad y_{pos} + (x_{pos}k_{\tan})k_{rad}$$

References: [1]

Required Parameters:

- Center Point
- Radius
- Segments

### 2.6.3. Interface

**Algorithm 7: Body Selection** This algorithm will work in a similar way to the collision detection code, but on the end of a single body instead of two. By getting the current position of the cursor in the world by using the function `gluUnProject()` and checking to see if the cursor is inside the radius of any body when a click occurs. Mouse input is handled by GLFW.

This function is used as it will use the current matrices that are used for manipulating what is displayed on screen on the cursor coordinates, providing a perfect mapping of what is happening on screen to the window system coordinates.

If a body is selected, a GUI will open which will allow the properties of that body to be modified, and then applied with a button on that GUI.

**Algorithm 8: Pan and Zoom** By holding down middle click and dragging, the user will be able to pan the view around the scene to get a better view of different parts. By scrolling, they will also be able to zoom into the position currently at the center of their screen, mouse coordinates and clicks are provided by GLFW.

The panning will be calculated by first waiting to see if the mouse has been held down for at least several hundred milliseconds using the GLFW timer, after which the change in cursor positions can be calculated per frame. Zoom will will modify a scale variable that will have set limits, preventing the user from zooming too far in and out.

These variables will then be passed into OpenGL specific functions that will perform transformation and scales on the OpenGL ModelView Matrix, giving the illusion of movement and zooming.

**Algorithm 9: Scenario Saving and Loading** The scenario will be able to be saved by the user using elements provided on the UI. The user will be required to enter a file-name in order to save the scenario. The file extension (.sav) will be automatically added.

The file will contain a structured and human readable/editable version of the scenario data used by the program, it contains any scenario control variables and the properties of

any bodies present in the simulation. There will also be some utility for auto-generation features, such as providing bodies with orbital velocity relative to any other body in the simulation or the generation of large super-structures.

In order to load a save, the user can either drag and drop a .sav file into the window or they can enter the file name into the UI element and click on the 'load' button.

#### **2.6.4. Management**

**Algorithm 10: Thread Synchronisation and Shared Data Management** When shared data is accessed, other threads must be prevented from accessing (Read or Write) the data at the same time as it could cause corruption or deadlocks due to race conditions. Once the thread is done accessing the data it will unlock it to allow another thread to access it.

The simulation thread will get initially get the body storage and control variables from the shared storage area (Previously set up and sent to shared by the main thread). It will then perform a number of iterations based on iterations per frame and place the data in the shared area buffer as well as setting a variable to show that new data is available.

If the simulation completes another frames worth of iteration before the renderer has taken the data and unset the new data available flag the simulation thread will wait until that data has been taken.

If any changes are made to the input scenario, the main thread will update the shared area and notify the simulation thread that new data is available, the Sim thread will discard any data it is currently carrying and take on the new user changes.

## 2.7. Pseudocode

The several algorithms that are being used warrant having some high-level pseudocode written in order to better explain their operation in the programming, those that were omitted are either highly implementation specific or basic equations.

---

**Algorithm 2** Calculation of All Forces

---

```
1: for  $Body_A \leftarrow 1$  to Bodies do
2:   for  $Body_B \leftarrow (Body_A - 1)$  to Bodies do
3:     if  $Body_B \neq Body_A$  then
4:        $Matrix_F[Body_A][Body_B] \leftarrow calculateForce(Body_A, Body_B, x)$ 
5:        $Matrix_F[Body_B][Body_A] \leftarrow calculateForce(Body_A, Body_B, y)$ 
6:     end if
7:   end for
8: end for
```

---

---

**Algorithm 3** Summation of All Forces

---

```
1: for  $Body_A \leftarrow 1$  to Bodies do
2:   for  $Body_B \leftarrow 1$  to Bodies do
3:     if  $Body_B \neq Body_A$  then
4:       if  $Body_B < Body_A$  then
5:          $Body[xAccess]_F - \leftarrow Matrix_F[Body_B][Body_A]$ 
6:          $Body[yAccess]_F + \leftarrow Matrix_F[Body_A][Body_B]$ 
7:       else
8:          $Body[xAccess]_F + \leftarrow Matrix_F[Body_A][Body_B]$ 
9:          $Body[yAccess]_F - \leftarrow Matrix_F[Body_B][Body_A]$ 
10:      end if
11:    end if
12:  end for
13: end for
```

---

---

**Algorithm 4** Integration of New Position

---

```
1: if ScenarioChange then
2:   CalculateForceMatrix()
3:   SumAllForcesToBodies()
4:
5:   for  $bC \leftarrow 1$  to Bodies do
6:     {Acceleration}
7:      $Body[bC]_{ax} \leftarrow Body[bC]_{Fx} / Body[bC]_m$ 
8:      $Body[bC]_{ay} \leftarrow Body[bC]_{Fy} / Body[bC]_m$ 
9:   end for
10:
11:   ScenarioChange  $\leftarrow false$ 
12: end if
13:
14: for  $bC \leftarrow 1$  to Bodies do
15:   {Half-Velocity}
16:    $Body[bC]_{vx} + \leftarrow 0.5(Body[bC]_{ax} \times \Delta t)$ 
17:    $Body[bC]_{vy} + \leftarrow 0.5(Body[bC]_{ay} \times \Delta t)$ 
18:   {Position}
19:    $Body[bC]_{rx} + \leftarrow Body[bC]_{vx} \times \Delta t$ 
20:    $Body[bC]_{ry} + \leftarrow Body[bC]_{vy} \times \Delta t$ 
21: end for
22:
23: CheckCollisions()
24:
25: CalculateForceMatrix()
26: SumAllForcesToBodies()
27:
28: for  $bC \leftarrow 1$  to Bodies do
29:   {Acceleration}
30:    $Body[bC]_{ax} \leftarrow Body[bC]_{Fx} / Body[bC]_m$ 
31:    $Body[bC]_{ay} \leftarrow Body[bC]_{Fy} / Body[bC]_m$ 
32:   {Half-Velocity}
33:    $Body[bC]_{vx} + \leftarrow 0.5(Body[bC]_{ax} \times \Delta t)$ 
34:    $Body[bC]_{vy} + \leftarrow 0.5(Body[bC]_{ay} \times \Delta t)$ 
35: end for
```

---

---

**Algorithm 5** Collision Detection and Simulation

---

```
1: for  $bC_A \leftarrow 1$  to Bodies do
2:   for  $bC_B \leftarrow (bC_A - 1)$  to Bodies do
3:     if  $bC_B \neq bC_A$  then
4:        $Distance \leftarrow calculateDistance(bC_A, bC_B)$ 
5:       if  $Distance < (Body[bC_A]_r + Body[bC_B]_r)$  then
6:          $p_x \leftarrow ((Body[bC_A]_m \cdot Body[bC_A]_{vx}) + (Body[bC_B]_m \cdot Body[bC_B]_{vx}))$ 
7:          $p_y \leftarrow ((Body[bC_A]_m \cdot Body[bC_A]_{vy}) + (Body[bC_B]_m \cdot Body[bC_B]_{vy}))$ 
8:
9:          $Body[bC_A]_m \leftarrow (Body[bC_A]_m + Body[bC_B]_m)$ 
10:         $Body[bC_A]_{vx} \leftarrow (p_x / Body[bC_A]_m)$ 
11:         $Body[bC_A]_{vy} \leftarrow (p_y / Body[bC_A]_m)$ 
12:
13:         $DeleteBody(bC_B)$ 
14:      end if
15:    end if
16:  end for
17: end for
```

---

## 2.8. OO Data Structure

The object/class structure for this program is used primarily as the Thread-Local Storage for the separate render and simulation threads. Class Scenario contains a vector consisting of type body, as well as the functions required to manage, return and update the body storage and any other relevant variables, rdrTLC and simTLC (Thread Local Class) will inherit the scenario class and add on specific functions that pertain to that particular thread.

The sharedStage class will contain storage variables similar to the scenario class, however no active management must be carried out in the shared area, which is why it does not inherit the scenario class as it contains functions that allow for the creation or deletion of bodies, scenarios private variables will be 'protected' which allows them to be accessed by the sub-scenario classes that inherit the scenario class.

The preliminary classes have been written in the C++ syntax style as this enables me to display them with syntax highlighting, small modifications will likely be made during implementation. Having the classes written in a C++ style syntax will also allow me to quickly implement these sections into actual code.



