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Math for Machine Learning

Linear algebra - Week 3

Vectors

Matrices

Dot product

Matrix multiplication

Linear transformations



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Vectors and Linear Transformations

Machine Learning motivation

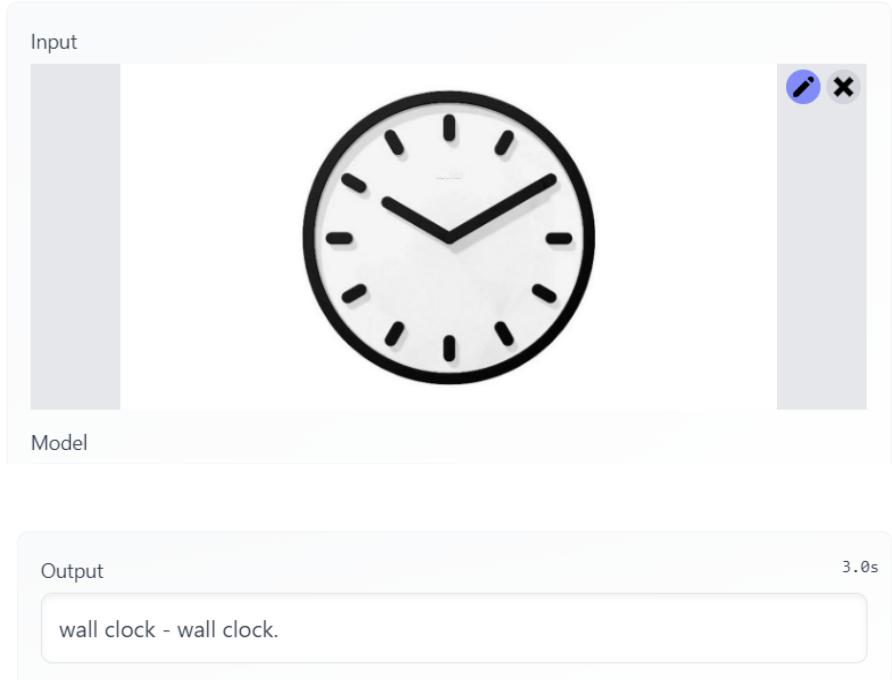
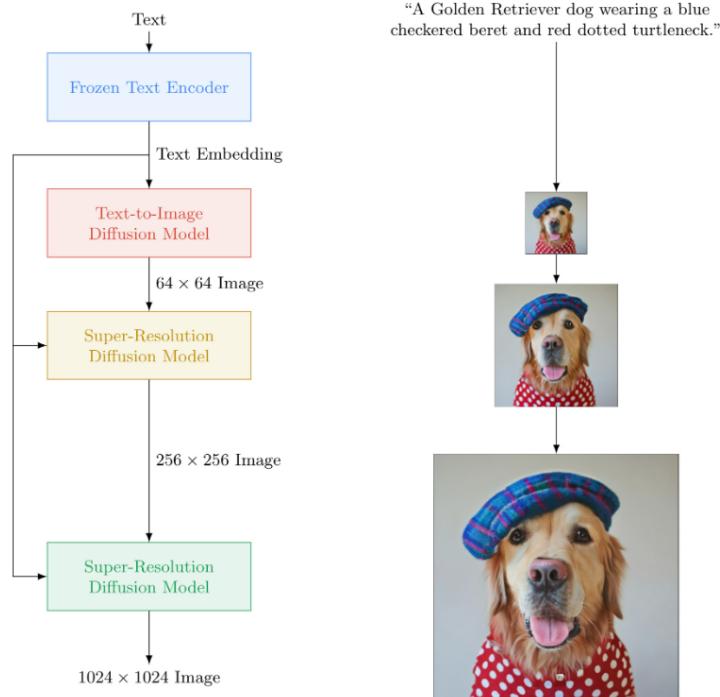
Neural Networks - AI generated images



AI-generated human faces.

- Generative learning: Generating realistic looking images.

Text-to-image and image-to-text generation



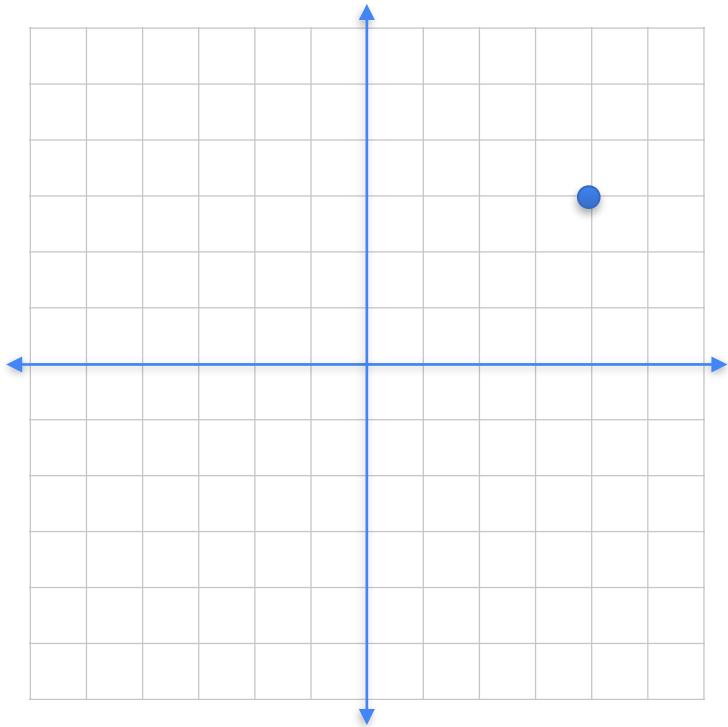


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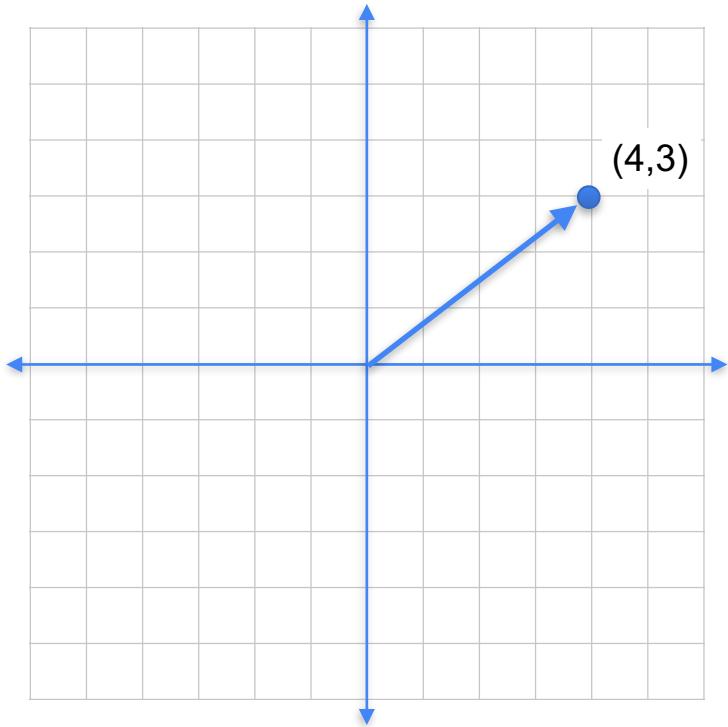
Vectors and Linear Transformations

Vectors and their properties

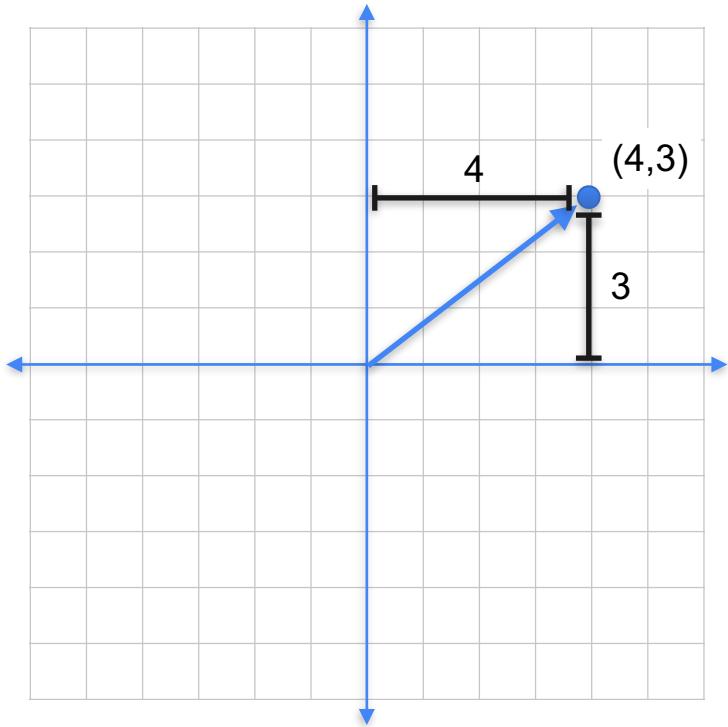
Vectors



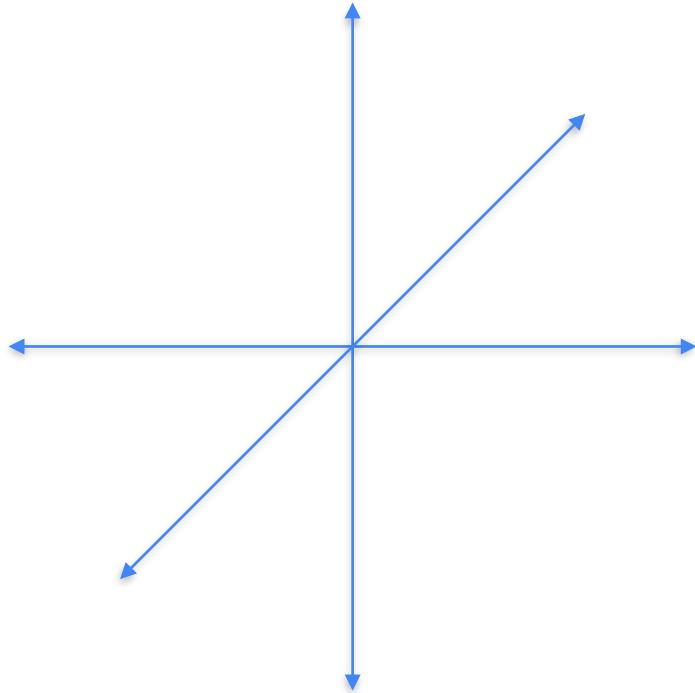
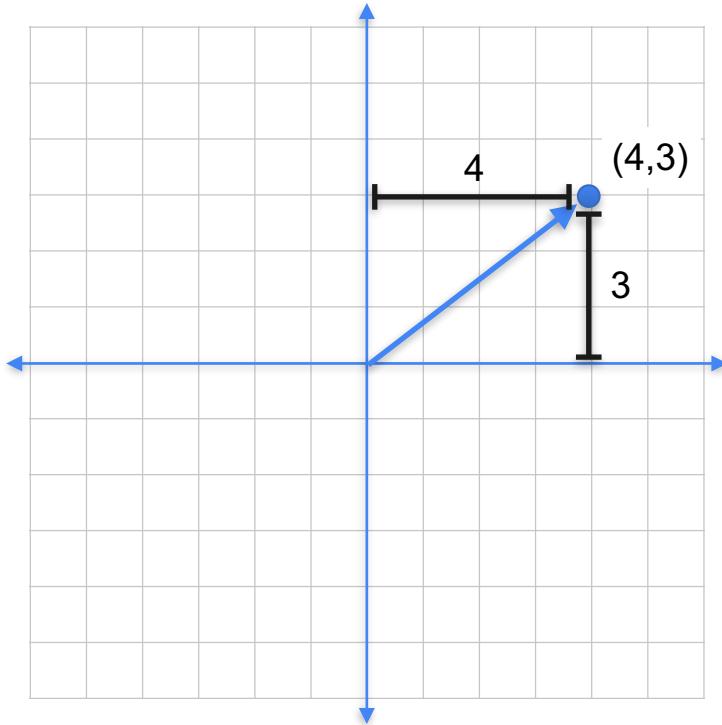
Vectors



