

## Homework #8

Due Monday 21 March, 2022 at 11:59pm.

Submit as a single PDF via Gradescope, linked from the Canvas page

[canvas.alaska.edu/courses/7017](https://canvas.alaska.edu/courses/7017)

Textbook Problems from Strang, *Intro Linear Algebra*, 5th ed. will be graded for completion. Answers/solutions to these Problems are linked at

[bueler.github.io/math314/resources.html](https://bueler.github.io/math314/resources.html)

The **P** Problems will be graded for correctness. When grading these Problems, I will expect you to write explanations using complete sentences!

*Put these Textbook Problems first on your PDF, in this order.*

**from Problem Set 3.5, pages 189–192:** # 1, 3, 4, 6, 7, 11, 13, 23

**from Problem Set 4.1, pages 201–204:** # 2, 4, 5, 6, 10, 11, 12, 13, 20

*Put these **P** Problems next on your PDF, in this order.*

**P36.** I have a  $3 \times 5$  matrix and I compute its `rref`:

$$A = \begin{bmatrix} 0 & 1 & 2 & 3 & 4 \\ 0 & 2 & 4 & 3 & 2 \\ 0 & 0 & 0 & 1 & 2 \end{bmatrix} \quad \rightarrow \quad R = \begin{bmatrix} 0 & 1 & 2 & 0 & -2 \\ 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Find a basis for each of the four subspaces associated to  $A$ :

row space  $C(A^\top)$ , column space  $C(A)$ , null space  $N(A)$ , left nullspace  $N(A^\top)$

**P37.** Suppose I have factored  $A$  as  $LU$ :

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 1 & 2 & 4 & 6 \\ 0 & 0 & 1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

Find a basis for each of the four subspaces associated to  $A$ :

row space  $C(A^\top)$ , column space  $C(A)$ , null space  $N(A)$ , left nullspace  $N(A^\top)$

**P38.** Let  $V$  be the subspace of  $\mathbb{R}^3$  spanned by  $(1, -1, 0)$  and  $(0, 2, 1)$ .

(a) Find a matrix  $A$  that has  $V$  as its row space.

(b) Find a matrix  $B$  that has  $V$  as its null space.

(c) Multiply  $AB^\top$ . Multiply  $BA^\top$ . Why do these come out so simple?

**P39.** Let  $I$  be the  $3 \times 3$  identity matrix and  $O$  be the  $3 \times 2$  zero matrix. For each of these matrices, find the dimensions of the four subspaces:

$$A = [0], \quad B = [I \quad 0], \quad C = \begin{bmatrix} I & I \\ 0^\top & 0^\top \end{bmatrix}$$

**P40. (a)** Prove that every  $\mathbf{x}$  in  $N(A)$  is perpendicular to every  $A^\top \mathbf{y}$  in the row space of  $A$  (i.e. in  $C(A^\top)$ ). *Hint. Start with  $A\mathbf{x} = \mathbf{0}$ . Now compute a dot product.*

**(b)** Prove that every  $\mathbf{y}$  in  $N(A^\top)$  is perpendicular to every  $A\mathbf{x}$  in the column space of  $A$  (i.e. in  $C(A)$ ).

**P41.** This system of equations  $A\mathbf{x} = \mathbf{b}$  has no solutions:

$$2x + 3y + 4z = 9$$

$$4x + 3y + 2z = 9$$

$$2x \quad \quad - 2z = 1$$

Find numbers  $c_1, c_2, c_3$  to multiply the equations so that they add to  $0 = 1$ . (*Hint. Do row operations, and keep track of them.*) You have found a vector  $\mathbf{y}$  in which subspace? Check its dot product:  $\mathbf{y}^\top \mathbf{b} = 1$ .

**P42.** Suppose  $S$  is spanned by the vectors  $(1, 2, 2, 3)$  and  $(1, 1, 1, 1)$ . Find two vectors that span  $S^\perp$ . This is the same as solving  $A\mathbf{x} = \mathbf{0}$  for which  $A$ ?