Introduction to Augmented Reality

Tutorial 5: Marker Tracking Part 5
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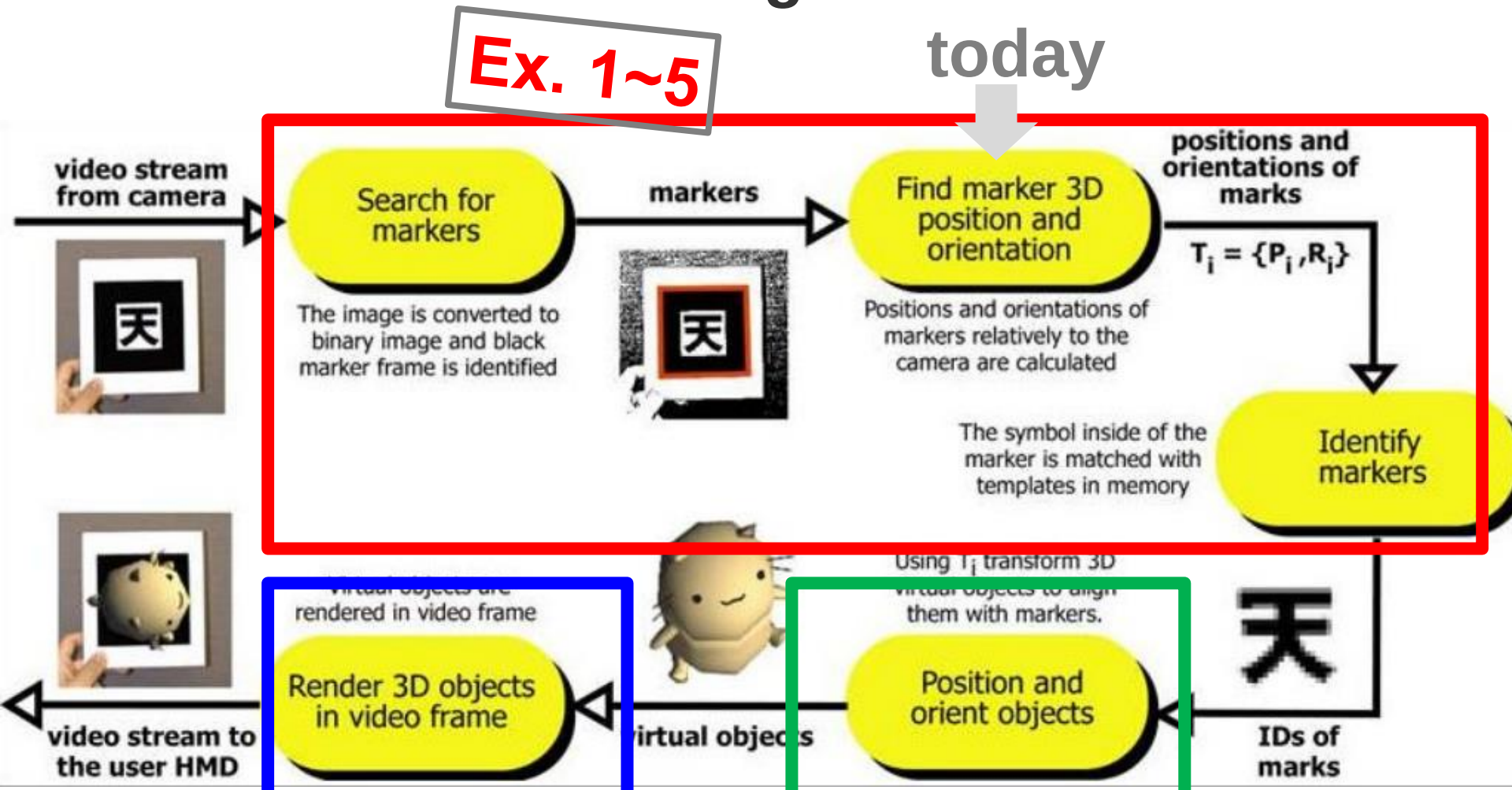
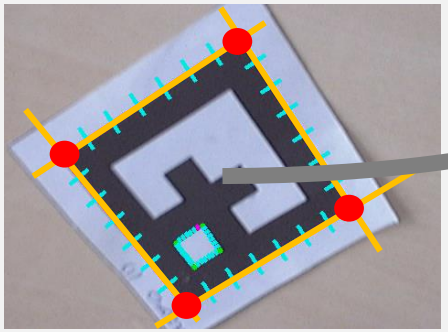
Marker-based Tracking



