Homework #8

Due Monday 21 March, 2022 at 11:59pm.

Submit as a single PDF via Gradescope, linked from the Canvas page
`canvas.alaska.edu/courses/7017`
Textbook Problems from Strang, *Intro Linear Algebra*, 5th ed. will be graded for completion. Answers/solutions to these Problems are linked at
`bueler.github.io/math314/resources.html`
The **P** Problems will be graded for correctness. When grading these Problems, I will expect you to write explanations using complete sentences!

Put these Textbook Problems first on your PDF, in this order.

from Problem Set 3.5, pages 189–192: # 1, 3, 4, 6, 7, 11, 13, 23

from Problem Set 4.1, pages 201–204: # 2, 4, 5, 6, 10, 11, 12, 13, 20

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*Put these **P** Problems next on your PDF, in this order.*

P36. I have a 3×5 matrix and I compute its `rref`:

$$A = \begin{bmatrix} 0 & 1 & 2 & 3 & 4 \\ 0 & 2 & 4 & 3 & 2 \\ 0 & 0 & 0 & 1 & 2 \end{bmatrix} \qquad \to \qquad R = \begin{bmatrix} 0 & 1 & 2 & 0 & -2 \\ 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Find a basis for each of the four subspaces associated to A:

row space $C(A^\top)$, column space $C(A)$, null space $N(A)$, left nullspace $N(A^\top)$

P37. Suppose I have factored A as LU:

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 1 & 2 & 4 & 6 \\ 0 & 0 & 1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

Find a basis for each of the four subspaces associated to A:

row space $C(A^\top)$, column space $C(A)$, null space $N(A)$, left nullspace $N(A^\top)$

P38. Let $\boldsymbol{V}$ be the subspace of $\mathbb{R}^3$ spanned by $(1, -1, 0)$ and $(0, 2, 1)$.

(a) Find a matrix A that has $\boldsymbol{V}$ as its row space.

(b) Find a matrix B that has $\boldsymbol{V}$ as its null space.

(c) Multiply $AB^\top$. Multiply $BA^\top$. Why do these come out so simple?

P39. Let I be the 3×3 identity matrix and O be the 3×2 zero matrix. For each of these matrices, find the dimensions of the four subspaces:

$$A = \begin{bmatrix} O \end{bmatrix}, \quad B = \begin{bmatrix} I & O \end{bmatrix}, \quad C = \begin{bmatrix} I & I \\ O^\top & O^\top \end{bmatrix}$$

P40. **(a)** Prove that every $\boldsymbol{x}$ in $N(A)$ is perpendicular to every $A^\top \boldsymbol{y}$ in the row space of A (i.e. in $C(A^\top)$). *Hint. Start with* $A\boldsymbol{x} = \mathbf{0}$. *Now compute a dot product.*

(b) Prove that every $\boldsymbol{y}$ in $N(A^\top)$ is perpendicular to every $A\boldsymbol{x}$ in the column space of A (i.e. in $C(A)$).

P41. This system of equations $A\boldsymbol{x} = \boldsymbol{b}$ has no solutions:

$$\begin{aligned} 2x + 3y + 4z &= 9 \\ 4x + 3y + 2z &= 9 \\ 2x \qquad\quad - 2z &= 1 \end{aligned}$$

Find numbers c_1, c_2, c_3 to multiply the equations so that they add to $0 = 1$. (*Hint. Do row operations, and keep track of them.*) You have found a vector $\boldsymbol{y}$ in which subspace? Check its dot product: $\boldsymbol{y}^\top \boldsymbol{b} = 1$.

P42. Suppose $\boldsymbol{S}$ is spanned by the vectors $(1, 2, 2, 3)$ and $(1, 1, 1, 1)$. Find two vectors that span $\boldsymbol{S}^\top$. This is the same as solving $A\boldsymbol{x} = \mathbf{0}$ for which A?