Homework #9

Due Friday 25 March, 2022 at 11:59pm.

Submit as a single PDF via Gradescope; see the Canvas page
canvas.alaska.edu/courses/7017
Textbook Problems from Strang, *Intro Linear Algebra*, 5th ed. will be graded for completion. Answers/solutions to these Problems are linked at
bueler.github.io/math314/resources.html
The **P** Problems will be graded for correctness. When grading these Problems, I will expect you to write explanations using complete sentences!

Put these Textbook Problems first on your PDF, in this order.

from Problem Set 4.2, pages 213–217: # 1, 3, 8, 13, 16, 17, 21, 22

from Problem Set 4.3, pages 228–231: # 1, 2, 3, 4, 8, 9

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*Put these **P** Problems next on your PDF, in this order.*

P43. For each part: *i)* Draw the projection $\boldsymbol{p}$ of $\boldsymbol{b}$ onto $\boldsymbol{a}$. *ii)* Compute it as $\boldsymbol{p} = \hat{x}\boldsymbol{a}$, where $\hat{x} = \dfrac{\boldsymbol{a}^\top \boldsymbol{b}}{\boldsymbol{a}^\top \boldsymbol{a}}$. *iii)* Compute the projection matrix $P = \dfrac{\boldsymbol{a}\boldsymbol{a}^\top}{\boldsymbol{a}^\top \boldsymbol{a}}$, and then $\boldsymbol{p} = P\boldsymbol{b}$.

(a) $\boldsymbol{b} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and $\boldsymbol{a} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

(b) $\boldsymbol{b} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$ and $\boldsymbol{a} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

(c) $\boldsymbol{b} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ and $\boldsymbol{a} = \begin{bmatrix} \cos\theta \\ \sin\theta \end{bmatrix}$ (*For your drawing, just pick a generic* θ*.*)

P44. For each part: *i)* Form and solve the normal equations $A^\top A\hat{\boldsymbol{x}} = A^\top \boldsymbol{b}$. *ii)* Compute the projection matrix $P = A(A^\top A)^{-1}A^\top$. (*You can use technology for the inverse.*) *iii)* Check that $P^2 = P$ and $P^\top = P$. *iv)* Compute $\boldsymbol{p} = P\boldsymbol{b}$, and check it matches $A\hat{\boldsymbol{x}}$ from the solution to the normal equations.

(a) $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$ and $\boldsymbol{b} = \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}$

(b) $A = \begin{bmatrix} 1 & 1 \\ 1 & 2 \\ 0 & 1 \end{bmatrix}$ and $\boldsymbol{b} = \begin{bmatrix} 2 \\ 8 \\ 6 \end{bmatrix}$

P45. An overdetermined system you cannot solve (4 equations in 2 unknowns):

$$\begin{aligned} x_1 + x_2 &= 1 \\ x_1 &= 0 \\ 2x_1 - x_2 &= 2 \\ 3x_1 + 4x_2 &= -1 \end{aligned}$$

(a) Each equation is a line in the x_1, x_2 plane. Plot all 4 lines in one plot. They do not meet in a single point. (*Feel free to use technology for this plot. Your plot box should at least include all the places where pairs of lines intersect.*)

(b) Write down the normal equations $A^\top A\boldsymbol{x} = A^\top \boldsymbol{b}$ for the above system "$A\boldsymbol{x} = \boldsymbol{b}$". Now form $A^\top A$ and $A^\top \boldsymbol{b}$.

(c) Solve the normal equations. (*Use technology as desired.*) Add the solution point to your plot in part **(a)**.

P46. **(a)** Consider the same $A, \boldsymbol{b}$ as in P45. Write out and simplify

$$E(x_1, x_2) = \|A\boldsymbol{x} - \boldsymbol{b}\|^2 = (A\boldsymbol{x} - \boldsymbol{b})^\top (A\boldsymbol{x} - \boldsymbol{b}).$$

(*Hint. This simplifies to a function which is quadratic in the two variables* x_1, x_2.)

(b) Compute and simplify the partial derivatives of E.

(c) Solve the linear system of two equations in two unknowns x_1, x_2:

$$\begin{aligned} \frac{\partial E}{\partial x_1} &= 0 \\ \frac{\partial E}{\partial x_2} &= 0 \end{aligned}$$

(*Hint. The solution is the same as in* ***P45 (c)****. The system is essentially the same.*)