Homework #10

Due Monday 4 April, 2022 at 11:59pm.

Submit as a single PDF via Gradescope; see the Canvas page
canvas.alaska.edu/courses/7017
Textbook Problems from Strang, *Intro Linear Algebra*, 5th ed. will be graded for completion. Answers/solutions to these Problems are linked at
bueler.github.io/math314/resources.html
The **P** Problems will be graded for correctness. When grading these Problems, I will expect you to write explanations using complete sentences!

Put these Textbook Problems first on your PDF, in this order.

from Problem Set 5.1, pages 253–256: # 1, 3, 4, 7, 10, 18, 19

from Problem Set 5.2, pages 265–271: # 2, 6, 7, 12

Put these ***P*** *Problems next on your PDF, in this order.*

P47. *This problem refers to ideas in section 4.4.* Suppose

$$Z = \begin{bmatrix} 1/\sqrt{3} & 1/\sqrt{6} & -1/\sqrt{2} \\ 1/\sqrt{3} & -\sqrt{2}/\sqrt{3} & 0 \\ 1/\sqrt{3} & 1/\sqrt{6} & 1/\sqrt{2} \end{bmatrix}$$

(a) Check, by computing dot products, that the columns of this matrix Z form an orthonormal basis.

(b) Check that $Z^\top Z = I$. This shows that Z is an orthogonal matrix.

(c) Explain briefly why calculations **(a)** and **(b)** are the same.

P48. **(a)** For each matrix A, do row reductions to reduce A to U, and then compute $\det A$ as the product of the pivots.

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 1 & 2 & 3 \end{bmatrix}, \qquad A = \begin{bmatrix} 2 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & 2 \end{bmatrix}$$

(b) Compute $\det A$ for the matrices in part **(a)** by using the cofactor formula.

P49. Assume A and B are square matrices of the same size. Answer true or false; give a reason if true or a 2 by 2 example if false.

(a) If A is not invertible then AB is not invertible.

(b) The determinant of A is always the product of its diagonal entries.

(c) The determinant of $A - B$ equals $\det A - \det B$.

(d) AB and BA have the same determinant.

P50. In both parts, assume A is an m by n matrix with $m \geq n$ and full column rank.

(a) Recall $P = A(A^\top A)^{-1}A^\top$ is the formula for the projection onto $C(A)$. Give an example of a 2 by 2 projection matrix P with determinant zero. (*Hint.* Any nontrivial example will work.)

(b) What is wrong with this proof that all projection matrices have $\det P = 1$?:

$$P = A(A^\top A)^{-1}A^\top \quad \text{so} \quad \det P = \det(A)\frac{1}{\det(A^\top)\det(A)}\det(A^\top) = 1.$$

Are there any matrices A for which this proof is correct?

P51. **(a)** The matrix Z in **P47** is orthogonal. Compute its determinant by using the big formula.

(b) If A is square and $A^\top A = I$ then we say A is an *orthogonal matrix*. Show that if A is *any* orthogonal matrix then $\det A = \pm 1$. (*Hint.* $\det$ of both sides.)