Operation tables

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In this activity you're going to fill out some *operation tables*. Each table will have some items listed across the top and side. For a given operation, your job is to fill each square with the result of that operation on the items in the corresponding row and column. For example, if we list 2, 3, and 5 along the side and top, and the operation is addition, then the operation table would look like this:

$+$	2	3	5
2	4	5	7
3	5	6	8
5	7	8	10

XOR

Fill in each spot in the table with the XOR of the two Boolean values.

$\oplus$	F	T
F		
T		

Addition

Fill in the table using addition.

$+$	0	1	2
0			
1			
2			

Addition modulo 3

Fill in the table using *addition modulo* 3. That is, imagine that the number line "wraps around" back to 0 when it gets to 3. If you get an answer bigger than 3, subtract 3 from it until you are back down under 3. For example, $2 +_3 2 = 1$ since $2 + 2 = 4$ and $4 - 3 = 1$.

$+_3$	0	1	2
0			
1			
2			

Addition modulo 5

$+_5$	0	1	2	3	4
0					
1					
2					
3					
4					

Max

Fill in the table using the max operation, which returns the largest of its two inputs.

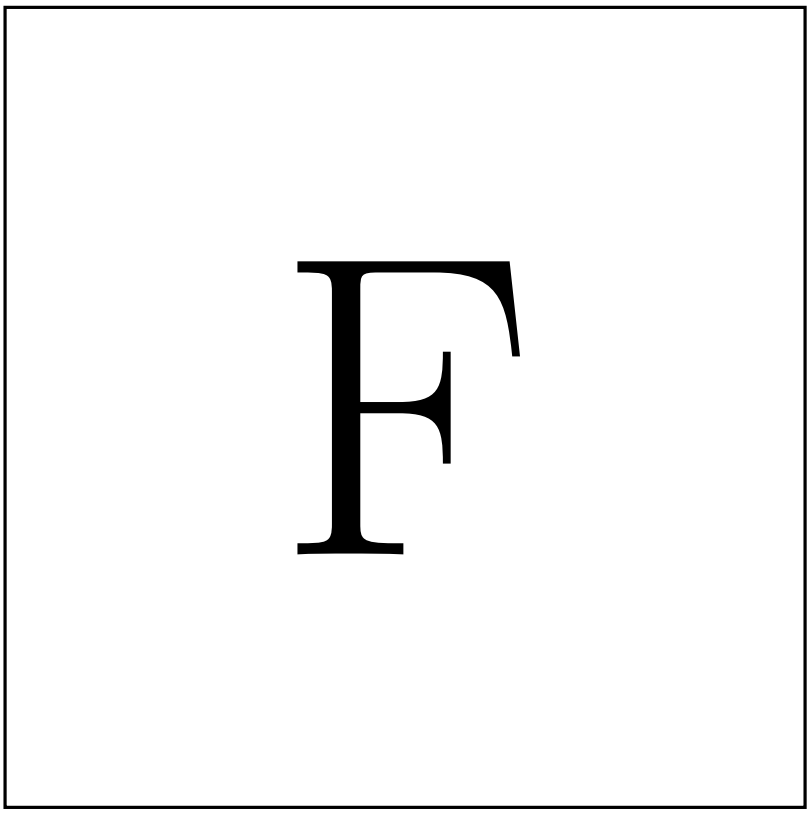
max	0	1	2
0			
1			
2			

Multiplication modulo 5

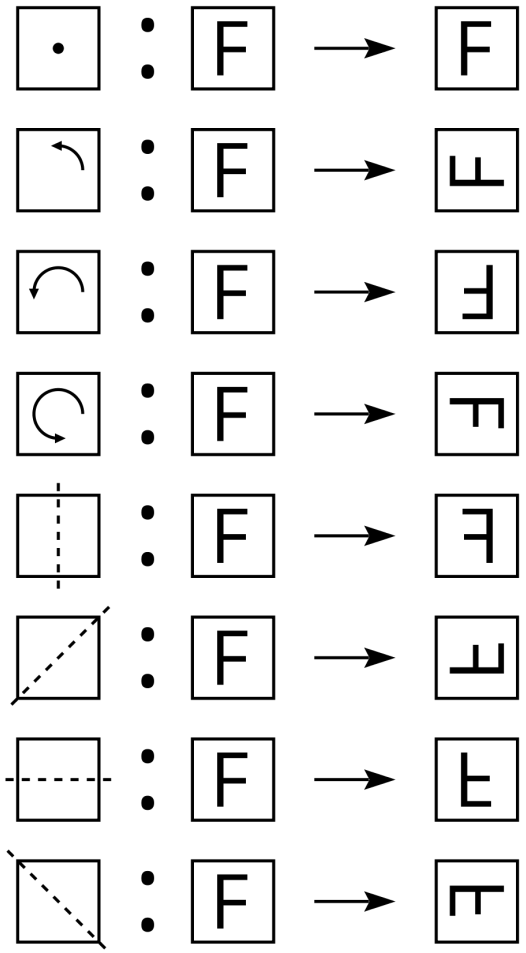
Multiplication modulo 5 works just like addition modulo 5, but with multiplication: to compute $a \times_5 b$, first multiply a and b, then keep subtracting 5 until you get something smaller than 5. For example, $3 \times_5 4 = 2$ since $3 \times 4 = 12$ and $12 - 5 - 5 = 2$.