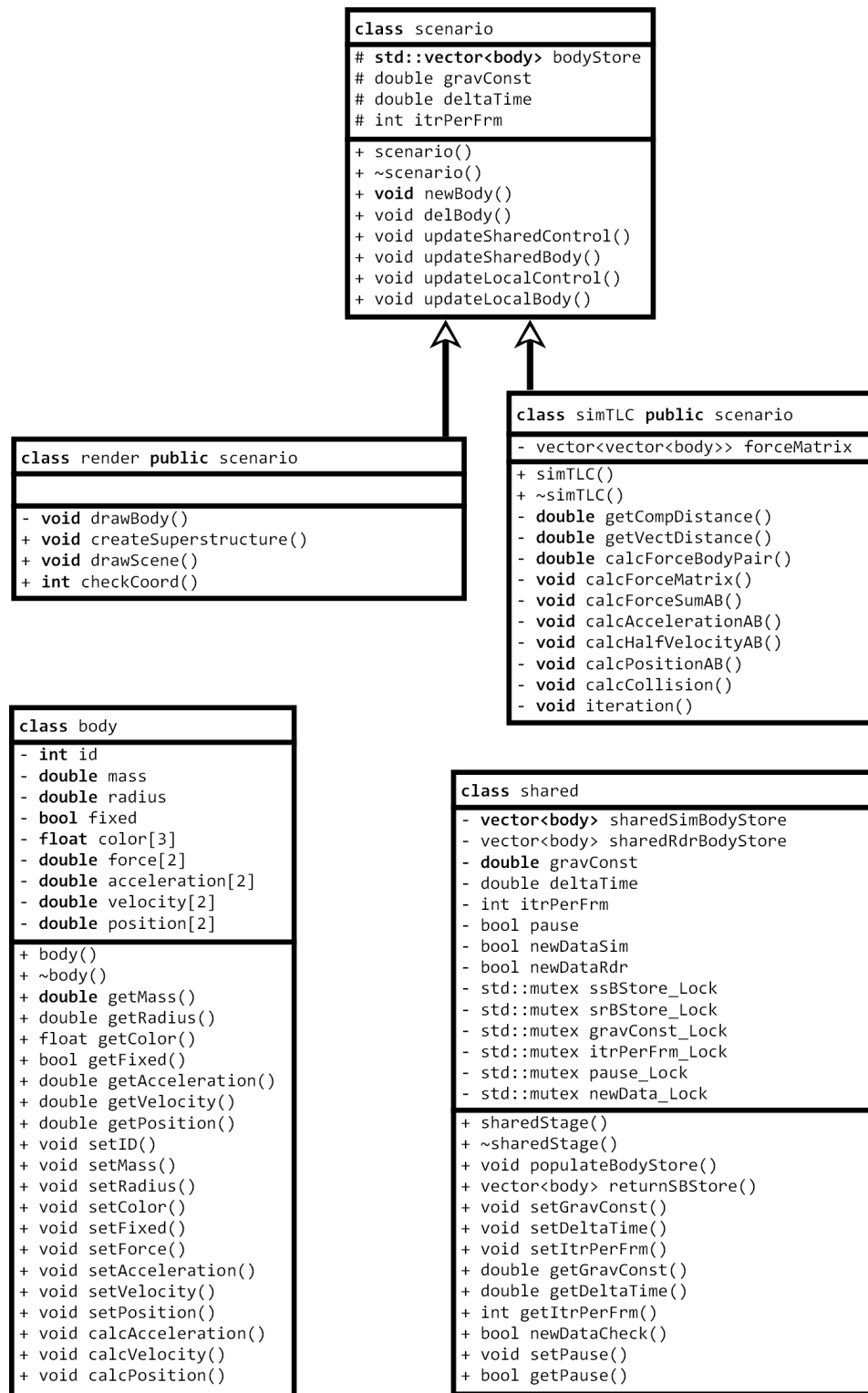


Figure 13: Object Inheritance Diagram

### 2.8.1. body

```
1 class body {
2 private:
3     // Parameters
4     int id;
5     double mass;
6     double radius;
7     float colour[3];
8     bool fixed;
9     // Properties
10    double force[2];
11    double acceleration[2];
12    double velocity[2];
13    double position[2];
14
15 public:
16     // Constructor
17     body(double p_mass, double p_radius, double p_position[2], double p_velocity[2]);
18     // Destructor
19     ~body();
20
21     // Encapsulation Methods
22     // Get
23     double getMass();
24     double getRadius();
25     float getColor(int p_cpIndex);
26     bool getFixed();
27     double getAcceleration(int p_xyIndex);
28     double getVelocity(int p_xyIndex);
29     double getPosition(int p_xyIndex);
30     // Set
31     void setID(int p_id);
32     void setMass(double p_mass);
33     void setRadius(double p_radius);
34     void setColor(float p_r, float p_g, float p_b);
35     void setFixed(bool p_fixed);
36     void setForce(double p_force, int p_xyIndex);
37     void setAcceleration(double p_acceleration, int p_xyIndex);
38     void setVelocity(double p_velocity, int p_xyIndex);
39     void setPosition(double p_position, int p_xyIndex);
40     // Calculate
41     void calcAcceleration(int p_xyIndex);
42     void calcVelocity(int p_xyIndex);
43     void calcPosition(int p_xyIndex);
44 }
```

Interface

### 2.8.2. scenario

```
1 class scenario {
2 protected:
3     // Body Storage
4     vector<body> bodyStore
5
6     // Status Register (Set Internally)
7     uint8_t sreg;
8
9     // Main Sim Control Variables
10    double gravConst;
11    double deltaTime;
12    int itrPerFrm;
13
14 public:
15     scenario();
16     ~scenario();
17
18     // Returns Bit 0-7 on sreg.
19     bool getSReg(int p_bit);
20
21     // Body Management
22     void newBody(double p_mass, double p_radius, double p_position[2], double p_velocity[2]);
23     void delBody(int p_bodyID);
24
25     // Shared Access Methods
26     void updateSharedControl(sharedStage* p_sharedAccess);
27     void updateSharedBody(sharedStage* p_sharedAccess);
28     void updateLocalControl(sharedStage* p_sharedAccess);
29     void updateLocalBody(sharedStage* p_sharedAccess);
30 };
```

The scenario class is designed to be a class template for the Thread-Local Class or Thread-Local Storage for the simulation and rendering threads, the specific classes for which will inherit this class.

Other than a body storage vector and the simulation controls, this class also has a status register, this is an 8 bit variable that will be used in a similar fashion to registers in embedded programming, each bit will represent a true / false state and will be used to describe the current state of the executing thread, this is mainly useful for debugging purposes. (It will likely be removed during implementation if there is no need for it.)

### 2.8.3. simTLC

```
1 class simTLC : public scenario {
2 private:
3     double calcCompDistance(int bodyID_A, int bodyID_B, int xy);
4     double calcVectDistance(double distX, double distY);
5
6     // 2D Body Matrix
7     vector<vector<body>> forceMatrix;
8     double calcForceBodyPair(int bodyID_A, int bodyID_B, double distV);
9     void calcForceMatrix();
10    void calcForceSumAB();
11
12    void calcAcceleraitonAB();
13    void calcHalfVelocityAB();
14    void calcPositionAB();
15    void calcCollision();
16
17 public:
18     simTLC();
19     ~simTLC();
20     void itteration();
21 };
```

### 2.8.4. rdrTLC

```
1 class rdrTLC : public scenario {
2 public:
3     rdrTLC()
4     ~rdrTLC()
5     // Render-Specific Functions Here
6     void drawBody(int bodyID);
7     void drawScene(void);
8     // Check Coordinates - Return Body ID if any under cursor.
9     int checkCoord(double x, double y);
10 };
```

Both simTLC and rdrTLC inherit the scenario class, giving them both a body storage vector, simulation control variables and basic public management functions for these variables to keep encapsulation. rdrTLC is very lightweight, including functions that link into the graphics portion, drawBody will draw any particular body given the ID, while draw scene calls drawBody for every body in the store. checkCoord will return the bodyID of a body should one be clicked on.

By contrast, simTLC is more complicated, containing functions for dealing with the simulation portion of the program, almost in its entirety. Methods for returning the

distance and forces between bodies are the only methods that will return variables, while all other methods make changes to other variables directly.

Forces for every single body relationship for the current iteration will be stored in a single 2D vector matrix, this will be reset and resized at the beginning of every iteration. The forces are stored for both X and Y, split by the diagonal of the same-same relationship. `calcForceSum` takes all of these relationships and adds them together for each body, putting them into the respective body object in the vector. (scenario) Force calculation is one of the main computational costs in the program.

Most actions, such as calculations for velocity, acceleration and position, are all calculated in the body object itself, the functions present in this class are loops to call functions for every body.

