Assignment 3

Due Friday 22 September 2023, at the start of class

Please read Lectures 3,4,5,6 in the textbook *Numerical Linear Algebra* by Trefethen and Bau. This Assignment mostly covers norms (Lecture 3) and the SVD (Lectures 4 & 5).

DO THE FOLLOWING EXERCISES from Lecture 2:

- **Exercise 2.3**

DO THE FOLLOWING EXERCISES from Lecture 3:

- **Exercise 3.2**
- **Exercise 3.3**

DO THE FOLLOWING EXERCISES from Lecture 4:

- **Exercise 4.3** Use the `svd` command on A. Write a MATLAB (or other) `function` of the form `vismat(A)`. Start by checking that the input matrix A is in fact 2×2, and that its entries are real. Correctness of the program is more important than figure appearance.

DO THE FOLLOWING ADDITIONAL EXERCISES.

P7. On page 21 of the textbook, equation (3.10) gives a formula for the ∞-norm of an $m \times n$ matrix. Prove it:

$$\|A\|_\infty = \max_{1 \le i \le m} \|a_i^*\|_1.$$

P8. Use by-hand calculations to determine SVDs of the following matrices. Note that in the decomposition $A = U\Sigma V^*$, the factor Σ is unique but the factors U, V are not. Thus there will be more than one correct answer. (*Hints. First, think. Then use Theorem 5.4 or Theorem 5.5 if needed. When in doubt, check (and show) that the unitary factors are indeed unitary. Check that the singular values are in the proper order and that $A = U\Sigma V^*$ is actually true.*)

(a) $\begin{bmatrix} -3 & 0 \\ 0 & 7 \end{bmatrix}$ **(b)** $\begin{bmatrix} 0 & 0 \\ 0 & 3 \\ 0 & 0 \end{bmatrix}$ **(c)** $\begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix}$ **(d)** $\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$