Recommender Systems || Lecture 12

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Today

Today

1. Ranking loss function and metrics

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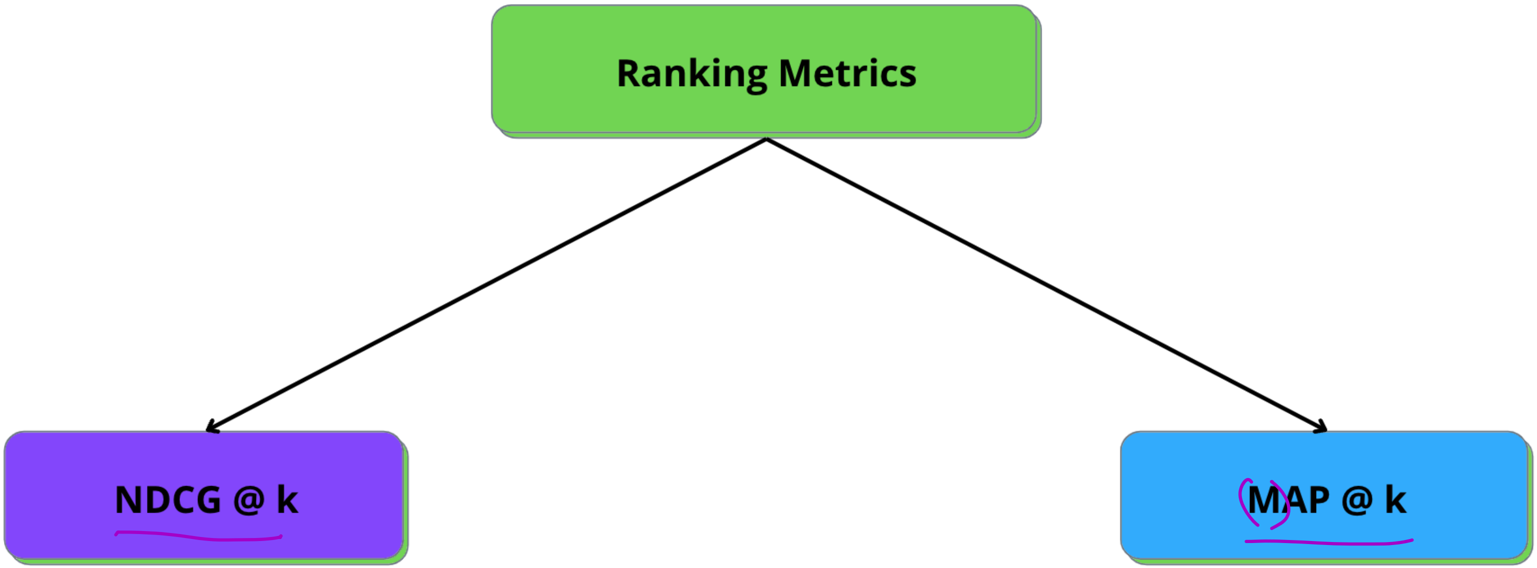
1. Ranking loss function and metrics
2. Page Ranking and Ranking Algorithms

Ranking Metrics



AP – Avg. Precision
(Average of Precision)

MAP – multiple queries
(mean of AP over queries)

Learning to Rank Survey

Ranking Metrics Example

Recommendations (model)

N=5

Compare

Purchases (Ground Truth)

N=6

Metrics

Let's compute all the metrics we know!

`Precision`, `Recall`, `Precision @ k`,
`Average Precision @ k`, `NDCG @ k`

Ranking Metrics Example

Recommendations

1 0 1 0 1

Purchases

Precision ?

$= 3/5$ → #Relevant recommendation / → #recommendations $= 0.6$

Assume:-
(care about the exact flavor/product)

Ranking Metrics Example

Recommendations

1 0 1

Purchases

Precision @ 3 ?

$$= \frac{2}{3} = 0.67$$

Ranking Metrics Example

vit. water zero

Recommendations

1 1 1

Purchases

Category Precision @ 3 ? $= \frac{3}{3} = 1$

Assume:- We don't care about the particular flavor

Model Metrics
Business Metrics
Gap?!

Ranking Metrics Example

Recommendations

} 5

Purchases
(Ground Truth)

} 6

Recall ?

$\frac{3}{6} \rightarrow$ # purchases (Ground Truth)

ICE # 1

Recommendations

Purchases

Assume:- Flavor doesn't matter for a match

Recall @ 3 ?

1. 0.3
2. 0.4
3. 0.5
4. 0.6

$\frac{3}{6}$ → Great ~~Good~~ Recall@3

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Average Precision

Recommendations

might be lesser than K summation

$$AP@K = \frac{\sum_{i:\, i \text{ is relevant}} P@i}{K}$$

Purchases

Average Precision ?

Relevant!

$$AP@2 = \frac{\sum_{i:\, i \text{ is relevant}} P(i)}{2} = \frac{P@1 + P@2}{2} = \frac{1 + 2/2}{2} = 1$$

ICE # 2: Average Precision @ 4

Recommendations

?

Purchases

Average Precision @ 4 ?

