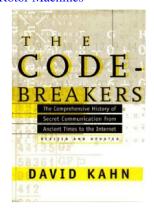
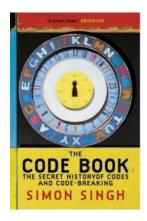
# 2 Historical Ciphers

# **Historical Ciphers**

- Shift Cipher
- Substitution Cipher
- Vigenère Cipher
- Vernam Cipher
- Rotor Machines





# 2.1 Shift Cipher

## **Shift Cipher**

Identify each letter with a number

- $\bullet$  **A** = 0
- $\bullet \ \mathbf{B} = 1$
- C = 2
- •
- Z = 25

The key k is a number in the range 0-25

• Encryption: add *k* to each letter (modulo 26).

Julius Caesar used the key k = 3.

• ATTACK AT DAWN becomes DWWDFN DW GDZQ

We break a shift cipher by using the statistics of the underlying language.

## **English Letter Frequencies**

| Α | 8.05  | J | 0.10 | S | 6.59 |
|---|-------|---|------|---|------|
| В | 1.62  | K | 0.52 | T | 9.59 |
| C | 3.20  | L | 4.03 | U | 3.10 |
| D | 3.65  | M | 2.25 | V | 0.93 |
| E | 12.31 | N | 7.19 | W | 2.03 |
| F | 2.28  | О | 7.94 | X | 0.20 |
| G | 1.61  | P | 2.29 | Y | 1.88 |
| Н | 5.14  | Q | 0.20 | Z | 0.09 |
| I | 7.18  | R | 6.03 |   |      |

Most common bigrams: TH,HE,IN,ER,AN,RE,ED,ON,ES,ST,EN,AT,TO,NT,HA Most common trigrams: THE,ING,AND,HER,ERE,ENT,THA,NTH,WAS,ETH,FOR

## **Shift Cipher**

Take the following example cipher text:

BPMZM WVKM EIA IV COTG LCKSTQVO EQBP NMIBPMZA ITT ABCJJG IVL JZWEV IVL BPM WBPMZ JQZLA AIQL QV AW UIVG EWZLA OMB WCB WN BWEV OMB WCB, OMB WCB, OMB WCB WN BWEV IVL PM EMVB EQBP I YCIKS IVL I EILLTM IVL I YCIKS QV I NTCZZG WN MQLMZLWEV BPIB XWWZ TQBBTM COTG LCKSTQVO EMVB EIVLMZQVO NIZ IVL VMIZ JCB IB MDMZG XTIKM BPMG AIQL BW PQA NIKM VWE OMB WCB, OMB WCB, OMB WCB WN PMZM IVL PM EMVB EQBP I YCIKS IVL I EILLTM IVL I YCIKS IVL I DMZG CVPIXXG BMIZ

We need to compare the frequency distribution of this text with standard English

## **Letter Frequencies**

| Α | 2.59  | J | 1.44  | S | 1.73 |
|---|-------|---|-------|---|------|
| В | 10.37 | K | 2.59  | T | 3.46 |
| C | 5.48  | L | 6.63  | U | 0.29 |
| D | 0.58  | M | 10.09 | V | 8.36 |
| E | 4.61  | N | 2.31  | W | 6.63 |
| F | 0.00  | О | 3.46  | X | 1.15 |
| G | 2.59  | P | 4.03  | Y | 1.15 |
| Н | 0.00  | Q | 4.03  | Z | 4.90 |
| I | 11.53 | R | 0.00  |   |      |

### **Cracking the Cipher**

The shift of E seems to be either 4, 8, 17, 18 or 23 The shift of A seems to be either 1, 8, 12, 21 or 22

Hence the key is probably equal to 8

We can now decrypt the cipher text to reveal:

THERE ONCE WAS AN UGLY DUCKLING WITH FEATHERS ALL STUBBY AND BROWN AND THE OTHER BIRDS SAID IN SO MANY WORDS GET OUT OF TOWN GET OUT, GET OUT, GET OUT OF TOWN AND HE WENT WITH A QUACK AND A WADDLE AND A QUACK IN A FLURRY OF EIDERDOWN THAT POOR LITTLE UGLY DUCKLING WENT WANDERING FAR AND NEAR BUT AT EVERY PLACE THEY SAID TO HIS FACE NOW GET OUT, GET OUT, GET OUT OF HERE AND HE WENT WITH A QUACK AND A WADDLE AND A QUACK AND A VERY UNHAPPY TEAR

## 2.2 Substitution Cipher

### **Substitution Cipher**

The problem with the shift cipher is that the number of keys is too small.

