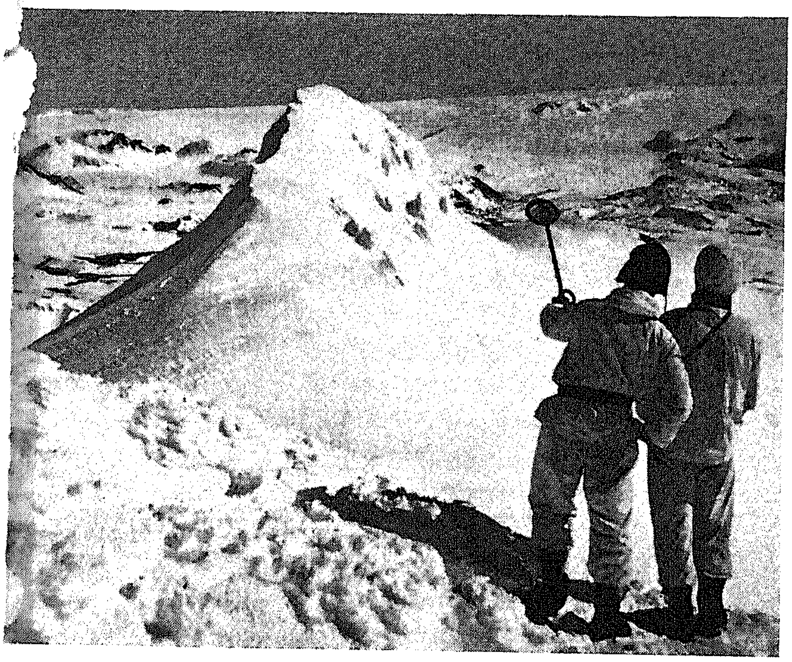

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5. ÁR REYKJAVÍK 1955

On the Flow of Ice-Sheets and Glaciers

by
GUNNAR BODVARDSSON[1])

ABSTRACT: Differential equations for the form of a flowing thin ice-sheet are derived by the boundary condition that the shear stress on the bed is proportional to the product of the thickness and the gliding velocity. The general mathematical character of the equations, stability of solutions and the response of ice-sheets to climatic variations are discussed.

INTRODUCTION

The difficulties encountered in the mathematical treatment of the flow of ice-sheets and glaciers can be listed as follow:

(I) *The general non-linear character of the basic equations of free-surface flow.* Some simplification can be obtained by the assumption of a static distribution of pressure within the ice, and by the assumption of flow-lines parallel to the bed.

(II) *The complex rheological character of ice and its low strength.* It is known that ice does not flow as a Newtonian fluid but shows a rather complex relation of shear stress to the rate of strain. (Glen, 1952). This implies a further non-linearity of the basic equations of flow. The low strength furthermore implies that planes of discontinuity are encountered which leads to a great complication. Approximations by the treatment of ice as a Newtonian fluid with constant viscosity have been made (Somigliana, 1921).

(III) *The unknown relation of the shear stress on the bed to the gliding velocity.* This uncertainty in the boundary conditions at the bed is probably at present the most serious difficulty encountered in the numerical treatment of the flow problems. Constant shear stress has been used by some authors (Nye, 1952).

(IV) *Complex three-dimensionl contours of the bed and variations in space and time of the rate of accumululation and ablation.* In the treatment of the flow of actual ice-sheets it is consequently necessary to collect a great deal of data on the contôurs of the bed and on the conditions in the accumulation and the ablation regions.

The purpose of the present paper is the dis-

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cussion of an approximate treatment of the general character of the flow of ice-sheets by the means of equations derived by the assumption of special boundary conditions at the bed. A short discussion of the stability and response of ice-sheets to climatic variations is included.

The following terminology will be used. The height of an ice-sheet moving on a horizontal plane (x, y) will be denoted by h, which is a function of the coordinates in the plane and of the time t. If the plane is not horizontal but characterized by the equation $z = f(x, y)$ the actual thickness of the ice-sheet will be $(h - f(x, y))$.

