

Assignment #6

Due Wednesday 22 October, at the start of class

Please read Lectures 10, 11, 12, and 13 in the textbook *Numerical Linear Algebra*, SIAM Press 1997, by Trefethen and Bau.

DO THE FOLLOWING EXERCISES FROM THE TEXTBOOK:

- **Exercise 11.3**
- **Exercise 12.2**
- **Exercise 13.4**

*Any computer you touch has $\epsilon_{\text{machine}} \approx 10^{-16}$, so do part (a) with floating point numbers as usual. I was able to get Matlab to do part (b) by starting with $x = \text{sym}('0')$ and then doing Newton's method the usual way at the command line. For instance, if I were approximating $\sqrt{2}$ by Newton's method then I might start with $x = \text{sym}('1')$ and then repeat $x = x - (x^2 - 2) / (2 * x)$. What is Trefethen's point in showing you this calculation?*

DO THE FOLLOWING ADDITIONAL PROBLEMS.

The Matlab built-in `qr()` computes the QR factorization using Householder reflectors (Lecture 10). Generally you should use it when QR is needed, but of course you can check it against `mgs()`, which you wrote on Assignment #5 from Lecture 8.

P14. By applying Matlab's backslash command, reproduce Figure 11.1. Then, by applying Algorithm 11.2, using the `qr` and backslash commands, reproduce Figure 11.2. Please make at least a modest effort to capture the appearance of these Figures. (*Hints.* Note `axis off` creates a clean picture without ticks and axes labels. You can put back the black axes using `plot([-6 6], [0 0], 'k')` and similar.)

P15. Suppose A is a 101×101 matrix with $\|A\|_2 = 50$ and $\|A\|_F = 51$. Give the sharpest possible lower bound on the 2-norm condition number of A . (*Hint.* Write everything in terms of singular values, and then think about best cases for $\kappa_2(A)$.)