Translation rules for code generation from TLA$^+$ specifications

Contents

1 TLA$^+$ syntax

Identifiers I, C Values v Parameters p

Specification	$Spec$	::=	$Module\ M$ CONSTANTS $C_o,\ \ldots, C_n$ VARIABLES $V_o,\ \ldots, V_n$ $D_0,\ \ldots, D_n$
Definition	D	::=	$Action(p_0, \ldots, p_n) \triangleq \mathcal{A}$
Action	$\mathcal{A}$	::=	$A \mid P \mid \mathcal{A} \wedge \mathcal{A} \mid \mathcal{A} \vee \mathcal{A}$
Condition	P, Q	::=	$\neg P \mid P \wedge P \mid P \vee P \mid v_1 \in v_2$
		|	$v_1 = v_2 \mid v_1 \neq v_2 \mid$ ENABLED $\mathcal{A}$
Transition	T	::=	$I' = v \mid$ UNCHANGED $\langle I_0, \ldots, I_n \rangle$
Set	S	::=	$v \mid S_a \cup S_b$
Record	R	::=	$[k \mapsto v] \mid [I$ EXCEPT $![k] = v]$

2 Elixir syntax

Atoms $\texttt{i}, \texttt{k}$ Values $\texttt{x}, \texttt{y}$ Parmetros $\texttt{p}$

State	$\texttt{t}$	::=	$\texttt{action(variables, } \overline{\texttt{p}}\texttt{)}$
		|	$\texttt{variables} \mid \texttt{Map.merge(a, a)}$
		|	$\texttt{\%\{ i}_o : \texttt{x}_o\texttt{, ..., i}_n : \texttt{x}_n\texttt{ \}}$
Condition	$\texttt{c}$	::=	$\texttt{condition(variables, } \overline{\texttt{p}}\texttt{)}$
		|	$\texttt{not c} \mid \texttt{c and c} \mid \texttt{c or c}$
Definition	$\texttt{d}$	::=	`def action(variables, p) do`
			`...`
			`end`
		|	
`def decide([info])`			
Set	$\texttt{s}$	::=	$\texttt{MapSet.new([x])}$
		|	$\texttt{MapSet.union(s}_a\texttt{, s}_b\texttt{)}$
Record	$\texttt{r}$	::=	$\texttt{\%\{ k: x \}} \mid \texttt{Map.put(i, k, x)}$
Information	$\texttt{i}$	::=	$\texttt{\%\{ action: ``Name'', condition: c, state: a \}}$
		|	$\texttt{Enum.map(x, f (i) -> [info] end)}$

3 Top-level Spec translation

$$\boxed{\vdash \mathit{Spec} \rightarrowtail \mathit{code}}$$

$$\frac{\begin{array}{rll} \Gamma \vdash_{const} C_0, \ldots, C_n & \rightarrowtail & \texttt{const}_0\texttt{, ..., const}_n \\ \{C_0 : const, \ldots, C_n : const\} \vdash_{dec} Def_0 & \rightarrowtail & \texttt{def}_0 \\ \vdots & & \\ \{C_0 : const, \ldots, C_n : const\} \vdash_{dec} Def_n & \rightarrowtail & \texttt{def}_n \\ \{C_0 : const, \ldots, C_n : const\} \vdash_{next} Def_{next} & \rightarrowtail & \texttt{def\_next} \\ \{M : module, C_0 : const, \ldots, C_n : const\} \vdash_{init} Def_{init} & \rightarrowtail & \texttt{state} \end{array}}{\begin{array}{ll} \textsc{module } M & \texttt{defmodule M do} \\ \textsc{constants } C_0, \ldots, C_n & \texttt{@oracle spawn(Oracle, :listen, [])} \\ \textsc{variables } V_0, \ldots, V_n & \texttt{const}_0\texttt{, ..., const}_n \\ Def_0 & \texttt{def}_0\texttt{, ..., def}_n \\ \vdash \quad \vdots & \rightarrowtail \quad \texttt{def\_decide} \\ Def_n & \texttt{def\_main} \\ Def_{init} & \texttt{end} \\ Def_{next} & \\ & \texttt{M.main(state)} \end{array}} \text{(MOD)}$$