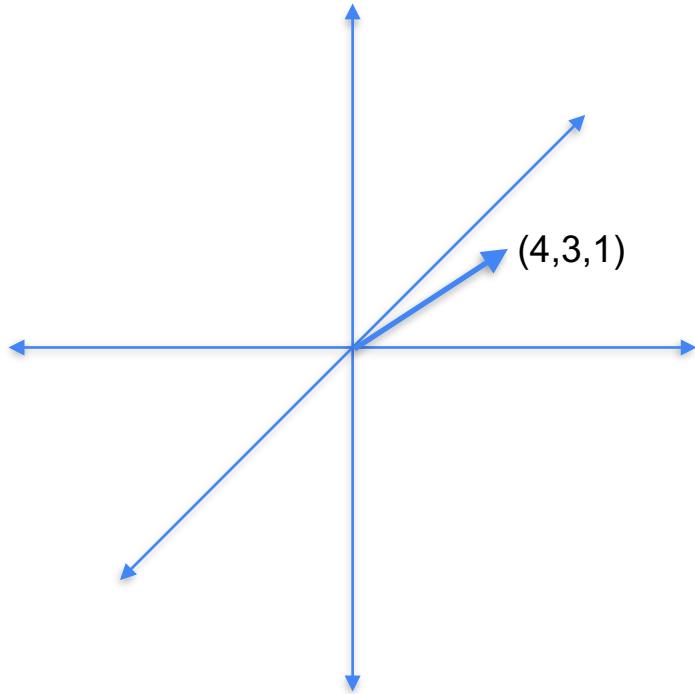
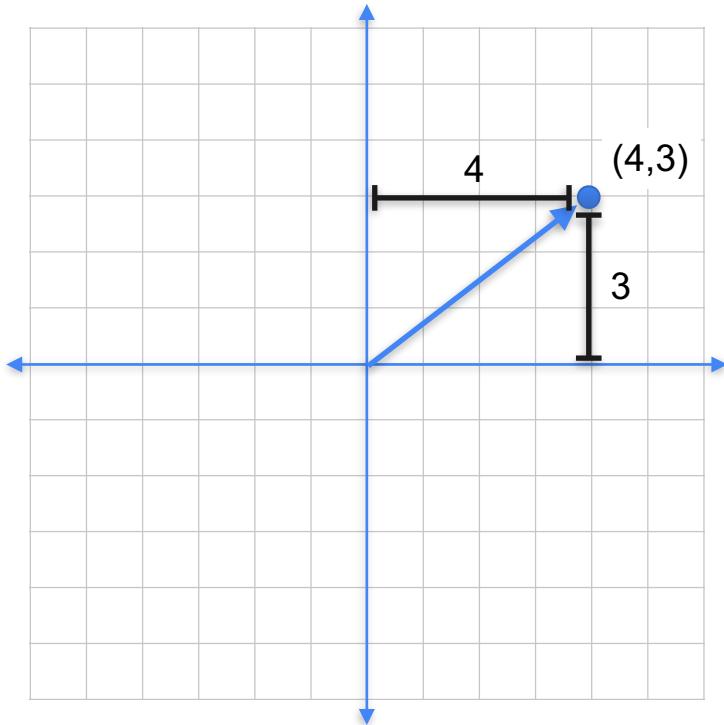
Vectors



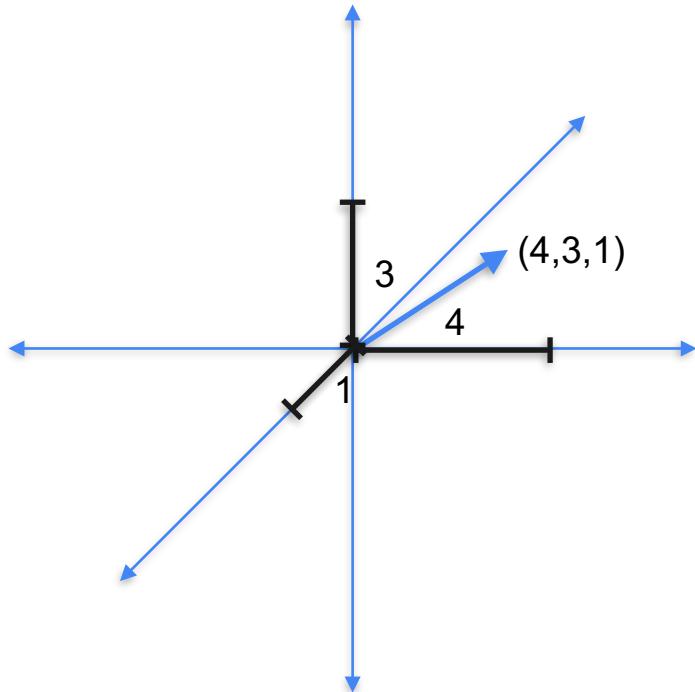
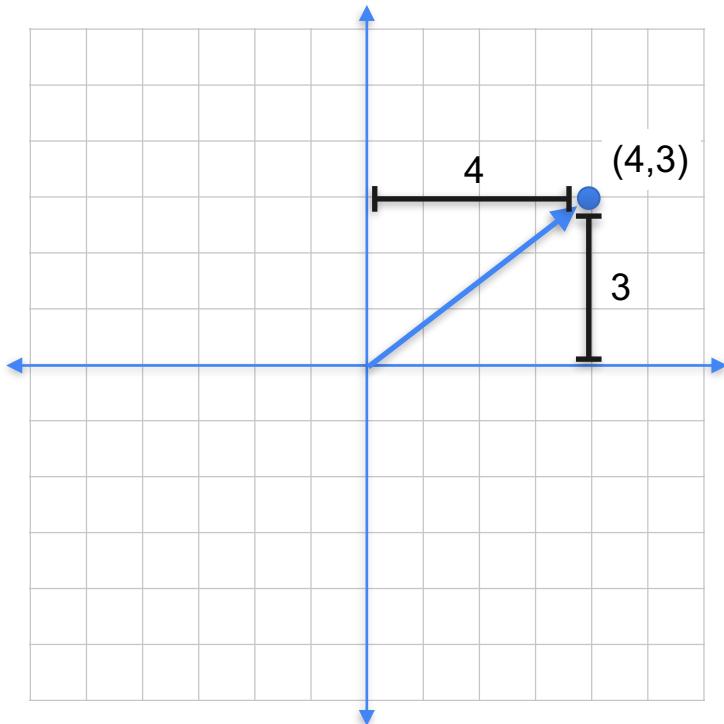
Vectors



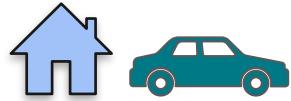
Vectors



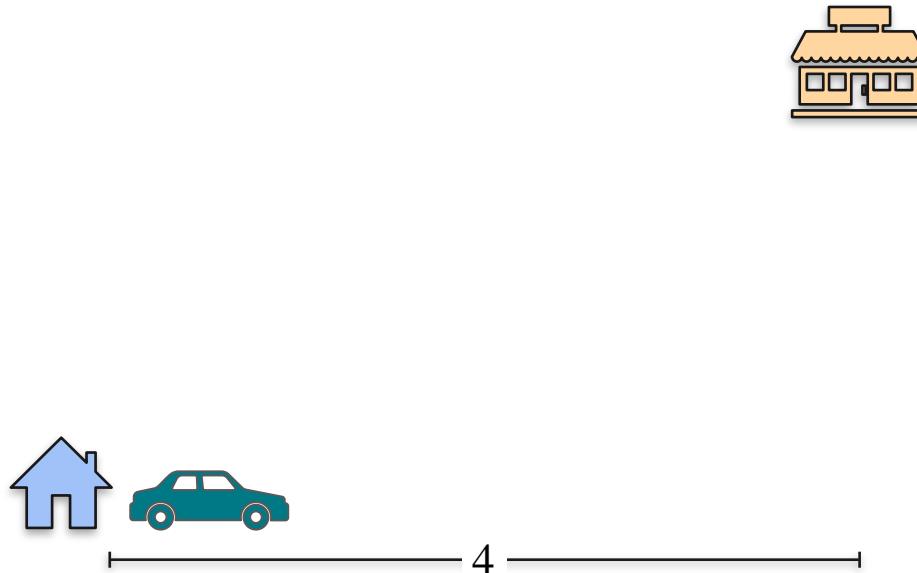
Vectors



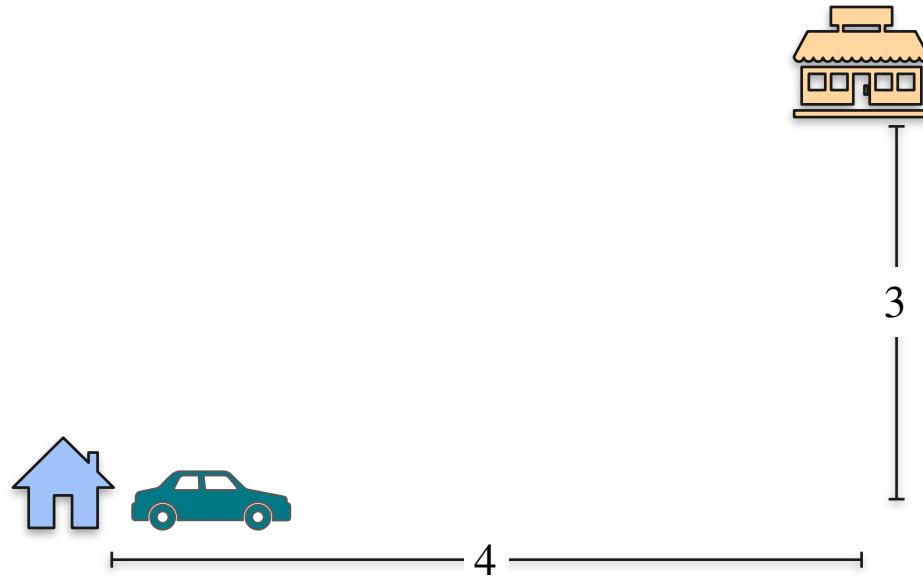
How to get from point A to point B?



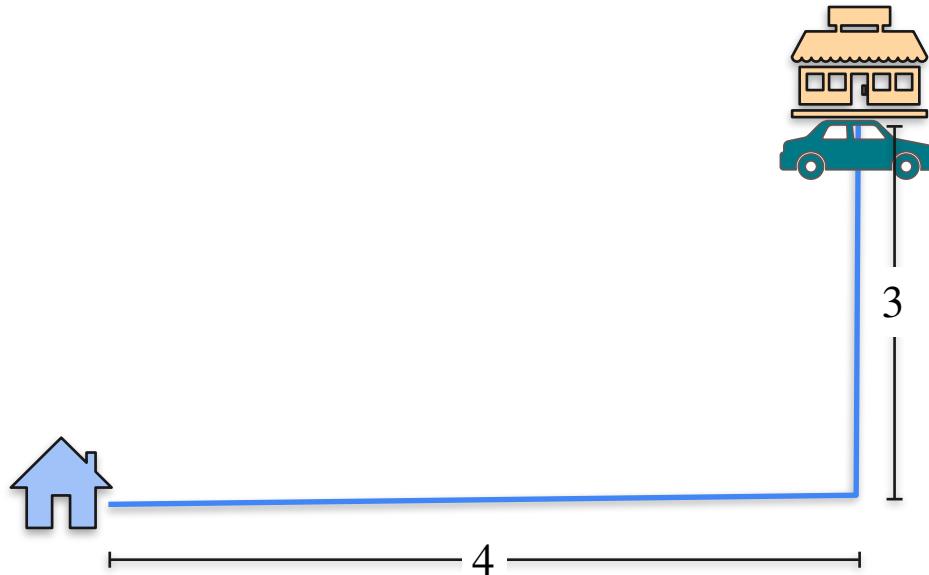
How to get from point A to point B?