Solution for the Previous Tutorial

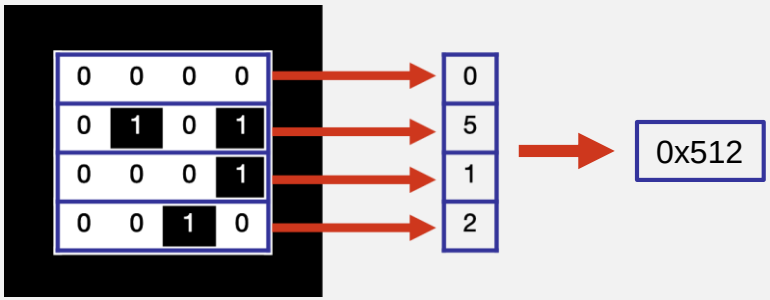
Ex. 4

Find marker corners **precisely**



RECTIFY marker

Identify marker ID



Code walkthrough

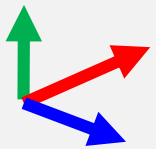
Today's Tutorial

Ex. 5

Marker-Pose Estimation

Pose = Position + Orientation

A code will be provided



3D Transformations Revisited

Homogeneous notation in R^3 → 4x4 matrix

Euclidean

Similarity

Translation, Rotation, Scaling

$$\begin{bmatrix} X' \\ Y' \\ Z' \\ 1 \end{bmatrix} \approx \begin{bmatrix} R_{00} & R_{01} & R_{02} & s_0 t_0 \\ R_{10} & R_{11} & R_{12} & s_1 t_1 \\ R_{20} & R_{21} & R_{22} & s_2 t_2 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

Scaling

Scaling: glScale* (s_x,s_y,s_z)

$$\begin{bmatrix} s_x & 0 & 0 & 0 \\ 0 & s_y & 0 & 0 \\ 0 & 0 & s_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \bullet \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} = \begin{bmatrix} s_x x \\ s_y y \\ s_z z \\ w \end{bmatrix}$$

Translation

Translation: glTranslate* (t_x,t_y,t_z)

$$\begin{bmatrix} 1 & 0 & 0 & t_x \\ 0 & 1 & 0 & t_y \\ 0 & 0 & 1 & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \bullet \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} = \begin{bmatrix} x + wt_x \\ y + wt_y \\ z + wt_z \\ w \end{bmatrix}$$

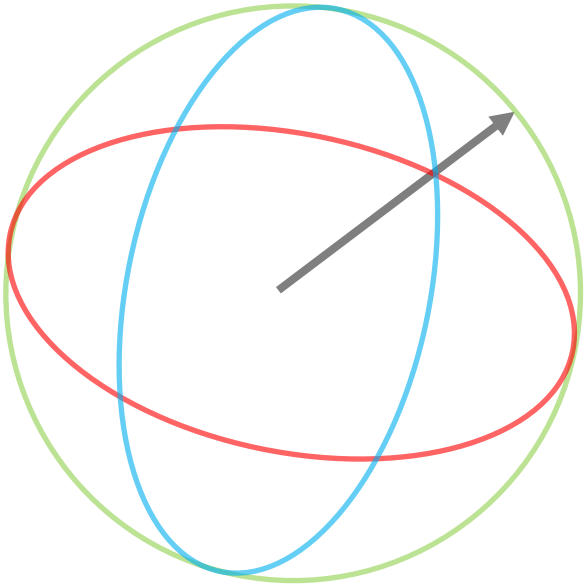
Rotation

$$R = \begin{bmatrix} R_{00} & R_{01} & R_{02} \\ R_{10} & R_{11} & R_{12} \\ R_{20} & R_{21} & R_{22} \end{bmatrix}$$

→ $I = RR^T$

(Unitary matrix)

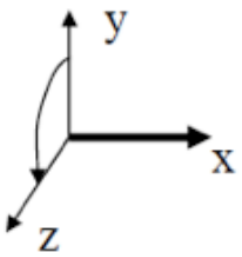
How to create a desired rotation?



e.g. Euler Angles (Around-axis Rotations)

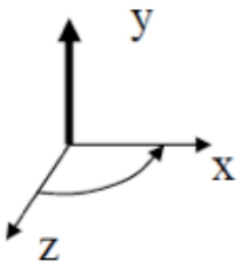
Rotation: glRotate* (a,ex,ey,ez)