• We only have 26 possible keys

To increase the number of keys a substitution cipher was invented. Encryption involves replacing each letter by its permuted version. Decryption involves use of the inverse permutation.



Hence ATTACK AT DAWN encrypts to TXXTKE TX GTHA

## **Substitution Cipher**

Number of keys is  $26! \approx 4.03 \times 10^{26} \approx 2^{88}$ 

• Feasible to only run a computer on a problem which takes under 2<sup>80</sup> steps.

This is far too large a number to brute force search using modern computers. Still we can break these ciphers using statistics of the underlying plaintext language.

### **Example**

XSO MJIWXVL JODIVA STW VAO VY OZJVCO'W LTJDOWX KVAKOAXJTXIVAW VY SIDS XOKSAVLVDQ IAGZWXJQ. KVUCZXOJW, KVUUZAIKTXIVAW TAG UIKJVOLOKXJVAIKW TJO HOLL JOCJOWOAXOG, TLVADWIGO GIDIXTL UOGIT, KVUCZXOJ DTUOW TAG OLOKXJVAIK KVUUOJKO. TW HOLL TW SVWXIAD UTAQ JOWOTJKS TAG CJVGZKX GONOLVCUOAX KOAXJOW VY UTPVJ DLVMTL KVUCTAIOW, XSO JODIVA STW T JTCIGLQ DJVHIAD AZUMOJ VY IAAVNTXINO AOH KVUCTAIOW. XSO KVUCZXOJ WKIOAKO GOCTJXUOAX STW KLVWO JOLTXIVAWSICW HIXS UTAQ VY XSOWO VJDTAIWTXIVAW NIT KVLLTMVJTXINO CJVPOKXW, WXTYY WOKVAGUOAXW TAG NIWIXIAD IAGZWXJITL WXTYY. IX STW JOKOAXLQ IAXJVGZKOG WONOJTL UOKSTAIWUW YVJ GONOLVCIAD TAG WZCCVJXIAD OAXJOCJOAOZJITL WXZGOAXW TAG WXTYY, TAG TIUW XV CLTQ T WIDAIYIKTAX JVLO IA XSO GONOLVCUOAX VY SIDS-XOKSAVLVDQ IAGZWXJQ IA XSO JODIVA.

# **English Letter Frequencies**

| A | 8.99 | J | 6.51  | S | 3.26 |
|---|------|---|-------|---|------|
| В | 0.00 | K | 4.81  | T | 7.60 |
| C | 2.95 | L | 4.34  | U | 3.57 |
| D | 3.10 | M | 0.62  | V | 8.06 |
| E | 0.00 | N | 1.40  | W | 7.13 |
| F | 0.00 | O | 11.63 | X | 7.75 |
| G | 3.72 | P | 0.31  | Y | 2.17 |
| Н | 0.78 | Q | 1.40  | Z | 2.17 |
| I | 7.75 | R | 0.00  |   |      |

Most common bigrams: TA,AX,IA,VA,WX,XS,AG,OA,JO,JV Most common trigrams: OAX,TAG,IVA,XSO,KVU,TXI,UOA,AXS

### **Analysis**

Since O occurs with frequency 11.63 we can guess O = ECommon trigrams are:

- $OAX = E^{**}$
- XSO = \*\*E

Common similar trigrams in English are:

- ENT, ETH
- THE

Hence likely to have:

- $\bullet$  X = T
- $\bullet$  S = H
- $\bullet$  A = N

## **Analysis**

From now on we only look at the first two sentences:

THE MJIWTVL JEDIVN HTW VNE VY EZJVCE'W LTJDEWT KVNKENTJTTIV NW VY HIDH TEKHNVLVDQ INGZWTJQ. KVUCZTEJW, KVUU ZNIKTTIVNW TNG UIKJVELEKTJVNIKW TJE HELL JECJEWENTEG, TLVNDWIGE GIDITTL UEGIT, KVUCZTEJ DTUEW TNG ELEKTJVNIK KVUUEJKE.

