Example

$$b_n = 4(2)^n - 3^n + (4n-1)(-1)^n$$

for all $n \geq 0$

Find a recurrence relation with initial conditions that defines b_n for all $n \geq 0$.

Solution

Look for a polynomial whose roots are $2, 3, -1, -1$

$$\begin{aligned}(y-2)(y-3)(y+1)(y+1) &= (y^2-5y+6)(y^2+2y+1)\\ &= y^4-3y^3-3y^2+7y+6\end{aligned}$$

If this is the characteristic polynomial then the recurrence for b_n should be

$$b_n - 3b_{n-1} - 3b_{n-2} + 7b_{n-3} + 6b_{n-4} = 0$$

for $n \geq 4$

How do we find out what b_0, b_1, b_2, b_3 are? From the formula (initial conditions)!

- $b_0 = 4(2^0) - 3^0 + (-1)(-1)^0 = 2$
- $b_1 = 2$
- $b_2 = 14$
- $b_3 = -6$

What about a recurrence relation that isn't homogeneous? - check notes.. main interest is because these come from generating series and the type that we get from generating series are homogenous.

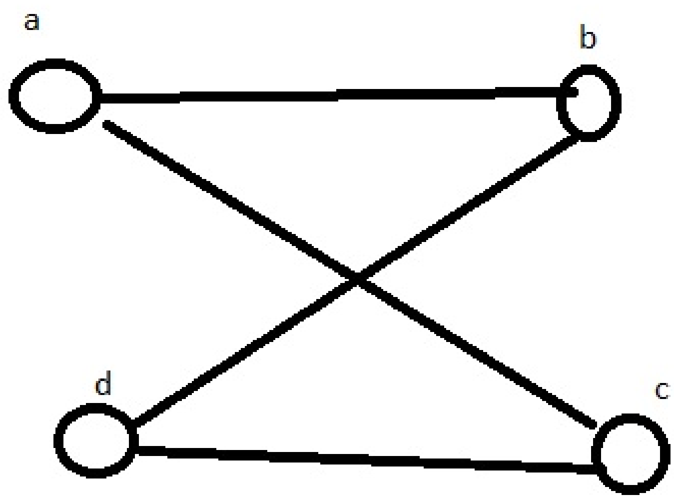

Figure 1: Sample graph

GRAPH THEORY ☺

Definition a *graph* G consists of a (finite) set of V *vertices* and a set of E *edges* where each *edge* in E is a subset of V of size 2.

Example

$$V = \{a, b, c, d\}$$

$$E = \{\{a, b\}, \{a, c\}, \{c, d\}, \{b, d\}\}$$

We can draw G by using a point for each vertex and a line segment joining a to b for each edge $\{a, b\}$.

(Note in a set there are only allowed to be unique elements - therefore no repeated edges. Also no loops)
If we wish to allow loops and multiple edges we will use the term *multigraph*.

