

The Variational Autoencoder exploits the methods described in the previous section to define a probabilistic model whose elbo is highly reminiscent of the objective optimised in a traditional autoencoder. In particular we define a generative model where we assume that the  $i^{th}$  observation was generated by first sampling a latent variable  $z_i \sim N(0, I)$  and that the each observation  $x_i \sim N(\mu_\theta(z_i), \sigma_\theta^2(z_i))$  if the observations are real-valued, or  $x_i \sim \text{Bernoulli}(f_\theta(z_i))$  if they are binary or valued on  $[0, 1]$ . In both cases the distribution parameters  $\mu_\theta(z_i), \sigma_\theta^2(z_i), f_\theta(z_i)$  are parameterised in terms of a multi-layer perceptron (MLP) whose input is  $z_i$ , which will be referred to as the “decoder MLP”. Specifically, define  $h_i$  to be the vector output at the final hidden layer of the decoder MLP when provided input  $z_i$ , then

$$\mu_\theta(z_i) = h_i W_\mu^{(q)} + b_\mu^{(q)}, \quad (1)$$

$$\log \sigma_\theta^2(z_i) = h_i W_\sigma^{(q)} + b_\sigma^{(q)}. \quad (2)$$

The recognition model is given by

$$q_\phi(z|x) = N(z | \mu_\phi(x_i), \sigma_\phi^2(x_i)), \quad (3)$$

where the distributional parameters  $\mu_\phi(x_i)$  and  $\sigma_\phi^2(x_i)$  are again given by an MLP whose input is  $x_i$ . Note that whenever a variance is parameterised by an MLP,