Module IN 2018

Introduction to Augmented Reality

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Sensor Fusion and Registration

SS 2018

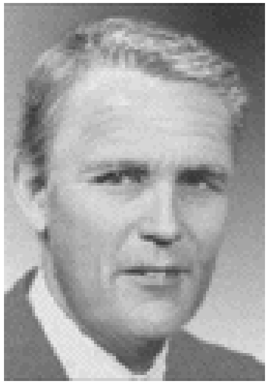
Agenda

1. The Kalman Filter
2. Sensor Fusion
3. Calibration and Registration

1. Kalman Filter

Literature

- G. Welch and G. Bishop, “An Introduction to the Kalman Filter”, SIGGRPAPH 2001 Course 8, http://www.cs.unc.edu/~welch/kalman
- A. Gelb (editor), “Applied Optimal Estimation”
- R.E. Kalman, “A new Approach to Linear Filtering and Prediction Problems”, Transactions ASME, 1960



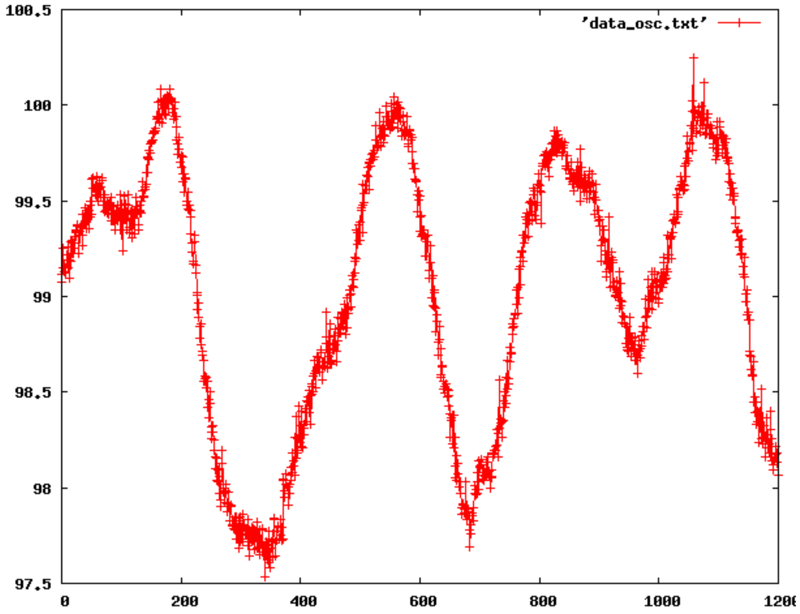
Rudolf Kálmán
https://de.wikipedia.org

1. The Kalman Filter

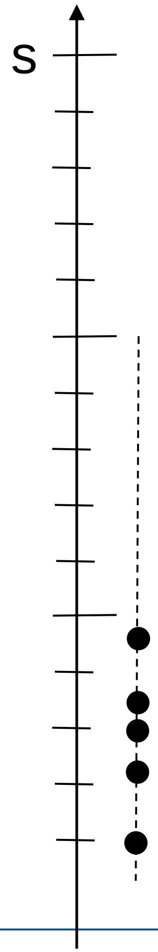
1.1 Motivation

Sensor measurements always subject to noise

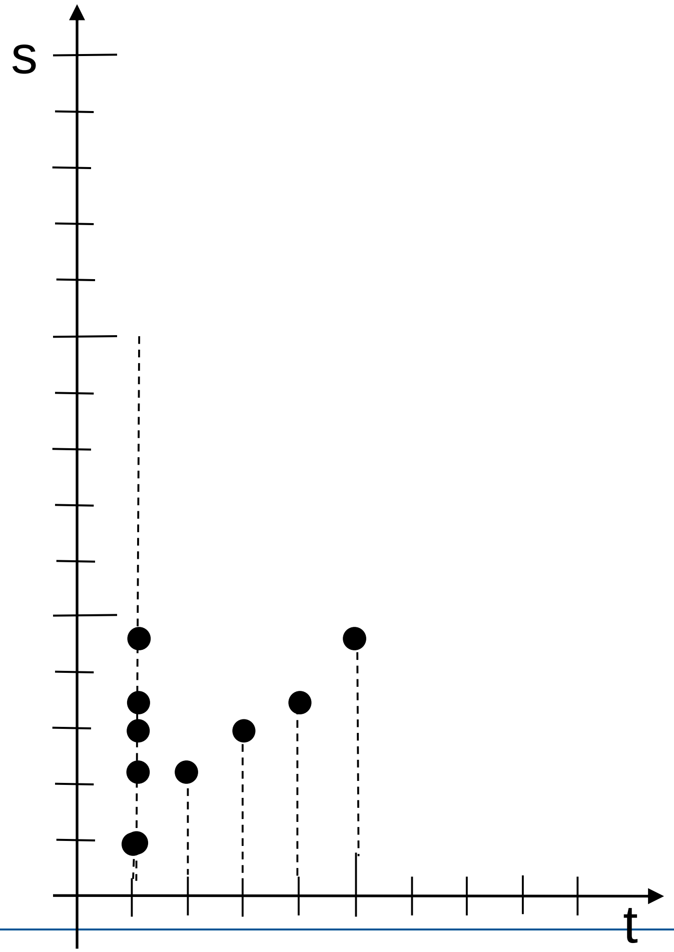
➡ Filtering



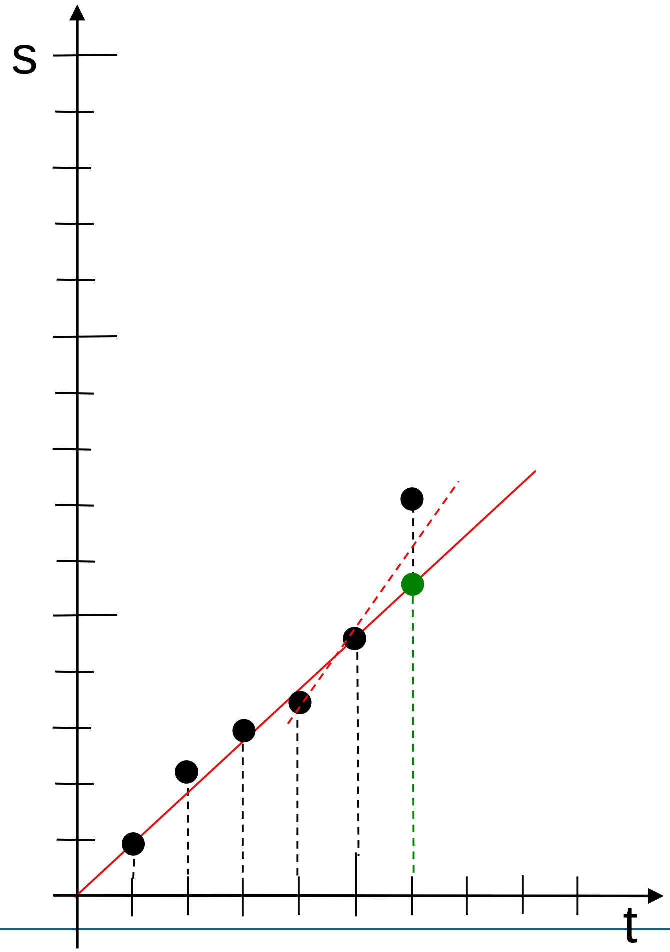
1.1 Motivation



1.1 Motivation



1.1 Motivation



- Sensor measurements
  - complex motion or noisy data?
- Motion model
  - $s(t) = \int v(t)\, dt$
  - e.g., constant speed: $s(t) = v \bullet t$
- Motion prediction
  - $s(t + \Delta t) = s(t) + v \bullet \Delta t$
- New measurement
  - update model "to some extent"

1.1 Motivation

Example

- Multiple sensors
  - Different ideas about time
  - Disagree on measured value

➡ Sensor-Fusion

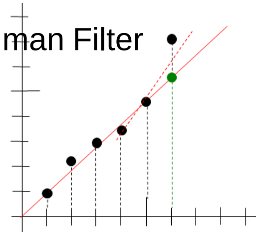
1. The Kalman Filter

1.2 Dynamic Process Model

- Model system as dynamic process
  - Estimation parameters
  - Noise parameters
- Model is application specific

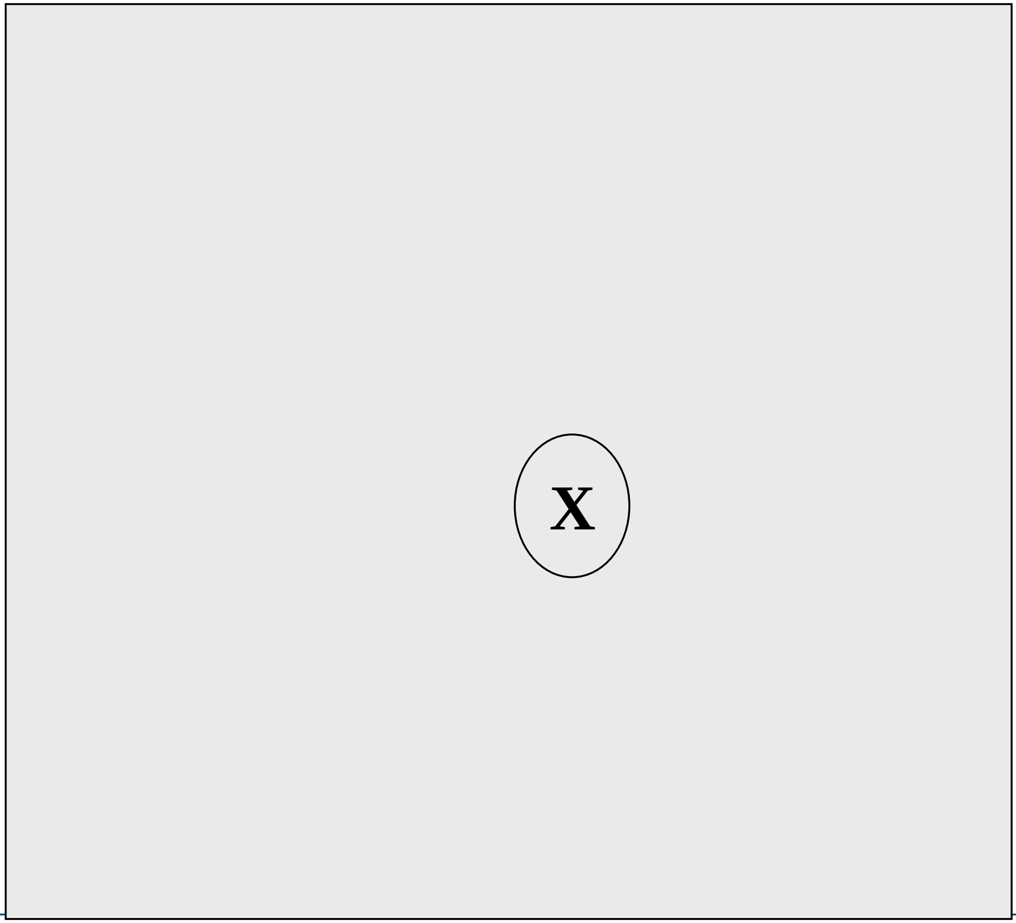
- Kalman Filter is a set of techniques
- Optimal estimator for linear, time-discrete, dynamic systems