## 2.8.5. sharedStage

```
1 class sharedStage {
2 private:
3     // Body Storage
4     vector<body> sharedSimBodyStore;
5     vector<body> sharedRdrBodyStore;
6
7     // Simulation Control
8     double gravConst;    // Simulation Universal Gravitational Constant
9     double deltaTime;    // Simulation Iteration Delta Time
10    int itrPerFrm;        // Simulation Iterations per Render Frame
11
12    // Runtime Control
13    bool pause;           // Pause Signal
14
15    // Flow Control
16    bool newDataSim;      // New Data for Sim
17    bool newDataRdr;      // New Data for Render
18
19    // Mutex Lock Objects
20    std::mutex sBStore_Lock;
21    std::mutex gravConst_Lock;
22    std::mutex deltaTime_Lock;
23    std::mutex itrPerFrm_Lock;
24    std::mutex pause_Lock;
25    std::mutex newData_Lock;
26
27 public:
28     sharedStage();
29     ~sharedStage();
30
31     // Set/Get Body Store (Render <-> Sim) (Locking)
32     void populateBStore(vector<body> p_BodyStore, int p_rs);
33     vector<body> returnSBStore(int p_rs);
34
35     // Set/Get Sim Control (Locking)
36     void setGravConst(double var);
37     void setDeltaTime(double var);
38     void setItrPerFrm(int var);
39     double getGravConst();
40     double getDeltaTime();
41     int getItrPerFrm();
42
43     // Return True if New Data Aavailable (Locking)
44     bool newDataCheck();
45
46     // Set/Get Pause (Locking)
47     void setPause(bool set);
48     bool getPause();
49 };
```

## 2.9. Human-Computer Interaction

Due to the interactive nature of this piece of software, the user experience is extremely important and ties in to both the design of the interface (Effective Layout) and the programming (Ensuring Responsiveness). It is extremely important that the application does not stutter or skip, miss user inputs due to hanging threads, in essence, the render thread should not hang for any considerable length of time, as this will greatly reduce the effective frame rate of the application.

### 2.9.1. Frame Rate

Frame rate of the application is something that should be kept as high as possible to ensure that the interface is always responsive. The main reason for separating the simulation from the render thread is so that the render loop could run independently of the simulation and not hang or stutter on large simulations.

24 FPS is the widely accepted frame rate at which the human eye will perceive smooth motion from a moving object on screen. While this a frame rate that is workable for a movie or other non interactive content, when it comes to an application that is taking human input the higher the frame rate, the better. Having a higher frame rate means that the user interface has the ability to be far more responsive, not least of which is that the input is actually tied to the frame as it is polled at the end of each frame.

60 FPS is the generally accepted frame rate for interactive media, this will be the frame rate that the operating system will run at as most PC monitors will display at 60Hz. By default, OpenGL sets a frame rate limit to the refresh rate of the monitor, this also brings the benefit of preventing screen tearing.

For an object travelling at a constant velocity, higher frame rate will end up providing more of the information in terms of the movement of the object. Resulting in fast moving objects being more visible, a display at 15FPS may only show 3 points while

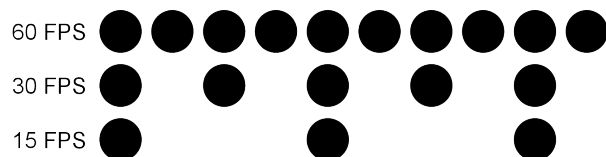


Figure 14: Frame Rate Example

60 FPS will show 12 points in the same period of time. Considering the simplicity of the graphics, 60FPS should be easy to obtain even on low end hardware.

### 2.9.2. Mock Interface

The following mock-up of the software interface represents the intended layout of the GUI. GLFW is responsible for the setup of the window frame itself, while the main renderer; OpenGL, draws and renders the graphics on a context inside the window. AntTweakBar is responsible for the management of the GUI itself, injecting itself into the OpenGL draw code and the GLFW Input Callbacks.

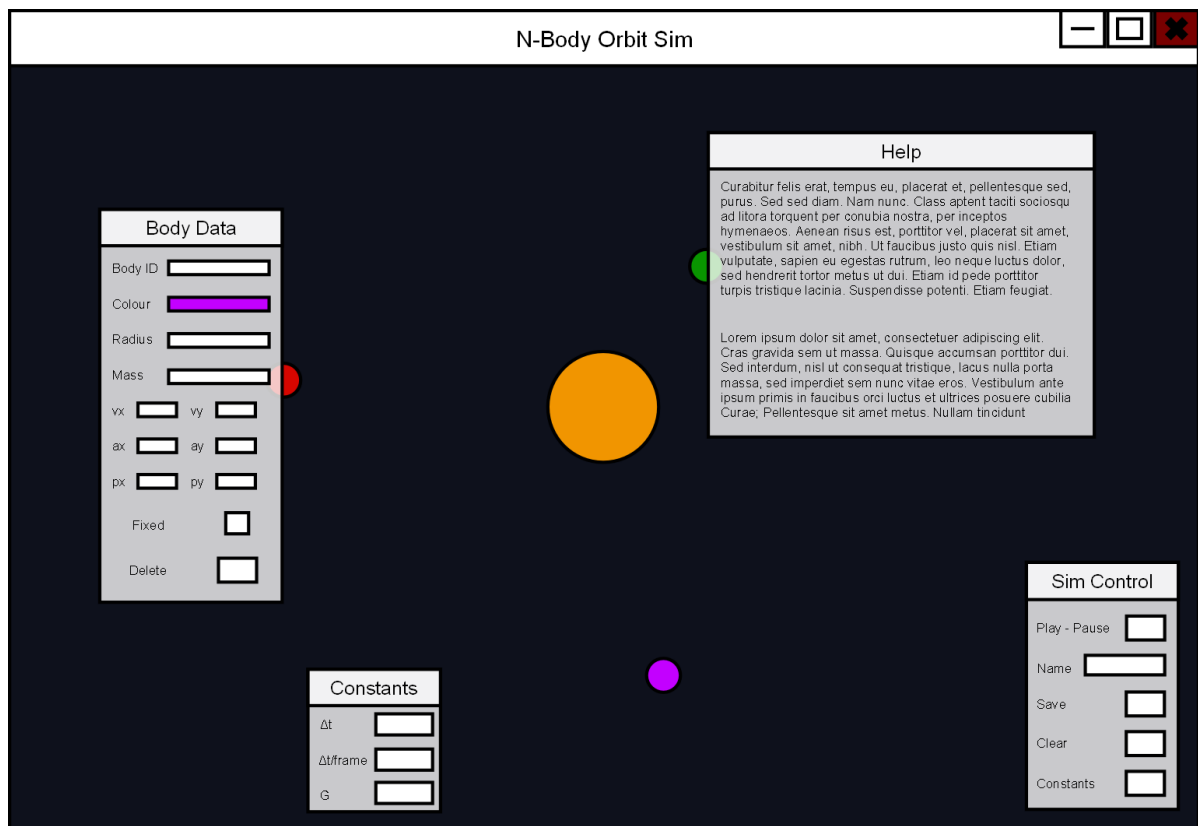


Figure 15: Initial UI Mockup

The AntTweakBar windows can be moved around within the OpenGL window, they will remain a constant size even if the main window is resized. The same goes for the OpenGL render, as it will stay a constant size even if the window is redrawn and will require the user to zoom in using the mouse wheel to change it.

All of the windows will be slightly transparent, but will be able to be closed using controls on the top of individual windows. The windows can also be opened with a



respective keyboard short-cut, with the exception of the body window, which opens when a particular body is selected, only one body can be selected at any time.

As mentioned previously, the circular bodies cannot be drawn as true circles, but instead must be drawn as a polygon, this will potentially cause objects to have very visible edges, it may be worthwhile to implement an algorithm that dynamically adjusts the number of vertices in the circle based on its radius in order to keep it visually smooth as an extension.

Another feature is that it is possible to drag and drop save files into the program from any directory rather than having the file inside the save file directory and typing in its name.

What this leads to is a very flexible, interactive interface, both the user input and output is combined and is completely interactive, user inputs make quick and very visible changes without necessarily needing to confirm the change, instead they can make changes, look at the result it gives and then unpaused to continue the simulation with the modifications made.

Due to the inherent complexity in this interface, it may be important to include a inbuilt help system in the program, giving a basic description of how to interact with the program, providing a very quick reference for first time users.