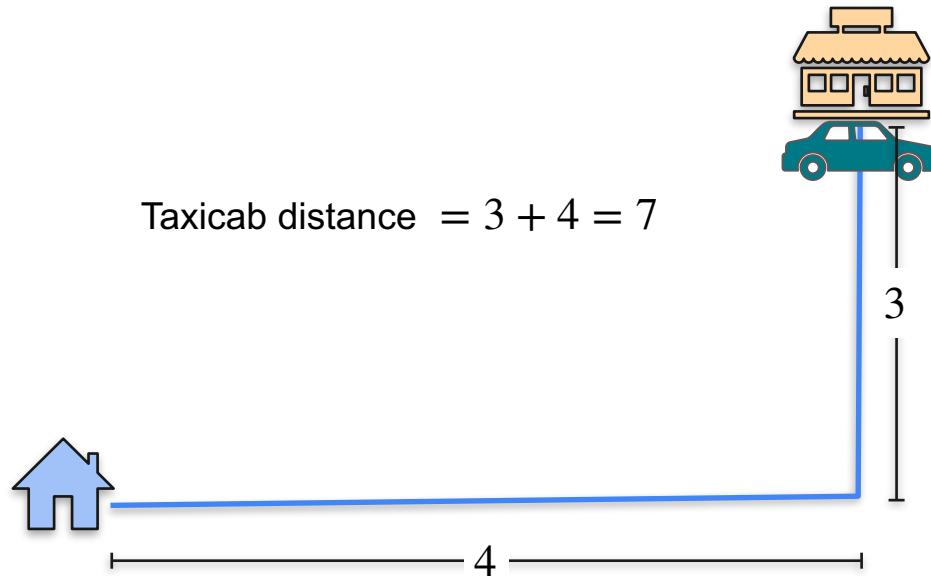
How to get from point A to point B?



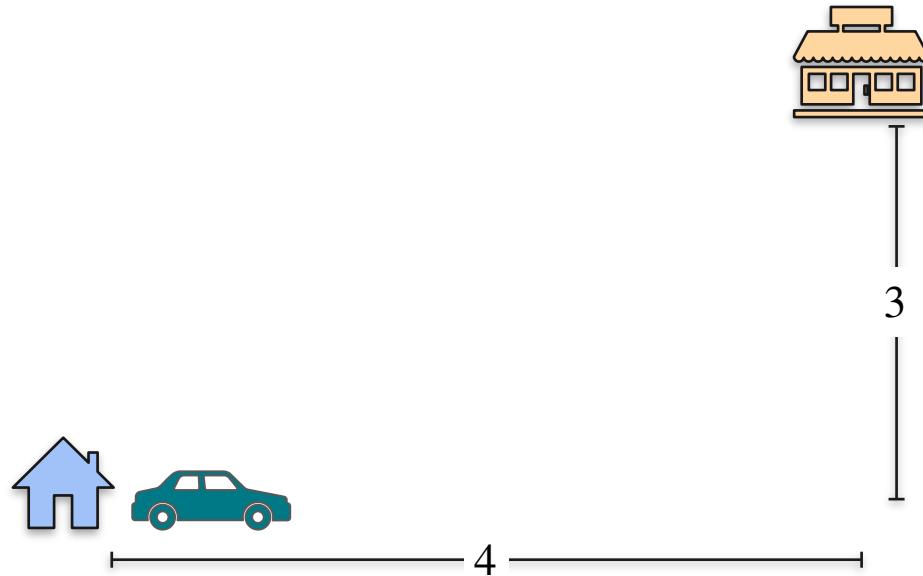
How to get from point A to point B?



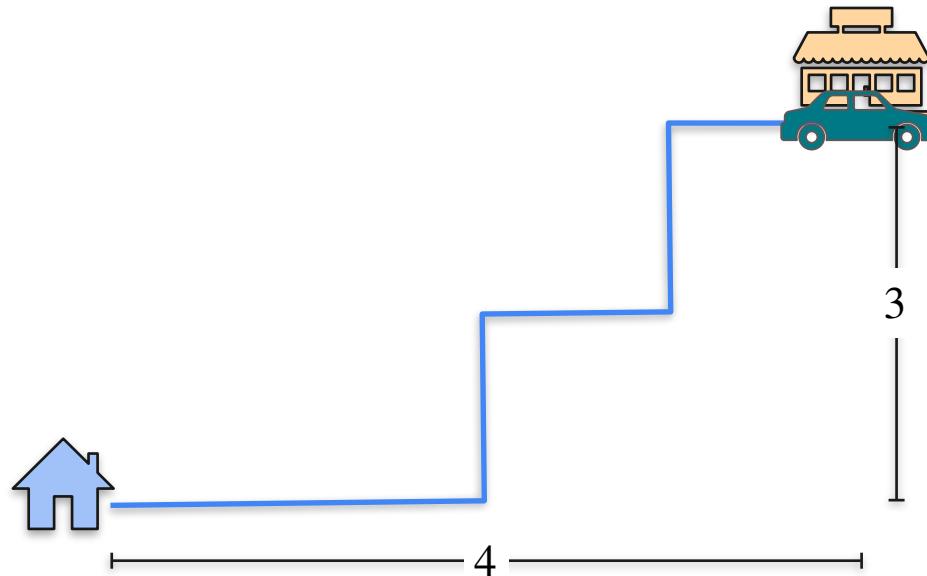
How to get from point A to point B?



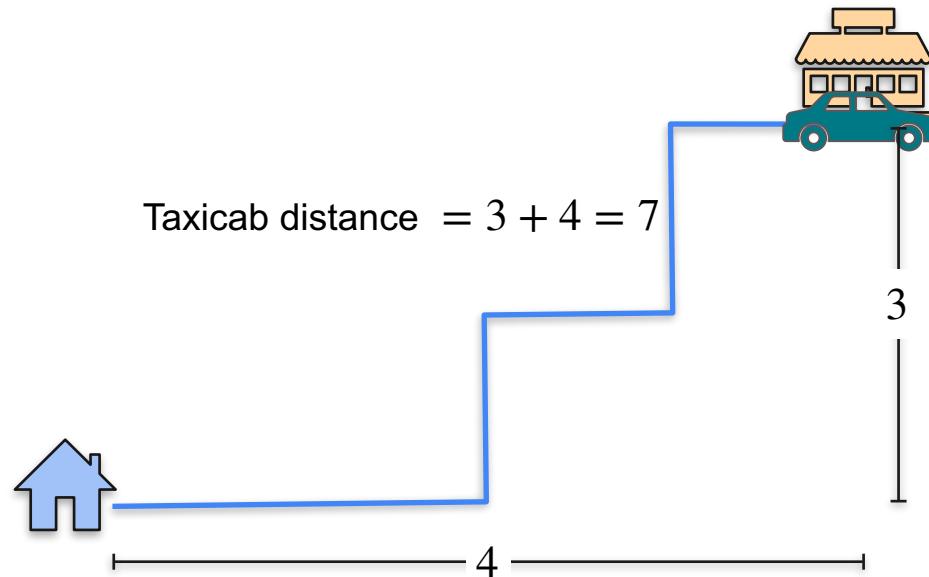
How to get from point A to point B?



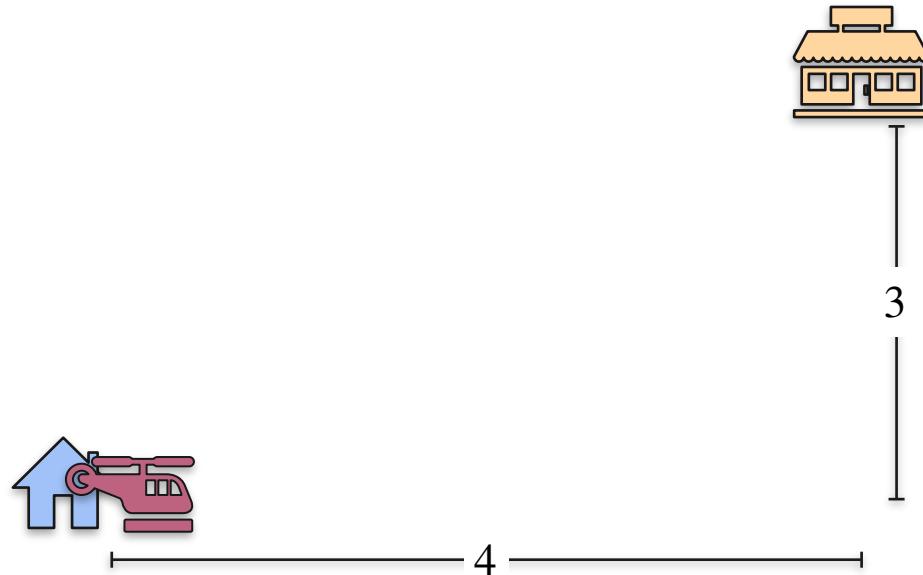
How to get from point A to point B?



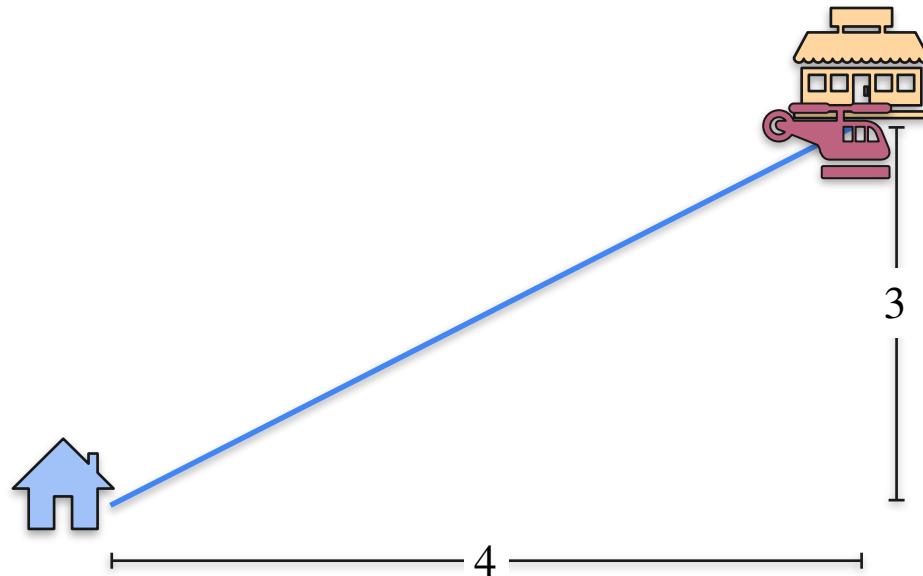
How to get from point A to point B?



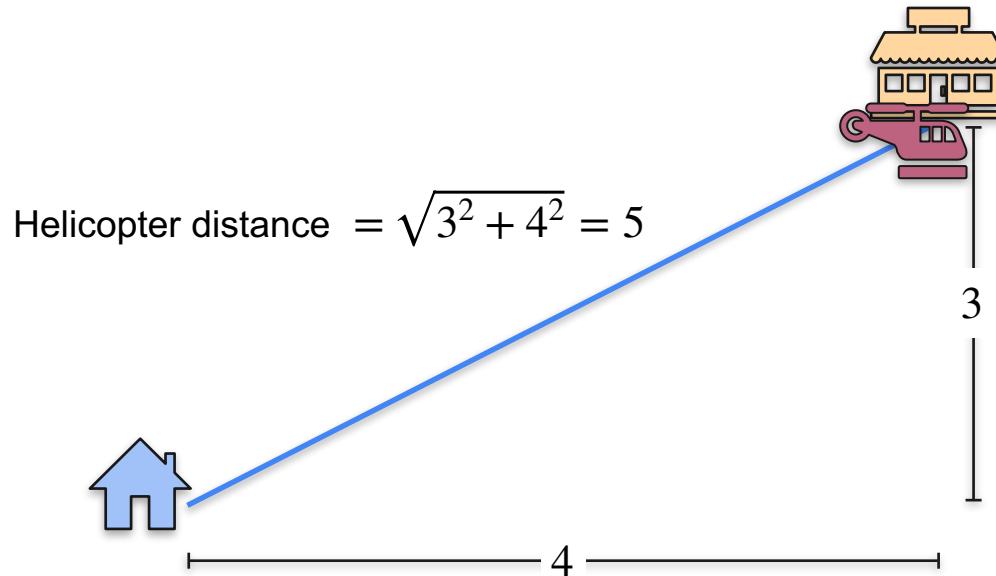
How to get from point A to point B?