1. Kalman Filter

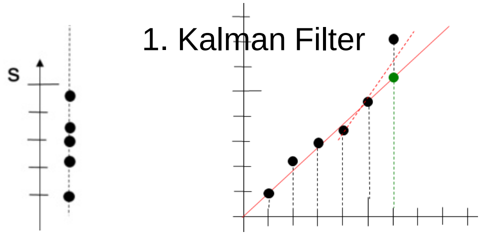


1.2 Dynamic Process Model

- State variable of process: $\mathbf{X}$
  (e.g: a motion model)
- Goal: Predict / approximate state
- Concrete characteristics dependent on application

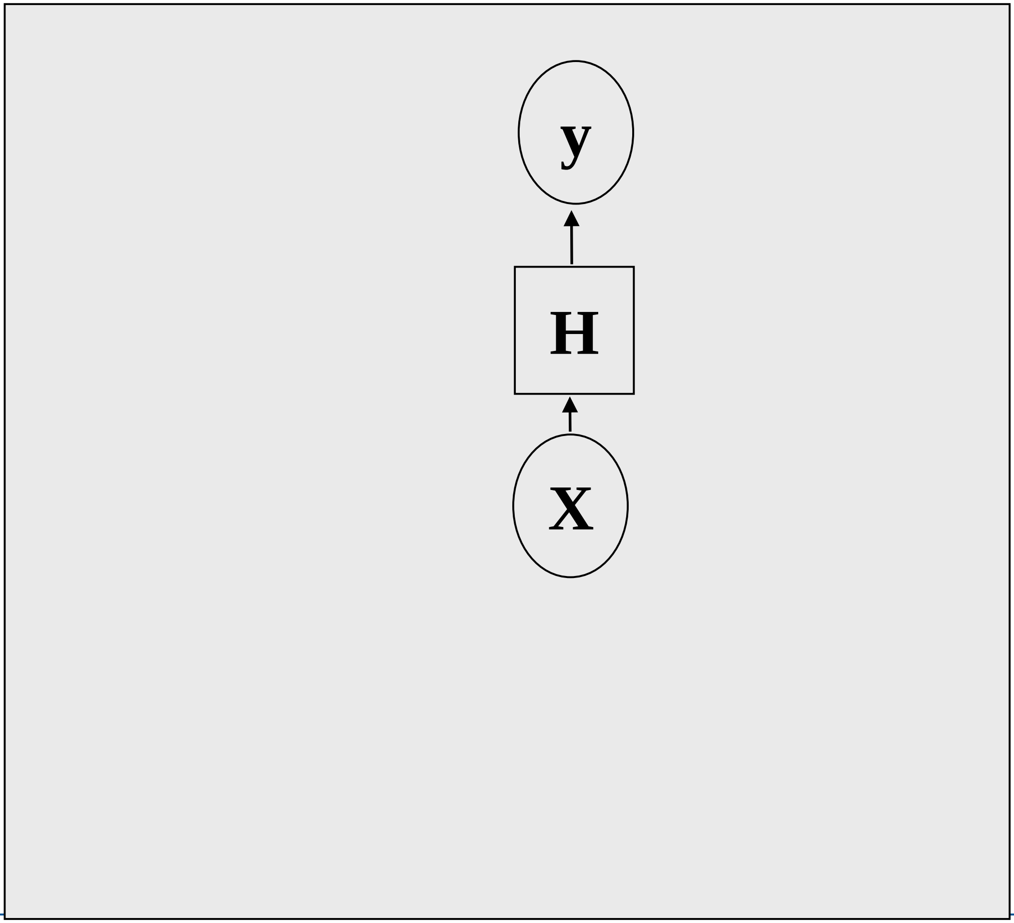


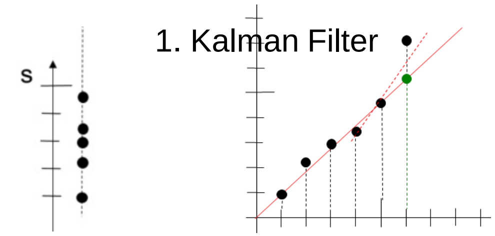
1.2 Dynamic Process Model



- What do we have to work with?
- Observations **y**

- Model: Linear projection **H** of state **X** to observable **y**

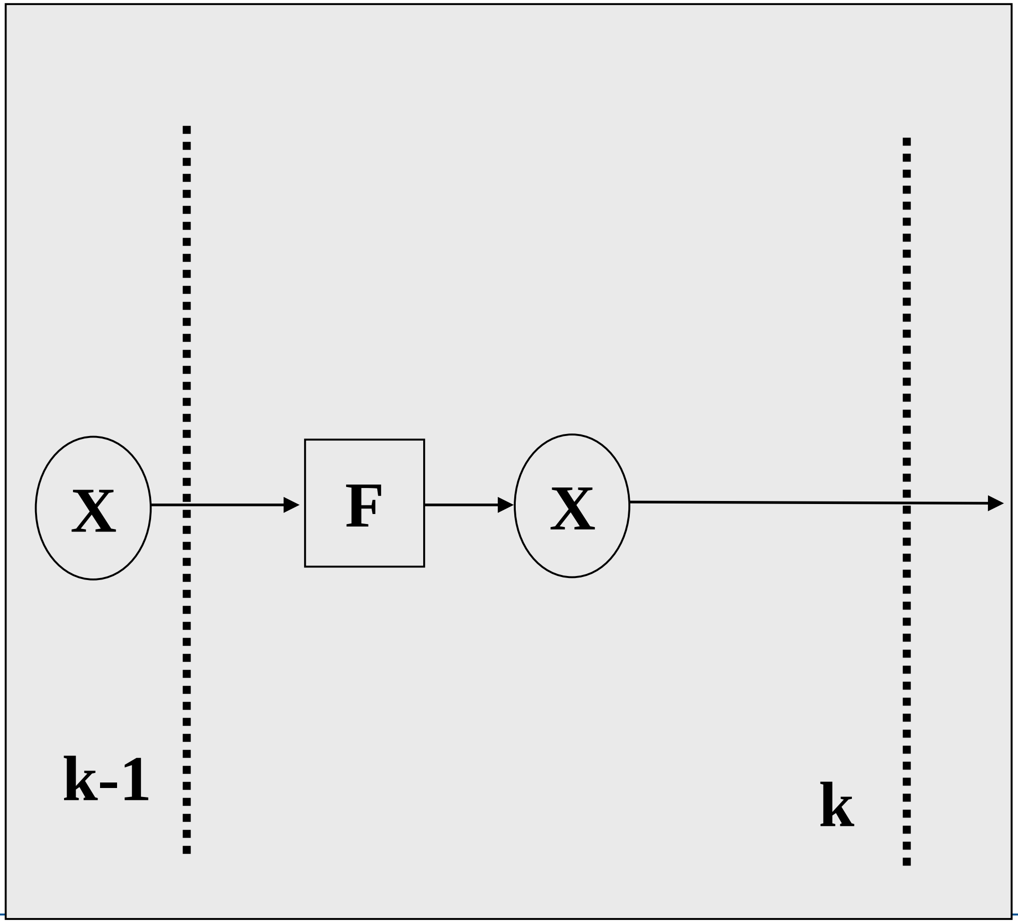




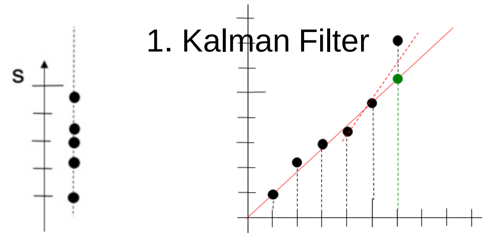
1.2 Dynamic Process Model

- State evolves on its own
  - Dynamic characteristics
  - User interaction

- Transition matrix **F**: Updates state **X** from time step k-1 to k

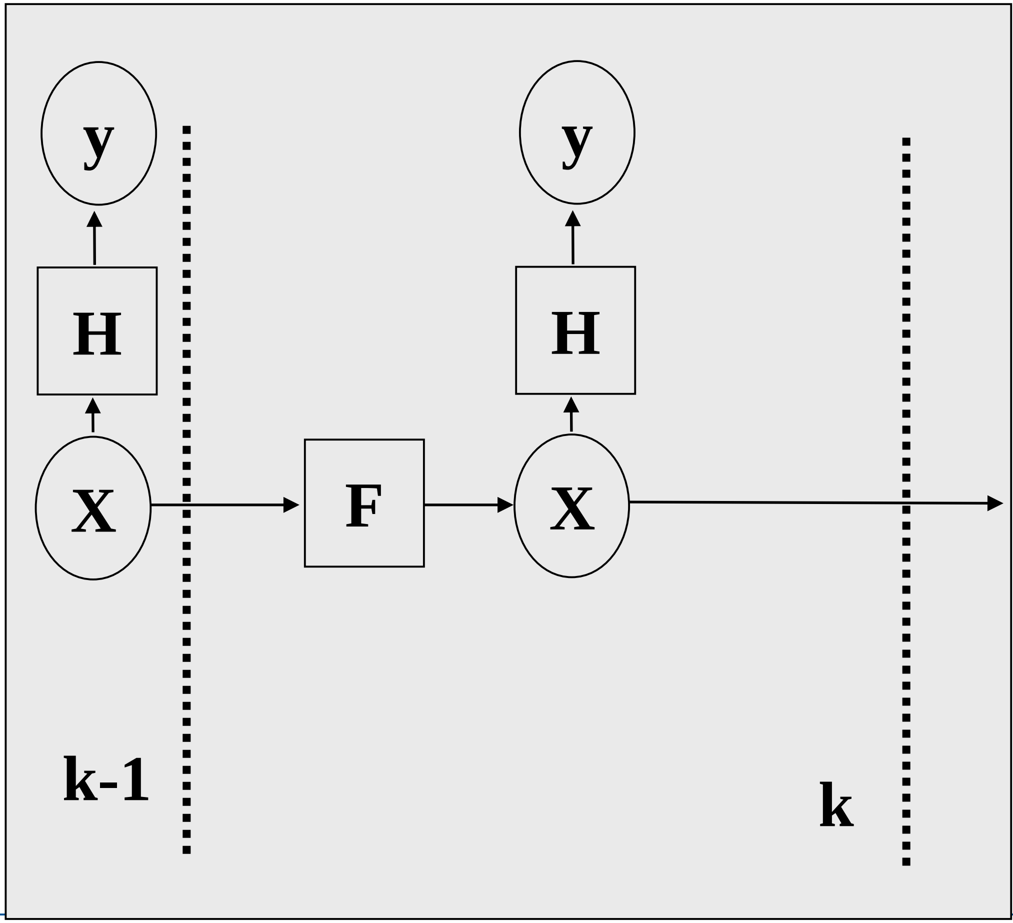


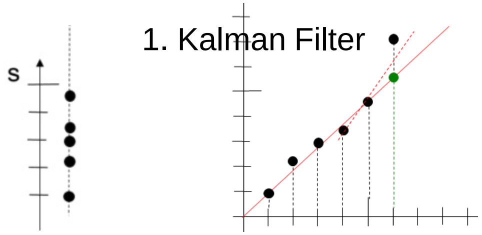
1.2 Dynamic Process Model



Summary thus far: Combination of

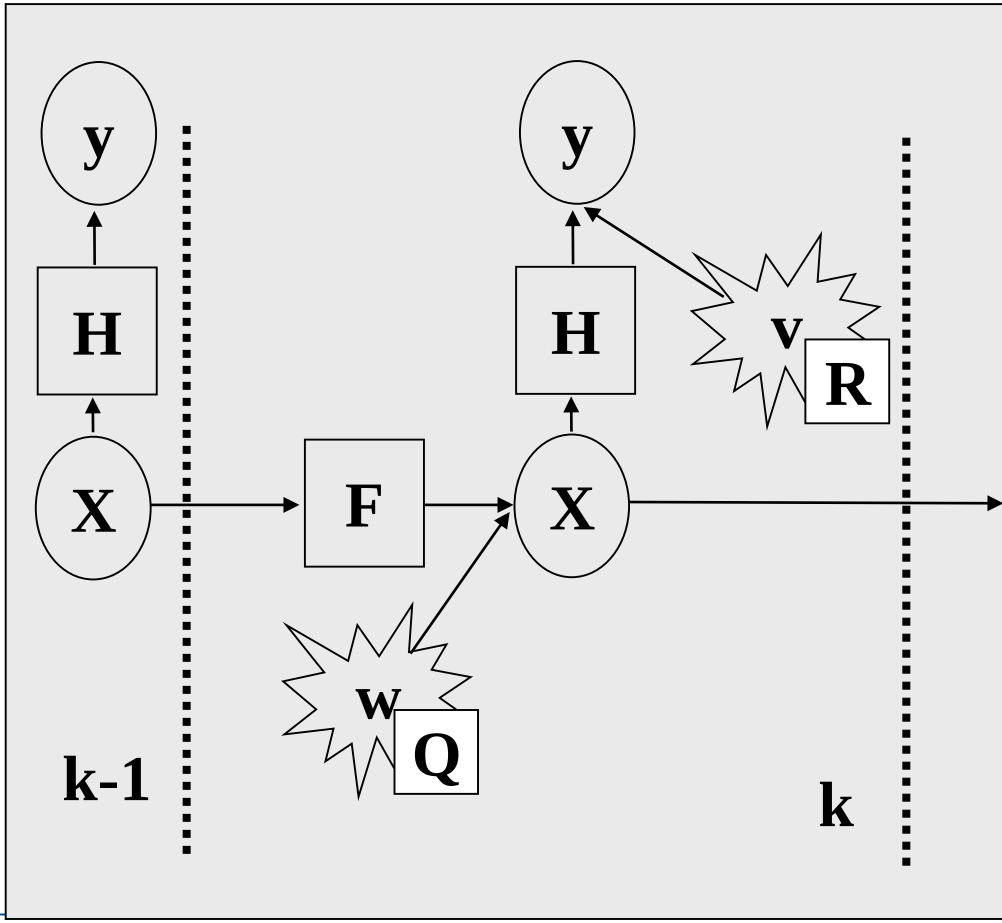
- State X_{k-1} at time step k-1
- Transition matrix F to transform X_{k-1} into X_k at time step k
- Projection matrix H to project internal system state X into observable "symptoms" y (at every time step …, k-1, k, …)





1.2 Dynamic Process Model

- Include two sources of noise
- Sensors are noisy
  - => Measurement noise $\mathbf{v}_k$ (described by covariance matrix R)
- Process is noisy
  - Indeterministic behavior
  - Unmodeled dynamic properties
  - Unmodeled external $\mathbf{w}_k$ influences
  - => Process Noise (described by covariance matrix Q)



1. The Kalman Filter

1.1 Motivation

1.2 Dynamic Process Model

→ 1.3 Mathematical Formulation

1.4 Outlook

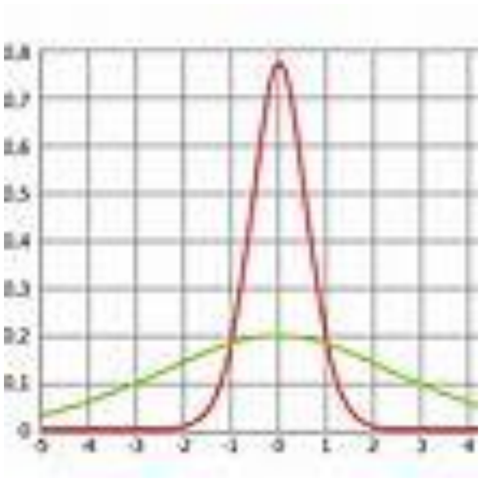
1.3 Mathematical Formulation

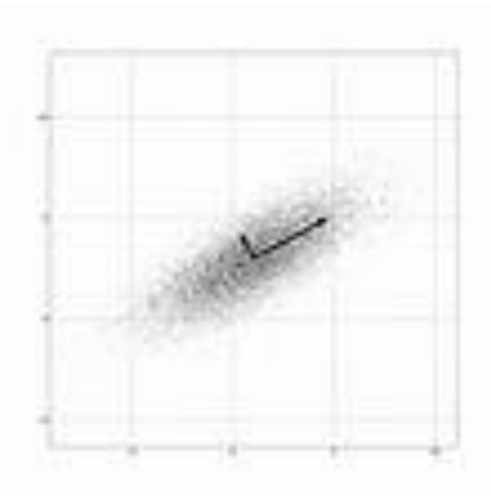