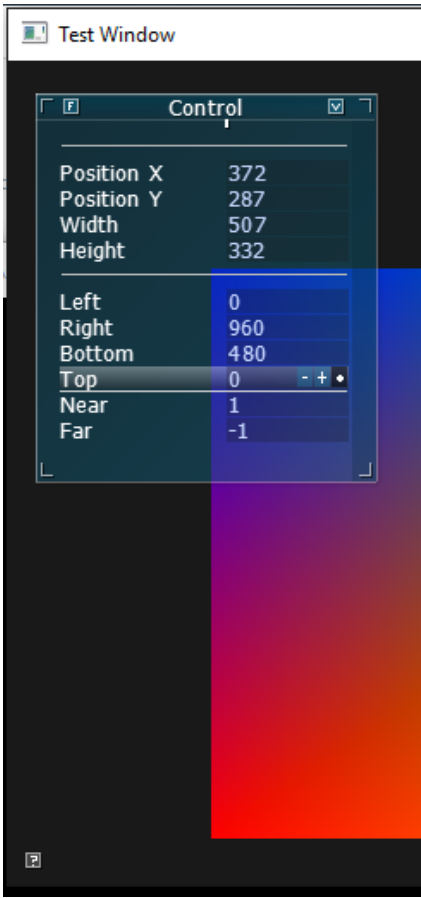


Figure 16: Example of AntTweakBar

## 2.10. Testing Strategy

Throughout both the implementation and testing stage there should be a range of different testing carried out to ensure that the program is functioning correctly, due to the complexity and 'locked-modular' construction of the multi-threaded program, it is somewhat difficult to test certain portions of the code using white-box testing, this means that a large proportion of the tests will be done using black-box testing.

Black-box testing is to be carried out in order to test the interface, this will be using the UI features that allow the user to add a body or modify a value of a body, checking that they work as intended from a user-centric point of view. Black-box testing will also be used in the performance profiling of the application, checking the performance by doing many runs with different body counts and timing the number of iterations completed in a set period of time. Some basic scenarios can be run to that will only work if the system is truly doing n-body style calculations, and would collapse if the system is not calculating the interaction between every body.

White-box testing will be used on several algorithms, such as the force calculation, checking that the relationships that are all calculated, rather than just relying on checking that a scenario runs as intended, actually verifying the loop that runs through the forces to be calculated and only runs the loop for the relationships that need to be calculated, if for example it was doing more than required the final simulation results would not be correct.

Trace tables will also be used in a limited capacity, because of the complexity of the system and the time it would take to do these calculations by hand a very simple scenario should be used where only the outer body is moving, even with this, the total movement of the body will be quite low over a small number of iterations.

## 2.11. Testing Plan

I have decided to split the testing plan into three main sections, System, containing basic black box tests that relate to the way memory is managed by the program, using Valgrind Memcheck in order to ensure that my code is not creating large memory leaks. As well as this, Valgrind Helgrind will be used to check that data locks are correctly implemented and data is not being accessed concurrently.

Interface testing concerns the use of the interface of the program, using basic white box testing to check that the mouse coordinates are being correctly converted from the window coordinates and that bodies can be selected by the user. The features in the UI interface library are also tested. The loss of rendering precision at the extents of single floating point precision is also showcased.

Simulation testing contains basic tests that should behave in very predictable ways allowing the checking of the implemented simulation algorithm, including a very basic trace table. Collision is also tested in a isolated fashion. The basic limitations that have been placed on the simulation, relating to the 'Laws of Physics' and will delete bodies that break the set limits.

The simulation tests also contain a basic benchmark test which will profile the performance of the simulation compared to the number of bodies used.

### 2.11.1. Test Scenarios

In order to speed up the testing of the program, some basic test scenarios have been defined in order to provide a known system that can be loaded into the application during testing in order to speed up the testing process, reducing the need to manually load the scenario every time.

1. **(Ref: TS1)** Single Body with no velocity.
  - Should stay completely stationary.
  - **IDT:** 0.1 (Simulation Delta Time)
  - **UGC:** 0.1 (Gravitational Constant)
  - **BC:** 1 (Body Count)
    - **Mass:** 1, **Radius:** 1, **Pos X:** 0 **Y:** 0, **Vel X:** 0 **Y:** 0
2. **(Ref: TS2)** Two body orbital system, orbital velocity.
  - Outer body has the velocity required for a circular orbit around the central body.
  - This is calculated using  $v_{circular} = \sqrt{\frac{G(M+m)}{r}}$ .
  - Orbital period can be found using  $T = \sqrt{\frac{4\pi^2 r^3}{Gm}}$ .
  - It should take 628.3 seconds or 6283 Iterations to complete one orbit. ( $T/IDT$ )
  - **IDT:** 0.1
  - **UGC:** 0.1
  - **BC:** 2
    - **Mass:** 1000, **Radius:** 10, **Pos X:** 0 **Y:** 0, **Vel X:** 0 **Y:** 0
    - **Mass:** 0.1, **Radius:** 1, **Pos X:** 100 **Y:** 0, **Vel X:** 0 **Y:** 1.00005
3. **(Ref: TS3)** Two body collision test, no gravity.
  - Both bodies start with velocity which will lead to a collision.
  - Gravitational constant is 0 to keep the velocity constant.
  - Bodies merge according to  $p = mv$ ,  $m_n = m_1 + m_2$ ,  $v_n = (p_1 + p_2)/m_n$ .
  - New radius is calculated by adding together circular areas  $\pi r^2$ .
  - **IDT:** 0.1
  - **UGC:** 0.0
  - **BC:** 2
    - **Mass:** 1, **Radius:** 1, **Pos X:** -10 **Y:** 0, **Vel X:** 1 **Y:** 0
    - **Mass:** 1, **Radius:** 1, **Pos X:** 10 **Y:** 0, **Vel X:** -1 **Y:** 0

4. (**Ref: TS4**) Defines a superstructure.
  - Starts the simulation with a superstructure, variable body count.
  - Relies on a separate variable to set body count.
  - **IDT:** 0.1
  - **UGC:** 0.1
  - **BC:** Variable
    - **Superstructure**
      - \* **Bodies:** Variable
      - \* **Central Mass:**  $1 \times 10^5$
      - \* **Outer Mass:** 0.1
      - \* **Central Radius:** 10
      - \* **Outer Radius:** 0.1
      - \* **Central Pos X:** 0
      - \* **Central Pos Y:** 0
      - \* **Central Vel X:** 0
      - \* **Central Vel Y:** 0
      - \* **Central Spacing:** 50
      - \* **System Radius:** 1000
5. (**Ref: TS5**) Body travels beyond simulation limits.
  - Should be destroyed once beyond simulation bounds.
  - **IDT:** 0.1 (Simulation Delta Time)
  - **UGC:** 0.1 (Gravitational Constant)
  - **BC:** 1 (Body Count)
    - **Mass:** 1, **Radius:** 1, **Pos X:**  $9.999 \times 10^{15}$  **Y:** 0, **Vel X:**  $1 \times 10^8$  **Y:** 0
6. (**Ref: TS6**) Body travelling faster than the speed of light.
  - Should be destroyed on iteration.
  - **IDT:** 0.1
  - **UGC:** 0.1
  - **BC:** 1
    - **Mass:** 1, **Radius:** 1, **Pos X:** 0 **Y:** 0, **Vel X:**  $3.00 \times 10^8$  **Y:** 0

7. (**Ref: TS7**) Grid of bodies for testing coordinate mapping and rendering precision.

- All bodies are fixed, velocity is ignored and not calculated.
- Bodies in center are positioned at 10 and 100 positions respectively.
- To the right of the system exists 4 bodies spaced an order of magnitude apart in order to show the loss of rendering precision.
- **IDT: 0.1**
- **UGC: 0.1**
- **BC: 20**
  - **Mass: 0.001, Radius: 1, Pos X: 10 Y: 0**
  - **Mass: 0.001, Radius: 1, Pos X: -10 Y: 0**
  - **Mass: 0.001, Radius: 1, Pos X: 0 Y: 10**
  - **Mass: 0.001, Radius: 1, Pos X: 0 Y: -10**
  - **Mass: 0.001, Radius: 1, Pos X: -10 Y: -10**
  - **Mass: 0.001, Radius: 1, Pos X: 10 Y: -10**
  - **Mass: 0.001, Radius: 1, Pos X: -10 Y: 10**
  - **Mass: 0.001, Radius: 1, Pos X: 10 Y: 10**
  - **Mass: 0.001, Radius: 1, Pos X: 100 Y: 0**
  - **Mass: 0.001, Radius: 1, Pos X: -100 Y: 0**
  - **Mass: 0.001, Radius: 1, Pos X: 0 Y: 100**
  - **Mass: 0.001, Radius: 1, Pos X: 0 Y: -100**
  - **Mass: 0.001, Radius: 1, Pos X: -100 Y: -100**
  - **Mass: 0.001, Radius: 1, Pos X: 100 Y: -100**
  - **Mass: 0.001, Radius: 1, Pos X: -100 Y: 100**
  - **Mass: 0.001, Radius: 1, Pos X: 100 Y: 100**
  - **Mass: 0.001, Radius: 1, Pos X:  $1 \times 10^5$  Y: 0**
  - **Mass: 0.001, Radius: 1, Pos X:  $1 \times 10^6$  Y: 0**
  - **Mass: 0.001, Radius: 1, Pos X:  $1 \times 10^7$  Y: 0**
  - **Mass: 0.001, Radius: 1, Pos X:  $1 \times 10^8$  Y: 0**

