Worksheet: Is it a subspace?

A *vector space* is a set of vectors with a defined addition operation and a scalar multiple operation. (Various sensible rules—p. 130—apply to those operations.) A *subspace* is a subset S of the vector space for which any linear combination of elements from S is in S.

You can verify that S is a subspace by checking if 0 is in S, if $\mathbf{v} + \mathbf{w}$ is in S whenever $\mathbf{v}$, $\mathbf{w}$ are in S, and finally if $c\mathbf{v}$ is in S whenever $\mathbf{v}$ is in S and c is any real number.

For each problem below, sketch S if possible, and otherwise describe it. Say whether S is a subspace or not. If so, provide a brief justification. If not, describe an element that is not in S but would be if S were a subspace.

1. Vector space: $\mathbf{R}^2$. S is the set of all points in the first quadrant of $\mathbf{R}^2$.

2. Vector space: all real-valued functions on the line. S is the set of all polynomials.

3. Vector space: $\mathbf{R}^3$. S is the set of all vectors $a\mathbf{i} + b\mathbf{j} + c\mathbf{k}$ where a, b, c are integers.

4. Vector space: $\mathbf{R}^2$. $S =$ all solutions to $A\mathbf{x} = \mathbf{0}$ where $A = \begin{bmatrix} 2 & -3 \\ 6 & 7 \end{bmatrix}$.

5. Vector space: $\mathbf{R}^2$. $S =$ all solutions to $A\mathbf{x} = \mathbf{b}$ where $A = \begin{bmatrix} 2 & -3 \\ -6 & 9 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} -1 \\ 3 \end{bmatrix}$.

6. Vector space: $\mathbf{R}^2$. $S =$ all solutions to $A\mathbf{x} = \mathbf{0}$ where $A = \begin{bmatrix} 2 & -3 \\ -6 & 9 \end{bmatrix}$.

7. Vector space: $\mathbf{R}^3$. $S =$ all solutions to $A\mathbf{x} = \mathbf{0}$ where $A = \begin{bmatrix} 3 & 2 & 1 \\ 0 & 1 & 1 \\ -6 & -3 & -1 \end{bmatrix}$.