In accordance with actual conditions in nature only thin ice-sheets will be treated, that is, ice-sheets where the thickness is very small compared to the other dimensions. The discussion is also restricted to almost level beds.

This admits the following basic approximations, (a) that the direction of flow within the ice can be assumed independent of z, which means that the direction of flow above each (x, y) is constant from $z = 0$ or $z = f(x, y)$ to $z = h$, and (b) that the flow lines are parallel to the bed, and finally (c) because of the very slow movement of the quasi-viscous ice that a static distribution of pressure prevails, that is, the pressure at the point z is $w(h - z)$, where w is the specific weight of the ice. The pressure on the bed is consequently $w(h - f(x, y))$, or wh in the case of the horizontal bed.

CONDITIONS AT THE BED

If the friction on the bed is an ordinary dry sliding friction the Coulomb friction law gives the shear stress on the bed $s = cw(h - f)$, where c is the coefficient of friction which would mainly depend on the character of the bed, but be approximately independent of the sliding velocity.

It is, however, known that ice melts under pressure when the temperature is near to zero C° and the friction on the bed can therefore not be expected to behave according to the Coulomb law. It is rather to be expected that the coefficient of friction will also depend on the sliding velocity as in the case of viscous friction. Consequently we may expect that the friction on the bed can be approximated by

$$s_b = kvw(h - f),$$

where v is the sliding velocity, h the thickness and k a new factor, probably more or less constant. If v is independent of z the above relation can be written

$$s_b = kwF = kG,$$

where F is the volume of flow per unit length and G the weight of flow per unit length.

There are further reasons for assuming this relation. According to Glen (1952) the rheological character of ice is expressed by a curve of the form A in Figure (1) showing the relation of shear stress to the rate of strain $d\gamma/dt$. The curve B represents the Newtonian fluid with linear behaviour, and C is the plastic body.

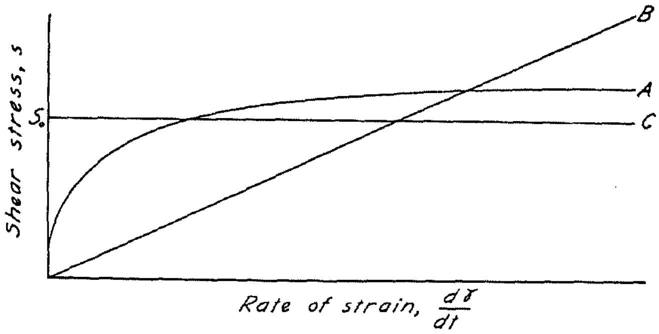

Fig.1. *Stress-strain relation for fluids and plastic bodies.*

Glen (1952) gives for ice the relation:

$$\frac{d\gamma}{dt} = \left(\frac{s}{S_0}\right)^n, \qquad (1)$$

where the rate of strain is expressed in units per year and s and S_0 in bar. For the temperature $-1.5°C$ the constants are $S_0 = 1.62$ and $n = 4.1$. According to this the behaviour of ice resembles that of plastic bodies, that is, when the shear stress is below a critical value, approximately S_0, then there is very little flow which, however, increases very much when the shear stress surpasses this value.

In flowing ice-sheets the greatest shear stress is at the bed and the rate of strain is, therefore, greatest there.

The ground moraine which the ice-sheet transports along the bed consists of pieces of rock which penetrate into the ice just above the bed. This forms a shallow *boundary layer* where the ice is broken by the inclusion of rock fragments and its strength consequently reduced. The ice in the boundary layer will therefore flow at an averaged shear stress lower than the above value of S_0, and this will greatly increase the rate of strain in the boundary layer.

Measurements on valley glaciers (Orvig, 1953) have actually shown that the maximun shear within the ice inferred from the surface slope is of the order of 0.5 to 1.5 bar, that is, a great deal lower than the above critical $S_0 = 1.6$ bar.

It is on the other hand to be expected that pressure increases the compactness and strength of the ice in the boundary layer and thus increases the critical shear stress there.