- Basic Approach: Recursive Filter
  - Predict next state from last state (Predict Step)
  - Update state estimate from measurement (Update Step)
- Optimality: Minimizes estimation error

1.3 Mathematical Formulation

- Reminder: Covariance matrix

$$Cov(X,Y) = E[(X - E[X])(Y - E[Y])] \qquad Var(X) = Cov(X,X)$$





[wikipedia.de]

1.3 Mathematical Formulation

- Reminder: Covariance matrix

$$Cov(X,Y) = E[(X - E[X])(Y - E[Y])] \qquad Var(X) = Cov(X,X)$$

- Generalization of variance to multivariate statistics

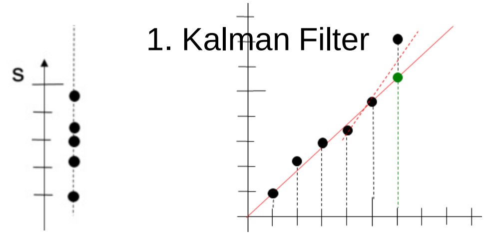
$$\underline{X} = \begin{bmatrix} X_1 \\ \vdots \\ X_n \end{bmatrix} \qquad \Sigma = \begin{bmatrix} Cov(X_1,X_1) & Cov(X_1,X_2) & \cdots & Cov(X_1,X_n) \\ Cov(X_2,X_1) & Cov(X_2,X_2) & \cdots & Cov(X_2,X_n) \\ \vdots & \vdots & \ddots & \vdots \\ Cov(X_n,X_1) & Cov(X_n,X_2) & \cdots & Cov(X_n,X_n) \end{bmatrix}$$

$$\Sigma = E[(\underline{X} - E[\underline{X}])(\underline{X} - E[\underline{X}])^T]$$

1.3 Mathematical Formulation

- Parameterizes multivariate distributions
- Modelled noise
  - Process Noise $\mathbf{w}_k \sim \mathrm{N}[0, \mathbf{Q}_k]$
  - Measurement Noise $\mathbf{v}_k \sim \mathrm{N}[0, \mathbf{R}_k]$
- Independent, white, Gaussian noise
- Q: modeling uncertainty
  - Larger Q => track large changes in data more closely
- R: how much to trust measurements
  - Large R => considers measurements as not very accurate
  - Smaller R => follow measurements more closely

1.3 Mathematical Formulation



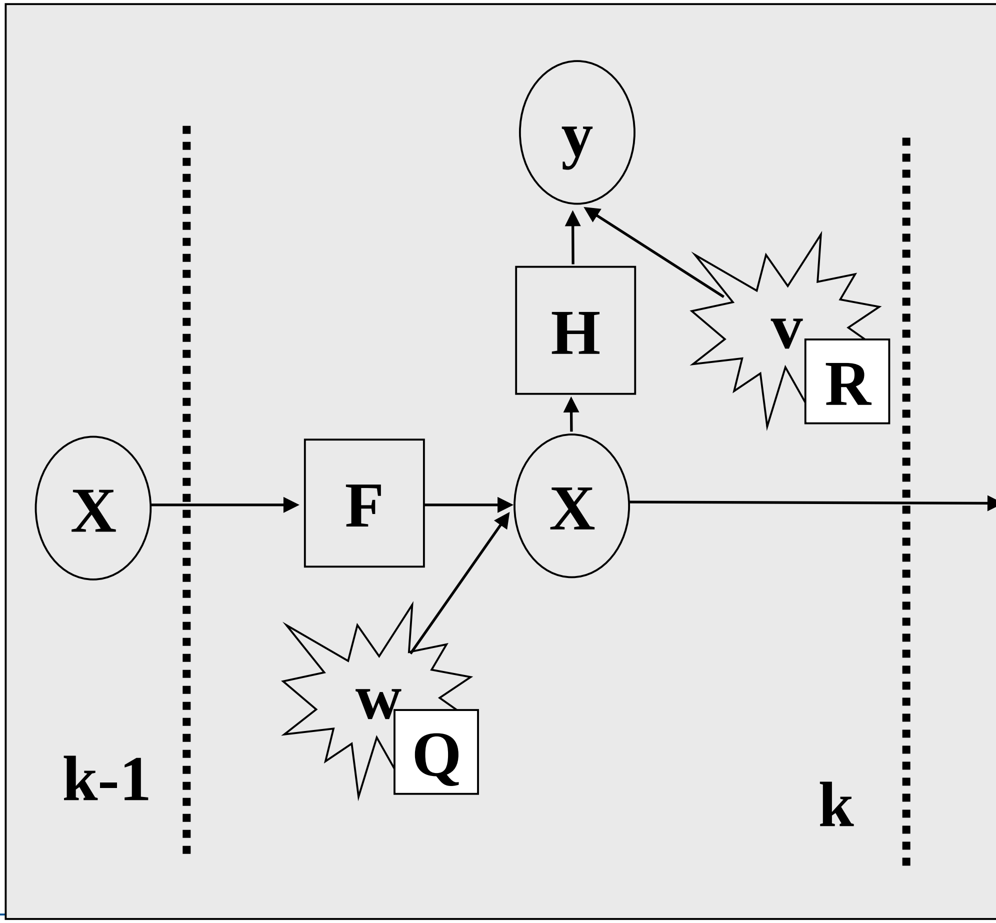
- Formulate model equations
  - Process equation

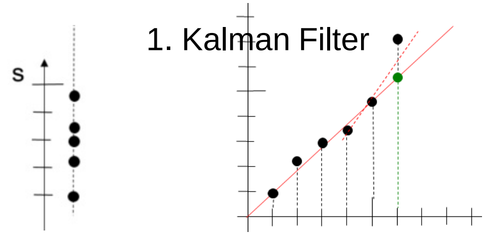
    $$\mathbf{x}_k = \mathbf{F}_{k,k-1}\mathbf{x}_{k-1} + \mathbf{w}_{k-1}$$