This was after the changes:

- $\bullet$  O = E
- $\bullet$  X = T
- $\bullet$  S = H
- $\bullet$  A = N

## **Analysis**

Since T occurs as a single letter we must have:

- T = I, or
- $\bullet$  T = A

T has probability of 8.07, which is the highest probability left Therefore, more likely to have:

 $\bullet$  T = A

The most frequent

- bigram is TA = AN
- trigram is  $TAG = AN^*$

Therefore highly likely that:

• G = D

### **Analysis**

THE MJIWTVL JEDIVN HAW VNE VY EZJVCE'W LAJDEWT KVNKENTJATIV NW VY HIDH TEKHNVLVDQ INDZWTJQ. KVUCZTEJW, KVUUZNIKATIVNW AND UIKJVELEKTJVNIKW AJE HELL JECJEWENTED, ALVNDWIDE DIDITAL UEDIA, KVUCZTEJ DAUEW AND ELEKTJVNIK KVUUEJKE.

This was after the additional changes:

- $\bullet$  T = A
- G = D

# **Analysis**

We now look at two letter words:

- IX = \*T
- Therefore I must be one of A, I due to English plaintext
- Already have A

Hence:

- $\bullet$  I = I
- XV = T\*
- Must have, due to English, V = O

## **Analysis**

More two letter words:

- **VY** = **O**\*
- Hence Y must be one of F, N, R due to English
- Already have N
- Y has probability 1.61
- F has probability 2.28
- R has probability 6.03

Hence Y = F

We also have:

- **IW** = **I**\*
- Therefore W must be one of F, N, S, T
- Already have F, N, T

Hence W = S

## **Analysis**

THE MJISTOL JEDION HAS ONE OF EZJOCE'S LAJDEST KONKENTJATIONS OF HIDH TEKHNOLODQ INDZSTJQ. KOUCZTEJS, KOUUZNIKATIONS AND UIKJOELEKTJONIKS AJE HELL JECJESENTED, ALONDSIDE DIDITAL UEDIA, KOUCZTEJ DAUES AND ELEKTJONIK KOUUEJKE.

This was after the additional changes:

- $\bullet$  I = I
- V = O
- $\bullet$  Y = F
- $\bullet$  W = S

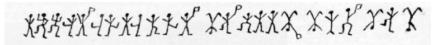
It is then easy to see what the underlying plaintext is.

## **Example Substitution Ciphers**

Edgar Alan Poe's "The Gold Bug":

53++!305))6\*;4826)4+.)4+);806\*;48!8'60))85;]8\*;;+\*8!83(88)5\*!; 46(;88\*96\*?;8)\*+(;485);5\*!2:\*+(;4956\*2(5\*-4)8'8\*;4069285);)6!8)4++;1(+9;48081;8:8+1;48!85;4)485!528806\*81(+9;48;(88;4(+?34;48)4+;161;:188;+?;

Sir Arthur Conan Doyle's "Adventure of the Dancing Men":



# 2.3 Vigenère Cipher

### Vigenère Cipher

The problem with the shift and substitution ciphers was that each plaintext letter always encrypted to the same ciphertext letter.

Hence underlying statistics of the language could be used to break the cipher.

From the early 1800s onwards cipher designers tried to break this link between the plain and cipher texts.

The most famous cipher used during the 1800s is the Vigenère Cipher

- Invented in 1553 by Giovan Batista Belaso.
- Believed to be unbreakable for a number of years.
- Actually easy to break.

### Vigenère Cipher

The Vigenère cipher again identifies letters with the numbers 0-25

The secret key is a short sequence of letters (e.g. a word).

Encryption involves adding the plaintext letter to a key letter, with the key letters used in rotation.

