Matrix World $\overline{\mathbf{M}}$ atrices $(m \times n)$ $A = U\Sigma V^{\mathrm{T}}$ A = CR $row \ rank = column \ rank$ SVD: orthonormal basis U, V Square Matrices $(n \times n)$ $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$ Invertible Singular $det(A) \neq 0$ all $\lambda \neq 0$ det(A) = 0one $\lambda = 0$ A = QR ---- Triangularize -- PA = LUGram-Schmidt → U has a zero row Diagonalizable $A = X \Lambda X^{-1}$ - Diagonalize $A = XJX^{-1}$ $A = \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}$ $A = \begin{bmatrix} 0 & 2 \\ 0 & 0 \end{bmatrix}$ Normal $A^{\mathrm{T}}A = AA^{\mathrm{T}}$ Symmetric $S \models S^{T} \text{ all } \lambda \text{ are real}$ $A = Q\Lambda Q^{\rm T}$ Positive $S = Q\Lambda Q^{\mathrm{T}}$ Semidefinite all $\lambda \geq 0$ all $A^{T}A$ $S = \begin{bmatrix} 1 & 2 \\ 2 & 0 \end{bmatrix}$ Orthogonal **Jordan** Projection $P^{2} = P = P^{T} \lambda = 1 \text{ or } 0$ $Q^{-1} = Q^{T}$ $J = \begin{bmatrix} \lambda_1 & 1 \\ 0 & \lambda_1 \end{bmatrix}$ all $|\lambda| = 1$ 0 $Q = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$ Diagonal $\Lambda \mid \Sigma$ Positive **Definite** all $\lambda > 0$

 $A^{-1} = V \Sigma^{-1} U^{\mathrm{T}} \quad \blacktriangleleft$

(v1.3) Drawn by Kenji Hiranabe

with the help of Prof. Gilbert Strang

 $A^+ = V \Sigma^+ U^{\mathrm{T}}$

pseudoinverse

for all A