$\times_5$	1	2	3	4
1				
2				
3				
4				

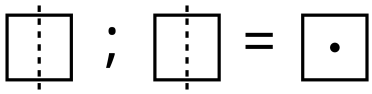
Cut out the square on this page. Ideally, you will also be able to see the letter F through the back of the paper. If you can't see it very well you are welcome to draw in a backwards F on the back that matches up with the F on the front.



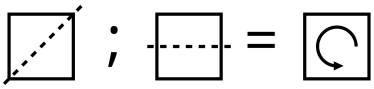
Let's list all the different things we can do to the square so that it ends up still being a square in the same orientation (that is, we want to list all the *symmetries* of the square). Each icon on the next page represents an operation we can do to the square. For example, the ⊡ in the first row means to do nothing; the ↰ in the second row means to rotate the square by 1/4 turn, and so on. On the right of the colon is a graphical depiction of the operation. If you start your square with the letter F facing you and perform the operation, it should end up looking like the square at the end. For example, the row for ↰ looks like ↰ : F — ⅃, meaning that if you start with F and perform the ↰ operation, the square ends up looking like ⅃. You should physically do all the operations with your square to make sure you understand how they work!



We can also *combine* two operations on the square by doing first one and then the other. In the end this will have the same effect as if we had done a single operation. For example, flipping the square vertically and then flipping it vertically again will put it back the way it started, so

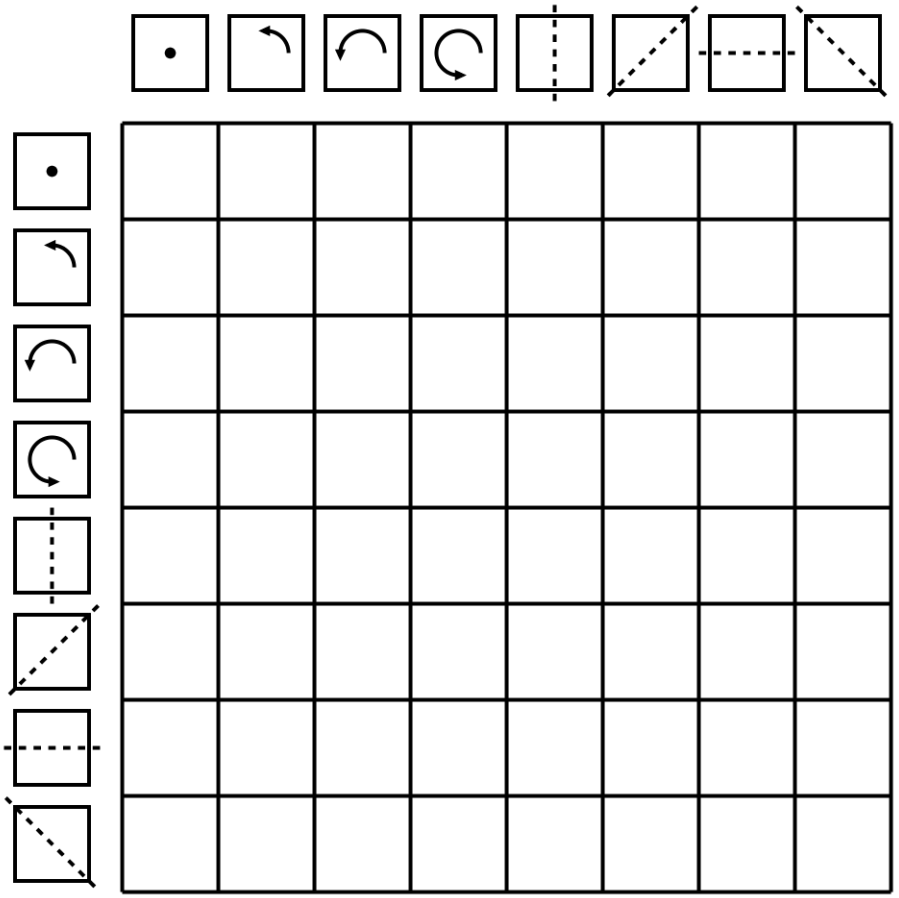


As another example,



(Try this with your physical square and make sure you understand it!)

Fill in the table below, using the combining operation explained on the previous page.



What patterns do you notice?