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Exercise Sheet 2
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Introduction to Augmented Reality

Exercise 4 (P,H) Finding Rectangles

In the last exercise, you have generated a thresholded black-and-white image.

(a) Expand your program using the functions `cvFindContours` and `cvApproxPoly` to first extract object boundaries and then approximate these with straight line segments.
Hint: for the second-to-last parameter of `cvApproxPoly`, try a value of `cvContourPerimeter(contours) * 0.02`.
Note: `cvFindContours` uses the OpenCV heap. Use `cvCreateMemStorage`, `cvClearMemStorage` and `cvReleaseMemStorage` to manage it.

(b) Traverse all found contours and determine each bounding box using `cv::boundingRect`. Skip all polygons with more or less than 4 corners or too small bounding boxes (experiment with constraint values). Use `cv::cvarrToMat` for converting (CvSeq*) to (cv::Mat).

(c) Mark the rectangles you have found with red lines using `cv::polyLine`. Do this in the original camera image and display it afterwards.

(d) Subdivide each edge of the rectangles into 7 parts of equal length and draw a small circle around each of the dividing points. You will need these in the next exercise.

Exercise 5 (H) Rotation matrices

Movement of objects in a 3-dimensional space can be described in various ways. An example is the description by translations and rotations. Rotations can be described by matrices. A point $\vec{x}$ in *2-dimensional* space can be rotated by an angle α (in mathematically positive direction) by means of a matrix.

$$R_\alpha = \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix}; \qquad \begin{pmatrix} x' \\ y' \end{pmatrix} = R_\alpha \cdot \begin{pmatrix} x \\ y \end{pmatrix}$$

(a) Calculate the point which results from rotation of the point $\binom{2}{1}$ by 90°.
Show that $R_\alpha \cdot R_\beta \cdot \vec{x} = R_\beta \cdot R_\alpha \cdot \vec{x}$ for any α, β. Why?

Each rotation in 3-dimensional space can intuitively be described by mapping the base vectors of $\mathbf{R}^3$ to three new base vectors. This new coordinate system is used to describe the object. This linear mapping corresponds to a 3×3 matrix, of which the columns are the new base vectors.

(b) Give the rotation matrix which maps the point $x = (5, 0, 0)^T$ to the point $(0, 5, 0)^T$.

(c) Are transformations possible which change only one coordinate axis?
If so, which ones?

According to *Eulers Theorem*, a transformation matrix which describes a *rigid* transformation, i.e. only translations and rotations, can be decomposed into a series of rotations around three linearly independent axes.

Here, ω is the rotation around the x axis,

$$R(\omega, 0, 0) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\omega & -\sin\omega \\ 0 & \sin\omega & \cos\omega \end{pmatrix}$$

φ the rotation around the y axis,

$$R(0, \varphi, 0) = \begin{pmatrix} \cos\varphi & 0 & \sin\varphi \\ 0 & 1 & 0 \\ -\sin\varphi & 0 & \cos\varphi \end{pmatrix}$$

and κ the rotation around the z axis.

$$R(0, 0, \kappa) = \begin{pmatrix} \cos\kappa & -\sin\kappa & 0 \\ \sin\kappa & \cos\kappa & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

(d) Show that $R(\omega, 0, 0) \cdot R(0, \varphi, 0) \cdot R(0, 0, \kappa) \cdot \vec{x} \neq R(0, 0, \kappa) \cdot R(0, \varphi, 0) \cdot R(\omega, 0, 0) \cdot \vec{x}$.
Why?

Exercise 6 (H) Homogeneous matrices

In most cases, more than one transformation is applied to a vector. In order to achieve this with a single linear transformation (matrix multiplication), homogeneous coordinates are used. An extra dimension is added to the vectors and matrices. For vectors, this additional element is supposed to have value 1 by default,

$$\begin{pmatrix} x_x \\ x_y \\ x_z \end{pmatrix} \mapsto \begin{pmatrix} x_x \\ x_y \\ x_z \\ 1 \end{pmatrix}$$

thereby allowing all possible transformations to be expressed as a 4×4 matrix:

$$x' = \begin{pmatrix} x'_x \\ x'_y \\ x'_z \\ 1 \end{pmatrix} = \begin{pmatrix} & R(\omega, \varphi, \kappa) & & t_x \\ & & & t_y \\ & & & t_z \\ 0 & 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} x_x \\ x_y \\ x_z \\ 1 \end{pmatrix}$$

(a) Give the homogeneous matrix which maps the point $x = (5, 0, 0)^T$ to the point $(0, 10, 0)^T$. Use the rotation matrix obtained at 5 (b).

(b) What is the meaning of the fourth row of the homogeneous matrix? What would happen if it's not $(0, 0, 0, 1)$?