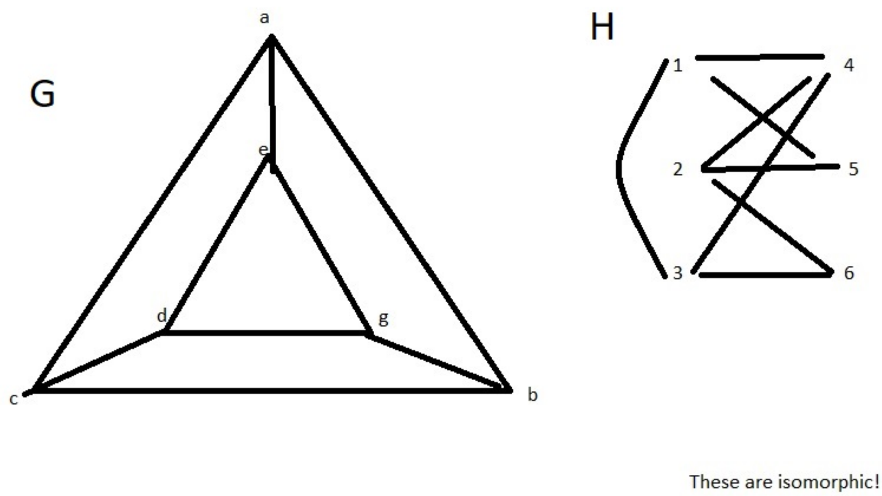

Figure 1: Isomorphic Grpahs

Consider figure 1: Isomorphic graphs.

Define $f : V(G) \to V(H)$ by:

- f(a) = 4
- f(b) = 3
- f(c) = 1
- f(d) = 5
- f(e) = 2
- f(g) = 6

Claim: f is an isomorphism.

1. f is a bijection as established
2. Must check the edges:

    - ab → 43
    - ae → 42
    - ac → 41
    - eg → 26
    - ed → 25
    - dc → 51
    - dg → 56
    - gb → 63
    - cb → 13

    Must check that all edges in G are in H and that all edges in H are in G.
    (each edge in H is the image of (exactly) one edge in G)

To show two graphs are isomorphic:
exhibit an isomorphism $f : G \to H$

But.. how do we prove to someone that graphs are *not* isomorphic? (open problem)

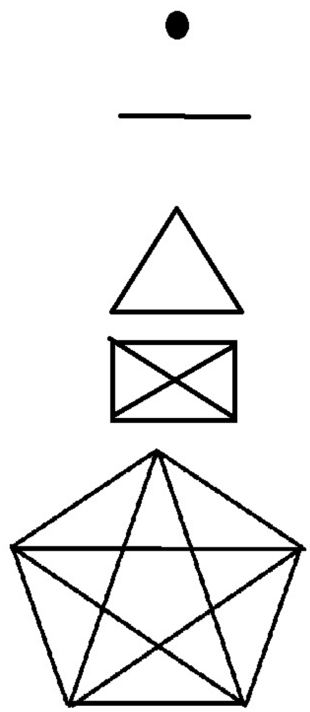

Figure 2: Complete Graphs

Theorem: Let G be a graph then

$$\sum_{v \in V(G)} deg(v) = 2|E(G)|$$

Since an edge must enter and exit some vertices, totalling all the degrees would explicitly double count.

Proof Since each edge in $E(G)$ has exactly two vertices it is counted exactly twice by $\sum_{v \in V(G)} deg(v)$, $\square$

Definition We say that a graph G in which deg(v) for each $v \in V(G)$ is *r-regular*

Corollary (Handshake Lemma): For any graph G the number of vertices with odd degree is even.

(since the sum of all degrees must be even. So can't have an odd number of odds otherwise the final result would be odd)

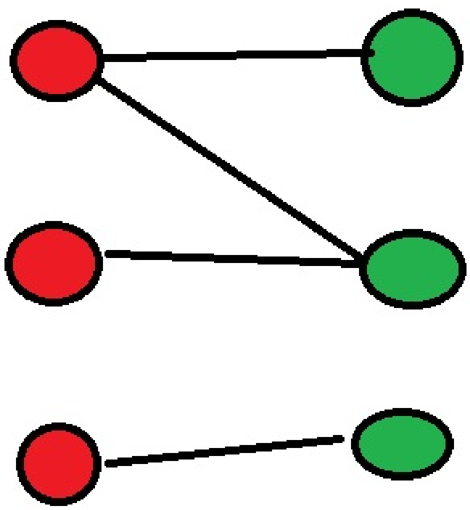

Figure 3: Bipartite Graphs

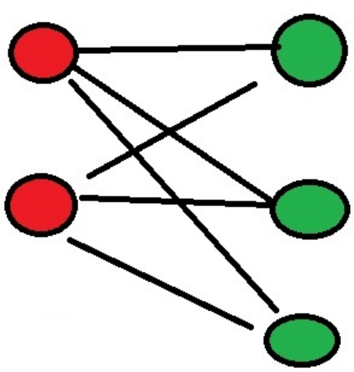

Figure 4: Complete Bipartite Graphs

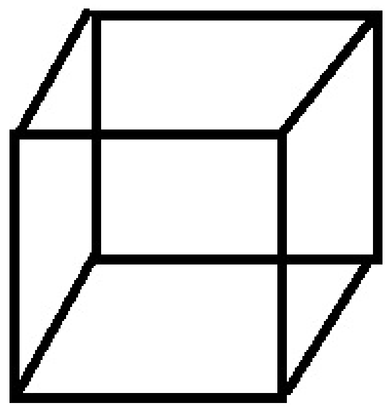

Figure 5: n-cube Q_n Graphs

Special Graphs

- The *complete graph* K_n with n vertices has a vertex set of size n, say $\{1, 2, ..., n\}$. Every pair of vertices is an edge! (See Figure 2)(More correctly: any graph isomorphic to this is a complete graph with n vertices)

$$|V(K_n)| = n$$

and

$$|E(K_n)| = \binom{n}{2}$$

and K_n is (n - 1)=regular

A graph G is called *bipartite* if there is a partition of $V(G)$ into two classes A and B such that each edge of G has a vertex in A and a vertex in B. (see Figure 3)

Note that a graph with an odd-length cycle cannot be bipartite.

The *complete bipartite* graph $K_{m,n}$ has vertex set partitioned into a class A of size m and a class B of size n and has *all* edges $\{ab : a \in A, b \in B\}$ (see Figure 4)

$$|E(K_{m,n})| = mn$$

The *n-cube* Q_n has vertex set $\{0,1\}^n$

Q_n has an edge between σ and σ^1 in $\{0,1\}^n$ if and only if σ and σ^1 differ in exactly one position. (see Figure 5)(looks like a cube)

We assert that a Q_n is bipartite for every n.
Let A be the set of vertices with an even number of 0s and let B be the set of vertices with an odd number of 0s.

Note each vertex in Q_n must have degree n since we must be given n chances to try to change each digit of a vertex.

So

$$|V(Q_n)| = 2^n$$

and

$$|E(Q_n)| = \frac{1}{2} \sum_{v \in V(Q_n)} deg(v) = \frac{1}{2}(2^n)n = 2^{n-1}n$$