Evolution Strategies

Applications of Evolution Strategies

Parallel Implementations of Evolution Strategies

Application of the Evolutionsstrategie to Discrete Optimization Problems

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Introduction

With 4 examples the present contribution will show that the Evolutionsstrategie is successfully applicable to discrete optimization problems although the theory has been formulated for continuous problems.

In many different optimization problems the Evolutionsstrategie turned out to be a useful problem-solving algorithm [1, 2, 3]. It was characteristic for all these optimization problems that they were described in a system of continuous variables. Small variations of these variables usually lead to small changes of the value of the quality function. This behavior is called strong causality. The validity of strong causality is a necessary condition for the applicability of the Evolutionsstrategie in continuous as well as in discrete optimization problems. In discrete optimization problems, therefore, small changes in the parameters must result in small changes of the quality function. If the original formulation of the problem does not comply with strong causality then creation of suitable mutation operators can often help.

The principal item of the theory of Evolutionsstrategie is the statement that a "window of evolution" exists. This says that to effectively move forward in the variable space it is necessary to have a stepsize which is perfectly adapted to the topology of the quality mountain. That is why Rechenberg [1, 2] developed a mutative stepsize control. A stepsize control is also necessary for discrete optimization problems. Stepsizes may be defined differently for each particular problem.

In the optimization problems described in the following, either $(\mu/\rho+\lambda)$-ES or $(\mu/\rho,\lambda)$-ES were applied. In these strategies the population consists of μ parents and λ offsprings and ρ parents are concerned at the recombination of each offspring. For each offspring these ρ parents are randomly chosen from the μ parents. For each parameter of the offspring it is again randomly decided, from which of the ρ parents it is taken. The stepsize of the offspring results from the mean of the ρ parent stepsizes. After generating the λ offspring every single parameter is being varied with a mean variation corresponding to the stepsize. The stepsize of each offspring is also randomly increased or decreased with a fixed multiplication factor. After the following valuation the μ best individuals become parents of the next generation. In the "+"-strategy these μ best individuals are selected from the whole population of $(\mu+\lambda)$ individuals, in the ","-strategy they are selected only from the λ offsprings.

The mimicry problem

As the most elementary example for a discrete optimization problem the mimicry problem may be mentioned: An imitator pattern is to be adapted identically to a prototype pattern. The prototype pattern consists of n points on the screen and each of them can take two states (black/white). By means of the Evolutionsstrategie the imitator pattern shall adapt to the prototype pattern.

The quality is determined by the hamming distance H between imitator and prototype which is the number of points being different in the two patterns. Specific mutation operators are not necessary in this simple example; the variation of a single point causes a minimum quality change. An adaptive stepsize S may be defined as the number of points to be mutated in each offspring. In the examined 500-points-problem the stepsize adapted to the value of S=1 during some generations; for that reason it was fixed to S=1 in the following simulations (**Table 1**).

With a $(\mu/\rho+\lambda)$-ES simulations with different values for μ (the number of parents), λ (the number of offsprings) and ρ (the number of parents concerned at the recombination) were performed. **Table 1** shows the results where every value is the mean of 200 simulations.

Simulation no.	Strategy	Generations
1	(1/1+1)	3072
2	(1/1+20)	341
3	(3/3+20)	247
4	(3/3+30)	192
5	(10/10+30)	167
6	(10/10+40)	134

Table 1

The result of G=3072 generations with a (1/1+1)-ES in simulation no. 1 corresponds to theory of probabilities, which delivers the value G=3050 generations for a 500-points-problem where 250 points are still not adapted. Lower generation numbers are attainable with higher imitative levels of biological evolution as shown in **Table 1**.

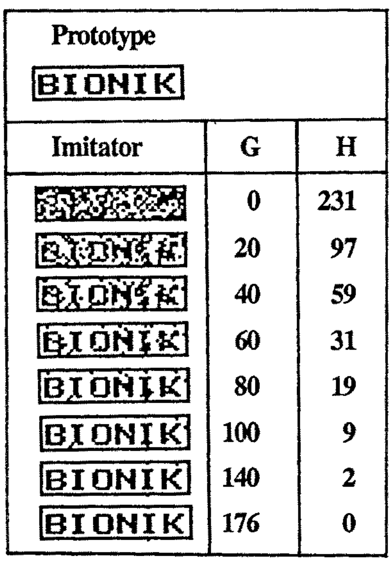

Figure 1 opposite shows the passing of simulation no. 5 where a (10/10+30)-ES was applied. Recombination means that more than one parent is involved in generating a new offspring. It can be observed that the influence of recombination is particularly strong at the beginning of the simulation. Between generation G=0 and G=20, the quality changes by 134 points. By means of variation, the quality can be improved only by one point per generation because the stepsize is fixed to S=1 so that in 20 generations, the maximum improvement in quality is 20 points. At least the remaining 114 points are correctly adapted by recombination. The same effect occurs between generation G=20 and G=40 and between G=40 and G=60. Here also the quality changes by more than the 20 points attainable by variation.