4 Definition translation

$$\boxed{\Gamma \vdash_{def} D \rightarrowtail code}$$

$$\frac{\Gamma \cup \{p_0 : param, \ldots, p_n : param\} \vdash_a \mathcal{A} \rightarrowtail (\{c_o, \ldots, c_n\}, \{a_0, \ldots, a_n\})}{\Gamma \vdash_{def} Action(p_0, \ldots, p_n) \triangleq \mathcal{A} \rightarrowtail \begin{array}{l}\text{def action\_condition(variables, } p_0, \ldots, p_n\text{) do} \\ \quad \text{Enum.all?([}\mathtt{c}_0\text{, ..., }\mathtt{c}_n\text{])} \\ \text{end} \\ \text{def action(variables, } p_0, \ldots, p_n\text{) do} \\ \quad \text{Map.merge(}a_0\text{, Map.merge(..., }a_n\text{))} \\ \text{end}\end{array}} \texttt{(DEF)}$$

5 Action translation

$$\boxed{\Gamma \vdash_a \mathcal{A} \rightarrowtail (\overline{c}, \overline{a})}$$

$$\frac{\Gamma \vdash_p P \rightarrowtail c}{\Gamma \vdash_a P \rightarrowtail (\{c\},\ \{\})}\ \texttt{(COND)} \qquad \frac{\Gamma \vdash_t A \rightarrowtail a}{\Gamma \vdash_a A \rightarrowtail (\{\},\ \{a\})}\ \texttt{(TRA)}$$

$$\frac{\begin{array}{c}\Gamma \vdash_a \mathcal{A}_0 \rightarrowtail (\overline{c_0},\ \overline{a_0}) \\ \vdots \\ \Gamma \vdash_a \mathcal{A}_n \rightarrowtail (\overline{c_n},\ \overline{a_n})\end{array}}{\Gamma \vdash_a \begin{array}{l}\wedge\ \mathcal{A}_0 \\ \vdots \\ \wedge\ \mathcal{A}_n\end{array} \rightarrowtail (\overline{c_o} \cup \cdots \cup \overline{c_n},\ \overline{a_o} \cup \cdots \cup \overline{a_n})}\ \texttt{(AND)}$$

$$\frac{\begin{array}{cc}\Gamma \vdash_a \mathcal{A}_0 \rightarrowtail (\overline{c_0},\ \overline{a_0}) & \Gamma \vdash_i \mathcal{A}_0 \rightarrowtail \overline{\texttt{i}_0} \\ \vdots & \vdots \\ \Gamma \vdash_a \mathcal{A}_n \rightarrowtail (\overline{c_n},\ \overline{a_n}) & \Gamma \vdash_i \mathcal{A}_n \rightarrowtail \overline{\texttt{i}_n}\end{array}}{\Gamma \vdash_a \begin{array}{l}\vee\ \mathcal{A}_0 \\ \vdots \\ \vee\ \mathcal{A}_n\end{array} \rightarrowtail \left(\begin{array}{c}\overline{c_o} \cup \cdots \cup \overline{c_n}, \\ \{\ \texttt{decide}(\overline{\texttt{i}_0} \cup \cdots \cup \overline{\texttt{i}_n})\ \}\end{array}\right)}\ \texttt{(OR)}$$

$$\frac{\begin{array}{c}\Gamma \vdash_v v_0 \rightarrowtail \texttt{x}_0 \\ \vdots \\ \Gamma \vdash_v v_n \rightarrowtail \texttt{x}_n\end{array}}{\Gamma \vdash_a \mathit{Action}(v_0, \ldots, v_n) \rightarrowtail (\{\ \text{action\_condition(variables, } \texttt{x}_0\text{, ..., } \texttt{x}_n)\ \}, \{\ \text{action(variables, } \texttt{x}_0\text{, ..., } \texttt{x}_n)\ \})}\ \texttt{(CALL)}$$

$$\frac{\begin{array}{c}\Gamma \vdash_p P \rightarrowtail c \\ \Gamma \vdash_a \mathcal{A}_t \rightarrowtail (\{\texttt{ct}_0, \ldots, \texttt{ct}_n\},\ \{\texttt{at}_0, \ldots, \texttt{at}_n\}) \\ \vdots \\ \Gamma \vdash_a \mathcal{A}_t \rightarrowtail (\{\texttt{ce}_0, \ldots, \texttt{ce}_n\},\ \{\texttt{ae}_0, \ldots, \texttt{ae}_n\})\end{array}}{\Gamma \vdash_a \begin{array}{ll}\text{IF} & P \\ \text{THEN} & \mathcal{A}_t \\ \text{ELSE} & \mathcal{A}_e\end{array} \rightarrowtail (\{\mathit{condition}\},\ \{\mathit{transition}\})}\ \texttt{(IF)}$$

where

condition =

```
if c do
  ct_0 and ... and ct_n
else
  ce_0 and ... and ce_n
end
```

transition =

```
if c do
  Map.merge(at_0, Map.merge(..., at_n))
else
  Map.merge(ae_0, Map.merge(..., ae_n))
end
```

