Fibonacci numbers

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The *Fibonacci numbers* are defined by

$$
\begin{aligned}
F_0 &= 0 \\
F_1 &= 1 \\
F_n &= F_{n-1} + F_{n-2} \qquad\qquad (n > 1)
\end{aligned}
$$

That is, the 0th Fibonacci number is 0; the 1st Fibonacci number is 1; and every number after that is formed by adding the two previous numbers. The first 17 Fibonacci numbers are therefore

$$0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987.$$

1. What is the next Fibonacci number?

2. Fill in the blanks below.

$$F_0 + F_1 = \underline{\hspace{2cm}}$$

$$F_0 + F_1 + F_2 = \underline{\hspace{2cm}}$$

$$F_0 + F_1 + F_2 + F_3 = \underline{\hspace{2cm}}$$

$$F_0 + F_1 + F_2 + F_3 + F_4 = \underline{\hspace{2cm}}$$

3. Keep going for three more steps.

4. Do you notice a pattern?

5. If $F_{28} = 317811$ and $F_{29} = 514229$, what is F_{27}?

6. In general, if you know F_n and F_{n-1}, how can you find F_{n-2}?

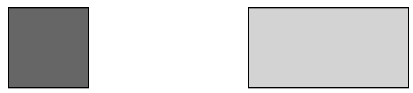
7. Using your answer to the previous questions, what should F_{-1} be (that is, the Fibonacci number before F_0)?

8. What about F_{-2}?

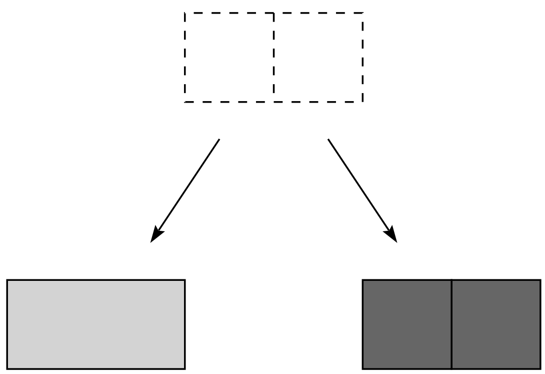
9. Write down the first ten Fibonacci numbers that should come before 0, that is, F_{-1} through F_{-10}.

10. What pattern do you notice?

There is a path that is 1 meter wide and n meters long, and you want to pave it with stones. You have two kinds of paving stones: square dark grey stones that are 1m $\times$ 1m and rectangular light grey stones that are 1m $\times$ 2m.



This means you might have some different choices in terms of how you pave your path. For example, if the path is 2 meters long, you have two choices: you could pave it with a single 1×2 stone, or you could pave it with two 1×1 stones.



11. If the path is 3 meters long, how many different ways do you have to pave it? Draw them here.

12. What if the path is 4 meters long?

13. What about 5 meters long?

14. Do you notice a pattern? Use the pattern to predict how many ways there are to pave a path that is 15 meters long.

Let's say a sequence of bits is *invalid* if it has two 0 bits right next to each other, and *valid* otherwise. For example, 1110100101 is invalid, since it has 00 in the middle. On the other hand, 1110110101 is valid since none of the 0s are right next to each other.

15. Is 110111010 valid or invalid?

16. What about 11111?

17. Write down all the valid sequences of two bits. How many are there?

18. Write down all the valid sequences of three bits. How many are there?

19. Write down all the valid sequences of four bits. How many are there?

20. Do you notice a pattern? How many valid sequences of 10 bits do you think there will be?