The magic square

An example from number theory represents the discovery of a true magic square. On a square grid with n x n fields the numbers from 1 to n^2 are being distributed at random. It is now the aim to find such an arrangement where all sums of columns, all sums of rows and all sums of diagonals yield the same value S. This arrangement is called a true magic square. The value S, the so called magic sum, results from $S = 0.5 \cdot n \cdot (n^2 + 1)$. For a magic square with n columns, n rows and 2 diagonals the quality of an offspring is calculated by adding the squared differences between the n sums of columns and S, the n sums of rows and S and the 2 diagonals and S.

For the variation, one field is selected at random. Afterwards this field is interchanged with one whose numerical value is only a little different. In this way the smallest changes of quality are obtained. In this example an adaptive stepsize is definable as the number of fields to be interchanged in each offspring or as the maximum difference of the both numbers being interchanged. A combination of both mechanisms leads to an effective stepsize control.

4	32	7	79	52	69	48	99	31	84
16	62	42	54	27	65	17	91	35	96
20	23	77	67	53	46	14	85	56	64
28	61	43	29	88	63	70	15	100	8
21	78	34	13	93	44	57	74	80	11
72	68	22	94	3	59	75	5	36	71
60	25	95	26	81	9	83	12	38	76
98	40	37	87	51	45	49	18	30	50
89	24	82	55	47	86	2	73	41	6
97	92	66	1	10	19	90	33	58	39

Figure 2

Figure 2 opposite shows a 10x10 magic square developed with the Evolutionsstrategie. All sums of columns, all sums of rows and both sums of diagonals yield the same value S=505. As an additional difficulty the fields for the date **1.10.1990** in the lowest row of the square had been declared to be fixed and not changeable. This magic square was found with a (1/1,30)-ES in 492 generations.

Rubik's Cube

Rubik´s Cube [5] is a three-dimensional logic game for which an application for a patent was made in 1975 by E. Rubik, professor of architecture and design in Budapest. It is altered from a random initial state to such a configuration where only monochromatic surfaces on the cube exist.

The cube consists of 26 partial-cubes, the 6 middle-cubes, 12 border-cubes and 8 corner-cubes. The visible faces of these partial-cubes are labelled with little colour tiles. The middle-cubes cannot change their position relative to each other. That is why the goal colour of each of the 6 surfaces of the cube is determined by the colour of the middle-cube.

The quality function $Q = Q_1 + Q_2 + Q_3$ for the evaluation of the state of the cube consists of three parts combined by addition. With Q_1, differences from the colour of the middle-cube are penalized for every surface of the cube. For every wrong colour tile, Q_1 is increased by 1. Q_2 and Q_3 penalize wrong positioned border- or corner-cubes. A border-cube is correctly positioned if it contains the two colours of the middle-cubes it touches. For every wrongly positioned border-cube, Q_3 is increased by 4. Similarly, a corner-cube is correctly positioned if it contains the three colours of the middle-cubes it touches. For every wrongly positioned corner-cube, Q_3 is increased by 6. In the case of correctly positioned but still wrongly turned border- or corner-cubes, Q_2 or Q_3 are not increased because these turnings are sufficiently taken into account by Q_1. Each of the three quality parts Q_1, Q_2 and Q_3 can take on a maximum value of 48 which means that Q can take on a maximum value of Q = 144.

For the variation of the cube, specific mutation operators were implemented each of them changing the quality only a little. It is, for example, not useful to take the turning of a cube-slice through 90° as a smallest variation operator because the quality changes in big steps and not continuously in small steps. The implemented mutation operators are two-border-turn, two-corner-turn, three-border-swap, two-border/two-corner-swap, three-corner-swap, double-border-swap and double-corner-swap. The operators three-border-swap, three-corner-swap and two-border/two-corner-swap are realized in two different directions so that there are 10 mutation operators available.