Thus if the key is SESAME, encryption works as follows:

```
T H I S I S A T E S T M E S S A G E Message
S E S A M E S E S A M E S E S A M E Keystream
L L A S U W S X W S F Q W W K A S I Ciphertext
```

This is a polyalphabetic substitution cipher

• A will encrypt to a different letter depending on where it is in the message.

#### Vigenère Cipher

### Ciphertext:

CJ UT WFCN LTTF VF AAHGKEE DNH VYC IPSPKGTMV EVLINFA, NC HXS SLGIX QNVYGEM VYYI MVG ZLUHFORRXHB-RIMRXGUZLV TBF KCAXQQDKJGWERRXHBU ICKHZWKGDGG PFU JGRGIUPR KKCJ RHBVZLJX JKXMGHIUCW XGHQ KFT MKGERN-YWTJR. LX WPKCGTQV RLS MFCEQPVH DP
BXKSEKGCZ TNFAZL CH UGVBHCC NPVYGKQ IHKCIBH XOEY MIAST KFGHIIY ANUSTJNPVS, ERPGRWPX JDOS PFRTL, RKXGITZ ERQW, TBF JCRKSV
TMGICTRRT WCELKTGHU. FSG ISTJMCTZ CEB TVCPFKXV ZKMCH KSNP KDKS CEB BHFG FL DNF CSGABHA KM AXH ULAW XHJVPTTZ ERPGBST GGVXCPJ KTWWCKC PM O FZQITBEV UWTH YV SHXR VF BD PWVY DPVS-VF-DPVS OVCIBBIJ, NPIST UMRNAGERH, TBF R DXKA JRLSLVCBC.
JGTQIRJGOVVJN, MVG KCRABKTYA PWBRPSKM GEYQEWPX PTFCVV ADEZCSMGTHKFLH BG HFSCWSF FL QKCCUAPLHKEE TOSTPRWBBI RQ HXEWVLRXG QW XTKCU RLS HBGJ RWTH QECH HKP UMV PCWCBCM FGTMVGWBV. UWTH KJ RD WWUKGCZIKJF P WWIZRPE RQCIPK KJVL XM
WU RQ TTGKCW GXDTFBJVWDCC PL HJV QEHYGE UDKR? JFU SH KG TMCOSTJC EKWXRRTEM YYCC XJGIW HRZNRZAX WU SMJGQGU MUY O
URRTEZKKC PGR UDCPKSF FTTK OP VLIBFG TMCMWPVLI?

## Vigenère Cipher

First we need to look for repeated sequences of characters.

- Recall English has a large repetition of certain bigrams
- These are likely to match up to the same two letters in the key every so often

• By looking for the distance between two repeated sequences we can guess the length of the keyword.

Each distance should be a multiple of the keyword.

• Taking the gcd of all distances between sequences should give the keyword length.

# Vigenère Cipher

First sentence is:

CJ UT WFCN LTTF VF AAHGKEE DNH VYC IPSPKGTMV EVLINFA, NC HXS SLGIX QNVYGEM VYYI MVG ZLUHFORRXHB-RIMRXGUZLV TBF KCAXQQD-KJGWERRXHBU ICKHZWKGDGG PFU JGRGIUPR KKCJ RHBVZLJX JKXMGHI-UCW XGHQ KFT MKGERN-YWTJR.

Distance between occurrences of RR: 30 Distance between occurrences of KG: 96, 46 We have:

 $\bullet$  gcd(30,96)=6, gcd(30,46)=2, gcd(96,46)=2

Unlikely to have a keyword of length 2

• Guess keyword is of length 6

### Vigenère Cipher

Now we take every sixth letter and look at the statistics just as we did for a shift cipher to deduce the first letter of the keyword.