Summing up the above considerations we can conclude that the rate of strain in the flowing ice-sheet will be by far greatest in the boundary layer, and that the ice above it will move more like a solid. The conditions will actually be somewhat similar to the gliding of a body on a slightly lubricated plane, and we may therefore infer that the shear stress at the top of the boundary layer can be expressed by the above relation

$$s_b = kvw\ (h - f),$$

where k is the coefficient of friction and $(h - f)$ the thickness. The dependence on the height is underlined by the effect of pressure on the strength of the boundary layer.

It is on the other hand to be stated that this relation does not hold at very low or zero velocity, but this is not harmful when we are dealing with glaciers which actually glide on the bed.

THE NEWTONIAN ICE

Although of theoretical interest only the first step will be the deduction of a differential equation for the form of an ice-sheet moving on a horizontal plane and behaving as a Newtonian fluid with constant viscosity.

If the vector of flow at (x, y) is denoted by F the following equation of continuity is obtained:

$$\operatorname{div} F + \frac{\partial h}{\partial t} = C - B \ , \qquad (2)$$

where C is the volume of accumulation and B the volume of ablation per unit area and unit time. The figures C and B are generally functions of the coordinates and of the time.

In accordance with the above approximations (a), (b) and (c) the differential equation for the velocity vector V can be written::

$$\mu \frac{d^2 v}{dz^2} = \operatorname{grad} p = w \operatorname{grad} h \ , \qquad (3)$$

where μ is the coefficient of viscosity for the ice. This equation has to be integrated with the boundary conditions $dV/dz = 0$ at $z = h$, and $kV_0wh = - wh$ (grad h) at the bed. The solution is therefore:

$$V = \frac{w}{\mu}\left(\frac{z^2}{2} - hz - \frac{\mu}{wk}\right) \operatorname{grad} h \ , \qquad (4)$$

if w is assumed constant, and this gives the flow:

$$F = \int_0^h V dz = -\left(\frac{wh^3}{3\mu} + \frac{h}{k}\right) \operatorname{grad} h \ . \qquad (5)$$

Inserting F from (5) into (2) we obtain the final approximate differential equation for the thin Newtonian ice-sheet with constant viscosity moving on a horizontal plane:

$$\operatorname{div}\left(\left(\frac{wh^3}{3\mu} + \frac{h}{k}\right) \operatorname{grad} h\right) + C - B = \frac{\partial h}{\partial t} \qquad (6)$$

which in the one-dimensional case becomes:

$$\frac{\partial}{\partial x}\left(\left(\frac{wh^3}{3\mu} + \frac{h}{k}\right)\frac{\partial h}{\partial x}\right) + C - B = \frac{\partial h}{\partial t} \ . \qquad (7)$$

It is to be underlined that the equations (6) and (7) are derived by the approximations (a) to (c), constant specific weight, and furthermore that the specific accumulation and ablation C and B are measured in volumes. Nevertheless, these are non-linear partial differential equations which are quite difficult to solve numerically.

THE REAL ICE

In the case of real ice the equation (1) has to be applied instead of the constant viscosity relation above. The treatment here will for simplicity at first be restricted to the one-dimensional ice-sheet moving on a horizontal plane as this furnishes all the essentials needed for the final result.

By the approximation (c) the equation (1) gives

$$\frac{\partial v}{\partial z} = \left(-\frac{w}{S_0}(h - z)\frac{\partial h}{\partial x}\right)^n \ , \qquad (8)$$

and by an integration:

$$v = \left(-\frac{w}{S_0}\frac{\partial h}{\partial x}\right)^n \frac{(h-z)^{n+1}}{-(n+1)} + C, \qquad (9)$$

where C is a constant which has to be fixed by the boundary condition at the bed $s_b = kvwh$, where v is the velocity at the bed. A simple calculation gives:

$$v = \left(\frac{-w\frac{\partial h}{\partial x}}{S_0}\right)^n \left(\frac{h^{n+1}}{n+1} - \frac{(h-z)^{n+1}}{n+1}\right) - \frac{1}{k}\frac{\partial h}{\partial x}, \qquad (10)$$

and the flow:

$$F = \int_0^h v\,dz = \frac{h^{n+2}}{n+2}\left(\frac{-w\frac{\partial h}{\partial x}}{S_0}\right)^n - \frac{h}{k}\frac{\partial h}{\partial x}, \qquad (11)$$

which can also be expressed in terms of the shear stress on the bed s_b:

$$F = \frac{h^2}{n+2}\left(\frac{s_b}{S_0}\right)^n + \frac{s_b}{kw}. \qquad (12)$$

By inserting F from (11) into (2) the following differential equation is obtained:

$$\frac{\partial}{\partial x}\left(\frac{-w^n}{(n+2)S_0^n}h^{n+2}\left(-\frac{\partial h}{\partial x}\right)^n + \frac{h}{k}\frac{\partial h}{\partial x}\right) + C - B = \frac{\partial h}{\partial t}. \qquad (13)$$

This equation is quite tedious and very difficult to handle especially because of the first term in the parenthesis. It is therefore fortunate that this term appears in most practical cases to be small compared to the second term.

The writer has investigated the numerical conditions at the Vatnajökull in Iceland and found that if the observed values of surface slope, which gives the shear stress at the bed s_b, thickness, accumulation and ablation, are used then the second term in equation (12) accounts for over 90% of the flow at least in the glaciers. In view of the general quality of the present treatment it therefore appears reasonable to cancel this term and write the equation (13) simply as follows:

$$\frac{\partial}{\partial x}\left(\frac{h}{k}\frac{\partial h}{\partial x}\right) + C - B = \frac{\partial h}{\partial t}. \qquad (14)$$

which is our final equation for the thin linear ice-sheet moving on a horizontal plane.

The last approximation actually means that the differential flow between top and bed accounts for only a small part of the total flow and that the real ice moves practically in solid blocks which slide on the boundary layer at the bed. The last approximation is, however, only valid for ice-sheets with a thickness less than 500 to 700 meters, somewhat depending on other conditions. This can be inferred from equation (12).

Nevertheless, in spite of the number of approximations made, the equation (14) reveals the general mathematical character of the problem and it can be used as an approximation under a number af circumstances.

Equation (14) which applies to the linear ice-sheet moving on a horizontal plane can easily be extended to the case of the thin ice-sheet moving in two dimensions on a slightly uneven surface defined by $z = f(x,y)$:

$$\mathrm{div}\left(\frac{h-f}{k}\,\mathrm{grad}(h+f)\right) + C - B = \frac{\partial h}{\partial t}. \qquad (15)$$

where it is assumed that the projections of the vectors grad h and grad f are parallel in the (x,y) plane. This induces certain restrictions, although very natural, on the distribution of the accumulation and the ablation.

Equations (14) and (15) are non-linear partial parabolic differential equations but considerably easier to handle than (13). The boundary conditions will be discussed below.

It is of interest to note that the first term of equation (14) is of the same form as encountered in the differential equation for the height of the surface of ground water moving in porous rock.

CHARACTER OF SOLUTIONS AND STABILITY

In order to study the solutions of the above equations we will turn to (14), that is, the differential equation for the linear thin ice-sheet moving on a horizontal plane, and restrict ourselves at first to the part below the firn line, that is, to the ablation region.

The volume of ablation per unit time and unit area will in practical cases for the most be a function of the height of the glacier, and we may write $B - C = a(H - h)$, where a is the ablation gradient and H the form of the firn line above the base of the glacier. Equation (14) is under these conditions:

$$\frac{\partial}{\partial x}\left(\frac{h}{k}\frac{\partial h}{\partial x}\right) = a(H-h) + \frac{\partial h}{\partial t}. \qquad (16)$$

which gives the time-dependent solution. The steady state solutions are consequently to be found by the equation:

$$\frac{d}{dx}\left(\frac{h}{k}\frac{dh}{dx}\right) = a(H-h) \quad . \tag{17}$$

The flow of ice through the section at the point x is $F = -\frac{h}{k}\, dh/dx$.

