Homework #1

Due Wednesday 19 January, 2022 at 11:59pm.

Submit as a single PDF by using Gradescope, via the course Canvas site
`canvas.alaska.edu/courses/7017`
Problems from the textbook (Strang, *Intro Linear Algebra*, 5th ed. 2016) will be graded for completion, while the "**P**" problems will be graded for correctness. Answers/solutions to textbook problems are linked at
`bueler.github.io/math314/resources.html`

from Problem Set 1.1, pages 8–10: # 2, 6, 8, 11, 13, 22, 31

from Problem Set 1.2, pages 18–21: # 1, 2, 3, 4, 6, 14, 21, 34

P1. If $\mathbf{v} - \mathbf{w} = \begin{bmatrix} 4 \\ 1 \end{bmatrix}$ and $\mathbf{v} + \mathbf{w} = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$, compute and draw the vectors $\mathbf{v}$ and $\mathbf{w}$.

P2. What linear combination $c \begin{bmatrix} 4 \\ 1 \end{bmatrix} + d \begin{bmatrix} 3 \\ 2 \end{bmatrix}$ produces $\begin{bmatrix} 7 \\ 8 \end{bmatrix}$? Express this question as two equations for the coefficients c and d, and find c, d.

P3. Two opposite corners of a unit cube in 4 dimensions are $(0, 0, 0, 0)$ and $(1, 1, 1, 1)$. All corners have coordinates that are either 0 or 1. How many corners are there? How many edges? How many "3D faces", which are themselves 3-dimensional cubes?

P4. Find nonzero vectors $\mathbf{v}$ and $\mathbf{w}$ which are perpendicular to $(-1, 0, 1)$ and to each other.

P5. True or false? If true give an explanation. If false give a counterexample:

(a) If $\mathbf{u}$ is perpendicular to $\mathbf{v}$ and $\mathbf{w}$, then $\mathbf{u}$ is perpendicular to $2\mathbf{v} - \mathbf{w}$.
(b) If $\mathbf{u}$ and $\mathbf{v}$ are perpendicular unit vectors then $\|\mathbf{u} + \mathbf{v}\| = \sqrt{2}$.
(c) If $\mathbf{u} = (-1, 1, -1)$ is perpendicular to $\mathbf{v}$ and $\mathbf{w}$, then $\mathbf{v}$ is parallel to $\mathbf{w}$.

P6. Draw a parallelogram with sides $\mathbf{v}$ and $\mathbf{w}$. Then show that the squared diagonal lengths $\|\mathbf{v} + \mathbf{w}\|^2 + \|\mathbf{v} - \mathbf{w}\|^2$ add to the sum of the four squared side lengths, that is, $2\|\mathbf{v}\|^2 + 2\|\mathbf{w}\|^2$.