This gives us the following:

| Α | 1.49  | J | 3.73 | S | 0.75 |
|---|-------|---|------|---|------|
| В | 1.49  | K | 8.96 | T | 7.46 |
| C | 8.96  | L | 0.00 | U | 8.21 |
| D | 1.49  | M | 0.00 | V | 8.21 |
| Е | 6.72  | N | 2.99 | W | 2.24 |
| F | 4.48  | O | 1.49 | X | 0.75 |
| G | 11.19 | P | 8.21 | Y | 1.49 |
| Н | 1.49  | Q | 4.48 | Z | 0.00 |
| Ι | 2.99  | R | 0.75 |   |      |

We look for a low value and then three high ones, corresponding to Q, R, S,  $T \Rightarrow Key$  is Q or Q

### Vigenère Cipher

Then we take every sixth letter starting from the second one and repeat to find the second letter of the keyword:

| A | 0.00  | J | 8.21  | S | 2.24  |
|---|-------|---|-------|---|-------|
| В | 0.75  | K | 9.70  | T | 3.73  |
| C | 5.22  | L | 2.24  | U | 3.73  |
| D | 1.49  | M | 0.75  | V | 10.44 |
| E | 7.46  | N | 1.49  | W | 0.75  |
| F | 11.19 | O | 0.00  | X | 2.99  |
| G | 0.75  | P | 2.24  | Y | 4.48  |
| H | 0.00  | Q | 0.00  | Z | 3.73  |
| I | 4.48  | R | 11.94 |   |       |

We look for a low value and then three high ones, corresponding to Q, R, S,  $T \Rightarrow Key$  is 17 or R

And so on to determine the six letters of the keyword: CRYPTO

## Vigenère Cipher

The underlying plaintext is then found to be:

AS WE DRAW NEAR TO CLOSING OUT THE TWENTIETH CENTURY, WE SEE QUITE CLEARLY THAT THE INFORMATION-PROCESSING AND TELECOMMUNICATIONS REVOLUTIONS NOW UNDERWAY WILL CONTINUE VIGOROUSLY INTO THE TWENTY-FIRST. WE INTERACT AND TRANSACT BY DIRECTING FLOCKS OF DIGITAL PACKETS TOWARDS EACH OTHER THROUGH CYBERSPACE, CARRYING LOVE NOTES, DIGITAL CASH, AND SECRET
CORPORATE DOCUMENTS. OUR PERSONAL AND ECONOMIC LIVES RELY MORE AND MORE ON OUR ABILITY TO LET SUCH ETHEREAL CARRIER
PIGEONS MEDIATE AT A DISTANCE WHAT WE USED TO DO WITH FACE -TO-FACE MEETINGS, PAPER DOCUMENTS, AND A FIRM HANDSHAKE. UNFORTUNATELY, THE TECHNICAL WIZARDRY ENABLING REMOTE COLLABORATIONS IS FOUNDED ON BROADCASTING EVERYTHING AS SEQUENCES
OF ZEROS AND ONES THAT ONES OWN DOG WOULDN'T RECOGNIZE. WHAT IS TO DISTINGUISH A DIGITAL DOLLAR WHEN IT IS A S EASILY REPRODUCIBLE AS THE SPOKEN WORD? HOW DO WE CONVERSE PRIVATELY WHEN EVERY SYLLABLE IS BOUNCED OFF A SATELLITE AND SMEARED
OVER AN ENTIRE CONTINENT?

# 2.4 Vernam Cipher

## Vernam Cipher (One-Time Pad)

- Gilbert Vernam patented this cipher in 1917 for encryption and decryption of telegraph messages.
- Used extensively during the first world war.
- To send a binary string m you need a key k as long as the message.
- Each key *k* is uniformly selected at random and can be used only once hence one-time pad.
- Encryption:  $c = m \oplus k$ .
- Decryption:  $m = c \oplus k$ .
- $\oplus$  is exclusive-or (XOR):

| $\oplus$ | 0 | 1 |
|----------|---|---|
| 0        | 0 | 1 |
| 1        | 1 | 0 |

# Vernam Cipher (One-Time Pad)

```
Plaintext:
                    n
                            e
                                     t
In binary: 01101111 01101110 01100101 01110100 01101001
        01011100 01010001 11100000 01101001 01111010
Plaintext:
                                             d
           m
                    e
                            p
        01101101 \ 01100101 \ 01110000 \ 01100001 \ 01100100
In binary:
Key:
         111111001 11000110 01011010 10110001 01110011
Ciphertext: 10010100 10100011 00101010 11010001 00010111
```

### **Vernam Cipher (One-Time Pad)**

Gives perfect secrecy, provided:

- Key is truly random.
- Key is at least as big as plaintext (not practical).
- Key is not reused e.g. consider a 'two-time' pad:

```
First encryption: c<sub>1</sub> = m<sub>1</sub> ⊕ k.
Second encryption: c<sub>2</sub> = m<sub>2</sub> ⊕ k.
c<sub>1</sub> ⊕ c<sub>2</sub> = m<sub>1</sub> ⊕ m<sub>2</sub>
```

- Vulnerable to frequency analysis.