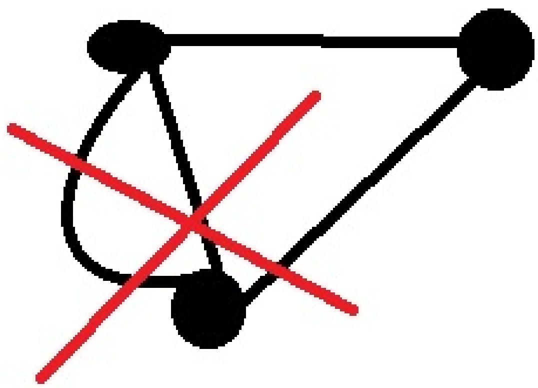

Figure 2: Graph with duplicate edges

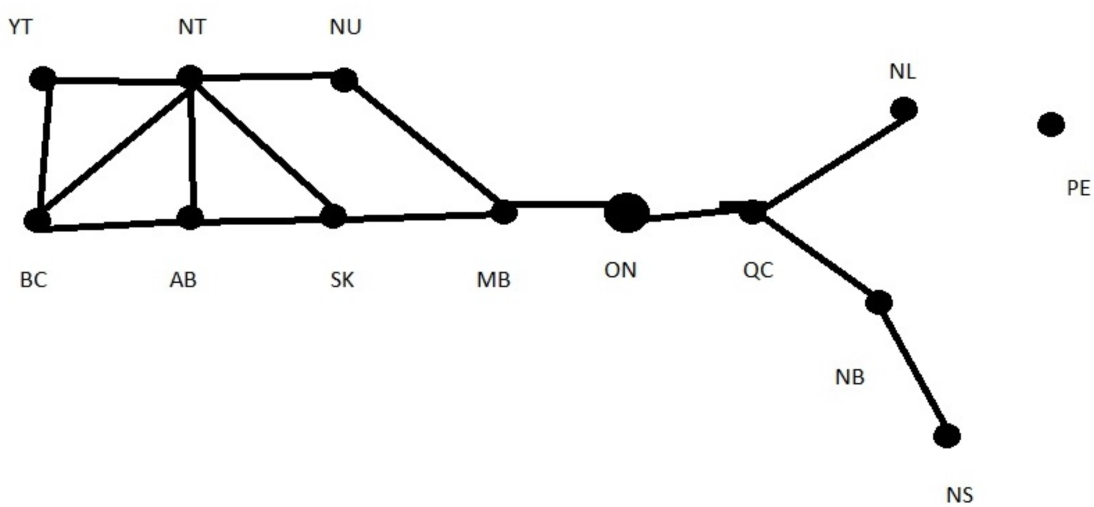

Figure 3: Province Graph

A *vertex* v (one element of the set of vertices) is said to be *adjacent* to a vertex w if $vw \in E$.
E.g. ON is adjacent to QC (and also to MB). (ref province graph)

We also say that "w is a *neighbour* of v "
We also say that edge vw "joins" v and w

The number of neighbours of a vertex v is called its *degree* and written deg(v).

A vertex of degree 0 is called an *isolated vertex*.

A vertex v is incident to each edge vw and v is an endpoint of vw.

If a graph is isomorphic then

There is a function $f : V(A) \rightarrow V(B)$, called an *isomorphism*.
Satisfying:

1. f is a *bijection* from $V(A)$ to $V(B)$
2. $f(a)f(b)$ is an edge of B if and onlify if ab is an edge of A

Essentially an isomorphic graphs just need to be relabled to be equivalent.