6 Predicate translation

$$\boxed{\Gamma \vdash_p P \rightarrowtail \texttt{c}}$$

$$\frac{\Gamma \vdash_v v_x \rightarrowtail x \quad \Gamma \vdash_v v_y \rightarrowtail y}{\Gamma \vdash_p v_x = v_y \rightarrowtail \texttt{x == y}}\ \texttt{(PRED-EQ)} \qquad \frac{\Gamma \vdash_v v_x \rightarrowtail x \quad \Gamma \vdash_v v_y \rightarrowtail y}{\Gamma \vdash_p v_x v_y \rightarrowtail \texttt{x != y}}\ \texttt{(PRED-NEQ)}$$

$$\frac{\Gamma \vdash_v v_e \rightarrowtail e \quad \Gamma \vdash_v v_l \rightarrowtail l}{\Gamma \vdash_p v_e \in v_l \rightarrowtail \texttt{Enum.member?(}l\texttt{, }e\texttt{)}}\ \texttt{(PRED-IN)}$$

$$\frac{\Gamma \vdash_p P \rightarrowtail c}{\Gamma \vdash_p \neg P \rightarrowtail \texttt{not c}}\ \texttt{(PRED-NOT)}$$

$$\frac{\begin{array}{c}\Gamma \vdash_p P_0 \rightarrowtail \texttt{c}_0 \\ \vdots \\ \Gamma \vdash_p P_n \rightarrowtail \texttt{c}_n\end{array}}{\Gamma \vdash_a \begin{array}{l}\vee\ P_0 \\ \vdots \\ \vee\ P_n\end{array} \rightarrowtail \texttt{Enum.any?([c}_0\texttt{, ..., c}_n\texttt{])}}\ \texttt{(PRED-OR)}$$

$$\frac{\Gamma \vdash_a \mathcal{A} \rightarrowtail (\texttt{c}_0, ..., \texttt{c}_n, \ \overline{\texttt{t}})}{\Gamma \vdash_p \text{ENABLED } \mathcal{A} \rightarrowtail \texttt{c}_0\ \texttt{and ...}\ \texttt{and c}_n}\ \texttt{(PRED-EN)}$$

7 Transition translation

$$\boxed{\Gamma \vdash_t T \rightarrowtail \texttt{t}}$$

$$\frac{}{\Gamma \vdash_t \text{UNCHANGED } \langle I_0, \ldots, I_n \rangle \rightarrowtail \begin{array}{l}\%\{I_0 : \ \texttt{variables[:}I_0\texttt{]} \\ \quad \vdots \\ I_n : \ \texttt{variables[:}I_n\texttt{]}\}\end{array}}\ \texttt{(TRA-UNCH)}$$

$$\frac{\Gamma \vdash_v v \rightarrowtail x}{\Gamma \vdash_t I' = v \rightarrowtail \texttt{\%\{ }I\texttt{: }x\texttt{ \}}}\ \texttt{(TRA-PRIM)}$$

8 Values translation

$$\boxed{\Gamma \vdash_v v \rightarrowtail x}$$

$$\frac{\{I : param\} \in \Gamma}{\Gamma \vdash_v I \rightarrowtail I}\ \texttt{(VAL-PARAM)} \qquad \frac{\{I : param\} \notin \Gamma \qquad \{I : const\} \notin \Gamma}{\Gamma \vdash_v I \rightarrowtail \texttt{variables[:I]}}\ \texttt{(VAL-VAR)}$$

$$\frac{\begin{array}{c}\{I : const\} \in \Gamma \\ \{M : module\} \notin \Gamma\end{array}}{\Gamma \vdash_v I \rightarrowtail \texttt{@I}}\ \texttt{(VAL-CONST)} \qquad \frac{\begin{array}{c}\{I : const\} \in \Gamma \\ \{M : module\} \in \Gamma\end{array}}{\Gamma \vdash_v I \rightarrowtail \texttt{M.I}}\ \texttt{(VAL-ATTR)}$$

$$\frac{\Gamma \vdash_v v_0 \rightarrowtail x_o \ \ldots\ \Gamma \vdash_v v_n \rightarrowtail x_n}{\Gamma \vdash_v \{v_0, \ldots, v_n\} \rightarrowtail \texttt{MapSet.new([x}_0, \ldots, \texttt{x}_n\texttt{])}}\ \texttt{(SET-LIT)}$$

$$\frac{\Gamma \vdash_v S_a \rightarrowtail s_a \quad \Gamma \vdash_v S_b \rightarrowtail s_b}{\Gamma \vdash_v S_a \cup S_b \rightarrowtail \texttt{MapSet.union(s}_a\texttt{, s}_b\texttt{)}}\ \texttt{(SET-UNION)}$$