1. 0.5
2. 0.75
3. 0.9
4. 1

Relevant Relevant Rel Not Relevant

$$\frac{P@1 + P@2 + P@3 + \cancel{P@4}}{4} \leftarrow (@4!)$$

Relevant

$$= \frac{1+1+1}{4} = 0.75$$

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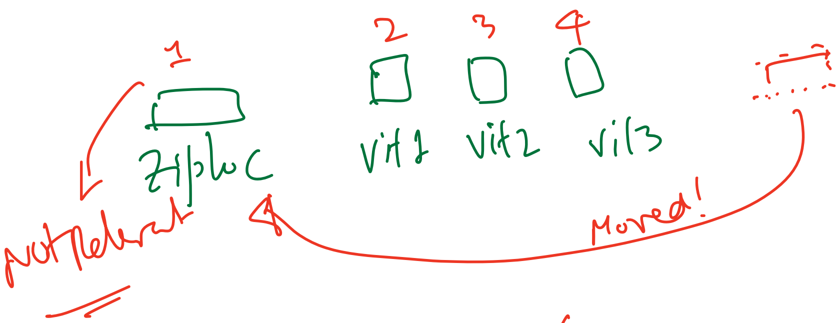


$$AP@4 = \frac{\cancel{P@1} + P@2 + P@3 + P@4}{4}$$

$$= \frac{1/2 + 2/3 + 3/4}{4}$$

$$= \frac{0.5 + 0.67 + 0.75}{4} = \frac{1.92}{4} \approx 0.5!!$$

AP@4 moved from 0.75 to 0.5 with a single shift in an recommended item

P@4 = 0.75 (still the same!!)

DCG

Discounted Cumulative Gain

$$\mathrm{DCG} = \underbrace{\sum_{i=1}^{N}}_{\text{Cumulative}} \frac{\overbrace{y(d_i)}^{\text{Gain}}}{\underbrace{\log_2(\mathrm{rank}(d_i) + 1)}_{\text{Discount}}}$$

Gain (explanational rating)

Ranking Metrics Example — NDCG @ 3

1 0.8 0.9 0 0.7 Gradation / Grade

1 1 1 | 0 Relevant or not 1

Recommendations

Purchases

NDCG @ 3

$$DCG@3 = \frac{2^{1}}{\log(1+1)} + \frac{2^{0.8}}{\log(2+1)} + \frac{2^{0.9}}{\log(3+1)}$$

$$\max DCG@3 = \frac{2^1}{\log(1+1)} + \frac{2^{0.9}}{\log(2+1)} + \frac{2^{0.8}}{\log(3+1)}$$

(corresponds to perfect ranking)

$$NDCG@3 = \frac{DCG@3}{\max DCG@3} = 0.995 \simeq 1$$

$0 \leq NDCG \leq 1$!

Ranking is close to perfect!!

Summary of Metrics

Metric	Description
Precision	
Recall	
Precision @ k	
F-score	
Average Precision @ k	
NDCG @ k	

$\frac{\#\text{ Relevant items}}{\#\text{ rec items}}$

$\frac{\#\text{ Relevant items Recomm}}{\#\text{ Relevant items}}$

Proxy to determine good Ranking

Learning to Ranking Formulation

Loss Algo (SGD) Search/Page

Given a query q and a set of documents $D = (d_1, \ldots, d_n)$:

Pointwise

Input: Single candidate

$x = (q, d_i)$

Loss: How accurate is the predicted score $s_i \approx y_i$?

Solution: Transform task into Regression.

Pairwise

Input: Pair of candidates

$x_i = (q, d_i)$ and $x_j = (q, d_j)$

→ Ground Truth

Loss: If $y_i > y_j$, then are the predicted scores $s_i > s_j$?

prediction

Solution: Transform task into Binary Classification.

Listwise

Input: Whole list of candidates

$x_1 = (q, d_1)$ .. $x_n = (q, d_n)$

Loss: Takes into account position of all retrieved docs.

Solution: Incorporate evaluation metrics (e.g. DCG) into loss.

1o item

True Ranking   Recommended ranking

Learning to Ranking Formulation

Point wise loss

predicted score

→ Regression Loss

$$L(\underline{s}, \underline{y}) = \sum_{i=1}^{n} (s_i - y_i)^2$$

Truth

Learning to Ranking Formulation

Drawback:
No position info included in Loss!!

Pairwise (Binary Cross Entropy loss)

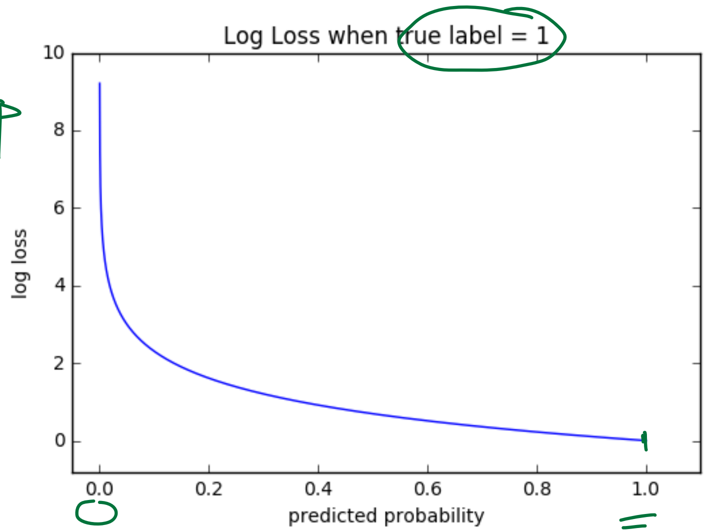
Let $y_{ij} = 1$

$$L(\underline{s}, \underline{y}) = -\sum_{i,j=1}^{n} y_{ij} \log(s_{ij}) + (1 - y_{ij}) \log(1 - s_{ij})$$