We write this as $A \approx B$

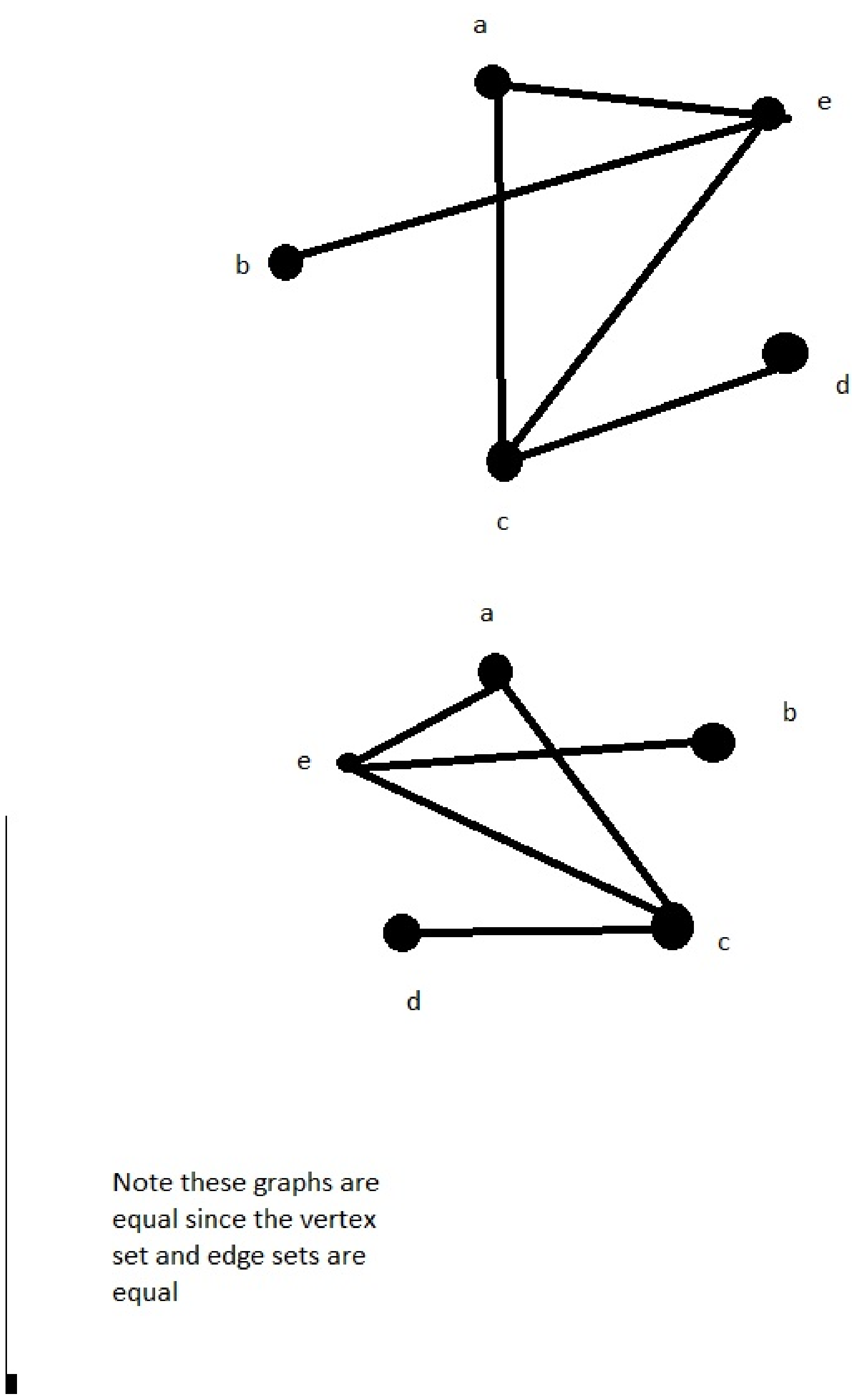

Figure 4: Identical Graphs

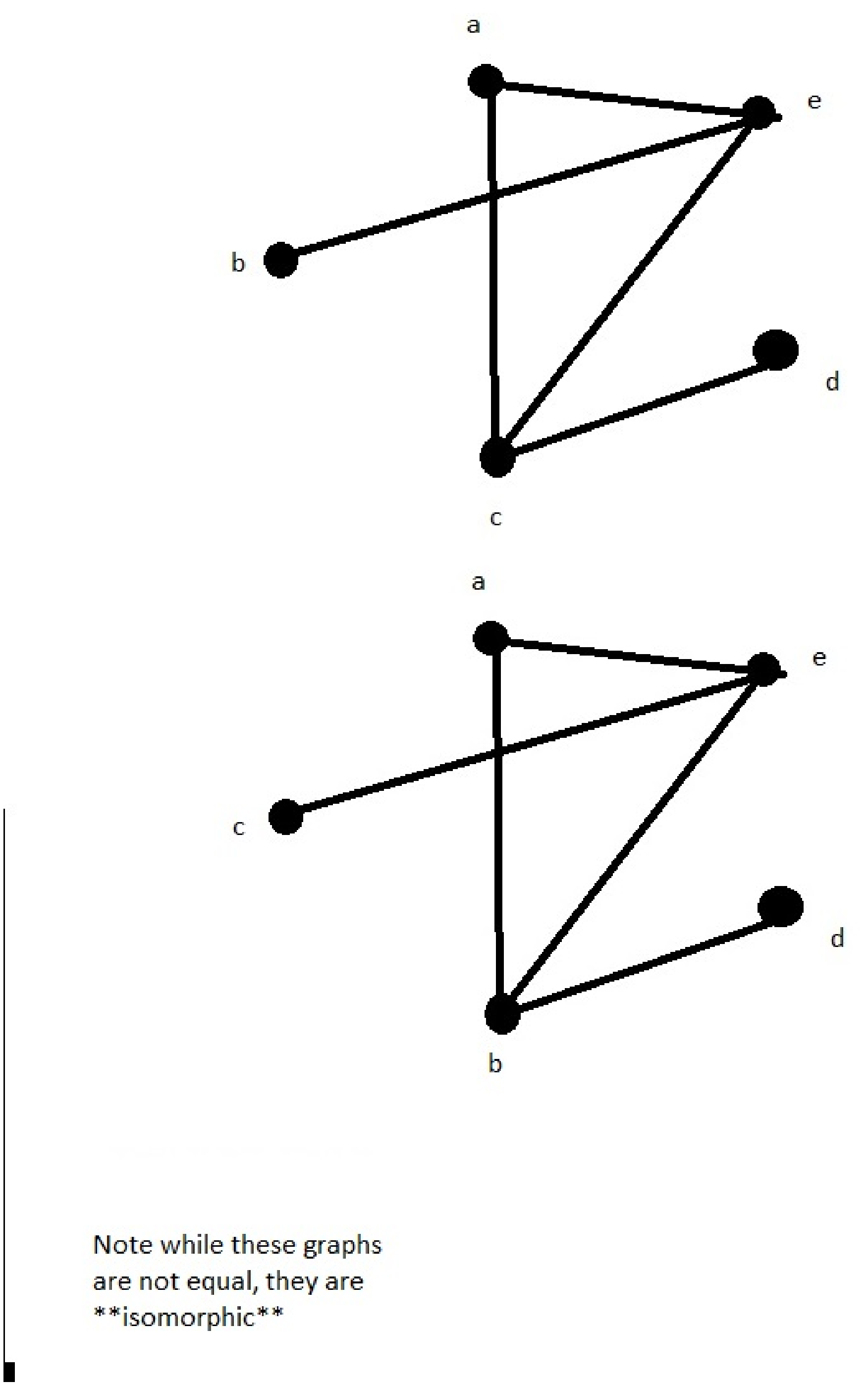

Figure 5: Isomorphic Graphs