8. (**Ref: TS8**) Three body system, Sun - Planet - Moon.
- An outer body orbits a massive central body.
  - Another body orbits the outer body.
  - **IDT:** 0.1
  - **UGC:** 0.1
  - **BC:** 3
    - **Mass:** 200, **Radius:** 5, **Pos X:** 0 **Y:** 0, **Vel X:** 0 **Y:** 0
    - **Mass:** 1, **Radius:** 0.5, **Pos X:** 20 **Y:** 0, **Vel X:** 0 **Y:** 1.00005
    - **Mass:** 0.001, **Radius:** 0.1, **Pos X:** 20 **Y:** 1, **Vel X:** 0.316 **Y:** 1.00005
9. (**Ref: TS9**) Simple two body system.
- This system is simple enough to be easy to create a trace table for.
  - The central body is fixed, making it much faster to run calculations.
  - A large time-step is used in order to make the numbers easier to handle.
  - **IDT:** 2
  - **UGC:** 0.1
  - **BC:** 2
    - **Mass:** 99, **Radius:** 1, **Pos X:** 0 **Y:** 0, **Fixed**
    - **Mass:** 1, **Radius:** 1, **Pos X:** 10 **Y:** 0, **Vel X:** 0 **Y:** 1

10. (**Ref: TS10**) Defines a simple two body system using a superstructure.
  - Creates a superstructure with the generation of a single body constrained to 100.
  - **IDT: 0.1**
  - **UGC: 0.1**
  - **BC: 2**
    - **Superstructure**
      - \* **Bodies: 1**
      - \* **Central Mass:  $1 \times 10^5$**
      - \* **Outer Mass: 0.1**
      - \* **Central Radius: 10**
      - \* **Outer Radius: 0.1**
      - \* **Central Pos X: 0**
      - \* **Central Pos Y: 0**
      - \* **Central Vel X: 0**
      - \* **Central Vel Y: 0**
      - \* **Central Spacing: 100.0**
      - \* **System Radius: 100.001**
11. (**Ref: TS11**) Stable three body system.
  - This system is a three-body system which remains stable. (Unless perturbed.)
  - The outer bodies have velocity such that they precess around each other.
  - Their presence applies an equal force to both sides of the central body, thus balancing to 0.
  - Any change will send the system into chaotic motion.
  - **IDT: 0.01**
  - **UGC: 0.1**
  - **BC: 3**
    - **Mass: 1, Radius: 1, Pos X: 0 Y: 0, Vel X: 0 Y: 0**
    - **Mass: 1, Radius: 1, Pos X: 10 Y: 0, Vel X: 0 Y: 0.11**
    - **Mass: 1, Radius: 1, Pos X: -10 Y: 0, Vel X: 0 Y: -0.11**



12. (Ref: TS12) Many Body System

- This system contains several bodies which will very clearly show the interactions occurring correctly between every body.
- **IDT:** 0.01
- **UGC:** 10
- **BC:** 6
  - **Mass:** 1, **Radius:** 1, **Pos X:** 10 **Y:** 2, **Vel X:** 0 **Y:** 0
  - **Mass:** 1, **Radius:** 1, **Pos X:** -4 **Y:** 4, **Vel X:** 0 **Y:** 0
  - **Mass:** 1, **Radius:** 1, **Pos X:** -18 **Y:** 16, **Vel X:** 0 **Y:** 0
  - **Mass:** 1, **Radius:** 1, **Pos X:** -8 **Y:** -12, **Vel X:** 0 **Y:** 0
  - **Mass:** 1, **Radius:** 1, **Pos X:** 6 **Y:** -13, **Vel X:** 0 **Y:** 0
  - **Mass:** 1, **Radius:** 1, **Pos X:** 8 **Y:** 11, **Vel X:** 0 **Y:** 0

### 2.11.2. System Tests

| Ref  | Name                                 | Description   | Data Set            | Data Type | Expected Result  |
|------|--------------------------------------|---|---------------------|-----------|--|
| SYS1 | Initial Memory Leak Check            | Run through valgrind, exit, check that own code is not leaking.   | Default             | Typical   | 72 Bytes are always 'definitely lost' in GLFW, Related to keyboard in Linux. |
| SYS2 | Memory Leak Delete All Bodies Check  | Run through valgrind, pause, delete all bodies, exit, check that own code is not leaking.                   | Default, User Input | Typical   | 72 Bytes are always 'definitely lost' in GLFW, Related to keyboard in Linux. |
| SYS3 | Memory Leak Add Body Check           | Run through valgrind, add a body, unpause, exit, check that own code is not leaking.                        | Default, User Input | Typical   | 72 Bytes are always 'definitely lost' in GLFW, Related to keyboard in Linux. |
| SYS4 | Memory Leak Delete, Add, Delete Body | Run through valgrind, pause, delete central body, add new body, unpause, pause again and delete body, exit. | Default, User Input | Typical   | 72 Bytes are always 'definitely lost' in GLFW, Related to keyboard in Linux. |
| SYS5 | Program Exits after Sim Thread Exits | When the application is closed, the simulation thread will exit before the program terminates.              | Default, User Input | Typical   | Program does not hang on exit. Use cerr stream used to check.                |
| SYS6 | No Concurrent Access to Shared Data  | Run in Helgrind, check that access is locking correctly.  | Default             | Typical   | No errors that indicate simultaneous access.                                 |

### 2.1.1.3. Interface Tests

| Ref   | Name                                  | Description   | Data Set                 | Data Type | Expected Result  |
|-------|---------------------------------------|---|--------------------------|-----------|--|
| GUI1  | Mouse to World Coordinates            | Using TS7, move the mouse pointer onto all of the points, they should map exactly to their positions in world space.  | TS7, Mouse Input         | Typical   | Every Point Maps to its correct position.                                |
| GUI2  | Mouse to World Coordinates (Panning)  | Using TS7, Pan the camera around to other positions, move the mouse pointer onto several of the points, they should map exactly to their positions in world space.                    | TS7, Mouse Input         | Typical   | Every point maps to its correct position.                                |
| GUI3  | Mouse to World Coordinates (Zooming)  | Using TS7, Zoom the camera in, move the mouse pointer onto several of the points, they should map exactly to their positions in world space. Repeat for zoom out.                     | TS7, Mouse Input         | Typical   | Every point maps to its correct position.                                |
| GUI4  | Mouse to World Coordinates (Pan+Zoom) | Using TS7, Zoom the camera in and pan to other positions, move the mouse pointer onto several points, they should map exactly to their positions in world space. Repeat for zoom out. | TS7, Mouse Input         | Typical   | Every point maps to its correct position.                                |
| GUI5  | Mouse to World Coordinates (Extreme)  | Using TS7, Pan the camera to the right, there is a large body at 100000 on the X Axis, check that the mapping is still correct.   | TS7, Mouse Input         | Typical   | Every point maps to its correct position.                                |
| GUI6  | Body Add (GUI)                        | If the user presses the 'New Body' button on the body UI and the simulation is paused, a body will be added at 0, 0, with mass of 1, radius of 1 and zero velocity.                   | Mouse Input              | Typical   | Body at 0, 0, becomes default selected.                                  |
| GUI7  | Body Add (Mouse)                      | If the user presses the right mouse button when the simulation is paused, a body will be added at that location with mass 1, radius of 1 and zero velocity.                           | Mouse Input, Right Click | Typical   | World coordinates of the mouse are already populated into body position. |
| GUI8  | Body Delete (GUI)                     | If the user presses the 'Delete Body' button when the simulation is paused, the currently selected body will be completely deleted from the simulation.                               | TS4                      | Typical   | Body count is reduced by one, correct body has been deleted.             |
| GUI9  | Body Over-Delete                      | If the user presses the 'Delete Body' button when no bodies exists in the simulation, the program does not crash. (Only when paused)  | Empty Scenario           | Extreme   | Body count remains at 0, program still runs                              |
| GUI10 | Delete All Bodies                     | If the user presses the 'Delete All Bodies' button when the simulation is paused, all bodies will be removed from the simulation.   | TS4                      | Typical   | Body count becomes 0.  |
| GUI11 | Superstructure Creation Dialogue      | If the user unminimises it, a superstructure creation window will appear allowing the creation of a large number of organised bodies.   | Mouse Input              | Typical   | Window opens, default values present                                     |