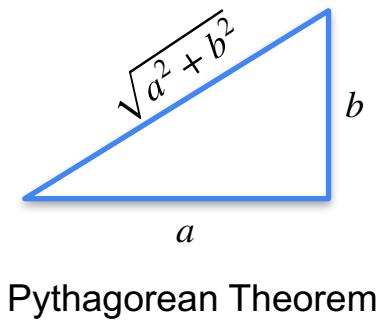
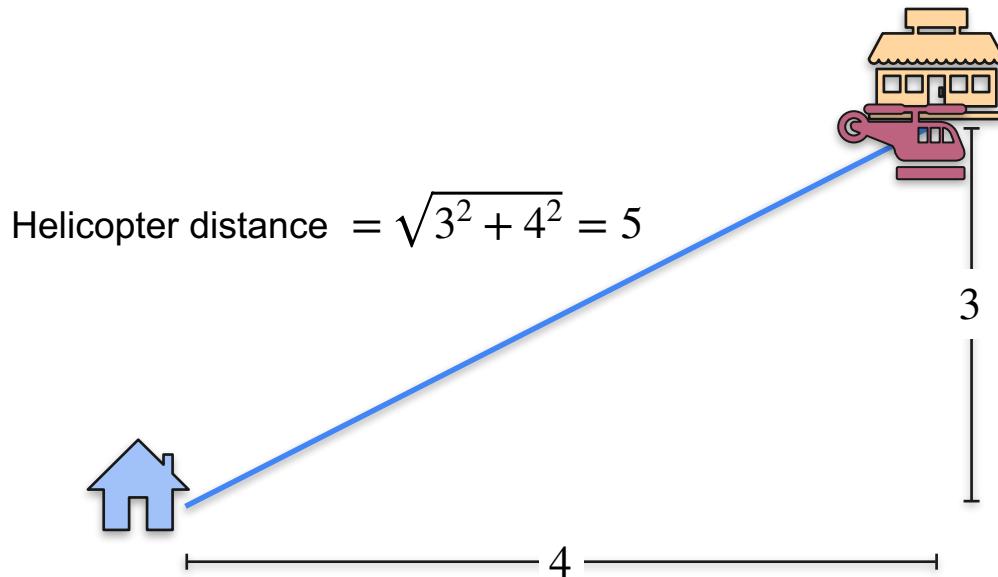
How to get from point A to point B?



How to get from point A to point B?

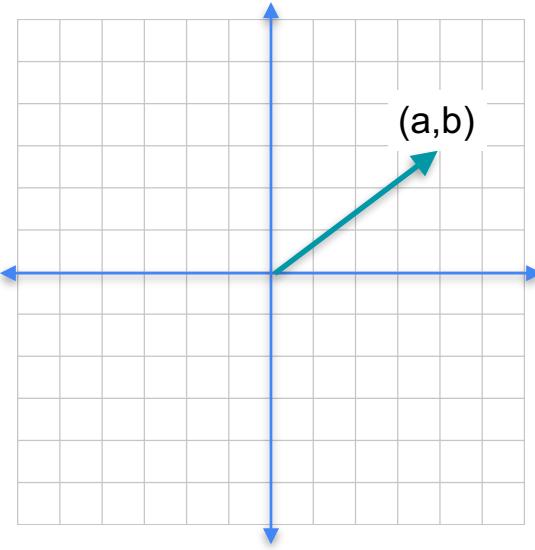


How to get from point A to point B?

