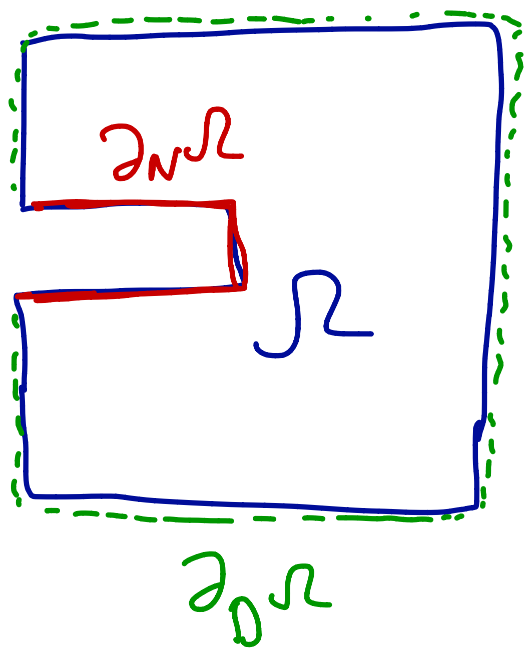
- suppose we want to model heat conduction in a very simplified engine block with a heater

- domain and problem



$$-\nabla^2 u = f \quad \text{on } \Omega$$

$$u = g_D \quad \text{on } \partial_D \Omega$$

$$\underbrace{\frac{\partial u}{\partial n}}_{= \nabla u \cdot \hat{n}} = g_N \quad \text{on } \partial_N \Omega$$

$\uparrow$ outward normal

- in terms of the model, if $g_N < 0$ on $\partial_N \Omega$ then heat is being applied along $\partial_N \Omega$

- as before, g_D gives temperature along $\partial_D \Omega$

- derive weak form:

$$-\nabla^2 u = f$$

$$\int_\Omega -(\nabla^2 u) v = \int_\Omega f v \qquad v \in H_0^1(\Omega)$$

$v = 0$ along $\partial_D \Omega$

$$\int_\Omega -\nabla \cdot ((\nabla u) v) + \nabla u \cdot \nabla v = \int_\Omega f v$$

divergence theorem

$$\int_\Omega \nabla u \cdot \nabla v \, dx = \int_\Omega f v \, dx + \int_{\partial\Omega} v \nabla u \cdot \hat{n} \, ds$$

$$= \int_\Omega f v \, dx + \int_{\partial_N \Omega} g_N v \, ds$$