Table 7: Interface Tests Continued

| Ref   | Name                            | Description   | Data Set                   | Data Type | Expected Result   |
|-------|---------------------------------|---|----------------------------|-----------|---|
| GUI12 | Superstructure Creation         | If the user presses the 'create' button on the superstructure creation window a superstructure is added to the simulation.  | - (Use default values)     | Typical   | Expected shape should be based on the parameters that are described in test scenario.                   |
| GUI13 | Superstructure Orbital Velocity | If the user presses the 'create' button on the superstructure creation window a superstructure is added to the simulation. (Start Paused)   | -                          | Typical   | All bodies in the superstructure have the correct velocity and orbit the central body. (Circular Orbit) |
| GUI14 | Body Modify Value (Accepted)    | Modify an attribute of a body while the simulation is paused, then unpauses the simulation. Eg: Position  | 0, 10, 1E3, 152            | Typical   | The change persists when unpaused.  |
| GUI15 | Body Modify Value (Erroneous)   | Modify an attribute of a body while the simulation is paused, then unpauses the simulation. Eg: Position  | 1E18, ABC, -1E17, asdfseaq | Erroneous | The body value does not change.   |
| GUI16 | IPF Change                      | Change Iterations Per Frame to check that the correct number are being carried out  | 1, 2, 5, 32                | Typical   | Correct number of iterations displayed on cerr stream.  |
| GUI17 | Loss of Rendering Precision     | Using TS7, Zoom out and pan the camera to the right, there are multiple bodies stretching far out, each an order of magnitude apart, observe the loss in precision in single precision floating point. (At each body) | TS7                        | Benchmark | The final body will not be visible at max zoom level.   |

### 2.11.4. Simulation Tests

| Ref   | Name  | Description  | Data Set  | Data Type           | Expected Result         |
|-------|---|--|---|---------------------|-------------------------|
| SIM1  | Stationary Body                             | Using TS1, simulate for 10 iterations, body should remain stationary.  | TS1   | Typical             | $\pm 0.0$ (XY Position) |
| SIM2  | Two-Body Force Check (B1)                   | Using TS2, read value calculated force value on X on smaller body and check that it is correct (Single ITR) ( $F = ma$ )   | TS2   | Typical             | 0.001 Newtons           |
| SIM3  | Two-Body Force Check (B2)                   | Using TS2, read value calculated force value on X on larger body and check that it is correct (Single ITR) ( $F = ma$ )  | TS2   | Typical             | 0.001 Newtons           |
| SIM4  | Circular Orbit Check (Complete)             | Using TS2, Simulate for 6283 Iterations, one full orbit should be completed, compare start position to end position  | TS2   | Typical (Deviation) | $\pm 1.0$ (XY Position) |
| SIM5  | Circular Orbit Check (Distance)             | Using TS2, Simulate for 6283 Iterations, one full orbit should be completed, check vector distance between two bodies, should remain constant within deviation over simulation.        | TS2   | Typical (Deviation) | $\pm 1.0$               |
| SIM6  | Two-Body Collision Test (Deletion/Creation) | Using TS3, Simulate to test collision of two bodies, Check Old Body Deleted and New Body Created. (No acceleration due to gravity)   | TS3   | Typical             | Body Count = 1          |
| SIM7  | Two-Body Collision Test (Mass)              | Using TS3, Simulate to test collision of two bodies, Check mass of new body is equal to combined mass of previous bodies. (And velocity changes accordingly)                           | TS3 [10, 1], [20, 100], [1000, 1000]            | Typical             | Dependant on input      |
| SIM8  | Two-Body Collision Test (Velocity)          | Start up the simulation, (paused), initial acceleration on the central body should be balanced. (0)  | TS3, [1, 1], [1, 2], [-150, 150]                | Typical             | Dependant on input      |
| SIM9  | Two-Body Collision Test (Radius)            | Using TS3, Simulate to test collision of two bodies, Check that the radius of the new body conforms to $\pi r^2 + \pi R^2$ (Adding the areas)  | TS3, [1, 10], [2, 10], [5, 5]                   | Typical             | Dependant on input      |
| SIM10 | High Body Count Benchmark                   | Using TS4, Simulate the interaction of a range of bodies and report how many iterations are completed within 20 seconds. (Remove Synchronisation, Collisions Off) (Time 20s and pause) | TS4 (Test 10, 20, 40, 80 in decades upto 10000) | Benchmark           | Should Not Crash        |
| SIM11 | Simulation Bounds Exit                      | Using TS5, Simulate a body on course to exit the simulation bounds, body should be deleted.  | TS5   | Extreme             | Body Count = 0          |

Table 8: Simulation Tests Continued

| Ref   | Name                               | Description   | Data Set | Data Type | Expected Result  |
|-------|------------------------------------|---|----------|-----------|--|
| SIM12 | Simulation Max Speed               | Using TS6, Attempt to populate body storage with a body that has a velocity higher than the speed of light.       | TS6      | Erroneous | Body Count = 0   |
| SIM13 | Simulation Trace Table (Two-Body)  | Run through algorithm using manual method, compare simulation results. (8 iterations)                             | TS9      | Typical   | Results should match                                     |
| SIM14 | Simulation Sun Planet Moon Test    | Run simulation scenario, moon should orbit planet, planet should orbit sun.                                       | TS8      | Typical   | -  |
| SIM15 | Three Body Stability               | Start up the simulation, (paused), initial acceleration on the central body should be balanced. (0)               | TS11     | Typical   | Acceleration on central should be 0, 0.                  |
| SIM16 | Three Body Chaos                   | Pause the simulation, add small velocity to central body on x, system will collapse into chaos.                   | TS11     | Typical   | Simulation becomes chaotic                               |
| SIM17 | Colour Mixing                      | Run a simulation using a large number of bodies, such as three super structures that are three different colours. | -        | Typical   | Colours will mix together correctly (Weighted Averaging) |
| SIM18 | Force Calculation Loop             | Run the simulation through a single iteration to show which forces are calculated.                                | TS11     | Typical   | 0-1, 0-2, 1-2  |
| SIM19 | Force Calculation Loop Trace Table | Calculate the forces for the initial state of objects for each relationship.                                      | TS12     | Typical   | Results should match                                     |

### 3. Implementation

## 4. Testing



## 5. Manual

## 6. Evaluation

## 7. Code Listing

### 7.1. Early Prototypes

#### 7.1.1. Euler (Trig)

```
1 #include <iostream>
2 #include <cmath>
3 #include <unistd.h>
4
5 #define GRAVCONST 1
6 #define TIMESTEP 1
7
8 using namespace std;
9
10 // Types
11 typedef struct {
12     // Properties
13     double mass = 0.0;
14     // State
15     double force[2] = {0.0, 0.0};
16     double acceleration[2] = {0.0, 0.0};
17     double velocity[2] = {0.0, 0.0};
18     double position[2] = {0.0, 0.0};
19 } sBody; // Single Body;
20
21 typedef sBody bodyPair[2];
22
23 // Prototypes
24
25 // Functions
26 int main() {
27     bodyPair currentBodies;
28     int i = 0, xy;
29     double currentAngle;
30     double forceCalc, distance, diffX, diffY;
31     double forceX, forceY;
32     double deltaVelX, deltaVelY;
33     double deltaPosX, deltaPosY;
34
35     double deltaVelXs, deltaVelYs;
36     double deltaPosXs, deltaPosYs;
37
38     // Setup
39     // M0
40     currentBodies[0].mass = 100;
41     currentBodies[0].position[0] = 0.0;
42     currentBodies[0].position[1] = 0.0;
43     // M1
44     currentBodies[1].mass = 1;
45     currentBodies[1].position[0] = 0.0;
46     currentBodies[1].position[1] = 40;
```

```

47 currentBodies[1].velocity[0] = 1.58;
48 currentBodies[1].velocity[1] = 0.0;
49
50 // Simulation
51 while (i < 500000) {
52     // Calculate Distance between Bodies
53     diffX = currentBodies[0].position[0] - currentBodies[1].position[0];
54     diffY = currentBodies[0].position[1] - currentBodies[1].position[1];
55     distance = sqrt(pow(diffX,2) + pow(diffY,2));
56
57     // Calculate Gravitational Force
58     forceCalc = -((GRAVCONST * currentBodies[0].mass * currentBodies[1].mass) / (pow(distance,2)));
59
60     // Find Angle Relative to X and resolve forces.
61     currentAngle = atan(diffY/diffX);
62     forceX = forceCalc * cos(currentAngle);
63     forceY = forceCalc * sin(currentAngle);
64     if(diffX < 0) {
65         forceX = -forceX;
66         forceY = -forceY;
67     }
68
69     // Put forces into body data
70     currentBodies[1].force[0] = -forceX;
71     currentBodies[1].force[1] = -forceY;
72
73     // Calculate Acceleration
74     currentBodies[1].acceleration[0] = currentBodies[1].force[0] / currentBodies[1].mass;
75     currentBodies[1].acceleration[1] = currentBodies[1].force[1] / currentBodies[1].mass;
76
77     // Calculate delta velocity
78     deltaVelX = currentBodies[1].acceleration[0] / TIMESTEP;
79     deltaVelY = currentBodies[1].acceleration[1] / TIMESTEP;
80
81     // Calculate position delta (S = ut + 0.5 * at^2)
82     deltaPosX = (currentBodies[1].velocity[0] * TIMESTEP + (0.5 * (currentBodies[1].acceleration[0] *
83         pow(TIMESTEP,2))));
84     deltaPosY = (currentBodies[1].velocity[1] * TIMESTEP + (0.5 * (currentBodies[1].acceleration[1] *
85         pow(TIMESTEP,2))));
86
87     // Update Velocity
88     currentBodies[1].velocity[0] += deltaVelX;
89     currentBodies[1].velocity[1] += deltaVelY;
90
91     // Update Position
92     currentBodies[1].position[0] += deltaPosX;
93     currentBodies[1].position[1] += deltaPosY;
94
95     cout << currentBodies[1].position[0] << ", " << currentBodies[1].position[1] << endl;
96     i++;
97 }

