Homework #2

Due Monday 24 January, 2022 at 11:59pm.

Submit as a single PDF by using Gradescope, via the course Canvas site
`canvas.alaska.edu/courses/7017`
Problems from the textbook (Strang, *Intro Linear Algebra*, 5th ed. 2016) will be graded for completion, while the "**P**" problems will be graded for correctness. Answers/solutions to textbook problems are linked at
`bueler.github.io/math314/resources.html`

from Problem Set 1.3, pages 29–30: # 1, 3, 8, 14

from Problem Set 2.1, pages 41–45: # 1, 4, 8, 13, 17, 18, 22, 23, 26, 34

P7. (a) Solve this equation $S\boldsymbol{y} = \boldsymbol{b}$ for $\boldsymbol{y}$. Note S is a *sum matrix*.

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \\ 8 \\ 9 \end{bmatrix}.$$

(b) Solve this equation $M\boldsymbol{y} = \boldsymbol{b}$ for $\boldsymbol{y}$. Note M is a *difference matrix*.

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ 4 \\ 1 \end{bmatrix}.$$

(c) If I take any vector $\boldsymbol{u}$ and first multiply it by M from part **(b)** to get $M\boldsymbol{u} = \boldsymbol{v}$, and then I multiply $\boldsymbol{v}$ by S from part **(a)** to get $S\boldsymbol{v} = \boldsymbol{w}$, what is $\boldsymbol{w}$?

P8. Here are three vectors:

$$\boldsymbol{v}_1 = \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}, \qquad \boldsymbol{v}_2 = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}, \qquad \boldsymbol{v}_3 = \begin{bmatrix} 5 \\ 6 \\ 7 \end{bmatrix}.$$

One may create the zero vector from the linear combination $x_1\boldsymbol{v}_1 + x_2\boldsymbol{v}_2 + x_3\boldsymbol{v}_3$ by choosing $x_1 = x_2 = x_3 = 0$, but that is obvious and boring. Instead, choose $x_1 = 1$ and find x_2 and x_3 so that the linear combination is again the zero vector. Does this show that the three vectors are independent or dependent? The three vectors lie in a ____________.
(Note that the matrix V formed from these vectors is *not invertible*.)

P9. **(a)** Compute this matrix-vector product by using dot products of the rows with the column vector:

$$\begin{bmatrix} 3 & -1 & 0 & 0 \\ -1 & 3 & -1 & 0 \\ 0 & -1 & 3 & -1 \\ 0 & 0 & -1 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 3 \\ 1 \\ 2 \end{bmatrix}.$$

(b) Compute the same matrix-vector product by a linear combination of the columns of the matrix.

P10. **(a)** What 2 by 2 matrix R rotates every vector counter-clockwise by $90°$? (Note R times $\begin{bmatrix} x \\ y \end{bmatrix}$ is $\begin{bmatrix} y \\ -x \end{bmatrix}$.)

(b) What 2 by 2 matrix S rotates every vector by $180°$?

(c) Show that for any vector $\boldsymbol{u}$, $R(R\boldsymbol{u}) = S\boldsymbol{u}$.