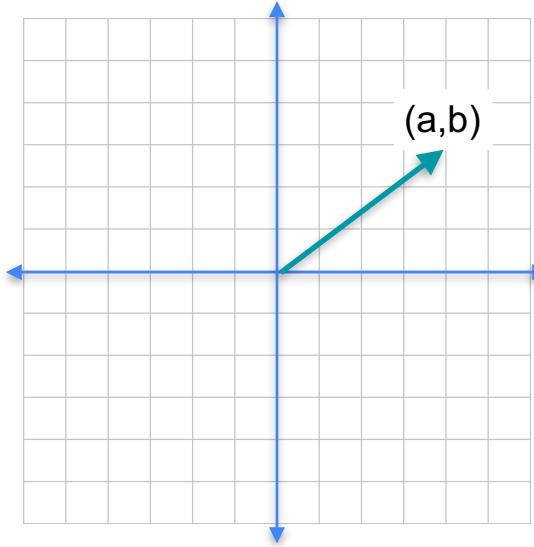


Pythagorean Theorem

Norms

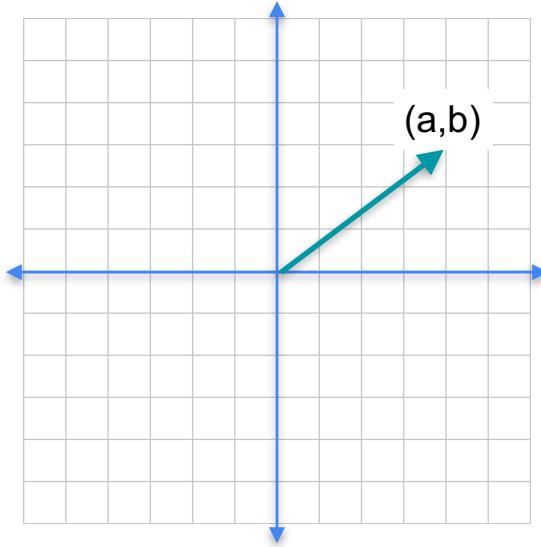


Norms

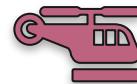


$$\text{L1-norm} = |(a, b)|_1 = |a| + |b|$$

Norms

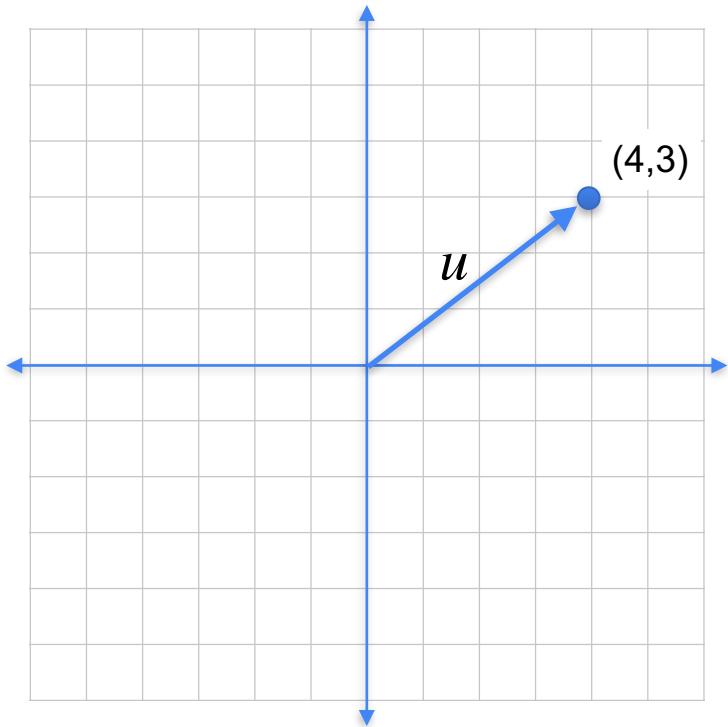


$$\text{L1-norm} = |(a, b)|_1 = |a| + |b|$$

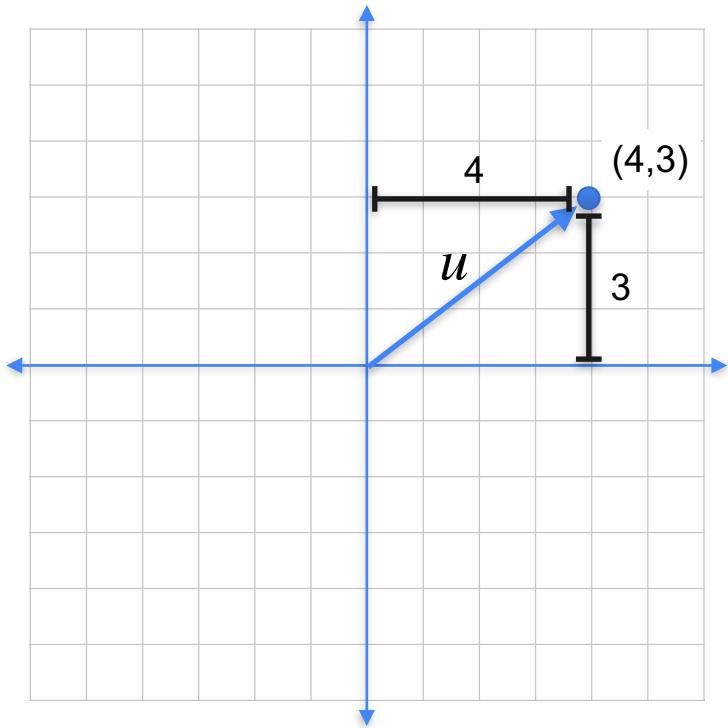


$$\text{L2-norm} = |(a, b)|_2 = \sqrt{a^2 + b^2}$$

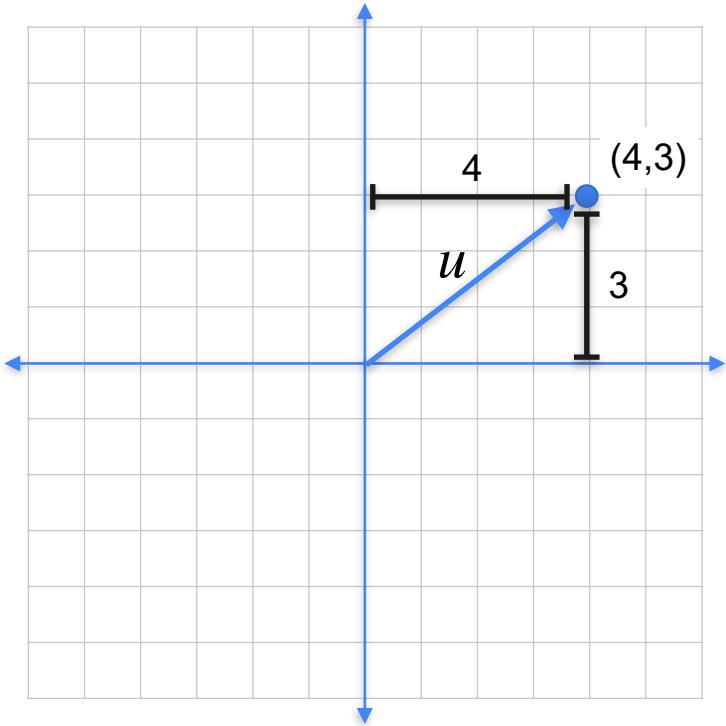
Norm of a vector



Norm of a vector

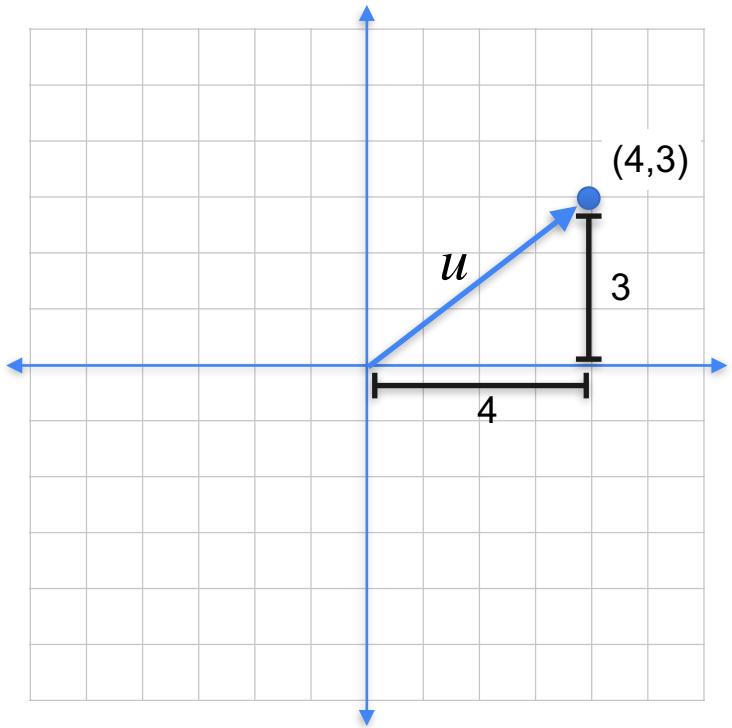


Norm of a vector

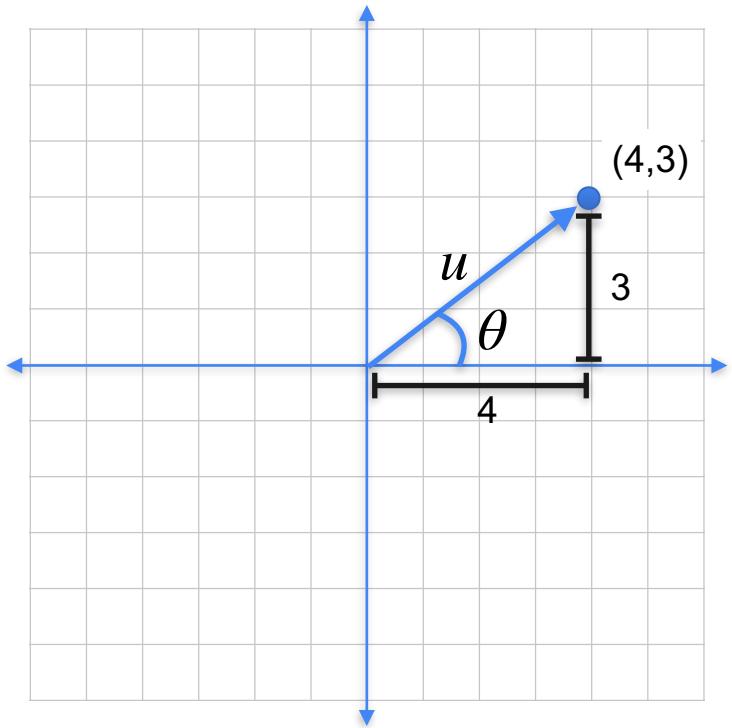


$$\sqrt{4^2 + 3^2} = \sqrt{25} = 5$$

