

## Homework #9

Due Sunday 27 March, 2022 at 11:59pm.  $\leftarrow$  REVISED!

Submit as a single PDF via Gradescope; see the Canvas page

[canvas.alaska.edu/courses/7017](https://canvas.alaska.edu/courses/7017)

Textbook Problems from Strang, *Intro Linear Algebra*, 5th ed. will be graded for completion. Answers/solutions to these Problems are linked at

[bueler.github.io/math314/resources.html](https://bueler.github.io/math314/resources.html)

The **P** Problems will be graded for correctness. When grading these Problems, I will expect you to write explanations using complete sentences!

*Put these Textbook Problems first on your PDF, in this order.*

**from Problem Set 4.2, pages 213–217:** # 1, 3, 8, 13, 16, 17, 21, 22

**from Problem Set 4.3, pages 228–231:** # 1, 2, 3, 4, 8, 9

*Put these **P** Problems next on your PDF, in this order.*

**P43.** For each part: *i)* Draw the projection  $\mathbf{p}$  of  $\mathbf{b}$  onto  $\mathbf{a}$ . *ii)* Compute it as  $\mathbf{p} = \hat{x}\mathbf{a}$ , where  $\hat{x} = \frac{\mathbf{a}^\top \mathbf{b}}{\mathbf{a}^\top \mathbf{a}}$ . *iii)* Compute the projection matrix  $P = \frac{\mathbf{a}\mathbf{a}^\top}{\mathbf{a}^\top \mathbf{a}}$ , and then  $\mathbf{p} = P\mathbf{b}$ .

(a)  $\mathbf{b} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$  and  $\mathbf{a} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

(b)  $\mathbf{b} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$  and  $\mathbf{a} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

(c)  $\mathbf{b} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$  and  $\mathbf{a} = \begin{bmatrix} \cos \theta \\ \sin \theta \end{bmatrix}$  (For your drawing, just pick a generic  $\theta$ .)

**P44.** For each part: *i)* Form and solve the normal equations  $A^\top A\hat{\mathbf{x}} = A^\top \mathbf{b}$ . *ii)* Compute the projection matrix  $P = A(A^\top A)^{-1}A^\top$ . (You can use technology for the inverse.) *iii)* Check that  $P^2 = P$  and  $P^\top = P$ . *iv)* Compute  $\mathbf{p} = P\mathbf{b}$ , and check it matches  $A\hat{\mathbf{x}}$  from the solution to the normal equations.

(a)  $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$  and  $\mathbf{b} = \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}$

(b)  $A = \begin{bmatrix} 1 & 1 \\ 1 & 2 \\ 0 & 1 \end{bmatrix}$  and  $\mathbf{b} = \begin{bmatrix} 2 \\ 8 \\ 6 \end{bmatrix}$

**P45.** An overdetermined system you cannot solve (4 equations in 2 unknowns):

$$x_1 + x_2 = 1$$

$$x_1 = 0$$

$$2x_1 - x_2 = 2$$

$$3x_1 + 4x_2 = -1$$

(a) Each equation is a line in the  $x_1, x_2$  plane. Plot all 4 lines in one plot. They do not meet in a single point. (*Feel free to use technology for this plot. Your plot box should at least include all the places where pairs of lines intersect.*)

(b) Write down the normal equations  $A^\top A\mathbf{x} = A^\top \mathbf{b}$  for the above system “ $A\mathbf{x} = \mathbf{b}$ ”.

(c) Solve the normal equations. (*Use technology as desired.*) Add the solution point to your plot in part (a).

**P46.** (a) Consider the same  $A, \mathbf{b}$  as in P45. Write out and simplify

$$E(x_1, x_2) = \|A\mathbf{x} - \mathbf{b}\|^2 = (A\mathbf{x} - \mathbf{b})^\top (A\mathbf{x} - \mathbf{b}).$$

(*Hint. This simplifies to a function which is quadratic in the two variables  $x_1, x_2$ .*)

(b) Compute and simplify the partial derivatives of  $E$ .

(c) Solve the linear system of two equations in two unknowns  $x_1, x_2$ :

$$\frac{\partial E}{\partial x_1} = 0$$

$$\frac{\partial E}{\partial x_2} = 0$$

(*Hint. The solution is the same as in P45 (c). The system is essentially the same.*)