### 2.5 Rotor Machines

### **Rotor Machines**

With the advent of the 1920s people saw the need for a mechanical encryption device. Taking a substitution cipher and then rotating it became seen as the ideal solution.

- This had actually been used during the American Civil War.
- But now this could be done more efficiently.

The rotors could be implemented using wires and then encryption can be done mechanically using an electrical circuit.

- By rotating the rotor we obtain a new substitution cipher.
- Transmission of message still done manually using Morse Code.

## **Rotor Machines**

To encrypt the first letter we use the substitutions:



To encrypt the second letter we use the substitutions:



To encrypt the third letter we use the substitutions:



and so on.

## Enigma

The most famous of these machines was the Enigma.



## Enigma

We shall describe the most simple version of Enigma. Used three such rotors chosen from a set of five:

- EKMFLGDQVZNTOWYHXUSPAIBRCJ Rotor One
- AJDKSIRUXBLHWTMCQGZNPYFVOE Rotor Two
- BDFHJLCPRTXVZNYEIWGAKMUSQO Rotor Three

- ESOVPZJAYQUIRHXLNFTGKDCMWB Rotor Four
- VZBRGITYUPSDNHLXAWMJQOFECK Rotor Five

The order of the rotors in the machine is important. Number of ways of choosing the rotors is  $5 \times 4 \times 3 = 60$ 

### Enigma

Each rotor has an initial starting position

• Number of starting positions is  $26^3 = 17576$ 

The first rotor would move on the second rotor on each full iteration under the control of a ring hitting a notch.

- The notches were fixed for each rotor but the rings were movable.
- Similarly the second rotor would move the third rotor.
- Number of positions of the rings is  $26^2 = 676$

The second rotor also had a kick associated to it

• Cycle length of three rotors was then  $26 \times 25 \times 26 = 16900$ 

A given plaintext letter would encrypt to a different ciphertext letter on each press of the keyboard.

## Enigma

A plugboard swapped letters twice in each decryption/encryption operation.

• This increased the complexity and gave another 10<sup>14</sup> keys.

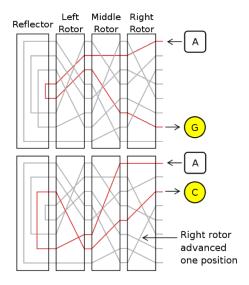
Total number of keys was then around  $10^{23}$ 

To make sure encryption and decryption were the same operation a reflector was used:

• This was a fixed substitution:



## Enigma



### **Enigma**

To use Enigma the Germans had a day setting of:

- Plugs to use
- Rotors to use, and their order
- Ring settings
- Initial rotor settings

However, we really need to change the key on each message.

- This is where the flaw in Enigma resulted.
- Not in the design, but in the use of the system

### **Enigma**

So sender of message would choose a new set of rotor positions for this message (A, F, G say) and then encrypt these twice using the day settings (G, H, K, L, P, T say). He would then set the machine to the message setting and encrypt the message. The receiver would decrypt the message key, reset his machine and then decrypt the rest of the message.

It is the repeating of the message rotor settings which led to to the Polish and British cryptographers being able to break Enigma in WWII.

## **Enigma**

However, it is not only bad usage which led to the breaking of Enigma.

The main problem with Enigma is that the plugboard and the rotors are orthogonal in some sense.

- In particular the plugboard acts on the permutation given by the rotors as conjugation.
- This means that many of the underlying statistics of the rotor settings are not disguised by the plugboard
- Once the rotor settings are found the plugboard settings can be found easily.