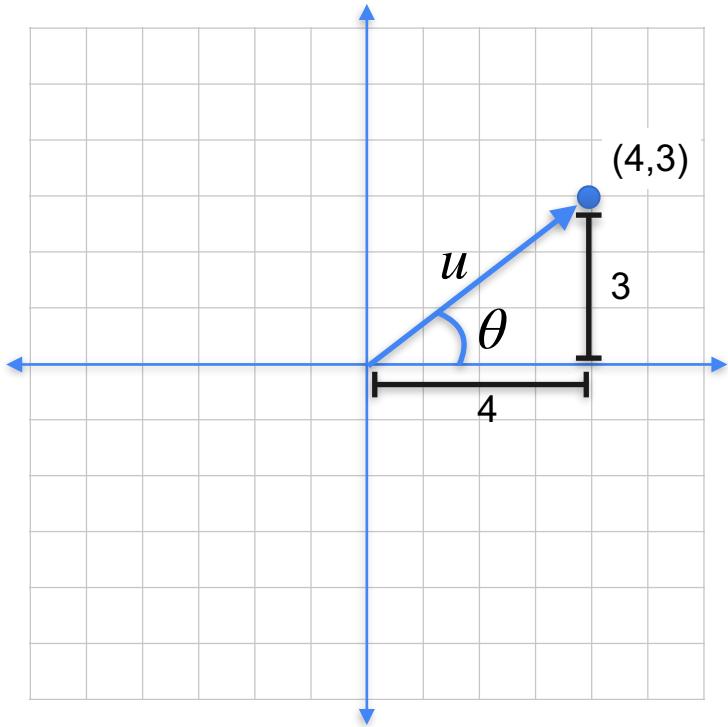
Direction of a vector



Direction of a vector

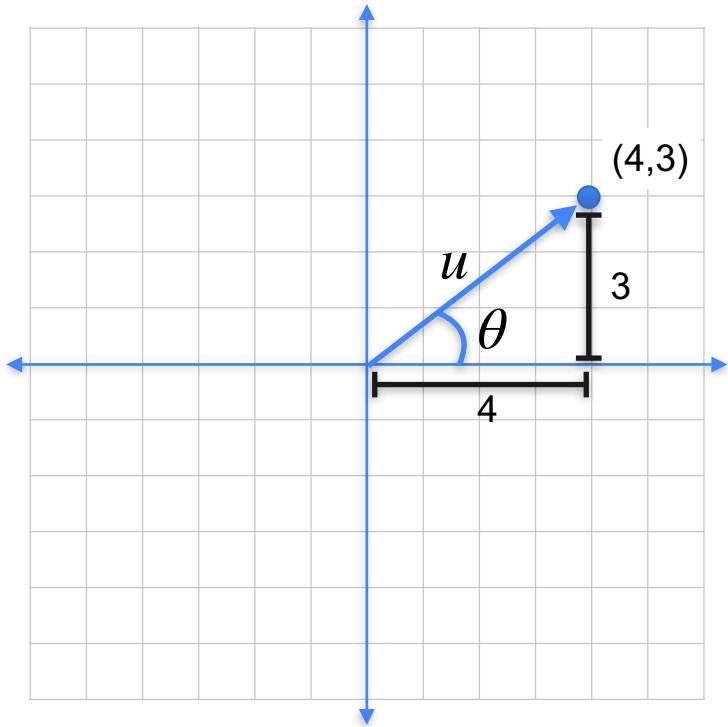


Direction of a vector



$$\tan(\theta) = \frac{3}{4}$$

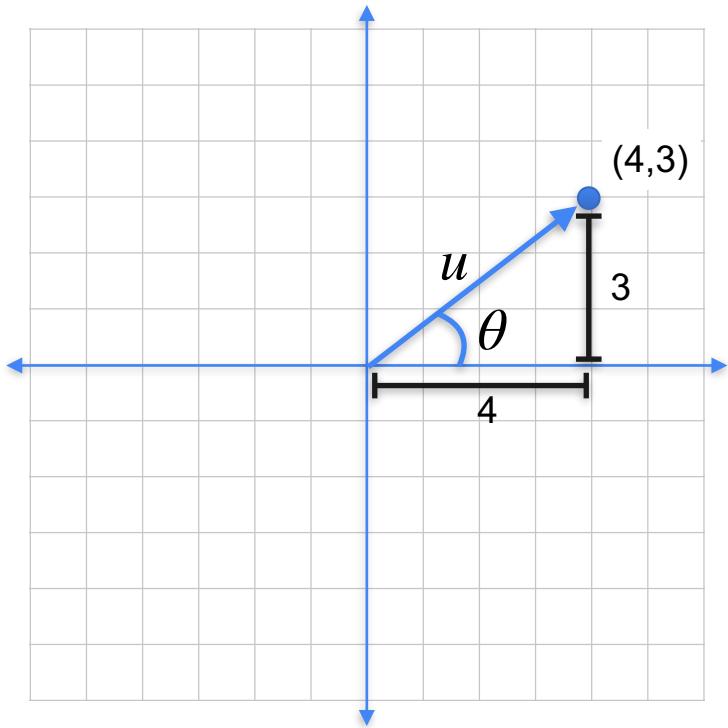
Direction of a vector



$$\tan(\theta) = \frac{3}{4}$$

$$\theta = \arctan(3/4) = 0.64$$

Direction of a vector



$$\tan(\theta) = \frac{3}{4}$$

$$\theta = \arctan(3/4) = 0.64 = 36.87^\circ$$

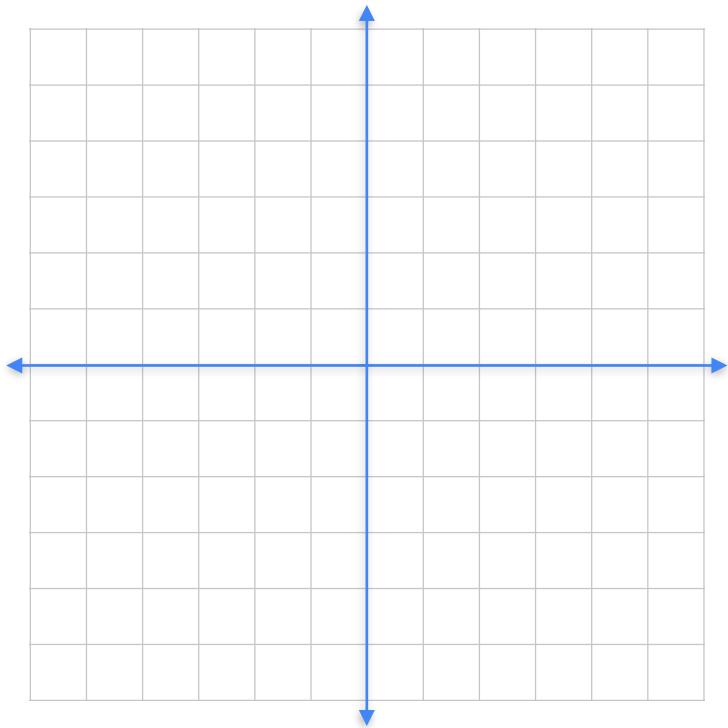


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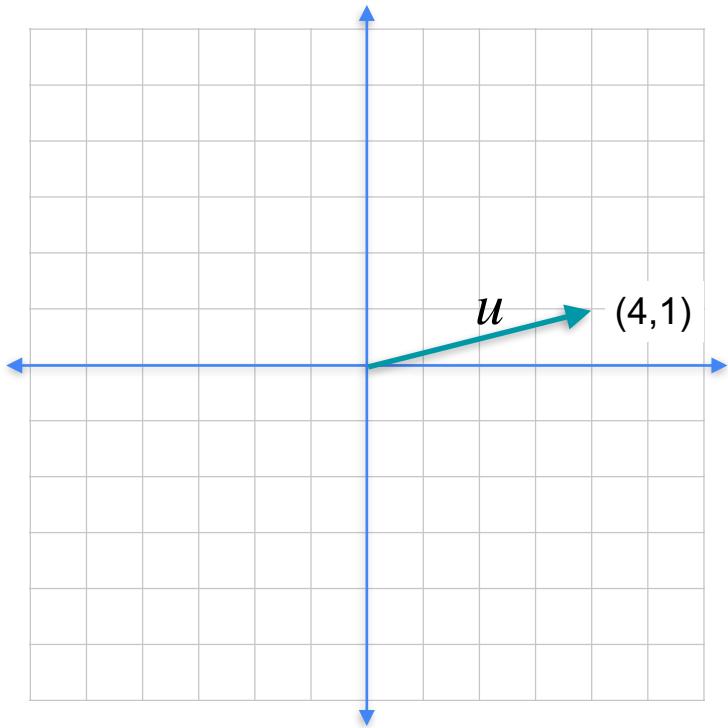
Vectors and Linear Transformations