Equation (17) is a non-linear differential equation of the second order. The general solution of such equations generally includes two arbitrary constants which are fixed by two boundary conditions.

The first boundary condition is obtained at the top of the glacier, which is here put at $x = 0$, where $h = H$, that is, the height of the top of the glacier has to be equal to the height of the firn line above the base.

The second boundary condition is obtained at the end of the glacier where the flow of ice is zero, that is, if the length of the glacier is L then we have the condition $F = 0$ at $x = L$. In the present case L is, however, not fixed but has to be determined from the condition $h = 0$. The second boundary condition is therefore $F = 0$ for $h = 0$, which actually means that the amount of ice flowing into the glacier at $x = 0$ has to be ablated from its surface.

In accordance with the character of equation (17) the above boundary conditions define one and only one solution which is admittable from the physical point of view. In other words, for each set of values of the constants a, k and H there exists in fact only one stable form of the thin linear glacier moving on a horizontal plane.

This implies that the flow of ice into the glacier at $x = 0$, is uniquely fixed by the constants a, k and H, and the stable glacier can consequently only dissipate a fixed amount of ice per unit time. This important fact not only holds for the special type of equation (17) but will be valid for a great group of similar equations, and it can thus be expected to be valid for glaciers in general although the physical behaviour differs in some degree from that described above.

The solution of (17), which is the only stable solution of (16) is very simple:

$$h = H\left(1 - 0.73\frac{x}{L} - 0.27\left(\frac{x}{L}\right)^2\right) , \tag{18}$$

where:

$$L = 1.27\sqrt{\frac{H}{ak}} , \tag{19}$$

and the flow of ice into the glacier at $x = 0$ is:

$$F = 0.46\, aHL = 0.58H\sqrt{\frac{aH}{k}} = 0.58H\sqrt{\frac{A}{k}} , \tag{20}$$

where $A = aH$ is the ablation at the end of the glacier.

As the boundary condition at the bed is $s_b = kvwh = -whdh/dx$, the velocity of the glacier becomes:

$$v = -\frac{1}{k}\frac{dh}{dx} = \frac{H}{kL}\left(0.73 + 0.54\frac{x}{L}\right) , \tag{21}$$

that is, proportional to the slope of the surface. Furthermore the flow through the section at x is:

$$F = \frac{H^2}{kL}\left(0.73 + 0.54\frac{x}{L}\right)\left(1 - 0.73\frac{x}{L} - 0.27\left(\frac{x}{L}\right)^2\right) . \tag{22}$$

The relations for the profile (18), velocity (21) and the flow (22) are in Figure 2 applied to a central section through the glacier Brúarjökull in the northern Vatnajökull in Iceland, where H is about 600 meters and $L = 20000$ m. This glacier appears to flow on a level surface. The total flow of ice into the glacier has been derived from recent measurements of the accumulation which give an average value of $C - B = 1.5$ meters of water in the accumulation region. The average density of the ice in the glacier is assumed to be 0.8. The fit of the theoretical profile is quite good although the Brúarjökull is comparatively thick.

The finding of the stable solutions is quite more tedious in the case of uneven bed or variable k, and the stabilty conditions also become more obscure and require a more detailed analysis, which will not be discussed in the present paper .The same applies in a greater degree to the two-dimensional cases where equation (15) has to be applied.

