Homework #12

Due Monday 25 April, 2022 at 11:59pm.

Submit as a single PDF via Gradescope; see the Canvas page
canvas.alaska.edu/courses/7017
Textbook Problems from Strang, *Intro Linear Algebra*, 5th ed. will be graded for completion. Answers/solutions to these Problems are linked at
bueler.github.io/math314/resources.html
The **P** Problems will be graded for correctness. When grading these Problems, I will expect you to write explanations using complete sentences!

Put these Textbook Problems first on your PDF, in this order.

from Problem Set 6.4, pages 344–348: # 4, 5, 8

from Problem Set 8.1, pages 406–409: # 1, 3, 13, 17, 20, 24 (*Hint. Append columns.*)

from Problem Set 8.2, pages 417–419: # 1, 4, 5, 10, 11, 14, 27

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*Put these **P** Problems next on your PDF, in this order.*

P57. **(a)** By hand calculation, find an orthogonal matrix Q which diagonalizes

$$S = \begin{bmatrix} 1 & 0 & 2 \\ 0 & -1 & -2 \\ 2 & -2 & 0 \end{bmatrix}$$

(b) Check your calculation using Matlab's `eig` command

```
[Q,D] = eig(S)
```

Explain any differences between your Q and the computed `Q` from Matlab.

(*Hints. Recall that if X is an invertible matrix of eigenvectors of A then X diagonalizes A in the sense that $AX = X\Lambda$ or $A = X\Lambda X^{-1}$, where Λ is diagonal. Recall that the columns of an orthogonal matrix are orthonormal vectors: $Q^\top Q = I$. If $X = Q$ is orthogonal then $A = X\Lambda X^{-1} = Q\Lambda Q^\top$.*)

P58. **(a)** What matrix A transforms $(1, 0)$ and $(0, 1)$ to (r, s) and (t, u)?

(b) What matrix B transforms (a, b) and (c, d) to $(1, 0)$ and $(0, 1)$?

(c) What condition on a, b, c, d will make part **(b)** impossible?

(d) When $r = a$, $s = b$, $t = c$, and $u = d$ then A and B are matrix inverses. Confirm this.

P59. Consider the symmetric matrices

$$A = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}, \qquad B = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

(a) Which of these classes of matrices do A and B belong to?:

INVERTIBLE, ORTHOGONAL, PROJECTION, PERMUTATION, DIAGONALIZABLE

Explain, or show work which supports your answers.

(b) Which of these factorizations are possible for A and B?:

LU, $X\Lambda X^{-1}$, $Q\Lambda Q^\top$

(*As usual, L is lower triangular with ones on diagonal, U is upper triangular, X is invertible,* Λ *is diagonal, and Q is orthogonal.*) Explain, or show work which supports your answers.

(c) By hand calculation, find Q orthogonal and Λ diagonal so that $B = Q\Lambda Q^\top$. (*Hint. B has a repeated eigenvalue* $\lambda = 0$*, and you will need to find* two *orthogonal and normalized eigenvectors for this* λ*. Check your work in Matlab.*)

P60. In this problem we consider transformations from $\boldsymbol{V} = \mathbb{R}^2$ to $\boldsymbol{W} = \mathbb{R}^2$.

(a) For each of these transformations, is it linear? (*Show it is, or give a counterexample.*) In either case, give a simplified formula for $T(T(\boldsymbol{v}))$:

- $T(\boldsymbol{v}) = -\boldsymbol{v}$
- $T(\boldsymbol{v}) = \boldsymbol{v} + (1, 1)$
- $T(\boldsymbol{v}) = (\text{do } 90^\circ \text{ rotation on } \boldsymbol{v}) = (-v_2, v_1)$
- $T(\boldsymbol{v}) = (\text{projection}) = \frac{1}{2}(v_1 + v_2, v_1 + v_2)$

(b) Show that if T is linear, i.e. $T(a\boldsymbol{v} + b\boldsymbol{w}) = aT(\boldsymbol{v}) + bT(\boldsymbol{w})$, then $T(T(\boldsymbol{v}))$ is also linear.

(*Note that it is common in mathematics to write "*T^2*" for the composition* $T(T(\cdot))$ *of a transformation T with itself, even if T is not linear, and/or T is not already represented by a matrix.*)

P61. **(a)** Consider the vector space $\boldsymbol{M}$ of 2 by 2 matrices. Show that the transpose transformation T, on $\boldsymbol{M}$, is linear: $T(A) = A^\top$. (*Hint. Not much to do! Fits on one line.*)

(b) Try to find a 2 by 2 matrix B so that $T(A) = BA$. Show that no such matrix B exists! (*Hint. Show that* $BA = A^\top$ *being true for* all *matrices A is impossible.*)

(c) If we rearrange the entries of a 2 by 2 matrix A into a column vector $\boldsymbol{a}$ with four entries then we can do what is asked in **(b)**. That is, show that there is a 4×4 matrix C so that $C\boldsymbol{a}$ is a column vector which is the rearrangement of $A^\top$.