- Around x



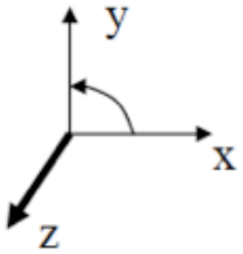
$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\alpha & -\sin\alpha & 0 \\ 0 & \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \bullet \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} = \begin{bmatrix} x \\ \cos\alpha\, y - \sin\alpha\, z \\ \sin\alpha\, y + \cos\alpha\, z \\ w \end{bmatrix}$$

- Around y



$$\begin{bmatrix} \cos\alpha & 0 & \sin\alpha & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\alpha & 0 & \cos\alpha & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \bullet \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} = \begin{bmatrix} \cos\alpha\, x + \sin\alpha\, z \\ y \\ -\sin\alpha\, x + \cos\alpha\, z \\ w \end{bmatrix}$$

- Around z



$$\begin{bmatrix} \cos\alpha & -\sin\alpha & 0 & 0 \\ \sin\alpha & \cos\alpha & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \bullet \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} = \begin{bmatrix} \cos\alpha\, x - \sin\alpha\, y \\ \sin\alpha\, x + \cos\alpha\, y \\ z \\ w \end{bmatrix}$$

We do use this for exercises with OpenGL, but…

Why Euler Angles are Evil

See video now!
http://www.youtube.com/watch?v=zc8b2Jo7mno

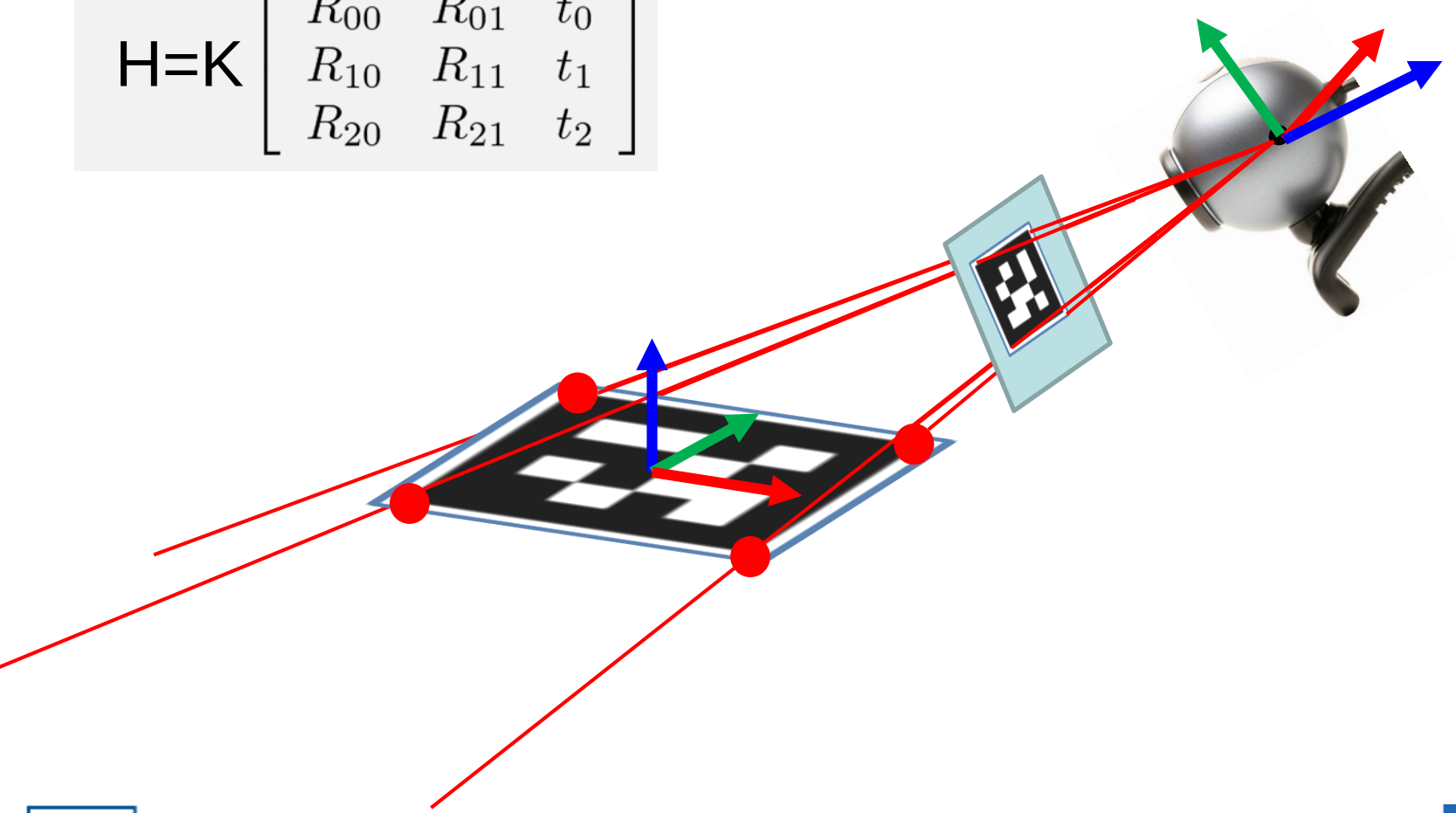
Pose Estimation via Homography

If H and K are known

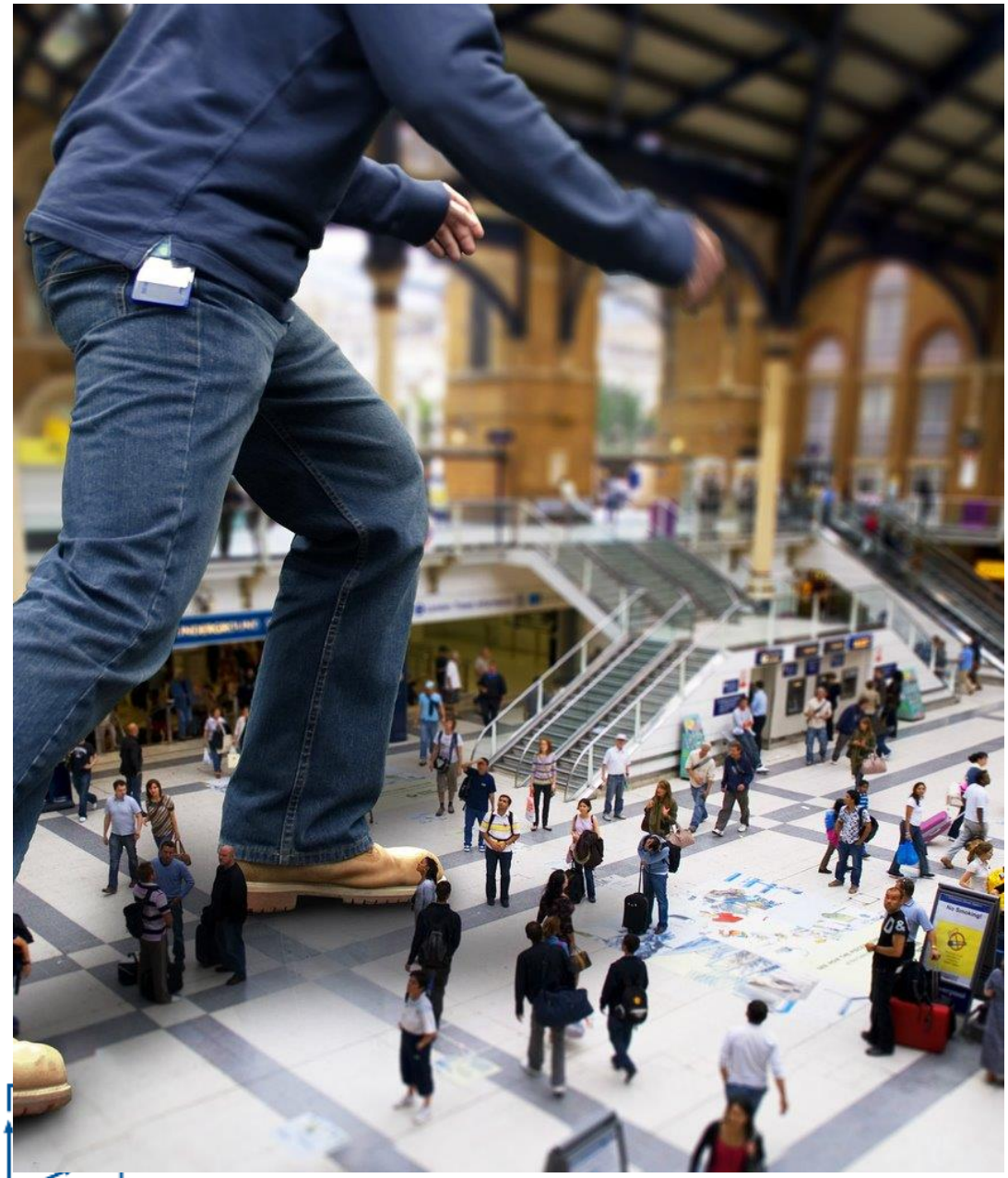
→(R, t)