  - Measurement equation

    $$\mathbf{y}_k = \mathbf{H}_k \mathbf{x}_k + \mathbf{v}_k$$

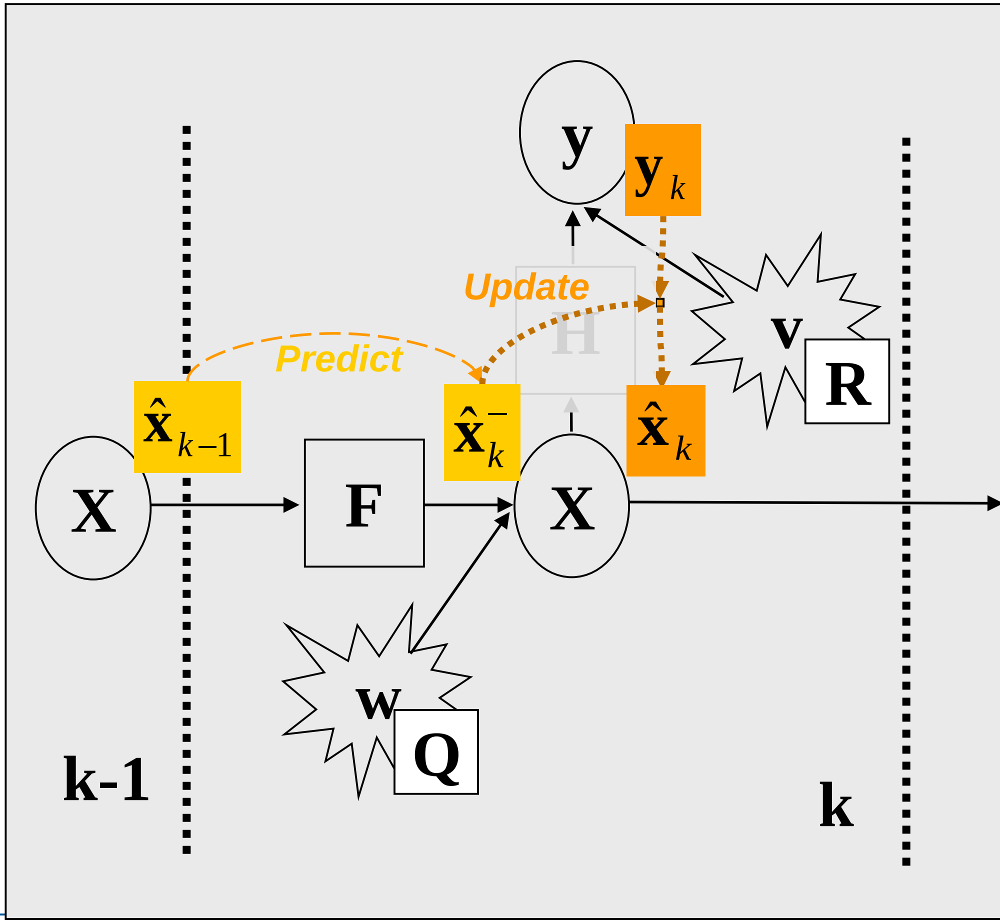
- With
  - State vector $\mathbf{x}_k$
  - Transition matrix $\mathbf{F}_{k,k-1}$
  - Observable $\mathbf{y}_k$
  - Measurement matrix $\mathbf{H}_k$
  - Noise $\mathbf{w}_k, \mathbf{v}_k$



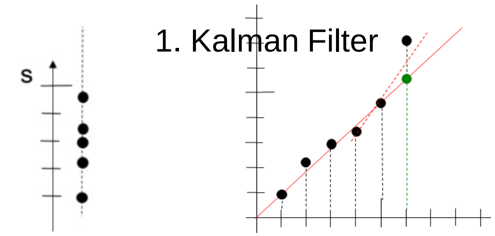


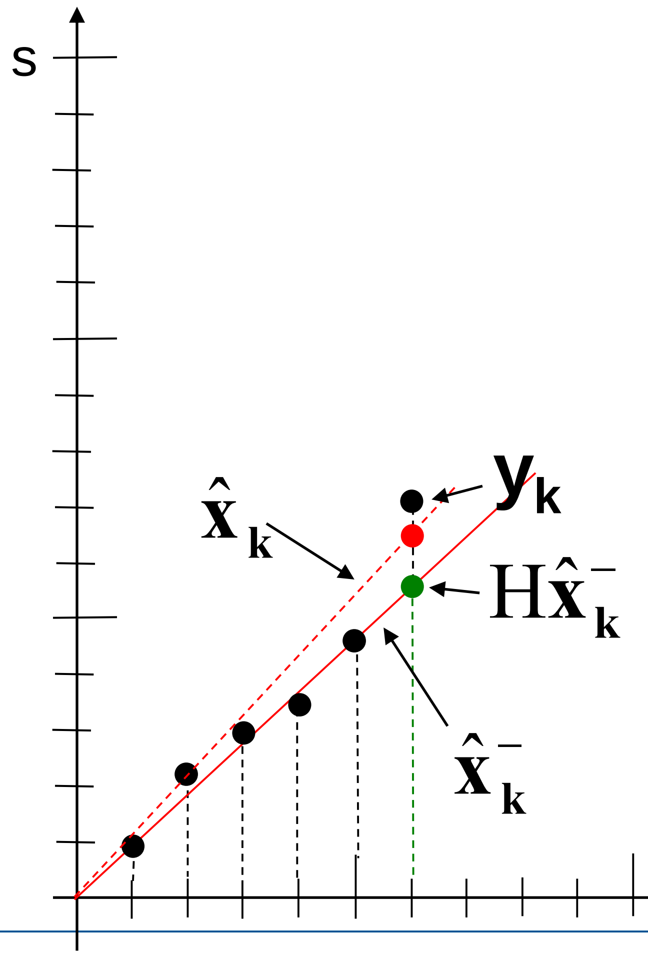
1.3 Mathematical Formulation

- Kalman Filter approximates true state $\mathbf{x}_k$ with estimate $\hat{\mathbf{x}}_k$
- At time step k:
  - A priori estimate using information of step k-1: $\hat{\mathbf{x}}_k^-$ (Prediction)
  - Measurement
  - => Improve a posteriori estimate (Update)
    $$\hat{\mathbf{x}}_k = \hat{\mathbf{x}}_k^- + \mathbf{G}_k(\mathbf{y}_k - \mathbf{H}\hat{\mathbf{x}}_k^-)$$
- Choose $\mathbf{G}_k$ such that error $\mathrm{E}[|\hat{\mathbf{x}}_k - \mathbf{x}_k|^2]$ is minimized
- $\mathbf{G}_k$ is called the Kalman-Gain



1.3 Mathematical Formulation





The a posteriori estimate $\mathbf{\hat{x}}_\mathbf{k}$ is a linear combination of the difference between measurement $\mathbf{y}_\mathbf{k}$ and measurement prediction $\mathrm{H}\mathbf{\hat{x}}_\mathbf{k}^-$ and the a priori state $\mathbf{\hat{x}}_\mathbf{k}^-$

$$\boxed{\mathbf{\hat{x}}_\mathbf{k} = \mathbf{\hat{x}}_\mathbf{k}^- + \mathrm{G}(\mathbf{y}_\mathbf{k} - \mathrm{H}\mathbf{\hat{x}}_\mathbf{k}^-)}$$

Kalman gain, blending factor

residual, innovation

1. The Kalman Filter

1.4 Outlook

- Non linear model with

$$\mathbf{x}_k = f(k, \mathbf{x}_{k-1}) + \mathbf{w}_{k-1} \qquad \mathbf{y}_k = h(k, \mathbf{x}_k) + \mathbf{v}_k$$

- Extended Kalman Filter (EKF)
  - Use functions where possible, linearize estimation around current estimate
  - Use Jacobians as transition/measurement matrices

$$\mathbf{F}_{k+1,k} = \frac{\partial f(k, \mathbf{x})}{\partial \mathbf{x}}\bigg|_{\mathbf{x}=\hat{\mathbf{x}}_k} \qquad \mathbf{H}_k = \frac{\partial h(k, \mathbf{x})}{\partial \mathbf{x}}\bigg|_{\mathbf{x}=\hat{\mathbf{x}}_k^-}$$

  - Notes:
    - F, H may change every step
    - Fundamental flaw: distributions of random variable no longer normal
- Unscented Kalman Filter
  - Uses deterministic sampling
  - Does not require calculation of Jacobians

Agenda

1. The Kalman Filter
2. Sensor Fusion
3. Calibration and Registration

2. Sensor Fusion

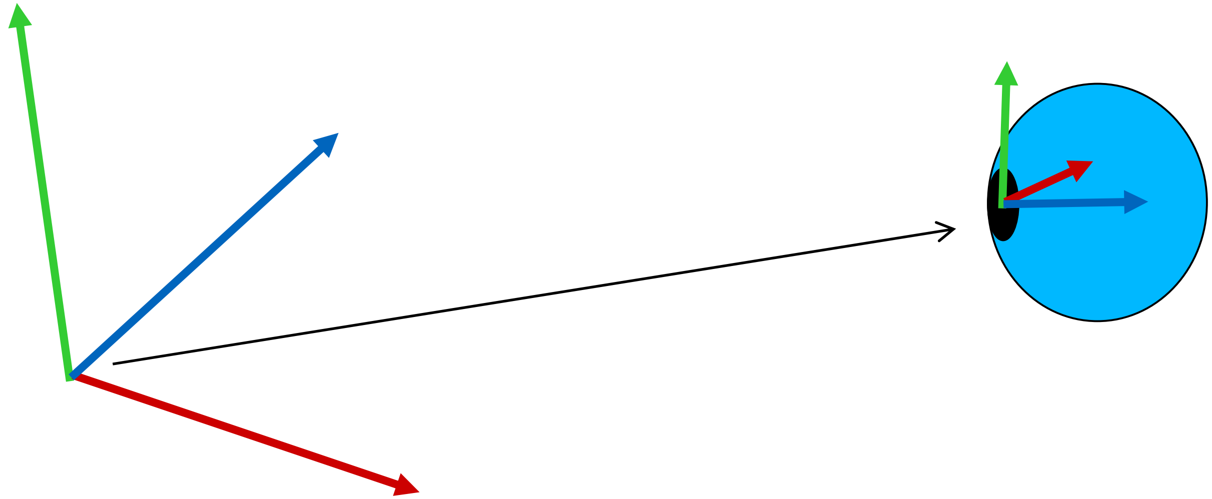
Literature

- Ronald Azuma and Gary Bishop. *Improving static and dynamic registration in an optical see-through HMD*. SIGGRAPH 1994, pages 197-204
- Greg Welch and Gary Bishop. *SCAAT: Incremental Tracking with Incomplete Information*. SIGGRAPH 1997, pages 333-344

2. Sensor Fusion

2.1 6DOF Tracking: Problem Definition

- Objective: 6DOF tracking of an object (e.g. HMD)
- Object pose expressed as transformation in 3-space
- Tracker measurements are noisy!