For a cube in a starting position with quality Q=118 with a (1,50)-ES the optimum was found in a mean of 38.7 generations. A mean of only 1935 from more than 43 trillion possible cube positions were calculated.

The travelling salesman problem

A frequent problem in operations research is the travelling salesman problem (TSP) [6]. A salesman has to visit n towns one after another on a tour with minimum costs and every town may be visited only once. In our implementationt he costs are caused only by the length of the tour. Moreover, this example deals only with symmetric TSP, i.e. the distance between two towns is independant of the direction.

The problem to find the tour with minimum costs can be solved with the Evolutionsstrategie [7]. Again it is necessary to ensure by implementation of specific mutation operators that the strong causality is valid, i.e. that small changes of the tour lead to small changes of the costs. Four different mutation operators were implemented: Inversion of a segment of the tour, insertiôn of a town at another point of the tour, reciprocal exchange of two towns and displacement of a segment of the tour. **Figure 3** shows examples for the effect of the different operators on a given parent tour. The order of the numbers represents the order in which the towns are visited by the salesman. After visiting the last town (no.12) he returns to the first town (no.1) again.

1 2 3 4 5 6 7 8 9 10 11 12	**Parents-Tour**
1 5 4 3 2 6 7 8 9 10 11 12	**Inversion** of the tour segment 2-3-4-5
1 3 4 5 2 6 7 8 9 10 11 12	**Insertion** of town no.2 between no.5 and no.6
1 6 7 2 3 4 5 8 9 10 11 12	**Displacement** of the tour segment 2-3-4-5
1 5 3 4 2 6 7 8 9 10 11 12	**Reciprocal Exchange** of towns no.2 and no. 5

Figure 3 The mutation operators for the TSP

If these operators are applied to any part of the tour, then strong causality is probably not fulfilled. Distances of towns from each other must be considered when applying the mutation operators. This means that, for example in the application of the inversion operator, only tour segments between adjacent towns may be inverted. Here in the TSP, an adaptive stepsize may also be used. It determines up to what neighbour a mutation operator may be applied. In addition to the mutation operators a specific recombination operator was implemented in the TSP. This one is different to the recombination operators usually used in the Evolutionsstrategie because here it must be taken into consideration that the recombination may produce only valid tours. Therefore this operator does not have a completely random character; it also contains deterministic components:

One of the ρ parents being recombined is chosen as a starting parent at random. In the tour of this starting parent, a town is selected at random. This is the first town in the tour of the offspring (we can call the town A). For each of the ρ parents, the left and right neighbour of this town (A) are examined. The nearest in distance is selected from the total $2{\cdot}\rho$ possibilities, and this becomes the second town (B) in the tour. The same process is applied to the neighbours of B to yield C and so on until the end of the tour. If the $2{\cdot}\rho$ possible neighbours are already built into the tour of the offspring, the nearest town still avaiable is taken.

The following **Figure 4** shows some interesting parts of the simulation with a (10/2,30)-ES for a 84-town-TSP.

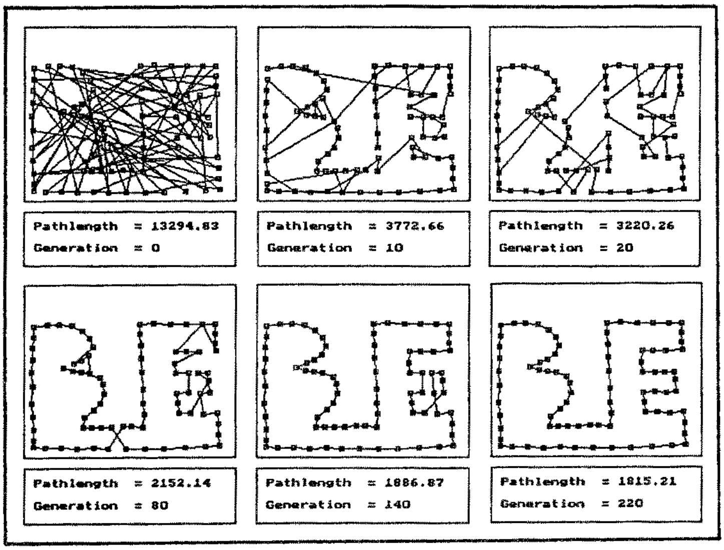

Figure 4

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