(Investigate Tsai's and Zhang's methods for details)

$$H = K \begin{bmatrix} R_{00} & R_{01} & t_0 \\ R_{10} & R_{11} & t_1 \\ R_{20} & R_{21} & t_2 \end{bmatrix}$$



2D Image Does not Give 3D Scale



(without prior knowledge)

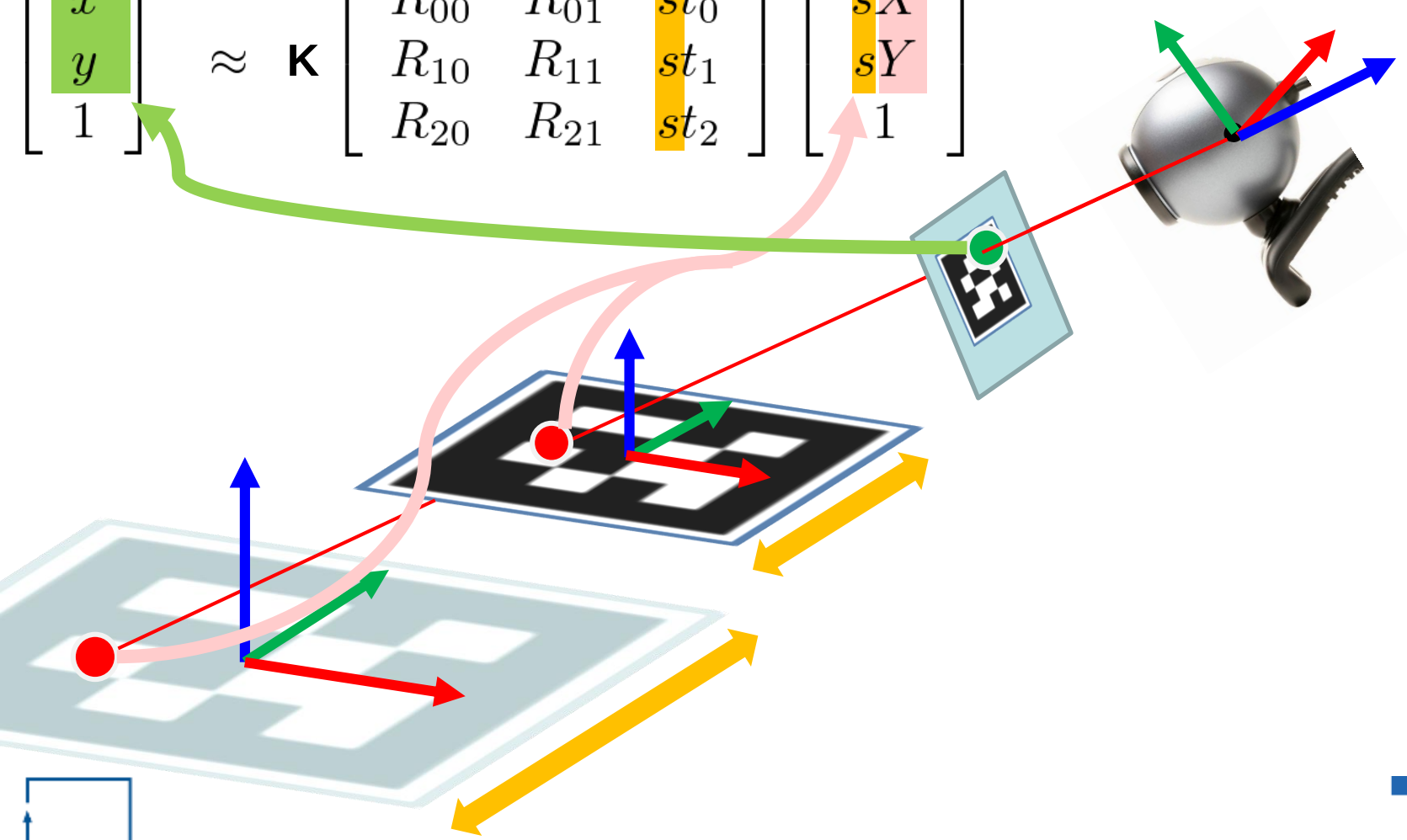
by Jamie Durrant

Scale Disambiguation

If **the marker size** is known

→ s (**s*t** gives physically correct distance)