2. Sensor Fusion

2.2 State Vector

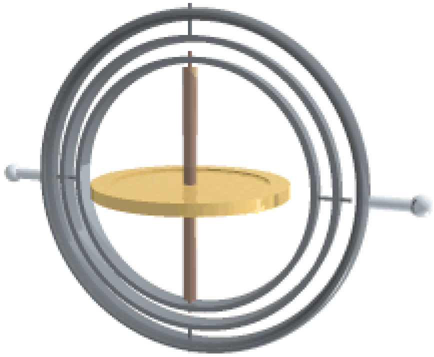
General Issue

- What do we need to know about the system?
  - Which parameters to estimate?
- Position
- Orientation
- Dynamic system
  - Translation velocity (positional changes, 1rst derivative)
  - Rotation velocity
- More derivatives possible
  - Accelerations (velocity changes, 2nd derivative) w.r.t. position and/or orientation

2.2 State Vector

Representation of Rotations

- So far: Pose represented as 4x4-Matrix
- 16 elements, but only 6 DOF → impracticable in state vector
- Separate translation (3 elements) and rotation
- Choices for rotation
  - Euler angles
    - Problems: gimbal lock, non-trivial multiplication
  - 3-element Axis-Angle
    - Vector direction: axis
    - Vector magnitude: angle
    - Problem: non-trivial multiplication
  - Quaternions



2.2 State Vector

Quaternions

- 4-element representation of rotation
- Generalization of complex numbers with imaginary $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$
- Only quaternions with $|\mathbf{q}|=1$ are pure rotations
  - All rotations lie on a 4D hyper-sphere
  - May require frequent normalization
- Inverse rotation: $\mathbf{q}^*$ (conjugate)
- Simple multiplication $\mathbf{q}_1 \cdot \mathbf{q}_2$
  - Consists only of sums and products
- Rotation of vectors: $\mathbf{x}' = \mathbf{q} \cdot \mathbf{x} \cdot \mathbf{q}^*$
  - Extend vector with 0 as real to construct a quaternion

2.2 State Vector

Quaternions and Axis-Angle

- Given
  - Rotation axis $\mathbf{x}$
  - $|\mathbf{x}| = 1$
  - Rotation angle θ
- The corresponding quaternion is

$$\mathbf{q} = (s\, x_1,\, s\, x_2,\, s\, x_3,\, c)$$

- where

$$s = \sin(\theta / 2)$$
$$c = \cos(\theta / 2)$$

2.2 State Vector

Complete State Vector

- Position
  - 3-Vector $\mathbf{p}$
- Translational velocity
  - 3-Vector $\mathbf{v}$
- Orientation
  - Quaternion $\mathbf{r}$
  - Sometimes requires normalization
- Rotation velocity
  - Axis-angle 3-vector $\mathbf{w}$
  - Allows velocities higher than 360° /s

$$\mathbf{x} = \begin{pmatrix} \mathbf{p} \\ \mathbf{v} \\ \mathbf{r} \\ \mathbf{w} \end{pmatrix}$$

$$\mathbf{x}_k = \mathbf{F}_{k,k-1}\mathbf{x}_{k-1} + \mathbf{w}_{k-1}$$

2. Sensor Fusion

2.3 Motion Model

Example: Simple Linear Model for Translation at Constant Speed

- Simple linear model

  $$\hat{\mathbf{p}}_k^- = \hat{\mathbf{p}}_{k-1} + \Delta t \hat{\mathbf{v}}_{k-1}$$

  $$\hat{\mathbf{v}}_k^- = \hat{\mathbf{v}}_{k-1} \qquad \text{(constant speed)}$$

- Jacobian
  - Linear model → same as state transition **F** matrix for KF
  - Only on $\mathbf{x}_p$=($\mathbf{p}$, $\mathbf{v}$)

$$\begin{bmatrix} 1 & 0 & 0 & \Delta t & 0 & 0 \\ 0 & 1 & 0 & 0 & \Delta t & 0 \\ 0 & 0 & 1 & 0 & 0 & \Delta t \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\mathbf{x}_k = \mathbf{F}_{k,k-1}\mathbf{x}_{k-1} + \mathbf{w}_{k-1}$$

$$\begin{bmatrix} p_{x_k} \\ p_{y_k} \\ p_{z_k} \\ v_{x_k} \\ v_{y_k} \\ v_{z_k} \end{bmatrix}_k = \begin{bmatrix} 1 & 0 & 0 & \Delta t & 0 & 0 \\ 0 & 1 & 0 & 0 & \Delta t & 0 \\ 0 & 0 & 1 & 0 & 0 & \Delta t \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \circ \begin{bmatrix} p_{x_{k-1}} \\ p_{y_{k-1}} \\ p_{z_{k-1}} \\ v_{x_{k-1}} \\ v_{y_{k-1}} \\ v_{z_{k-1}} \end{bmatrix}_{k-1} + \mathbf{w}_{k-1}$$

2.3 Motion Model

Example: Rotation at Constant Rotational Speed

- Time update function

$$\hat{\mathbf{r}}_k^- = \hat{\mathbf{r}}_{k-1} \cdot \mathbf{q}(\Delta t \hat{\mathbf{w}}_{k-1})$$

$$\hat{\mathbf{w}}_k^- = \hat{\mathbf{w}}_{k-1}$$

- $\mathbf{q}(\mathbf{x})$ converts axis-angle to quaternion
  - Non-linear → EKF required
- Non-trivial Jacobian
  - Compute using symbolic math software (e.g. Mathematica, Maple)

$$\begin{bmatrix} \mathbf{J}_{r1} & \mathbf{J}_{r2} \\ 0 & \mathbf{I} \end{bmatrix}$$

2.3 Motion Model

Combined Time Update Equations

$$\hat{\mathbf{x}}_k^- = \begin{pmatrix} \hat{\mathbf{p}}_k^- \\ \hat{\mathbf{v}}_k^- \\ \hat{\mathbf{r}}_k^- \\ \hat{\mathbf{w}}_k^- \end{pmatrix} = \mathbf{f}(\hat{\mathbf{x}}_{k-1}) = \begin{pmatrix} \hat{\mathbf{p}}_{k-1} + \Delta t \hat{\mathbf{v}}_{k-1} \\ \hat{\mathbf{v}}_{k-1} \\ \hat{\mathbf{r}}_k^- \cdot \mathbf{q}(\Delta t \hat{\mathbf{w}}_{k-1}) \\ \hat{\mathbf{w}}_{k-1} \end{pmatrix}$$

$$\mathbf{P}_k^- = \mathbf{A}_k \mathbf{P}_{k-1} \mathbf{A}_k^T + \mathbf{Q}_k$$

$$\mathbf{A}_k = \left.\frac{\delta \mathbf{f}(\hat{\mathbf{x}})}{\delta \hat{\mathbf{x}}}\right|_{\hat{\mathbf{x}}=\hat{\mathbf{x}}_{k-1}} = \begin{bmatrix} \mathbf{I} & \Delta t\,\mathbf{I} & 0 & 0 \\ 0 & \mathbf{I} & 0 & 0 \\ 0 & 0 & \mathbf{J}_{r1} & \mathbf{J}_{r2} \\ 0 & 0 & 0 & \mathbf{I} \end{bmatrix}$$

2. Sensor Fusion

2.4 Measurement Update

6DOF-Tracker Integration

- Absolute 6DOF tracker directly measures $\mathbf{p}_\mathrm{k}$ and $\mathbf{r}_\mathrm{k}$

$$\hat{\mathbf{y}}_k^- = \mathbf{h}(\hat{\mathbf{x}}_k^-) = \mathbf{h}\begin{pmatrix} \hat{\mathbf{p}}_k^- \\ \hat{\mathbf{v}}_k^- \\ \hat{\mathbf{r}}_k^- \\ \hat{\mathbf{w}}_k^- \end{pmatrix} = \begin{pmatrix} \hat{\mathbf{p}}_k^- \\ \hat{\mathbf{r}}_k^- \end{pmatrix}$$

$$\mathbf{H} = \begin{bmatrix} I & 0 & 0 & 0 \\ 0 & 0 & I & 0 \end{bmatrix}$$

- Note: $\mathbf{v}$, $\mathbf{w}$ are not measured explicitly!

2.4 Measurement Update

Gyroscope Integration

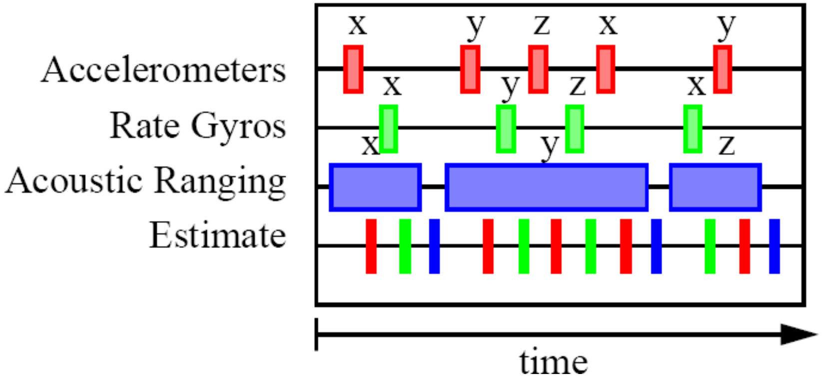
- Gyroscope directly measures $\mathbf{w}_\mathrm{k}$

$$\hat{\mathbf{y}}_k^- = \mathbf{h}(\hat{\mathbf{x}}_k^-) = \mathbf{h}\begin{pmatrix} \hat{\mathbf{p}}_k^- \\ \hat{\mathbf{v}}_k^- \\ \hat{\mathbf{r}}_k^- \\ \hat{\mathbf{w}}_k^- \end{pmatrix} = \left(\hat{\mathbf{w}}_k^-\right)$$

2.4 Measurement Update

The Actual Sensor Fusion Step

- Multiple different sensors
- For each measurement:
  1. Time update (to time of measurement)
  2. Measurement update (can be sensor-specific)
- Measurements should arrive in order!
- Requires timestamps!



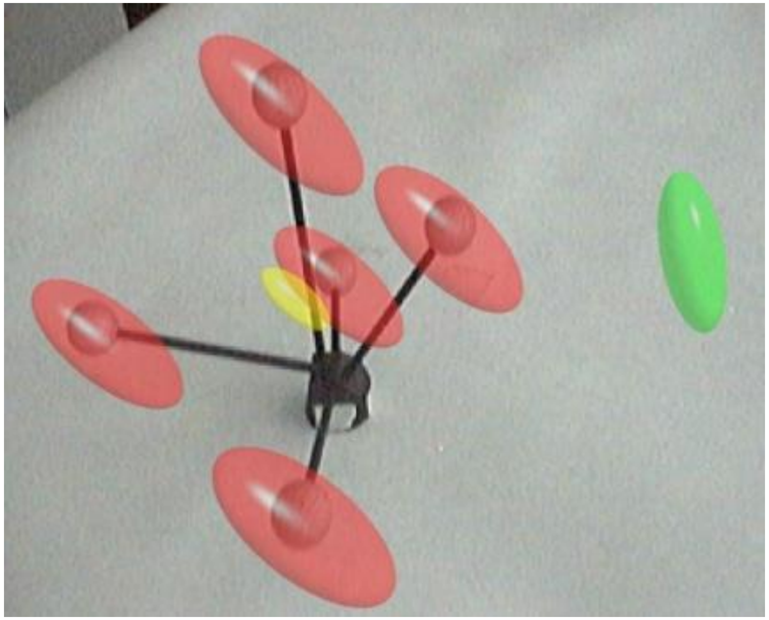
2.4 Measurement Update

Tracker Covariance

- How to set matrix $\mathbf{R}$ (measurement covariance)?
- Describes error distribution of tracker
- Trivial version:

$$\mathbf{R} = \sigma^2 \mathbf{I}$$

- Better: Compute $\mathbf{R}$ for each measurement based on the actual observations