```

### 7.1.2. Euler (Vector) with $\Delta E$

```
1 #include <iostream>
2 #include <cmath>
3
4 using namespace std;
5
6 int main() {
7     // Body Data
8     double position_r[2], velocity[2], acceleration[2];
9     // Time Variables
10    double deltaT, simTime;
11    const double outputTime = 0.01;
12    double tc = outputTime;
13
14    // Setup Initial Parameters
15    cerr << "Enter_Timestep(s):_";
16    cin >> deltaT;
17    cerr << "Enter_Simulation_Time(s):_";
18    cin >> simTime;
19
20    // Relative Position XY
21    position_r[0] = 1;
22    position_r[1] = 0;
23    // Velocity XY
24    velocity[0] = 0;
25    velocity[1] = 0.5;
26
27    // Calculate Initial System Total Energy
28    double e_k = 0.5 * (pow(velocity[0],2) + pow(velocity[1],2)); // Kinetic Energy
29    double e_p = - 1.0 / sqrt(pow(position_r[0],2) + pow(position_r[1],2)); // Potential Energy
30    double e_i = e_k + e_p; // Initial Energy
31
32    // Simulations
33    for (double t = 0; t < simTime; t += deltaT) {
34
35        // Calculate Relative Distance Squared
36        double r2 = pow(position_r[0],2) + pow(position_r[1],2);
37
38        // Calculate Acceleration
39        for (int c = 0; c < 2; c++) {
40            acceleration[c] = - position_r[c] / (pow(sqrt(r2), 3)); // a = p/r^3
41        }
42
43        // Calculate New Position and New Velocity
44        for (int c = 0; c < 2; c++) {
45            position_r[c] += velocity[c] * deltaT;
46            velocity[c] += acceleration[c] * deltaT;
47        }
48
49        // Output Data at Interval
50        if (t >= tc) {
51            cout << position_r[0] << "_ " << position_r[1] << endl;
```

```

52     //cerr << t << endl;
53     tc += outputTime;
54 }
55 }
56
57 // Calculate Final System Total Energy
58 e_k = 0.5 * (pow(velocity[0],2) + pow(velocity[1],2)); // Kinetic Energy
59 e_p = - 1.0 / sqrt(pow(position_r[0],2) + pow(position_r[1],2)); // Potential Energy
60 double e_f = e_k + e_p; // Final Energy
61
62 // Calculate Error
63 double error = (e_f-e_i)/e_i * 100;
64 cerr << "Percentage_Error:_" << error << "%." << endl;
65 return 0;
66 }

```

### 7.1.3. Leapfrog (Vector) with $\Delta E$

```
1 #include <iostream>
2 #include <cmath>
3
4 using namespace std;
5
6 int main() {
7     // Body Data
8     double position_r[2], velocity[2], acceleration[2];
9     // Time Variables
10    double deltaT, simTime;
11    const double outputTime = 0.01;
12    double tc = outputTime;
13
14    // Setup Initial Parameters
15    cerr << "Enter_Timestep(s):_";
16    cin >> deltaT;
17    cerr << "Enter_Simulation_Time(s):_";
18    cin >> simTime;
19
20    // Relative Position XY
21    position_r[0] = 1.0;
22    position_r[1] = 0.0;
23    // Velocity XY
24    velocity[0] = 0.0;
25    velocity[1] = 0.5;
26
27    // Calculate Initial System Total Energy
28    double e_k = 0.5 * (pow(velocity[0],2) + pow(velocity[1],2)); // Kinetic Energy
29    double e_p = - 1.0 / sqrt(pow(position_r[0],2) + pow(position_r[1],2)); // Potential Energy
30    double e_i = e_k + e_p; // Initial Energy
31
32    // Calculate Initial Acceleration
33    double r2 = pow(position_r[0],2) + pow(position_r[1],2);
34    for (int c = 0; c < 2; c++) {
35        acceleration[c] = - position_r[c] / (r2 * sqrt(r2));
36    }
37
38    // Simulations
39    for (double t = 0; t < simTime; t += deltaT) {
40        // Calculate New Velocity and Position
41        for (int c = 0; c < 2; c++) {
42            velocity[c] += 0.5 * acceleration[c] * deltaT;
43            position_r[c] += velocity[c] * deltaT;
44        }
45
46        // Calculate New Acceleration and Velocity
47        r2 = pow(position_r[0],2) + pow(position_r[1],2);
48        for (int c = 0; c < 2; c++) {
49            acceleration[c] = - position_r[c] / (r2 * sqrt(r2));
50            velocity[c] += 0.5 * acceleration[c] * deltaT;
51        }
52    }
```

```

52
53 // Output Data at Interval
54 if (t >= tc) {
55     cout << position_r[0] << " " << position_r[1] << endl;
56     cerr << t << "\r";
57     tc += outputTime;
58 }
59 }
60
61 // Calculate Final System Total Energy
62 e_k = 0.5 * (pow(velocity[0],2) + pow(velocity[1],2)); // Kinetic Energy
63 e_p = - 1.0 / sqrt(pow(position_r[0],2) + pow(position_r[1],2)); // Potential Energy
64 double e_f = e_k + e_p; // Final Energy
65
66 // Calculate Error
67 double error = (e_f-e_i)/e_i * 100;
68 cerr << "Percentage_Error:" << error << "%." << endl;
69 return 0;
70 }

```



# Appendices

## A. Dialogue

I had several brief conversations with my teacher that were not documented during pre-planning stages of the project, these conversations consisted of outlining the idea for a teaching tool that could simulate a scenario set-up by a teacher in order to accurately demonstrate the fundamentals of circular and orbital motion, as a physics student I already have some understanding of what would be required from a project like this.

---

TO: Mr Snowden

In terms of my A2 Computing Project, I have a basic idea which I think would be workable and fit what you described previously, please let me know if you have any more ideas to expand the program. The program will feature relatively simple visuals, probably 2D, meaning it should run well on low end computers in the school, the program will simulate individual objects at the appropriate level in sped-up real time. You will be able to set up scenarios by introducing objects, varying the mass and velocity of the object before placement, multiple objects can be placed and they will interact with each other. (n-body problem could be interesting)

Creating the user interface for this will be somewhat of a challenge, likely I will try to integrate it all into a single window as it would remove the need for a secondary library for the user interface, information about what state an object is going to be placed down in (Mass, Velocity, Size, Fixed) As well as simulation state. (Time, Simulation Speed), A right click menu could contain certain options (Presets) but keyboard controls would be the primary method of control.

I have also considered implementing a system which would allow the saving and loading of different scenarios, allowing them to be set-up, saved and loaded at other times, which could be useful in a classroom situation, I would be interested to know if you would find this feature useful.

If you have any questions, let me know, I need to have a bit of a dialogue going in order to establish some groundwork, as the project progresses I will have questions for you in regards to particular decisions or compromises that need to be made along the way.

Many Thanks

Byron Theobald.

---

TO: Byron

A really useful innovation might be to have a graphical way of representing initial velocity, e.g. an arrow extending from each mass, representing the vector for initial velocity. I like the idea of being able to save the scenario for later on, it would definitely increase the value of the tool.

A very difficult (but very useful) feature might be to be able to tick a box to display both gravitational field lines and lines of equipotential. This is an area of Physics which you haven't covered yet, but is fairly straightforward...

Thanks for trying a Physics application!

SDS

---

TO: Mr Snowden

Something I wanted to find out is how you would like the orbital simulator to handle collisions.

Based on my understanding of momentum, simply adding the mass of the two bodies and recalculating the velocity based on conservation of momentum would make sense.

Happy to hear your thoughts on this.

---

TO: Byron

Yes, provided momentum is conserved, everything will be fine. Remember that they will be inelastic collisions (ie KE not conserved).

Your idea below assumes no fragments are ejected - a reasonable assumption to avoid hideous complexity.

Best wishes, SDS

---

TO: Mr Snowden

I have attached a copy of the objectives that I will be adhering to during the development and implementation of the current system.