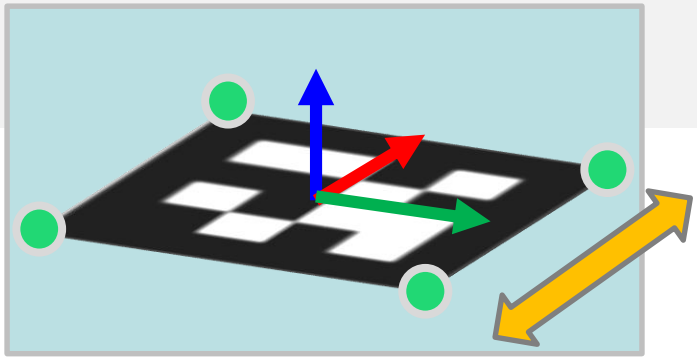
$$\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \approx K \begin{bmatrix} R_{00} & R_{01} & st_0 \\ R_{10} & R_{11} & st_1 \\ R_{20} & R_{21} & st_2 \end{bmatrix} \begin{bmatrix} sX \\ sY \\ 1 \end{bmatrix}$$



Our Pose Estimation Function

```
/**
* computes the orientation and translation of a square
* @param result result as 4x4 matrix
* @param p2D coordinates of the four corners in clock-wise order.
* the origin is assumed to be at the camera's center of projection
* @param markerSize side-length of marker. Origin is at marker center.
*/
void estimateSquarePose
                ( float* result, const CvPoint2D32f* p2D,float markerSize );

void estimateSquarePose
                ( float* result, const cv::Point2f* p2D, float markerSize );
```