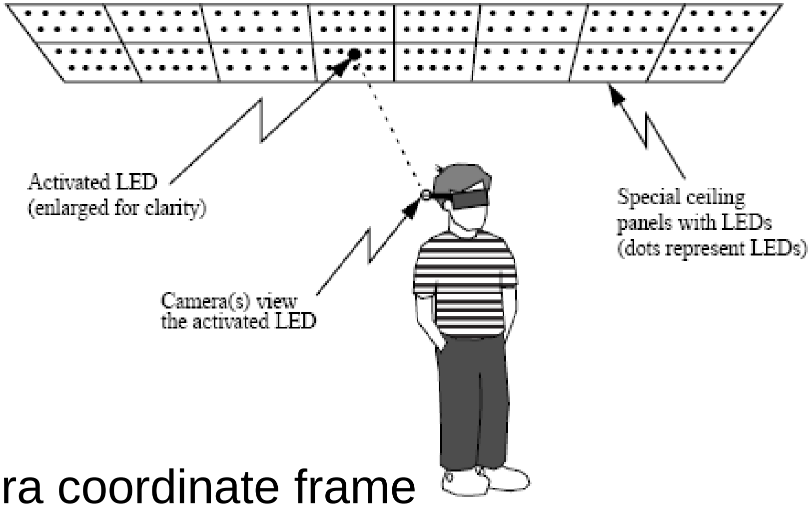


2. Sensor Fusion

2.5 Further Applications

2D Image Measurements

- EKF can directly integrate 2D image measurements



- Measurement equation

$$h(\hat{\mathbf{x}}_k^-) = \frac{\left(\mathbf{K}(\hat{\mathbf{r}}_k^- \mathbf{a} \hat{\mathbf{r}}_k^{-*} + \hat{\mathbf{t}}_k^-)\right)_{x,y}}{\left(\mathbf{K}(\hat{\mathbf{r}}_k^- \mathbf{a} \hat{\mathbf{r}}_k^{-*} + \hat{\mathbf{t}}_k^-)\right)_z}$$

1. Rotates the world point $\mathbf{a}$ into camera coordinate frame
2. Applies intrinsic camera matrix $\mathbf{K}$
3. Computes perspective division (de-homogenization)
   → Non-linear!

2.5 Further Applications

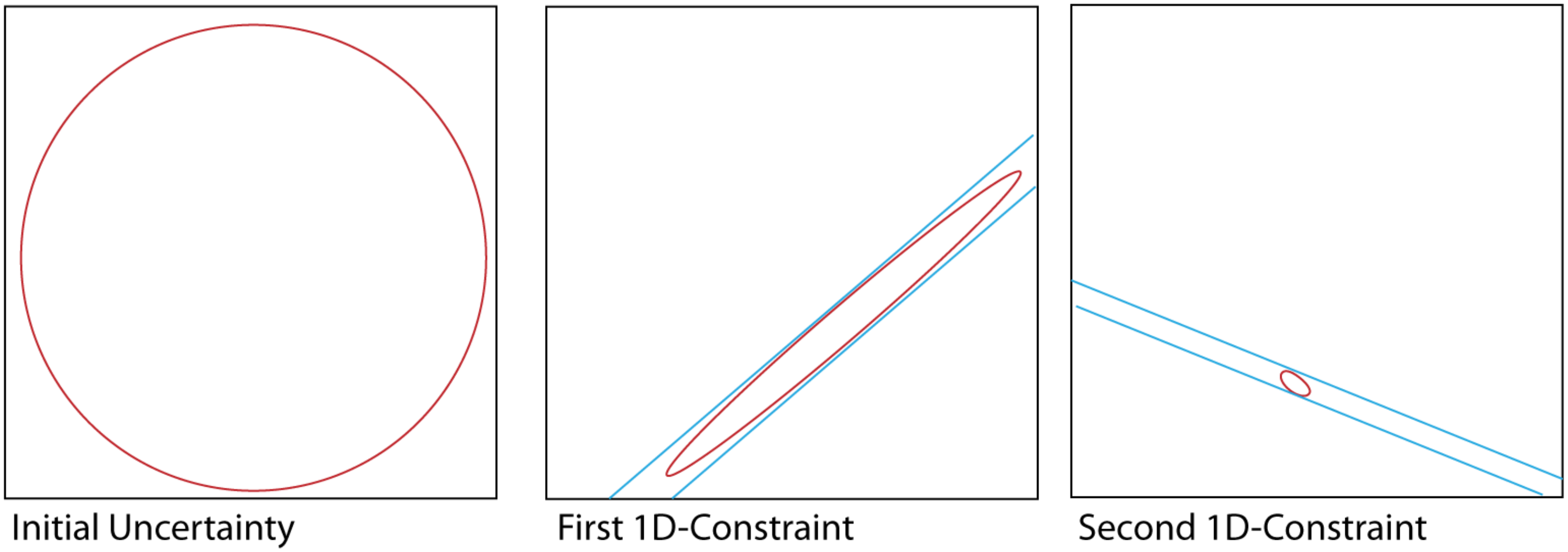
SCAAT

- State vector has 13 DOF
- (E)KF can compute all 13 elements given only one of (a large set of) 2D observations at a time
  - → Single Constraint At A Time (SCAAT)
- However: System must be observable
  - Needs different points

2.5 Further Applications

SCAAT – Why does it work?

- Example: Observation of a 2D point by two 1D cameras



2.5 Further Applications

Auto-Calibration / SLAM

- What if world point $\mathbf{a}$ is not known exactly?
- Idea: Integrate $\mathbf{a}$ into state vector

$$\mathbf{x} = (\mathbf{p}, \mathbf{v}, \mathbf{r}, \mathbf{w}, \mathbf{a}_1, \ldots, \mathbf{a}_n)^T$$

- Measurement equation $\mathbf{h}$ same as before

$$h(\hat{\mathbf{x}}_k^-) = \frac{\left(K(\hat{\mathbf{r}}_k^- \mathbf{a}_i \hat{\mathbf{r}}_k^{-*} + \hat{\mathbf{t}}_k^-)\right)_{x,y}}{\left(K(\hat{\mathbf{r}}_k^- \mathbf{a}_i \hat{\mathbf{r}}_k^{-*} + \hat{\mathbf{t}}_k^-)\right)_z}$$

- Jacobian H includes derivative of h w.r.t. a
- Kalman covariance matrix P stores uncertainty of each individual point and the dependencies between them!
- If new points are added while running
  - Simultaneous Localization and Mapping (SLAM)

2.5 Further Applications

Agenda

1. The Kalman Filter
2. Sensor Fusion
3. Calibration and Registration

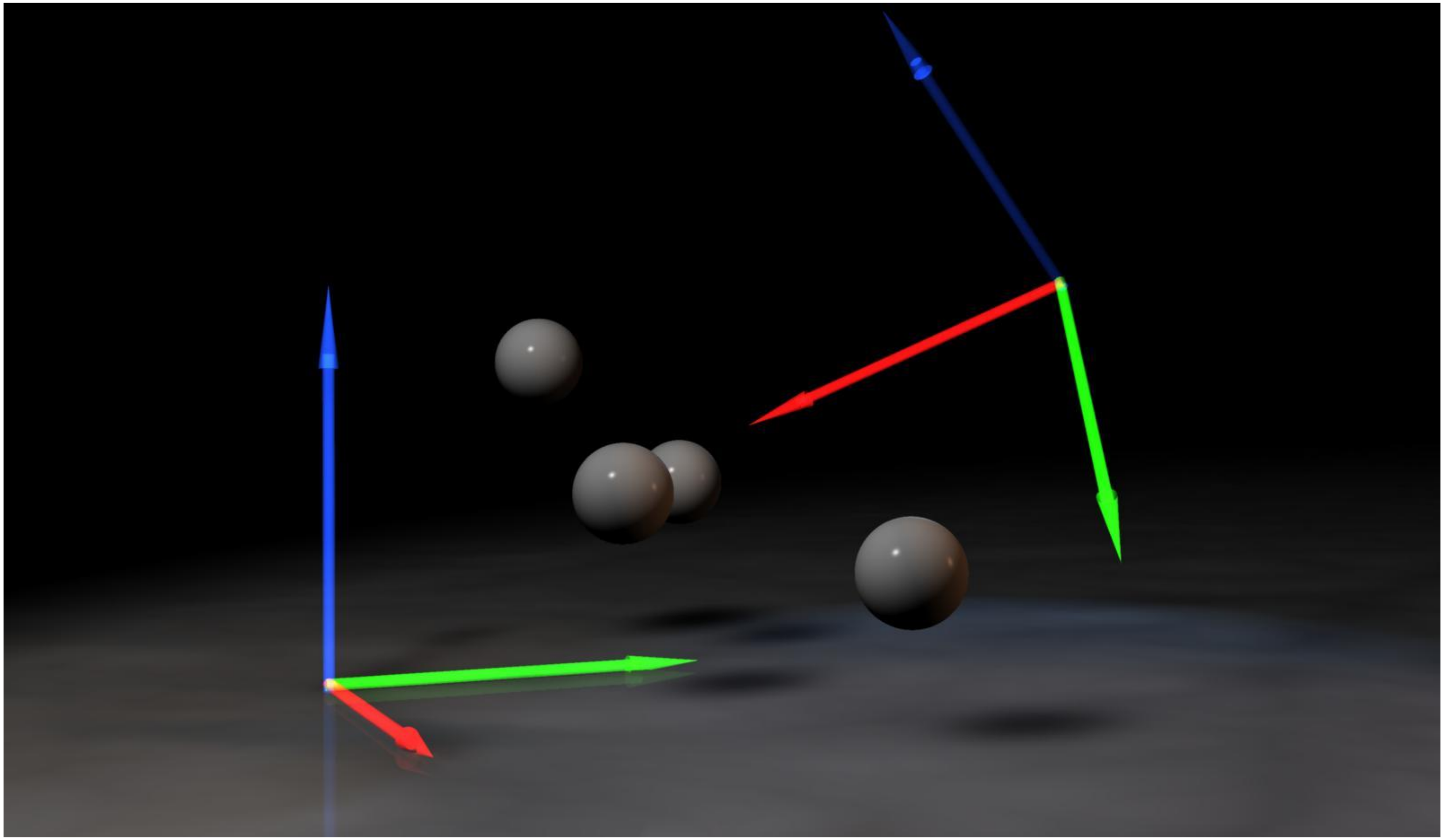
3. Calibration and Registration

→ 3.1 Absolute Orientation

3.2 Hand-Eye Calibration

3.1 Absolute Orientation

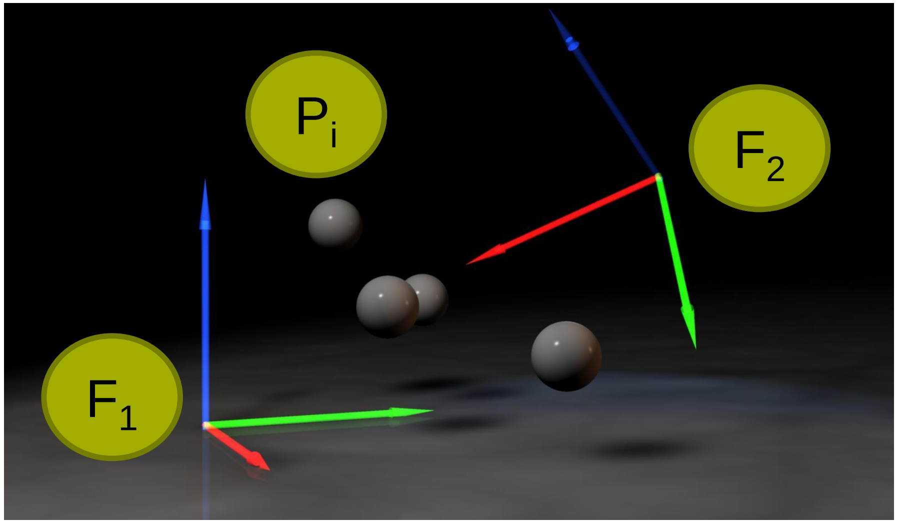
3.1 Absolute Orientation



3.1 Absolute Orientation

Setup and Motivation

- Setup
  - Two different coordinate frames, F_1 and F_2
  - Given pairs of coordinates of points in both frames P_i (i = 1..n)