If you could review these and let me know your thoughts.

Many Thanks

Byron Theobald

---

TO: Byron

All of these look good, nothing that I can think of that isn't on the list.

Best wishes, SDS

---

TO: Mr Snowden

I wanted to update you on the current progress of the orbital simulator, apologies for infrequency in that regard.

In its current state the code is sitting at just over 850 lines total.

So far the simulation portion is mostly complete, aside for the implementation of body collision detection and handling.

I have modified the simulation interface to allow it to be completely dynamic, meaning that it is much easier to define as many bodies as necessary in the code. While I have not tested the limits of this, aside from the memory and performance requirements there is no reason why there should be any limit to the number of bodies. (Part of my test plan revolves around benchmarking the performance of large numbers of bodies, up to 64. Back of the envelope calculations suggest that at this number, each frame would take more than 250ms to compute, this is a severe reduction in performance but you are unlikely to need to use more than a few bodies.)

I have also nearly completed the interface to link together the multi-threaded nature of the program, allowing data to seamlessly be transferred between the renderer and the simulation, there is still some parts that need to be added, such as getting new changes made by the user sent to the simulation, ignoring any previously stored simulation data, user changes must take priority.

The next major task is to implement the renderer, this involves some understanding of matrix operations due to the nature of OpenGL programming, however I do not expect this to be a major issue as it is mostly abstracted.

After the renderer is implemented I will then focus on the UI and other utilities.

Attached is an animation of the raw simulation output showing a model sun-planet-moon system

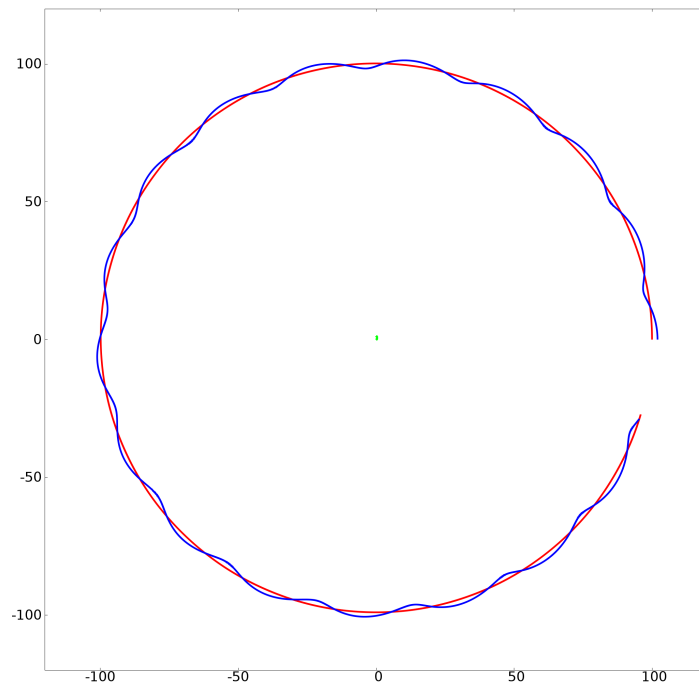


Figure 17: Three Body Sun-Planet-Moon System

---

TO: Mr Snowden

I implemented a basic version of the rendering module that i will be developing going forward.

Here is a preview:



Figure 18: N-Body Graphics Test

Performance is looking extremely promising, will test with a very large number of bodies once I am able to create a distribution system.

---

TO: Byron

Thanks for the updates. this all looks very promising indeed - keep up the good work and keep me posted!

Best wishes, SDS

---

Prototype testing of the simulation showed that it can still support a massive amount of bodies and run smoothly, several orders of magnitude more than expected.

I had a few discussions with Mr Snowden about the potential applications of having large numbers of bodies, I agreed to implement the ability for the user to generate these superstructures; similar to the one shown below, through the interface.

This could allow the software to be used to simulate a very low body count galaxy collision.

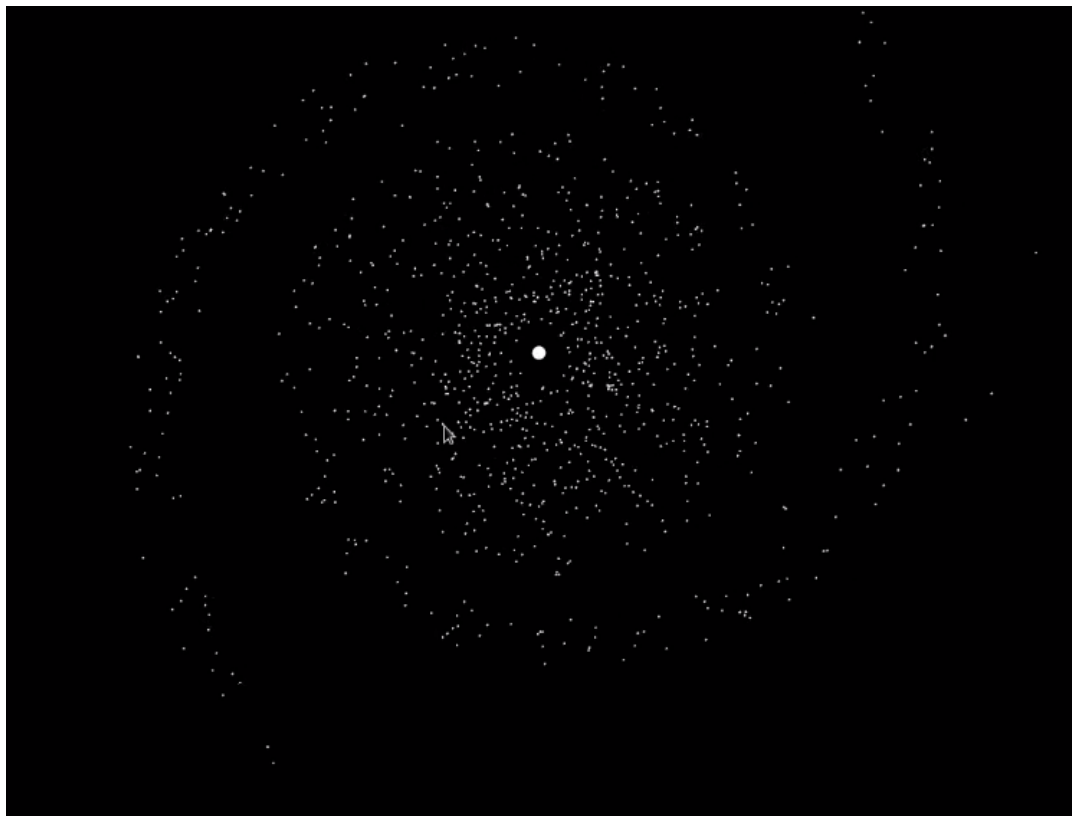


Figure 19: 1000 Body Superstructure

Something that we also decided on was the ability to turn on and off body collisions and combinations, as not having collisions results in the simulation staying more stable for longer.

While the earlier intention was to only consider the simulation for a few bodies, I will now consider the simulation for larger numbers of bodies, the benefit is that if the simulation is correct for 3 bodies, it will be correct for any number.

The other benefit that this brings is the showcase aspect of it, something like this 'looks' really good to anybody who is studying or considering studying Physics, It is also far more likely to move into the area of inspiration for younger people, increasing their interest in Physics and academia in general. Spectacle is something that can easily affect the way that people think.

TO Mr Snowden

As of the week before last I have put a freeze on major programming for the project.

During the weekend (week before last) I decided to rewrite the entire program from scratch, as the way that certain core functions of the program were written made it extremely difficult to add critical user interface controls for adding and deleting bodies, among other things.

I also rewrote major algorithms that stored the forces for each relationship and then summed them on the individual bodies after they had all been calculated, the issue with this is that it was a relatively large memory hog in the program, while it wasn't going to affect its normal use, I still felt that it was worth doing differently.

The current system is now much more efficient, as forces are no longer directly stored in the bodies themselves and instead, each relationship is calculated, converted to acceleration and then summed onto the body object, this is done for both axis and for both bodies in the relationship.

After the full rewrite there where substantial performance gains to the simulation, a 2000 body simulation runs about as well as a 1000 body simulation did previously.

Unfortunately, due to time constraints several features have not been included in the project at this current time, the main omission is the ability to save and load pre-made scenarios.

Regardless of that fact, I feel that in its current state the program is more than usable, I still do need to compile a version that can run on Windows, however that is not priority at the moment.

## References

- [1] Circle Drawing [http://slabode.exofire.net/circle\\_draw.shtml](http://slabode.exofire.net/circle_draw.shtml)
- [2] Maya Open Lab, **Moving Stars Around**, School Series, Piet Hut and Jun Makino  
<http://www.artcompsci.org/kali/pub/msa/volume1.pdf>