In the case of the thin one-dimensional glacier moving on a bed with a slope down the glacier, the height of the firn line H to be applied in the

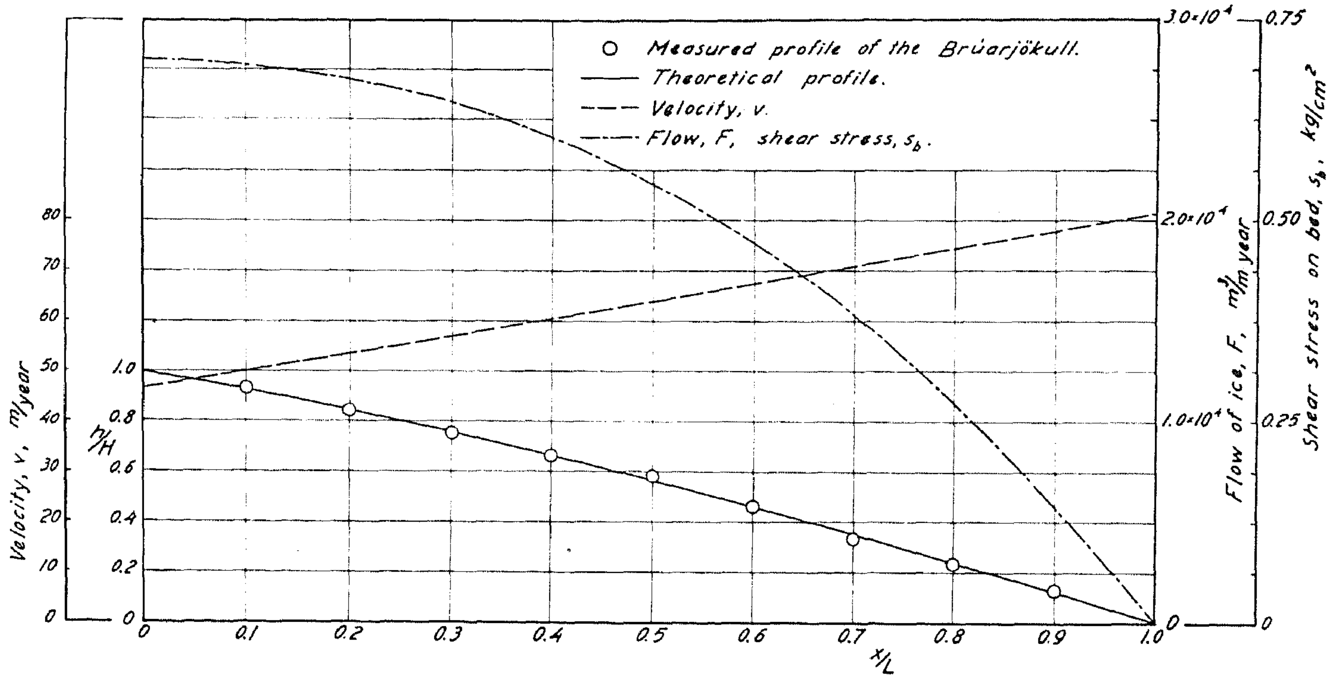

Figure 2. *Section through Brúarjökull, a glacier in the northern Vatnajökull in Iceland. This is approximately a central section with $H = 600$ m and $L = 20$ km. The velocity v and flow F are theoretical values from equations (21) and (22).*

above equations is not constant but varies with the length of the glacier L, that is, the longer glacier can dissipate more ice. This means that the glacier is more stable, and it can in special cases be expected that the glacier can be stable for a whole interval of lengths L. An outward slope is consequently a stabilizing factor whereas an inward slope has the reverse effect.

Another stabilizing factor is the spreading of the lower parts of the glaciers as in the case of piedmont-glaciers. By the spreading the ablation area increases rapidly with the length.

It is consequently to be expected that actual glaciers are in most cases more stable than the theoretical case treated here, but the stabilizing factors can be taken into account by a more elaborate treatment.

RESPONSE TO CLIMATIC VARIATIONS

A further factor interfering with the stability is the variability of the meterological factors especially of the height of the firn line H. These variations cause fluctuations of the height and movement of the ice-sheets and glaciers. The fluctuations present one af the most interesting problems of glaciology and the present paper will therefore be concluded by a short investigation of the influence of climatic variations on the ice-sheets described by the equations above. The method to be used is the method of small perturbations which is much used in other branches of mechanics and physics.