**Sum and difference of
vectors**

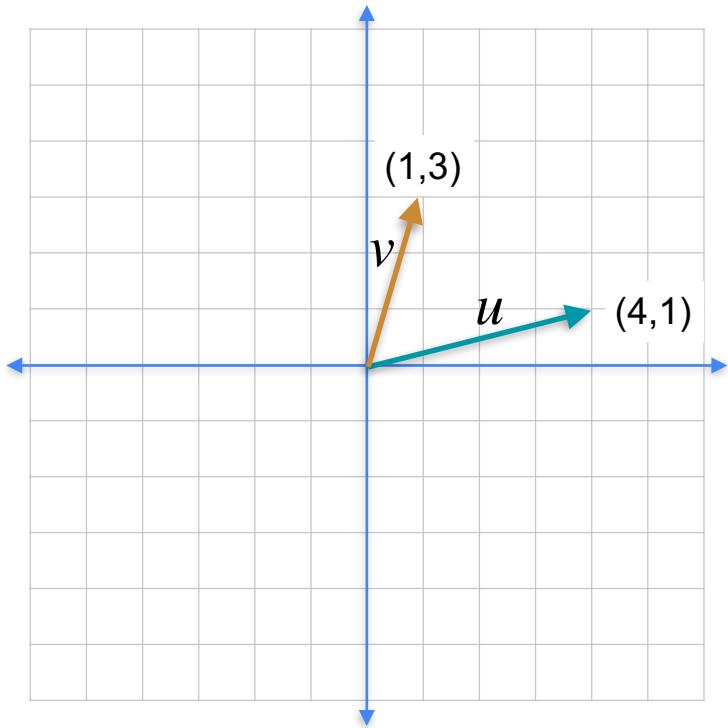
Sum of vectors



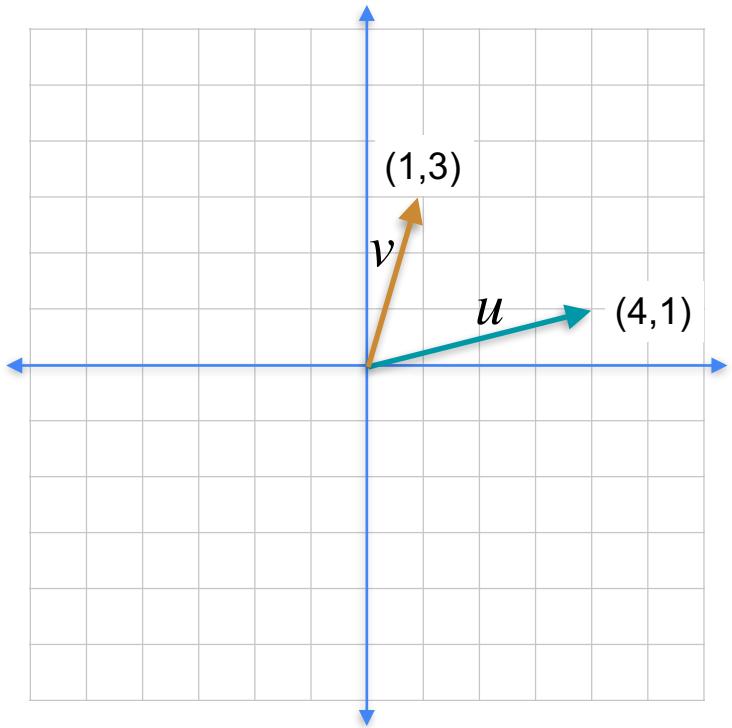
Sum of vectors



Sum of vectors

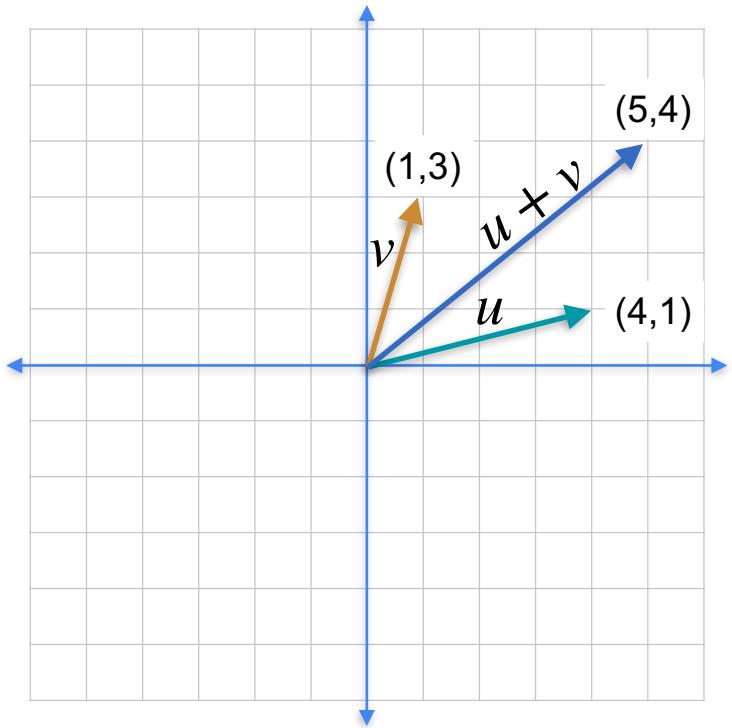


Sum of vectors



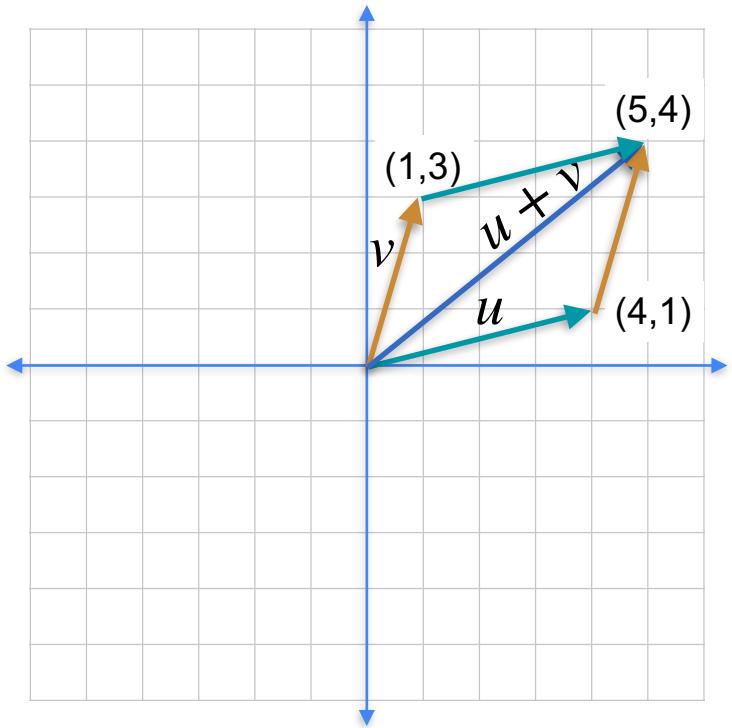
$$u + v = (4 + 1, 1 + 3) = (5, 4)$$

Sum of vectors



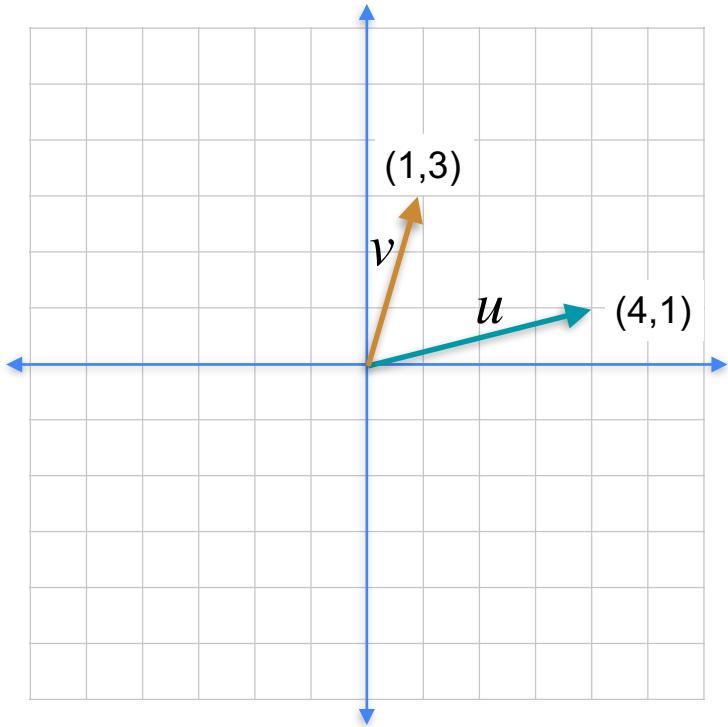
$$u + v = (4 + 1, 1 + 3) = (5, 4)$$

Sum of vectors

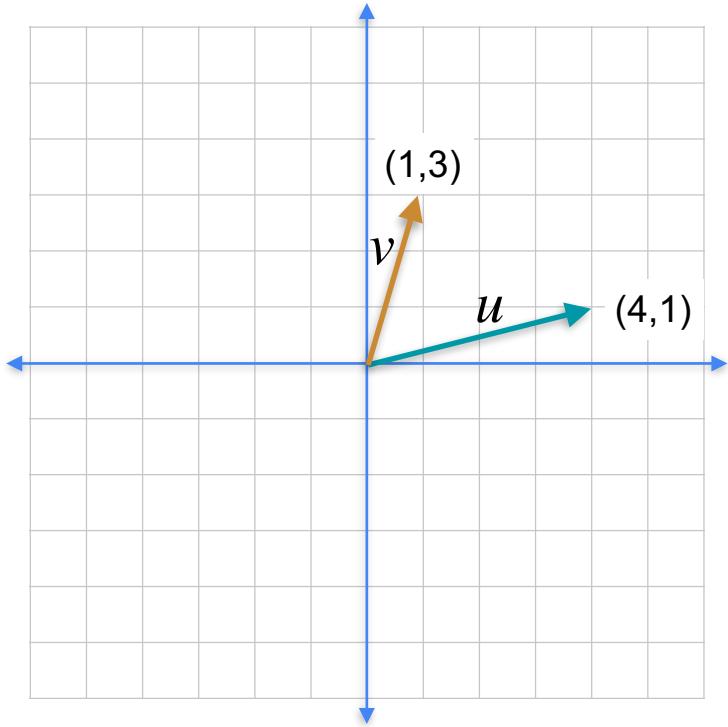


$$u + v = (4 + 1, 1 + 3) = (5, 4)$$

Difference of vectors

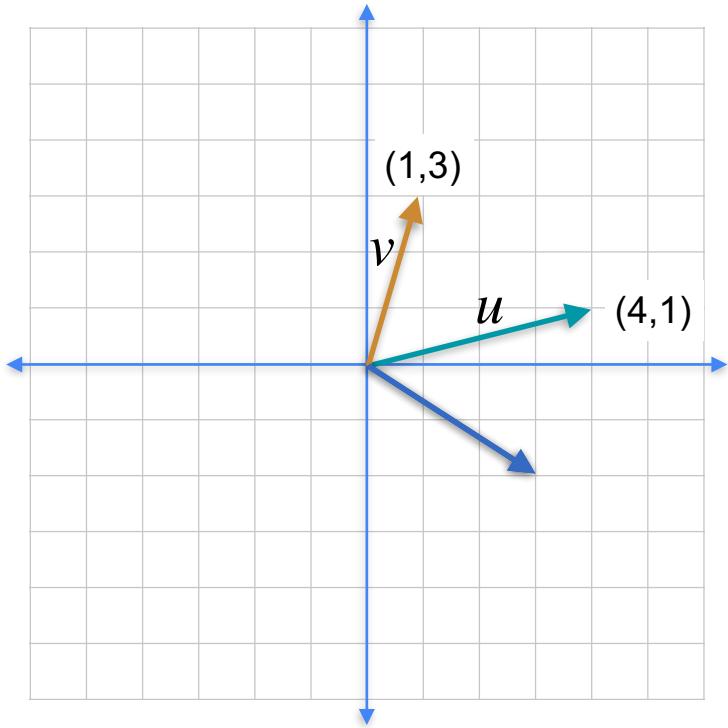


Difference of vectors



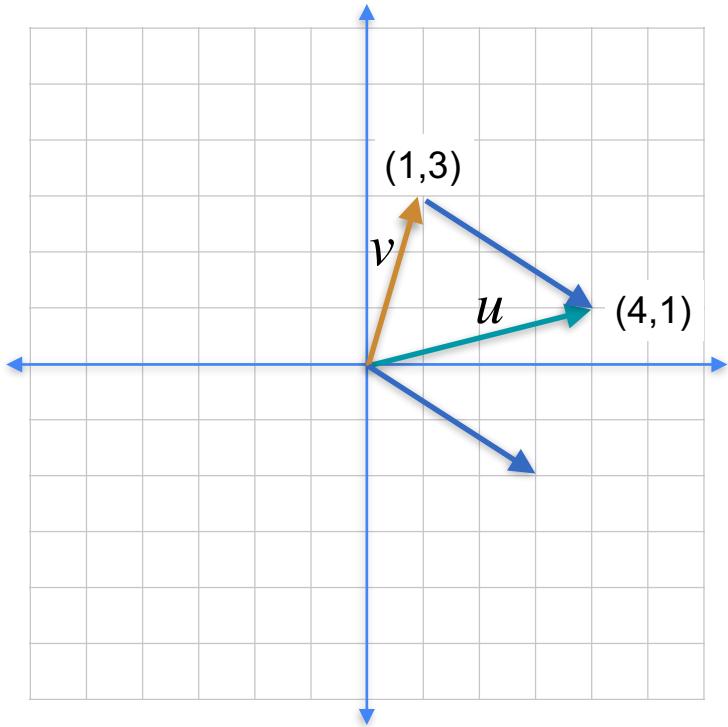
$$u - v = (4 - 1, 1 - 3) = (3, -2)$$

Difference of vectors



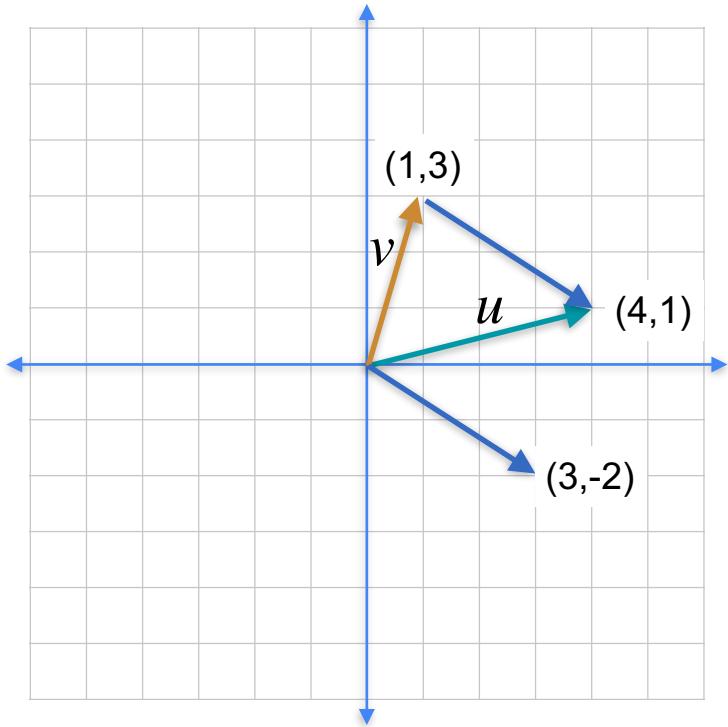
$$u - v = (4 - 1, 1 - 3) = (3, -2)$$

Difference of vectors



$$u - v = (4 - 1, 1 - 3) = (3, -2)$$

Difference of vectors



$$u - v = (4 - 1, 1 - 3) = (3, -2)$$

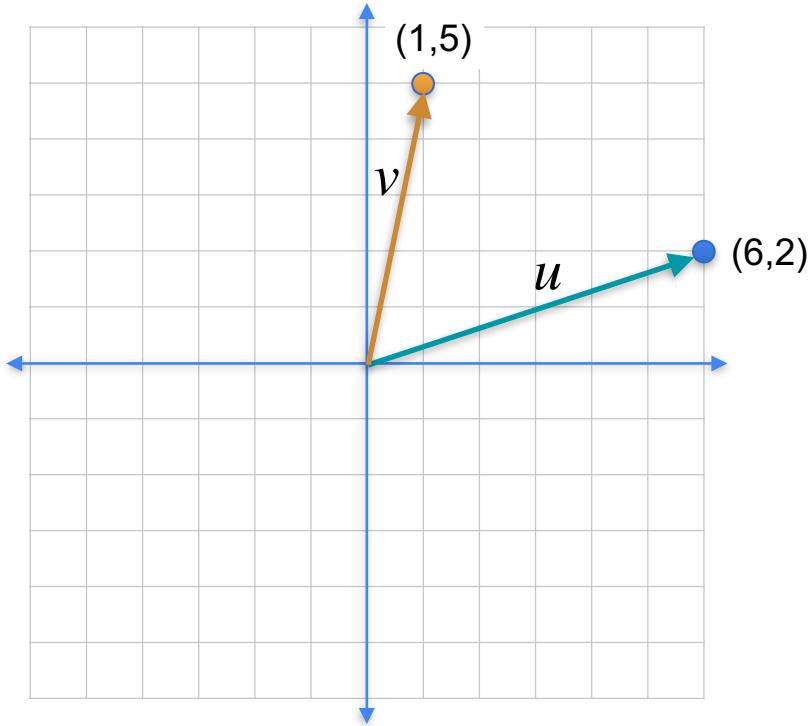


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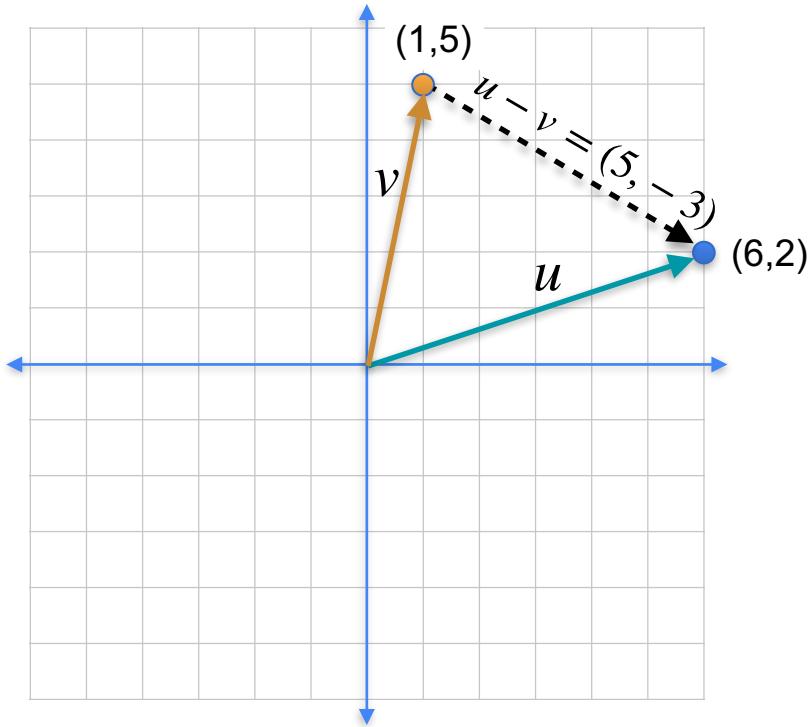
Vectors and Linear Transformations