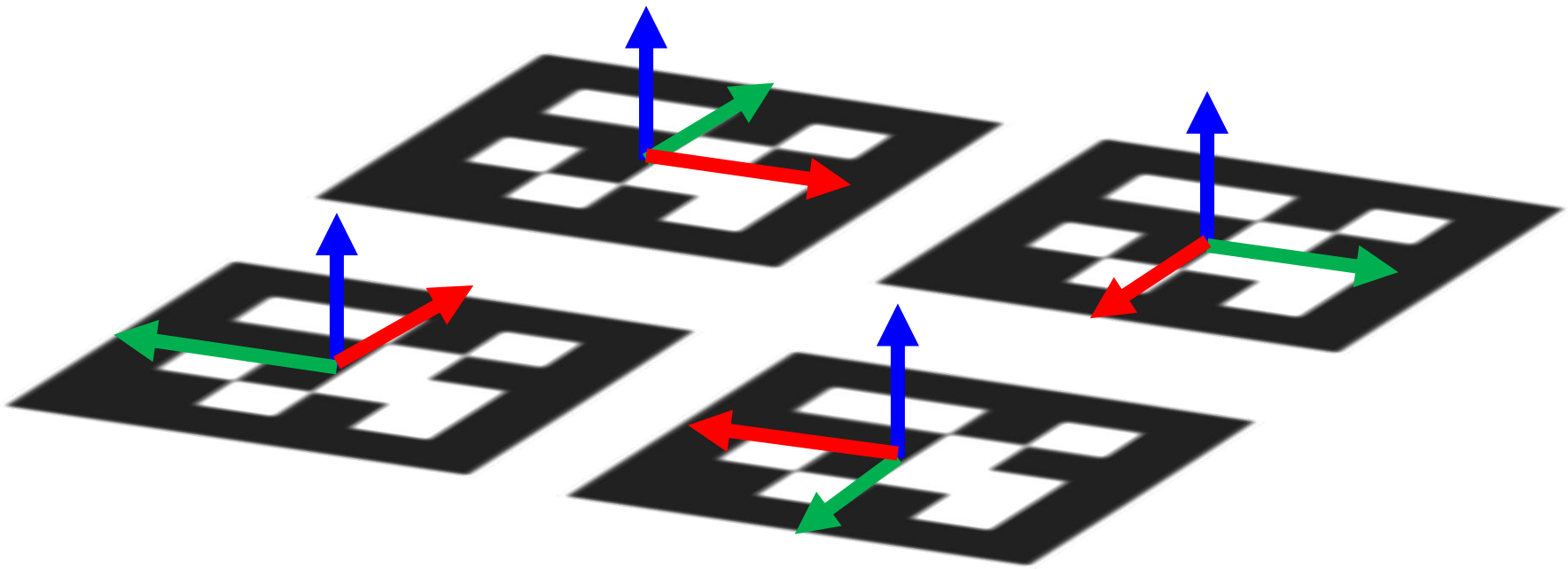


Pose Estimation

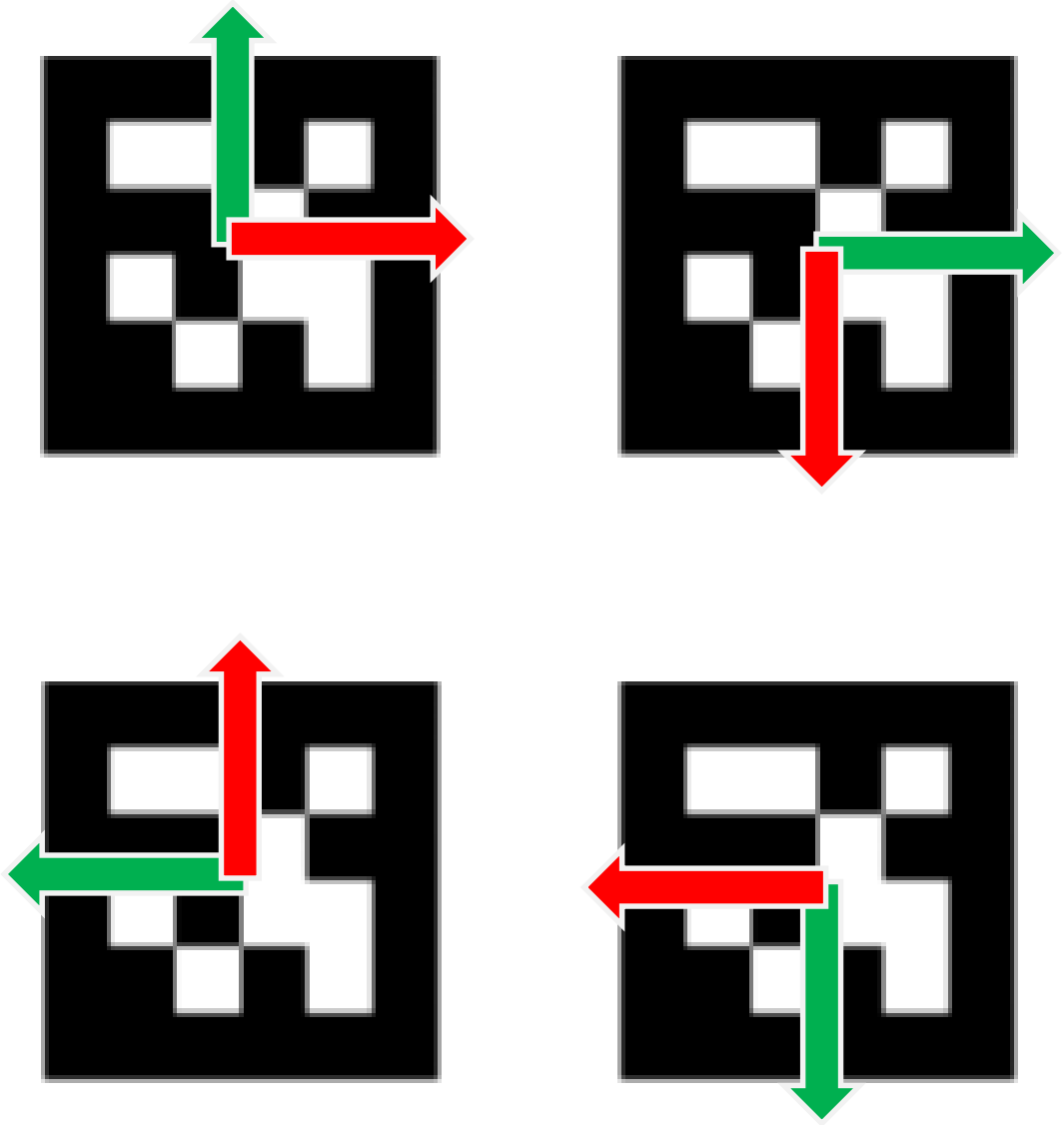
```
float* result = [  0,  1,  2,  3,
                   4,  5,  6,  7,
                   8,  9, 10, 11,
                  12, 13, 14, 15];
```

$$\begin{bmatrix} R_{00} & R_{01} & R_{02} & t_0 \\ R_{10} & R_{11} & R_{12} & t_1 \\ R_{20} & R_{21} & R_{22} & t_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