$$\frac{\Gamma \vdash_v v_0 \rightarrowtail x_o \ \ldots\ \Gamma \vdash_v v_n \rightarrowtail x_n}{\Gamma \vdash_v [k_0 \mapsto v_o, \ldots, k_n \mapsto v_n] \rightarrowtail \texttt{\%\{ k}_0\texttt{: x}_0, \ldots, \texttt{k}_n\texttt{: x}_n\ \texttt{\}}}\ \texttt{(REC-LIT)}$$

$$\frac{\Gamma \vdash_v v_v \rightarrowtail \texttt{x} \quad \Gamma \vdash_v v_i \rightarrowtail \texttt{i}}{\Gamma \vdash_v [v_i \text{ EXCEPT } ![k] = v_v] \rightarrowtail \texttt{Map.put(i, k, x)}}\ \texttt{(REC-EXCEPT)}$$

$$\frac{\Gamma \vdash_v v \rightarrowtail x}{\Gamma \vdash_v v[k] \rightarrowtail \texttt{x[k]}}\ \texttt{(REC-INDEX)}$$

$$\frac{\Gamma \vdash_v [\overline{k- > v}] \rightarrowtail \texttt{x}_l \quad \Gamma \vdash_v v_r \rightarrowtail \texttt{x}_r \quad \Gamma \vdash_v v_v \rightarrowtail \texttt{x}_v}{\begin{array}{c}\Gamma \vdash_v [I \in v_r- > v_v,\ \overline{k- > v}] \rightarrowtail \\ \texttt{x}_r\ \texttt{|> Enum.map(fn i -> \{i, x}_v\texttt{\} end) |> Enum.into(x}_l\texttt{)}\end{array}}\ \texttt{(REC-EX)}$$

9 Initial state translation

$$\boxed{\Gamma \vdash_{init} P \rightarrowtail \texttt{t}}$$

$$\frac{\Gamma \vdash_v v \rightarrowtail \texttt{x}}{\Gamma \vdash_{init} I = v \rightarrowtail \texttt{\%\{ i: x \}}}\ \texttt{(INIT-EQ)}$$

$$\frac{\begin{array}{c}\Gamma \vdash_{init} P_0 \rightarrowtail \texttt{t}_0 \\ \vdots \\ \Gamma \vdash_{init} P_n \rightarrowtail \texttt{t}_n\end{array}}{\Gamma \vdash_{init} \begin{array}{l}\wedge\ P_0 \\ \vdots \\ \wedge\ P_n\end{array} \rightarrowtail \begin{array}{l}\text{Map.merge(t}_0\text{,} \\ \quad \text{Map.merge(..., t}_n\text{))}\end{array}}\ \texttt{(INIT-AND)}$$

10 Next state action translation

$$\boxed{\Gamma \vdash_{next} \mathit{Action} \triangleq \mathcal{A} \rightarrowtail \mathit{code}}$$

$$\frac{\Gamma \vdash_a \mathcal{A} \rightarrowtail (\{\},\ \mathtt{t}_0, \ldots, \mathtt{t}_n)}{\Gamma \vdash_{next} \mathit{Action} \triangleq \mathcal{A} \rightarrowtail} \texttt{(NEXT)}$$

```
def main(variables) do
  IO.puts (inspect variables)

  main(
    Map.merge(t_0, Map.merge(..., t_n))
  )
end
```

11 Information extraction

$$\boxed{\Gamma \vdash_i \mathcal{A} \rightarrowtail \texttt{info}}$$

$$\frac{\Gamma \vdash_d \mathcal{A} \rightarrowtail (\{c_0, \ldots, c_n\},\ \{\ a_0, \ldots, a_n\})}{\Gamma \vdash_i \mathcal{A} \rightarrowtail} \texttt{(INFO-DEF)}$$

```
%{
  action: show([a_0, ..., a_n])
  condition: c_0 and ... and c_n
  state: Map.merge(a_0, Map.merge(..., a_n))
},
```

$$\frac{\begin{array}{c}\Gamma \vdash_v v \rightarrowtail x \\ \Gamma \vdash_i \mathcal{A}_0 \rightarrowtail \mathtt{i}_0 \\ \vdots \\ \Gamma \vdash_i \mathcal{A}_n \rightarrowtail \mathtt{i}_n\end{array}}{\Gamma \vdash_d \exists I \in v : \mathcal{A}_0 \vee \cdots \vee \mathcal{A}_n \rightarrowtail} \texttt{(INFO-EX)}$$

```
Enum.map(x, fn (i) -¿ [
    i_0,
    ...
    i_n
  ]
end
```