(i,j) d_i, d_j

Cross Entropy behavior

Loss $\to \infty$ at 0



$y_{ij} = 1 \Rightarrow d_i$ is more relevant than document j

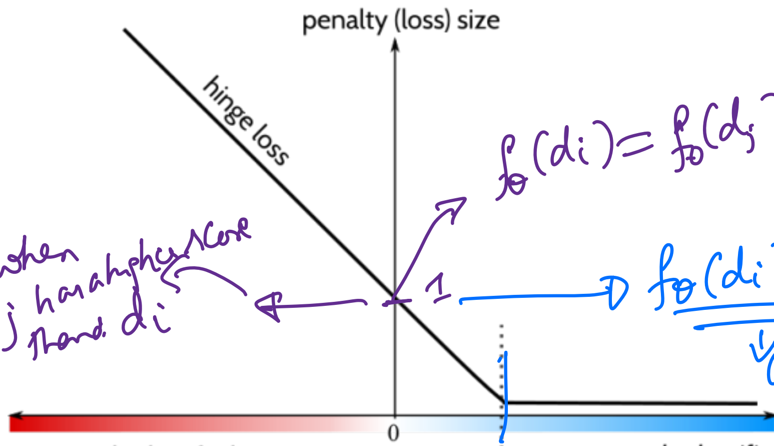
Pros/ Advantage:-
Pairwise comparison
Easy to obtain Ground Truth!

Learning to Ranking Formulation

Pairwise Hinge loss

$$\mathcal{L}_{\textsf{pairwise}} = \sum_{\underbrace{y(d_i) > y(d_j)}_{y_{ij}=1}} \max\Big(0,\ 1 - \big(f_\theta(d_i) - f_\theta(d_j)\big)\Big)$$

Hinge Loss Behavior



when d_j has a higher score than d_i

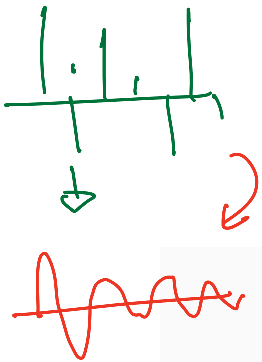
1

$f_\theta(d_i) = f_\theta(d_j)$

$f_\theta(d_i) > f_\theta(d_j)$

Good place

DCG



Incorporate position in Learning to Rank

Approx

Smoother ver.

$$\text{DCG} = \sum_{i=1}^{N} \frac{y(d_i)}{\log_2(\text{rank}(d_i) + 1)}$$

Not Differentiable

rank(di) = f(Sorting(Scores))

not differentiable!!

$\Rightarrow$ Cannot compute gradient

$\Downarrow$

Cannot use SGD!

Lambda Rank Proxy Loss

Heuristic

SOTA

Logistic loss (Logistic Regression)

Proxy for Gradient

$$\underbrace{\mathcal{L}_{LambdaRank}}_{\text{Model Algo}} = \sum_{y(d_i)>y(d_j)} \log\left(1 + e^{f_\theta(d_j)-f_\theta(d_i)}\right) \cdot \underbrace{|\Delta \mathrm{DCG}|}_{\text{Accounts for position}}.$$

Doesn't account for position

Learning to Ranking Formulation

Given a query q and a set of documents $D = (d_1, \ldots, d_n)$:

Pointwise

Input: Single candidate
$x = (q, d_i)$

↓

Loss: How accurate is the predicted score $s_i \approx y_i$?

↓

Solution: Transform task into Regression.

Pairwise

Input: Pair of candidates
$x_i = (q, d_i)$ and $x_j = (q, d_j)$

↓

Loss: If $y_i > y_j$, then are the predicted scores $s_i > s_j$?

↓

Solution: Transform task into Binary Classification.

Listwise

Input: Whole list of candidates
$x_1 = (q, d_1)$ .. $x_n = (q, d_n)$

↓

Loss: Takes into account position of all retrieved docs.

↓

Solution: Incorporate evaluation metrics (e.g. DCG) into loss.

Mapping Loss Functions to Ranking Models

Loss Function Type	**Loss Function**	**Ranking Model**
Pointwise	Quadratic	
Pairwise	Binary Cross-Entropy/ Hinge Loss	RankNet, LambdaRank, LambdaMart
Listwise		SoftRank

Optimizes Approx DCG

Next Class

Chris Burges

1. Deeper look into Rank models including LambdaRank
2. Neural models for Search Ranking and Re-ranking