

Assignment 7

Due Monday 30 October 2023, at the start of class

Please read Lectures 11,12,13,14,15 in the textbook *Numerical Linear Algebra* by Trefethen and Bau. This Assignment covers least squares, conditioning, and floating point.

DO THE FOLLOWING EXERCISES from Lecture 11:

- **Exercise 11.3**

DO THE FOLLOWING EXERCISES from Lecture 13:

- **Exercise 13.2** *Do parts (a) and (b) only.*

DO THE FOLLOWING ADDITIONAL EXERCISES.

P13. Suppose A is a 100×100 matrix with $\|A\|_2 = 20$ and $\|A\|_F = 21$. Give the sharpest possible lower bound on the 2-norm condition number of A . (*Hint. Write everything in terms of singular values, and then think about best cases for $\kappa_2(A)$.*)

P14. For each problem, compute the absolute condition number $\hat{\kappa}$ and the relative condition number κ ; generally both formulas will involve x .¹ Choose the most convenient norm, but make your choice explicit.² Comment on when the problem is well-conditioned or ill-conditioned; generally this answer also depends on x .

- Compute x^3 for $x > 0$.
- Compute $\cos x$ for real x .
- For $x \in \mathbb{C}^2$ compute $x_1 x_2$, the product of the entries.
- Fix $a \in \mathbb{R}^m$, a column vector. Compute the inner product $a^* x$ for $x \in \mathbb{R}^m$.

P15. Consider the polynomial

$$\begin{aligned} p(x) &= (x - 3)^{10} \\ &= x^{10} - 30x^9 + 405x^8 - 3240x^7 + 17010x^6 - 61236x^5 \\ &\quad + 153090x^4 - 262440x^3 + 295245x^2 - 196830x + 59049 \end{aligned}$$

- For $x = 2.85 : 0.01 : 3.15$, plot $p(x)$ by evaluating it via its coefficients $1, -30, 405, \dots$
- Plot $p(x)$ again on the same interval and same graph, using expression $(x - 3)^{10}$.
- In two or three sentences, compare and contrast the bad behavior here with the ill-conditioning phenomenon in Example 12.5 on page 92, i.e. Wilkinson's example.

¹You can use formulas (12.3) and (12.6) without justification.

²For **a)** and **b)** just use absolute values for the norm.

P16. *This is a reading assignment. Actually read it! It's good.*

Please read the following 12 page encyclopedia entry:

L. N. Trefethen, *Numerical Analysis*, in W. T. Gowers, editor, Princeton Companion to Mathematics, Princeton U. Press, 2008.

people.maths.ox.ac.uk/trefethen/NAessay.pdf

Answer the following questions with a sentence or two at most:

- (i) Give a one-sentence version of Trefethen's definition of "numerical analysis."
- (ii) Is analysis of rounding errors the main business of numerical analysis? If not, what is?
- (iii) Gaussian elimination with pivoting is a matrix factorization. State it.
- (iv) Trefethen refers to Householder triangularization, Algorithm 10.1 in our text-book, as "QR factorization". But then what does the "QR algorithm" do?
- (v) What is the "central dogma" of numerical linear algebra?
- (vi) Fill the blank: "The discovery of _____ came quickly, but its theoretical analysis has proved astonishingly hard."
- (vii) What is the "the biggest unsolved problem in numerical analysis"?