By the treatment of this problem it becomes necessary to treat the ice-sheet as a whole, that is, look for the perturbations of the combined accumulation and ablation areas.

We will again turn to equation (17) and ask for solutions for the combined accumulation and ablation areas, that is, extend the above solution (18) above the firn line. It is then assumed that the total net accumulation above the firn line is a(h—H), which is a rather natural relation although other relations may apply in many practical cases, but this will not interfere with the main results.

The solution of (17) for the total thin linear ice-sheet moving on a horizontal plane is very simple:

$$h_s = 1.5H\left(1-\left(\frac{x}{L}\right)^2\right), \qquad (23)$$

where L_0 is the total length of the accumulation and the ablation areas, that is, the length of the stable ice-sheet situated as in Figure (3).

The height of the top is $h_0 = 3H/2$ and the total length:

$$L_0 = 3\sqrt{\frac{H}{ak}} \quad . \tag{24}$$

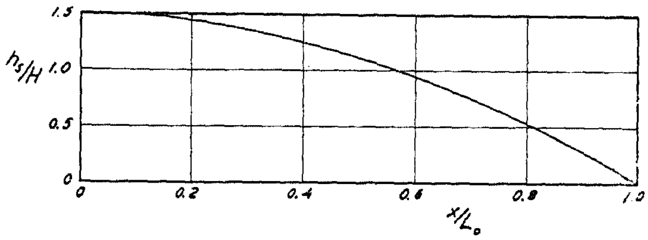

Fig. 3. *Theoretical form of a thin one-dimensional ice-sheet, moving on a horizontal plane, according to equation (23).*

Comparing this with equation (17) we see that the ratio of the length of the accumulation area to that of the ablation area is 1.73/1.27 = 1.36.

As equation (23) represents the only stable form of the ice-sheet we can infer that perturbations of this form will lead to instability, either growth or decrease, depending on the character of the perturbation. If the three basic figures a, k and H are constants in space and time, then the removal of a small amount of material from the theoretical ice-sheet will bring it to a decrease or retreat ending in complete disappearance. On the other hand the adding of material will bring it to grow without limit. An increase of the height of the firn line will thus bring the ice-sheet to retreat, and a decrease of the height will bring it to grow.

The fact of main importance is that cause and effect are out of proportions, that is, a small cause results in a very great effect, and a small variation in the clima can, therefore, result in great changes of glaciers. The amount of change depends in actual cases on the various stabilizing factors mentioned above and also on the mass of the ice-sheet which is a further stabilizing factor.

The numerical problem to be treated here is the initial growth, or decrease, of the ice-sheet in Figure (3) by a change in the height of the firn line, that is, we will ask for the perturbation u of the stable form h_s in equation (23) when the height of the firn line changes from H to H + g(t), where g(t) is the time-dependent perturbation of the height of the firn line, which will be assumed small compared to H.

Inserting (h_s + u) for h, and (H + g(t)) for H in equation (16) we get:

$$\frac{\partial}{\partial x}\left(\frac{h_s+u}{k}\,\frac{\partial(h_s+u)}{\partial x}\right) = a(H+g(t)-h_s-u) + \frac{\partial u}{\partial t} \; , \tag{25}$$

but as h_s is the stable form corresponding to H and u is to be small compared to h, we can in the first approximation write by a rearrangement:

$$\frac{\partial}{\partial x}\left(\frac{h_s}{k}\,\frac{\partial u}{\partial x}\right) + au - \frac{\partial u}{\partial t} = ag(t) \; . \tag{26}$$

which is a linear partial differential equation for u. By the substitution:

$$u = q(x)\cdot e^{mt} + p(t) \; , \tag{27}$$

where q(x) is a function of x only, and p(t) a particular solution of (26), that is, solution of the equation:

$$ap - \frac{\partial p}{\partial t} = ag(t) \; , \tag{28}$$

we get the general solution of (26):

$$u = \sum_i C_i\, q_i(x) e^{-m_i t} + p(t) \; . \tag{29}$$

where the functions $q_i(x)$ are the orthonormal set of solutions of the Sturm-Liouville equation:

$$\frac{d}{dx}\left(\frac{h_s}{k}\,\frac{dq}{dx}\right) + (a+m)q = 0 \; , \tag{30}$$

and by the boundary conditions:

$$\frac{dq}{dx} = 0, \quad \text{for } x = 0 \quad \text{and } x = L_0 \tag{31}$$