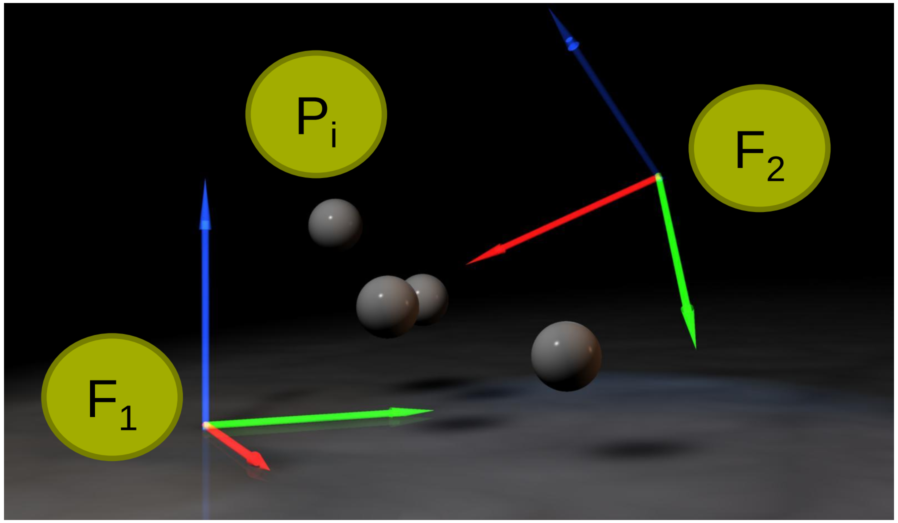


- Task
  - Determine the relationship between the two coordinate frames

3.1 Absolute Orientation

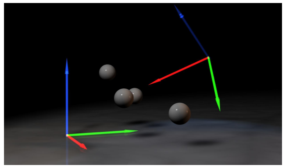
Setup and Motivation

- Possible Transformations
  - Rotation
  - Translation
  - Scaling (not expressible by pose)



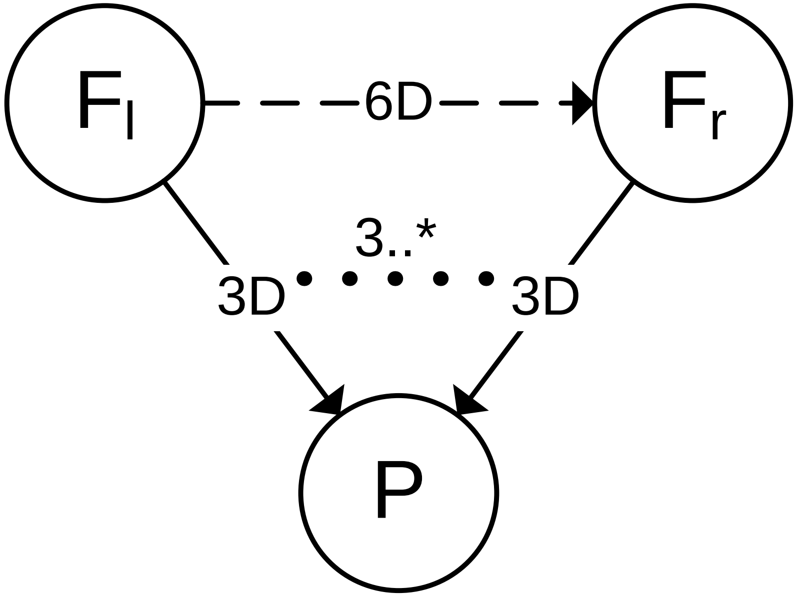
- Horn, B. K. P.
  Closed-form solution of absolute orientation using unit quaternions

3.1 Absolute Orientation



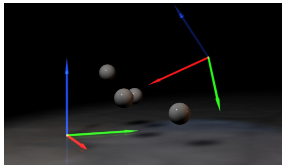
Setup and Motivation

- Spatial Relationship Graph
  - Tracked Relationships
    - $F_l \rightarrow P$
    - $F_r \rightarrow P$
  - Unknown Relationships
    - $F_l \rightarrow F_r$



3.1 Absolute Orientation

3.1 Absolute Orientation



Solution

- For any vector r_l in the left frame find parameters such that

  $$r_r = R(r_l) + t$$

  is the corresponding vector in the right frame
- Let the corresponding point coordinates be

  $$\{r_{l,i}\} \text{ and } \{r_{r,i}\}$$

- Strategy
  - First determine the rotation R
  - Translation follows easily

3.1 Absolute Orientation

Solution

- Calculate the centroids of the points

$$\bar{r}_l = \frac{1}{n}\sum_i r_{l,i} \qquad \bar{r}_r = \frac{1}{n}\sum_i r_{r,i}$$

- Normalize the coordinates

$$r'_{l,i} = r_{l,i} - \bar{r}_l \qquad r'_{r,i} = r_{r,i} - \bar{r}_r$$

- Note that given the rotation R we can calculate the translation

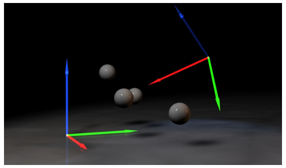
$$t = \bar{r}_r - R(\bar{r}_l)$$

3.1 Absolute Orientation

3.1.1 Setup and Motivation

3.1.2 Solution

→ 3.1.3 Applications



3.1 Absolute Orientation

Applications

- 3D-3D-Pose estimation

3. Calibration and Registration

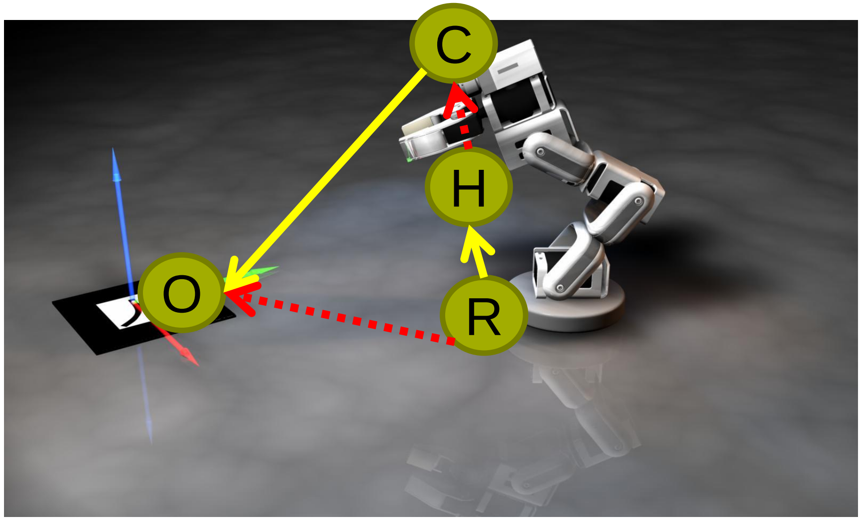
3.2 Hand-Eye Calibration

3.2 Hand-Eye Calibration

Setup and Motivation

Source: faro.com

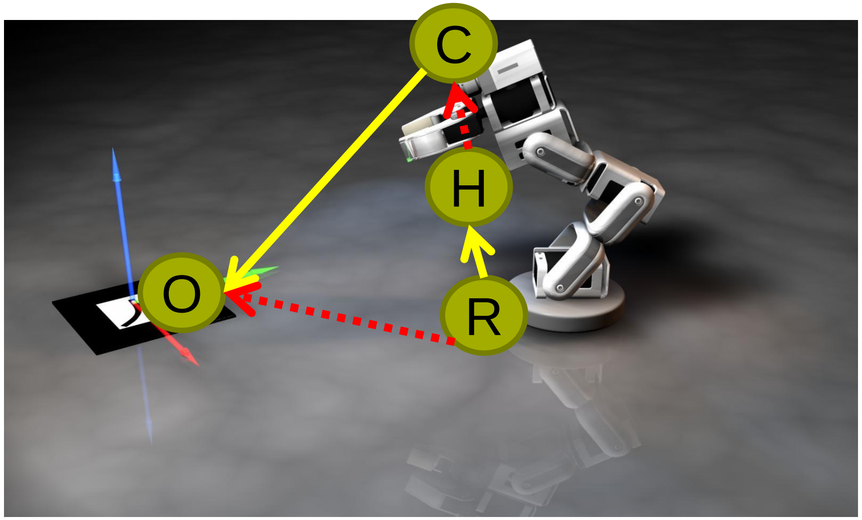
3.2 Hand-Eye Calibration



Setup and Motivation

- Involved Coordinate Frames
  - Robot-Base (R)
  - Hand (H)
  - Camera (C)
  - Object (O)
- Hand pose is known by forward kinematics
- Camera tracks Object
- Unknown:
  - Pose of Camera, relative to robot Hand
  - Pose of Object, relative to Robot base

3.2 Hand-Eye Calibration

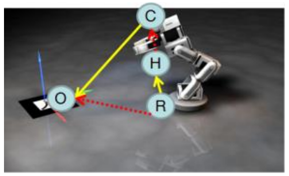


Setup and Motivation

- Task
  - Find the Object pose in the Robot base coordinate frame
- Problem
  - Transformation Camera to Hand is unknown