### **Enigma**

Reflector weakens Enigma: no difference between encryption and decryption.

- Problem 1: encryption becomes involutary, i.e. if  $K \to T$ , then  $T \to K$
- Problem 2: no letter is encrypted to itself (electricity can't go same way back)

Heavy reduction of encryption alphabet - violation of Kerckhoff's principle:

- Security of Enigma depended on wiring of rotors
- Wiring was part of algorithm, not part of key
- Wiring never changed from 1920s until 1945

### 2.6 Summary

### **Historical Ciphers**

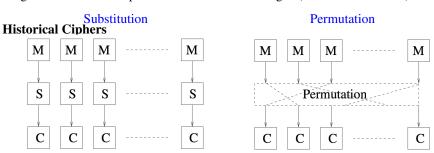
The historical ciphers we have studied can be categorised as follows:

- monoalphabetic: shift cipher, substitution cipher
- polyalphabetic: Vigenère, Enigma

Simplest ciphers could be easily broken unless encryption rule was kept secret (security by obscurity).

Simple ciphers could easily be broken because they did not conceal the language characteristics (encryption functions were not complicated enough).

Enigma was the first step towards more mature designs (secure and efficient).



- Substitution: provides confusion, i.e. it makes the relationship between key and ciphertext complex.
- Permutation: provides diffusion, i.e. each bit (symbol) of the ciphertext depends on many (if possible all) bits (symbols) of the plaintext.

# **Modern Ciphers**

Modern ciphers use both substitution and permutation.

They typically encrypt small blocks of plaintext at a time.

They can be efficiently implemented on different types of platforms.

Ciphers that apply an encryption function to small blocks of plaintext at a time are called block ciphers.

Ciphers that use substitution and permutation are called substitution-permutation networks.

# 2.7 Security

### **Computational Security**

A system is computationally secure if the best algorithm for breaking it requires *N* operations.

- Where N is a very big number
- No practical system can be proved secure under this definition.

In practice we say a system is computationally secure if the best known algorithm for breaking it requires an unreasonably large amount of computer time.

### **Computational Security**

Another practical approach is to reduce a well studied hard problem to the problem of breaking the system.

• For example, the system is secure if a given integer n cannot be factored.

Systems of this form are often called provably secure.

- However, we only have a proof relative to some hard problem.
- Not an absolute proof.

Essentially bounding the computational power of the adversary.

• Even if the adversary has limited (but large) resources they still will not break the system.

### **Computational Security**

When considering schemes which are computationally secure:

- We need to be careful about the key sizes.
- We need to keep abreast of current algorithmic developments.
- At some point in the future we should expect our system to be broken (may be many millennia hence though).

Most schemes in use today are computationally secure.

### **Unconditional Security**

For unconditional security we place no bound on the computational power of the adversary.

In other words, a system is unconditionally secure if it cannot be broken even with infinite computing power.

• Some systems are unconditionally secure.

Other names for unconditionally secure are:

- Perfectly secure
- Information-theoretically secure
- Semantically secure

### **Unconditional Security**

A cryptosystem has unconditional security iff:

$$p(P = m \mid C = c) = p(P = m)$$

for all  $m \in P$  and  $c \in C$ .

That is, the probability that the plaintext is m given that the ciphertext c is observed is the same as the probability that the plaintext is m without seeing c.

In other words knowing c reveals no information about m.

Unconditional security implies that:  $|keys| \ge |messages|$ .

The use of such large key spaces is impractical in practice.

## **Key Distribution**

Perfect secrecy implies length of key is at least length of plaintext.

Key distribution becomes the major problem.

Aim of modern cryptography is to design systems where:

- one key can be used many times
- a short key can encrypt a long message.

Such systems will not be unconditionally secure, but should be at least computationally secure.

# **Examples**

Of the ciphers we have seen or will see later on, the following are not computationally secure:

- Caesar cipher
- Substitution cipher
- Vigenère cipher

The following are computationally secure but not unconditionally secure:

- DES(?)-AES
- RSA

The Vernam cipher (one-time pad) is unconditionally secure if used correctly.