The values of the constants m_i are determined by the eigenvalues of (30) and (31), and the constants C_i are determined by the initial condition of (26):

$$u = \sum_i C_i\, q_i(x) + p(0), \quad \text{for } t = 0, \tag{32}$$

where the values of u at t = 0 are to be inserted and the constants determined by the usual procedure in expanding given functions in a series of orthonormal functions.

In the present case the initial condition is $h = h_s$ that is, $u = 0$, and by putting $p(0) = 0$ we get simply $C_i = 0$ for all i. The initial perturbation of the stable form is therefore independent of x, which is a very simple result. The function u is thus obtained by the integration of (28).

Two cases will be treated here, that is, (1) the change of the height of the firn line by the constant g, and (2) the periodic change $g = Ae^{i\omega t}$, where A is a constant. The solutions of (28) are:

$$(1) \qquad g = constant, \quad u = g(1 - e^{at}) \qquad (33)$$

$$(2) \qquad g = Ae^{i\omega t}, \quad u = \frac{aA}{a - i\omega}(e^{i\omega t} - e^{at}) \qquad (34)$$

The instability of the ice-sheet is clearly revealed by these eqations.

As these relations are obtained by the linearized equation (26) they hold only as long as u is small compared to h, that is, they will not be true at the snout of the glacier.

In order to get some numerical data we will turn to the conditions at the Vatnajökull, where the total ablation gradient is approximately 0.01 per year. The equation (33) shows that an increase in the height of the firn line by a constant amount will in approximately 70 years have lowered the ice-sheet by the same amount.

On the other hand a climatic cycle of the amplitude 0.5°C and having the total period of 200 years, starting with an increase in the height of the firn line, will after one total period have lowered the ice-sheet by 150 meters.

The first approximation above leads naturally to a second step by the inserting of the u-values obtained into (25). This gives a new differential equation which is again linearized and a second u computed.

The response to climatic variations of large glaciers, as for instance the Vatnajökull, can theoretically be treated in the above manner, but the computational work will be very great and the practical aspects of this work will not be discussed in the present paper.

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Übersicht über die Eisrandlagen in Kringilsárrani von 1890-1955

von E. M. Todtmann, Hamburg

Nachdem ich das westliche Vorfeld des Brúarjökull zwischen Kringilsá um Kverká seit 1950, das östliche Vorfeld, Maríutungur, seit 1953 kannte, untersuchte ich 1955 das Vorfeld Kringilsárrani, das umgrenzt wird von der Kringilsá, der Jökulsá á Brú mit ihrem Nebenfluss Illakvisl und vom Eisrand.

Páll Hjarðar aus Hjarðarhagi im Jökuldal brachte uns mit dem Lastwagen, auf den wir ein kleines Boot geladen hatten, bis an die Kringilsá. Dicht südlich der Endmoräne von 1890 setzten wir mit dem Boot über den Fluss. Mit mir blieb Þorleifur Einarsson aus Reykjavik. Student der Geologie in Erlangen.

Das Schrifttum gibt nur geringe Kenntnisse über Kringilsárrani. Thoroddsen ist selber nicht dort gewesen, zu seiner Zeit lag der Eisrand noch an der Moräne von 1890. Th. Kjerulf (nach Thoroddsen) beobachtete 1890 eine gewaltige Spalte im Gletscher südwestlich vom Snæfell. Pálmi Hannesson, (nach Thorarinsson) besuchte Kringilsárrani 1933. Er sah im Moränenfeld radial verlaufende lange Rücken, die er als Spaltenausfüllungen deutet, eine reiche Vegetation und Anfänge von Palsen, *(dysjar).*