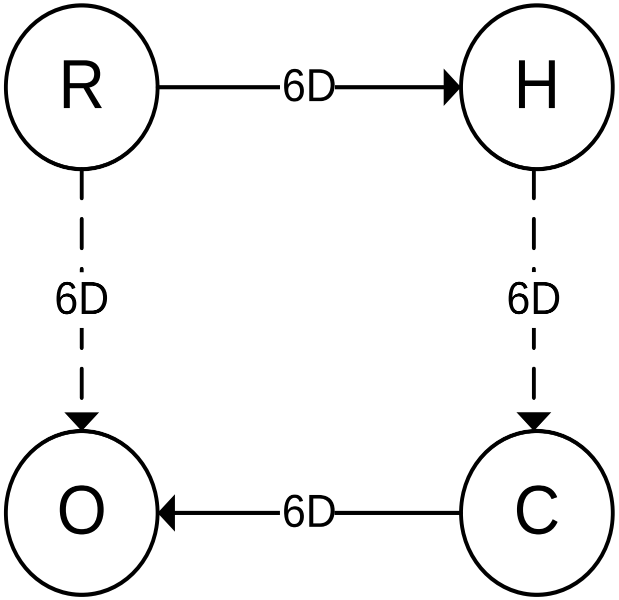
- Calibration (registration) problem

3.2 Hand-Eye Calibration



Setup and Motivation

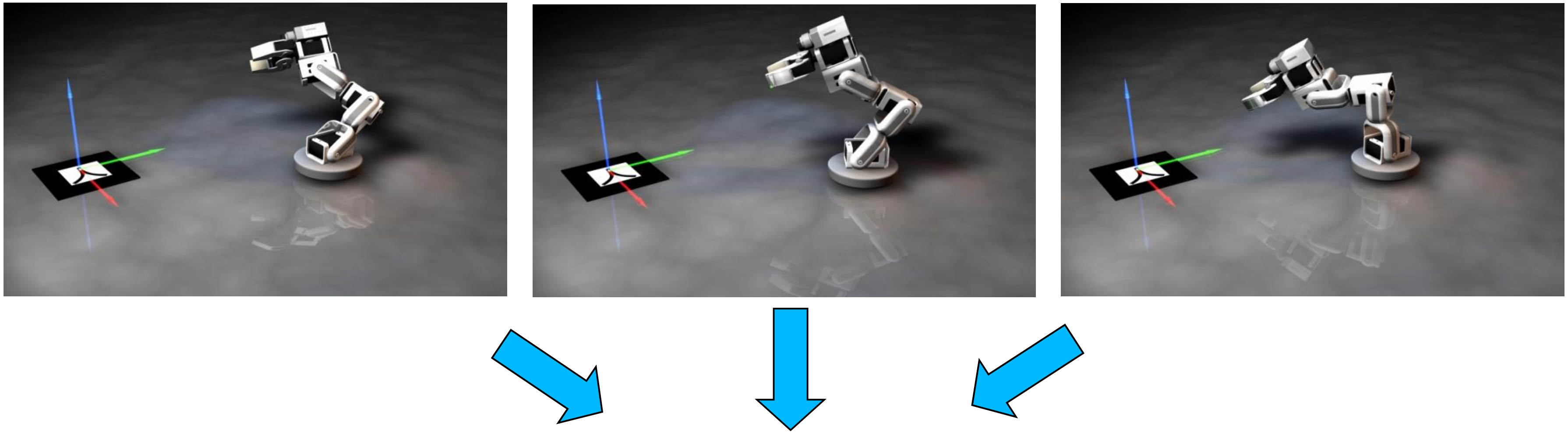
- Spatial Relationship Graph
  - Tracked Relationships
    - R → H
    - C → O
  - Unknown Relationships
    - H → C (Calibration)
    - R → O



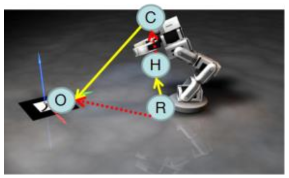
3.2 Hand-Eye Calibration

3.2 Hand-Eye Calibration

Basic Approach



- Move robot arm while keeping the object fixed; use several robot postures
- At least 3 different postures are needed

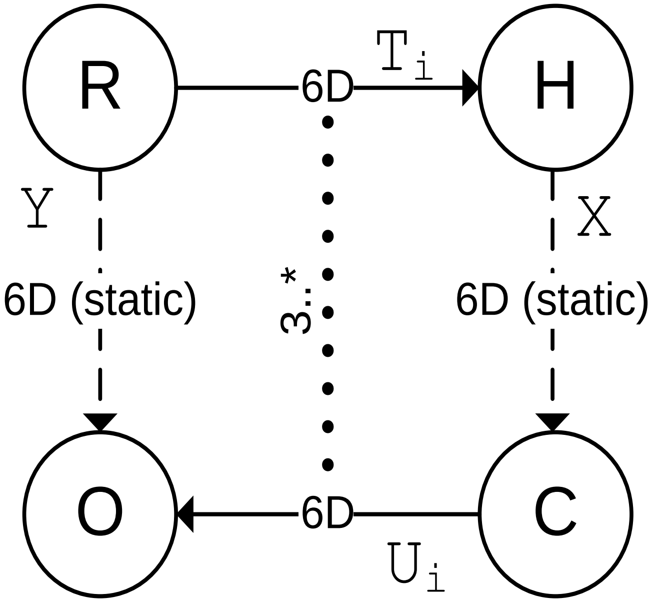


3.2 Hand-Eye Calibration

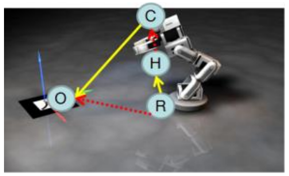
Basic Approach

- Represent poses as 4x4 matrices
- For the i^th robot configuration:

$$Y = T_i X U_i$$



3.2 Hand-Eye Calibration



Basic Approach

- For the i[th] robot configuration:

$$Y = T_i X U_i$$

- Combining two distinct configurations `i` and `j`:

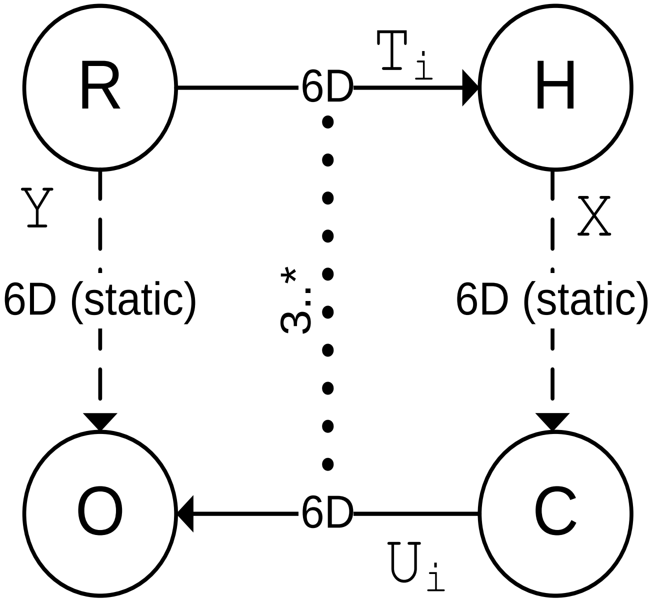
$$T_j X U_j = T_i X U_i$$

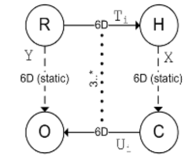
- Simplifying using

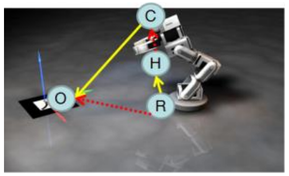
$$A = T_i^{-1} T_j \qquad B = U_i U_j^{-1}$$

- yields:

$$AX = XB$$







3.2 Hand-Eye Calibration

Basic Approach

$$AX = XB$$

- Solution of equation not unique
- One equation per pair i, j

- At least two equations simultaneously needed
  $\rightarrow$ ≥3 robot configurations

- System is over constrained
  - Solve for X minimizing error
  - e.g. Linear least squares

3.2 Hand-Eye Calibration

3.2 Hand-Eye Calibration

Solutions

$$AX = XB$$

- Using

$$A = \left[\begin{array}{ccc|c} & R_a & & T_a \\ \hline 0 & 0 & 0 & 1 \end{array}\right] \qquad B = \left[\begin{array}{ccc|c} & R_b & & T_b \\ \hline 0 & 0 & 0 & 1 \end{array}\right] \qquad X = \left[\begin{array}{ccc|c} & R_x & & T_x \\ \hline 0 & 0 & 0 & 1 \end{array}\right]$$

- we can split the equation into rotation and translation

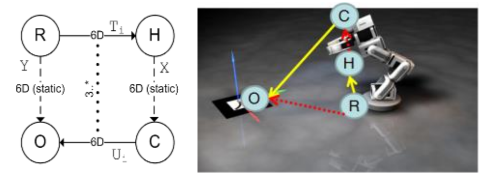
$$\begin{cases} R_a R_x = R_x R_b \\ R_a T_x + T_a = R_x T_b + T_x \end{cases}$$

3.2 Hand-Eye Calibration

Solution Strategies

- Solve for rotation first and determine translation
  - Tsai, R.Y., Lenz, R.K.
    Real Time Versatile Robotics Hand/Eye Calibration using 3D Machine Vision
  - Shiu, Y.C., Ahmad, S.
    Calibration of wrist-mounted robotic sensors by solving homogeneous transform equations of the form AX=XB
  - Chou, Jack C. K. and Kamel, M.,
    Quaternions Approach to Solve the Kinematic Equation of Rotation of a Sensor-Mounted Robotic Manipulator
- Solve for rotation and translation simultaneously
  - K. Daniilidis,
    Hand-eye calibration using dual quaternions

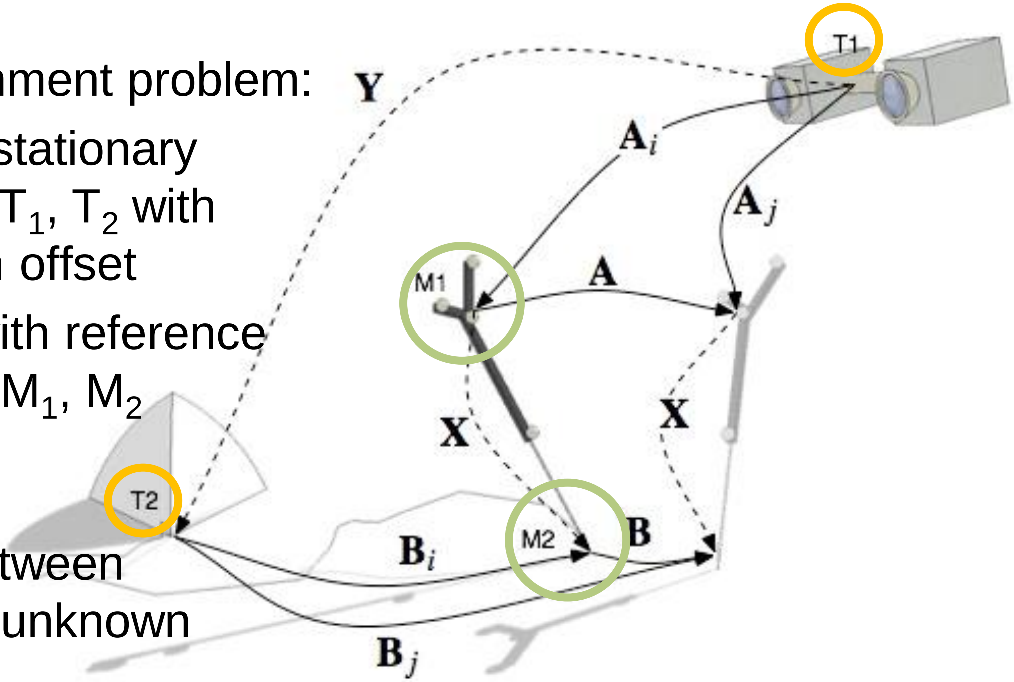
3.2 Hand-Eye Calibration

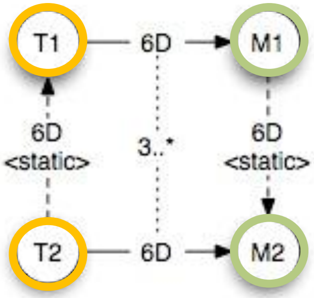


3.2 Hand-Eye Calibration

Applications

- Tracker alignment problem:
  - Several stationary trackers T_1, T_2 with unknown offset
  - Target with reference markers M_1, M_2 for each sensor; offset between markers unknown
- Examples:
  - Magnetic ↔ Optical
  - Inertial ↔ Optical





Thank you!