Midterm Review

Midterm Checklist
Enumeration

- Disjoint union of sets
- Cartesian product
- Cartesian power
- injective (one to one)
- surjective (onto)
- bijection
- inverse of a function
- binomial coefficient $\binom{n}{k}$
- weight function defined on a set S
- generating series for a set S with respect to a weight function w
- formal power series (over a field, e.g. $\mathbb{Q}$)
- addition, subtraction, multiplication of formal power series
- multiplicative inverse of a formal power series
- composition (or substitution) of formal power series $A(B(x))$
- composition of an integer
- parts of a composition (recall parts must be POSITIVE)
- empty composition (with 0 parts, which is a composition of 0)
- binary string (of length n)
- empty binary string ϵ
- concatenation of many binary strings
- concatenation of many sets of binary strings
- substring
- block
- deompositions for sets of binary strings (e.g. 0-decomposition, 1-decomposition, block-decomposition, recursive decomposition)

- unambiguous
- A^* where A is a set of binary strings (e.g. $\{0,1\}^*$)
- rational expression $\frac{g(x)}{f(x)}$ with f, g polynomials
- partial fraction
- (linear homogeneous) recurrence relation for a sequence $\{a_n\}_{n\geq 0}$ (e.g. for the sequence of coefficients of a generatings series)
- characteristic polynomial of a recurrence relation

Graph

- vertex
- edge
- adjacent
- incident
- neighbour
- isomorphism
- degree
- bipartite
- n-cube
- complete graph K_n
- complete bipartite graph $K_{m,n}$
- subgraph
- spanning subgraph
- walk
- path
- cycle
- Hamilton cycle
- connected / disconnected
- component

- maximal connected subgraph

Results proved in class may be used as tools. However, you must refer to them explicitly (by name, otherwsie state the result)

Named Theorems

- Binomial Theorem $(1+x)^n = \sum_{k=0}^{n} \binom{n}{k} x^k$
- Negative Binomial Theorem $(1-x)^{-m} = \sum_{k\geq 0} \binom{m+k-1}{m-1} x^k$
- Finite Geometric Series $1 + x + x^2 + \ldots + x^t = \frac{1-x^{t+1}}{1-x}$
- Sum, Product, $*$ Lemmas
- Decomposition theorems (0-, 1-, block)
- Handshake Lemma

How to

- Show function is a bijection (injection + surjection or find inverse)
- Find generating series
- Find coefficients in formal power series
- Find a recurrence relation for a sequence of coefficients
- Solve a recurrence relation
- Show decomposition is unambiguous
- find number of edges in a graph
- determine whether 2 graphs are isomorphic
- show a graph is (not) connected

Example (Show there is a bijection)

Let k and n be fixed. Let S be the set of all k-tuples $(a_1, a_2, ..., a_n)$. Such that $a_1 \in \geq \not\vdash$ for each i and $a_1 + a_2 + ... + a_k = n$

Let T be the set of all binary strings of length $n + k - 1$ with exactly $k - 1$ 1s.

Show there exists a bijection from S to T and hence conclude $|S| = |T| = \binom{n+k-1}{k-1}$

Define $S \to T$ by $f(a_1, ..., a_k) = \sigma_i$ where $\sigma = (a_1 : 0)1(a_2 : 0)1....1(a_k : 0)$

Then σ has length $a_1 + ... + a_k + k - 1 = n + k - 1$, and has exactly $k - 1$ 1s, so $\sigma \in T$.

Define g on T as follows. Each $\sigma \in T$ has the form $\sigma = b_1 1 b_2 ... 1 b_k$ where each B_i is a string of 0s (possibly empty).

Let $g(\sigma) = (a_1, a_2, ..., a_k)$ where a_i is the length of b_i. Then $g(\sigma) \in S$ since σ has length $n + k - 1$ and has exactly $k - 1$ 1s so $a_1 + ... + a_k = n$.

Then $g(f(a_1, ..., a_k)) = (a_1, ..., a_k)$ for each $(a_1, ..., a_k) \in S$.

Also note $f(g(\sigma)) = \sigma$.

So g is the inverse of f, so f is a bijection.

Graph Isomorphism

Informal idea: $G \approx H$ means you can re-label the vertices of G to get H