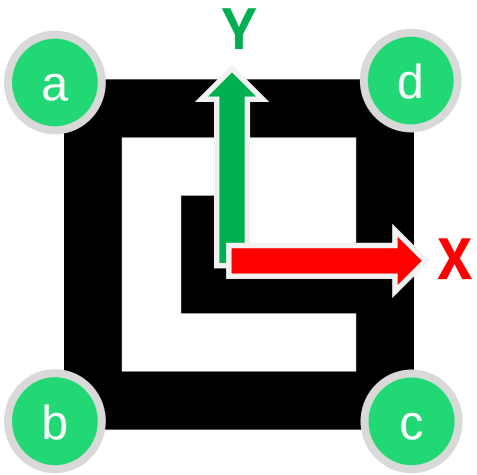
Ambiguity of the rotation around Z-axis



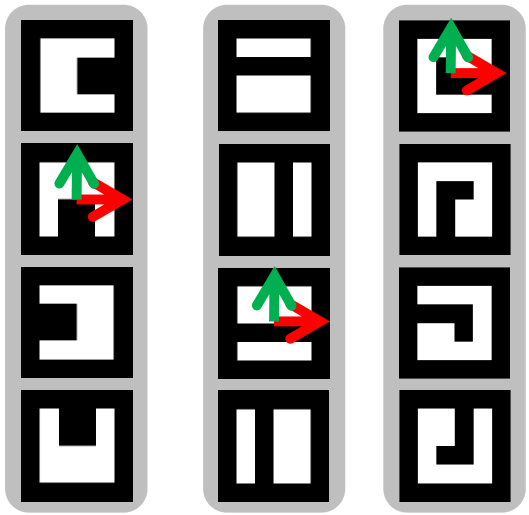
Ambiguity of the rotation around Z-axis



Preparation for Pose Estimation



Define a consistent
rotation around **Z-axis**



a-b-c-d

d-a-b-c

c-d-a-b

b-c-d-a

Adujst the order of

```
const cv::Point2f* p2D
```

`estimateSquarePose()`

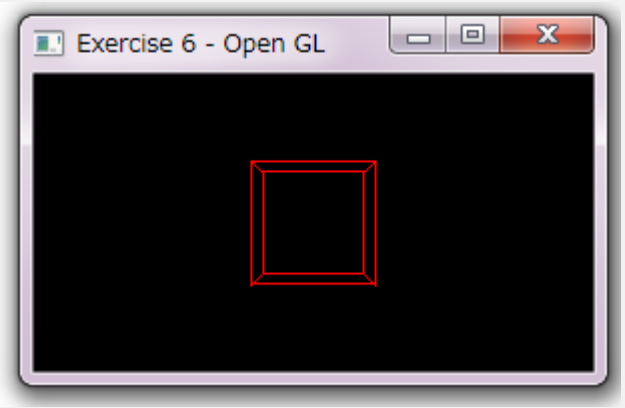
Homework

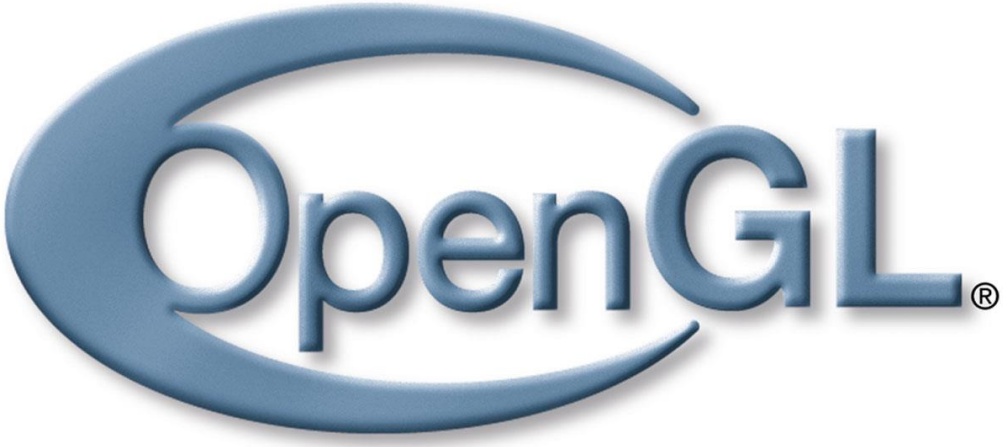
- Pose estimation: see PDF sheet on Moodle

Spoiler of the next tutorial

OpenGL basics

-with GLFW





That‘s it…

- Questions