Distance between vectors

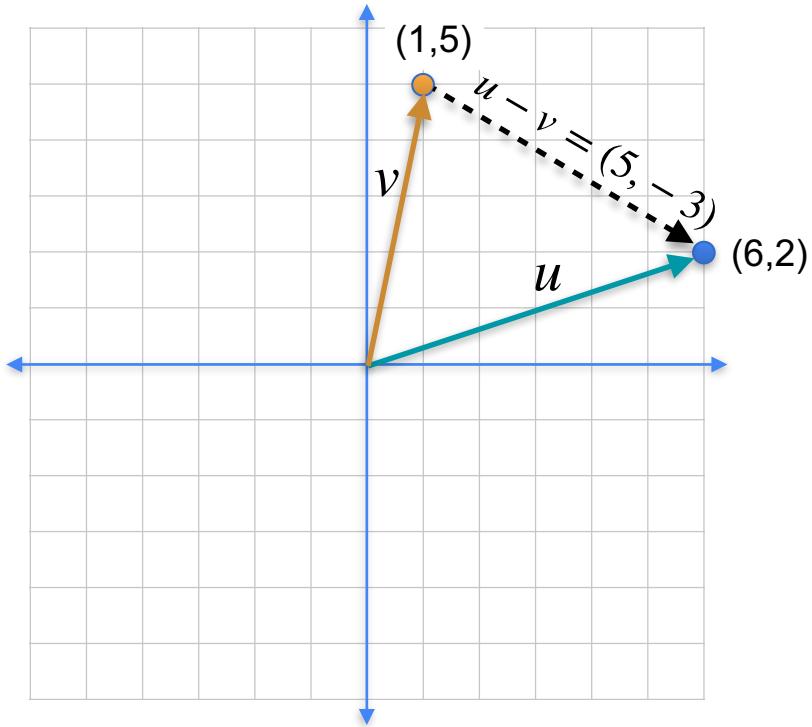
Distances



Distances



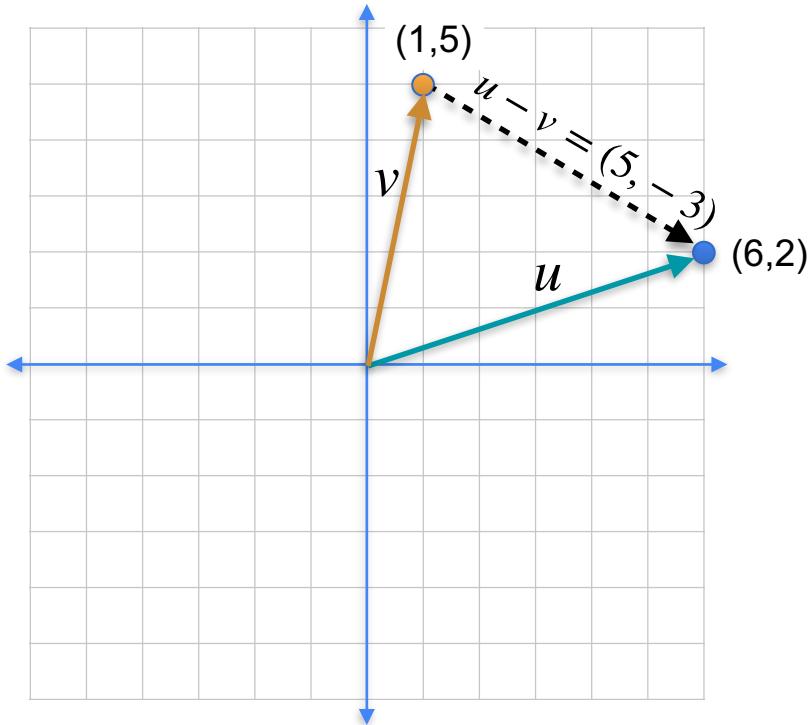
Distances



L1-distance

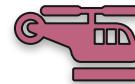
$$||u - v||_1 = |5| + |-3| = 8$$

Distances



L1-distance

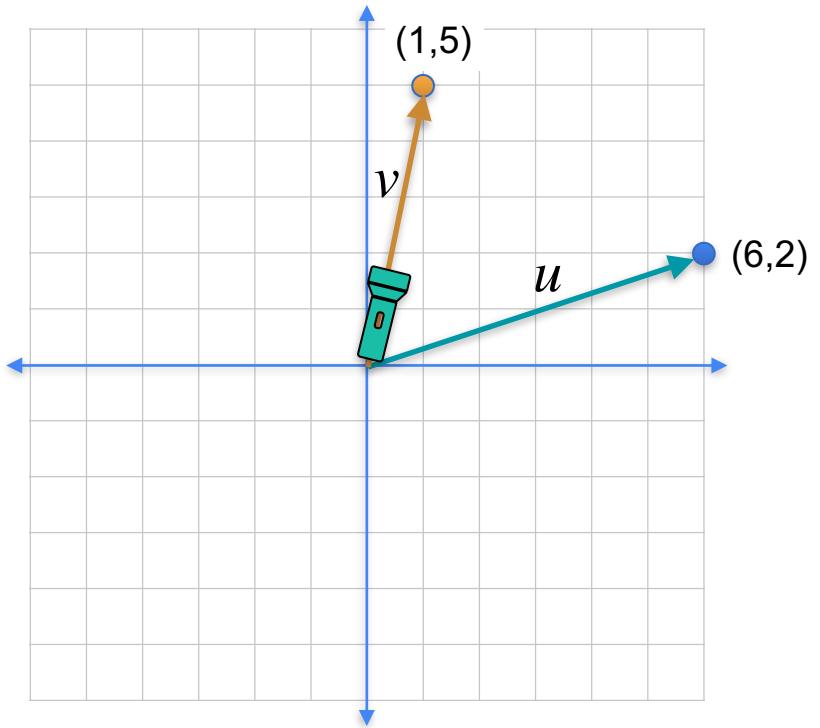
$$|u - v|_1 = |5| + |-3| = 8$$



L2-distance

$$|u - v|_2 = \sqrt{5^2 + 3^2} = 5.83$$

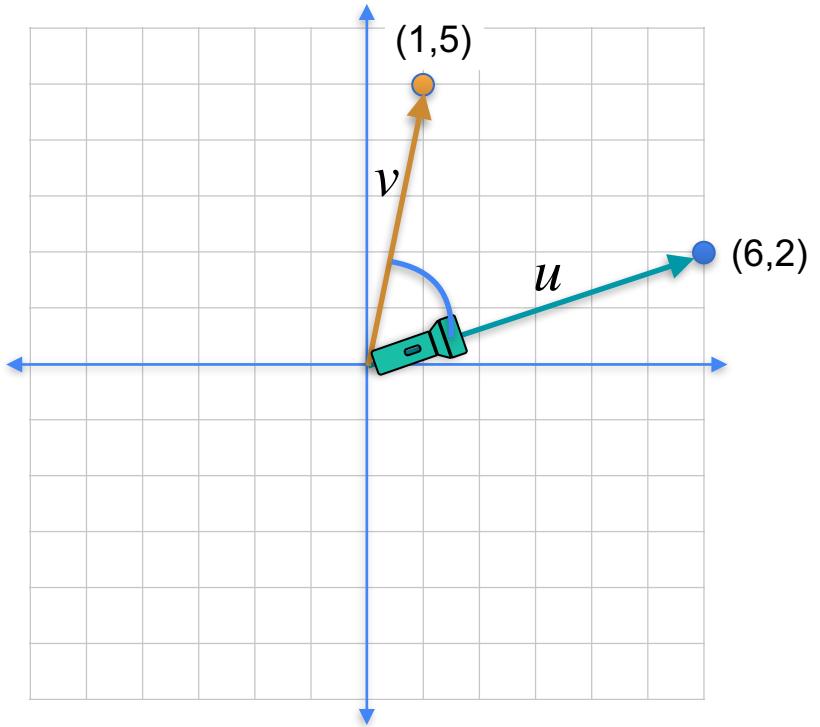
Distances



L1-distance $|u - v|_1 = |5| + |-3| = 8$

L2-distance $|u - v|_2 = \sqrt{5^2 + 3^2} = 5.83$

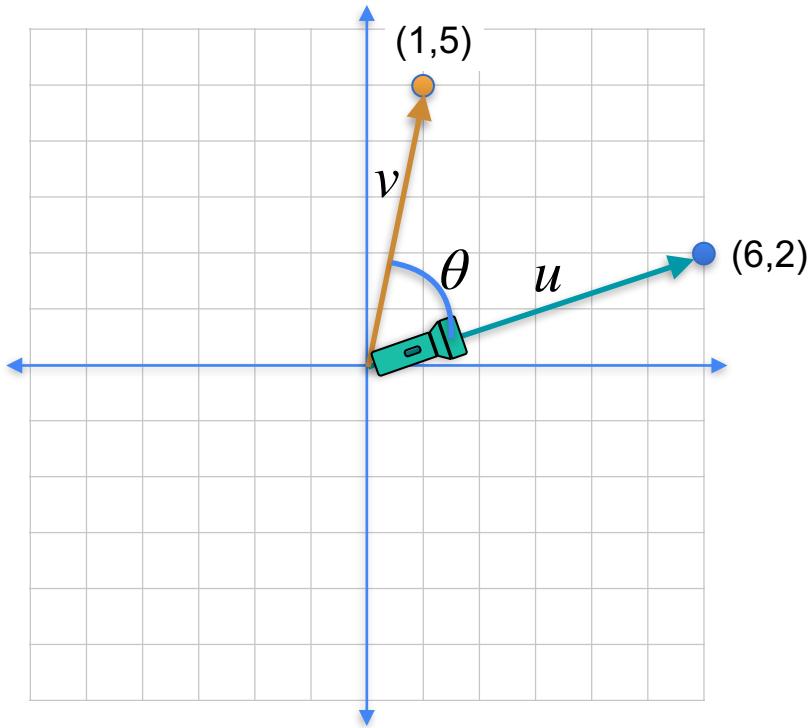
Distances



L1-distance $|u - v|_1 = |5| + |-3| = 8$

L2-distance $|u - v|_2 = \sqrt{5^2 + 3^2} = 5.83$

Distances



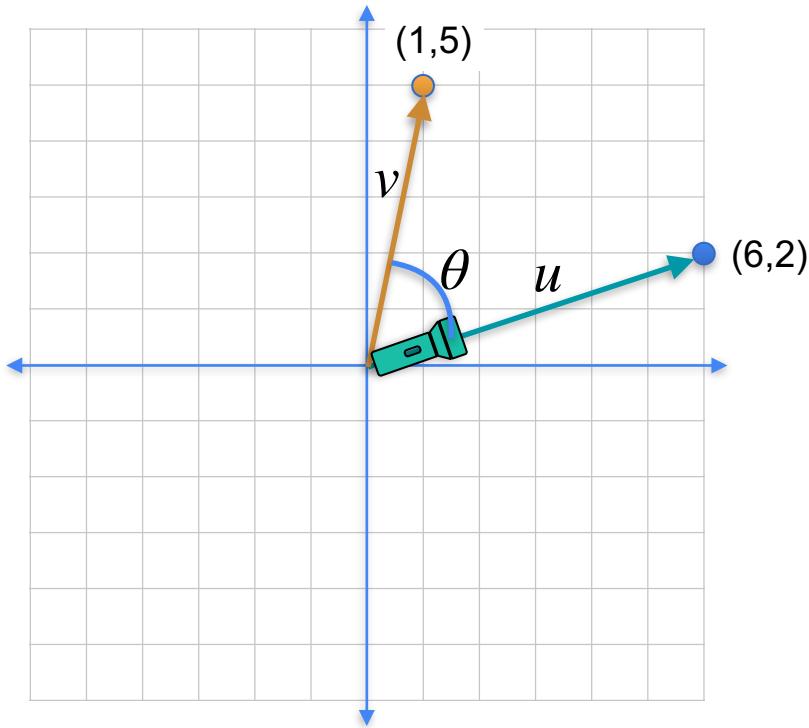


L1-distance $|u - v|_1 = |5| + |-3| = 8$



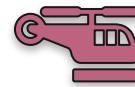
L2-distance $|u - v|_2 = \sqrt{5^2 + 3^2} = 5.83$

Distances



L1-distance

$$|u - v|_1 = |5| + |-3| = 8$$



L2-distance

$$|u - v|_2 = \sqrt{5^2 + 3^2} = 5.83$$



Cosine distance

$$\cos(\theta)$$

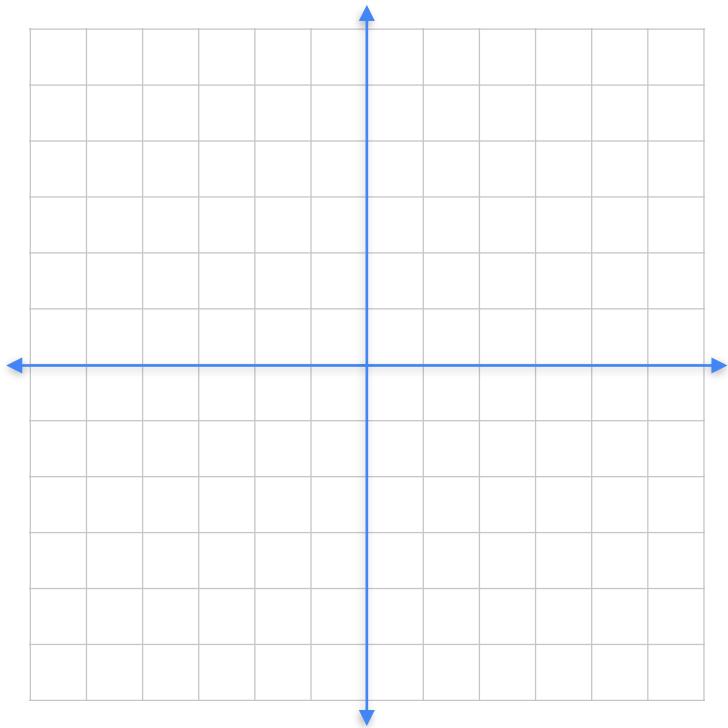


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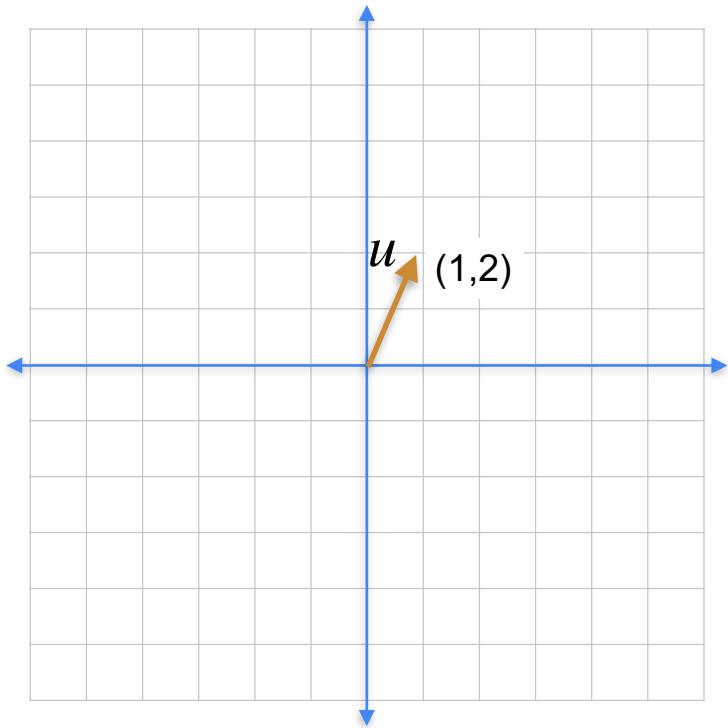
Vectors and Linear Transformations

Multiplying a vector by a scalar

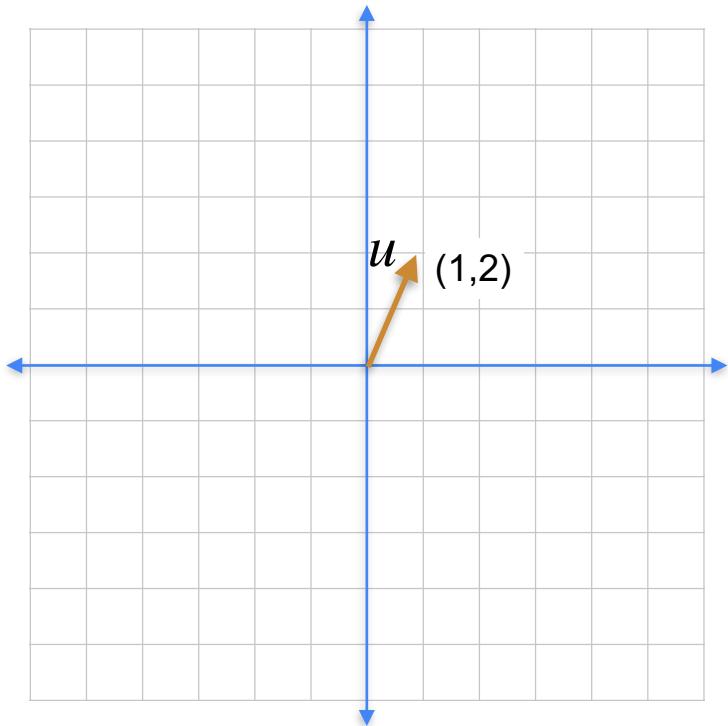
Multiplying a vector by a scalar



Multiplying a vector by a scalar

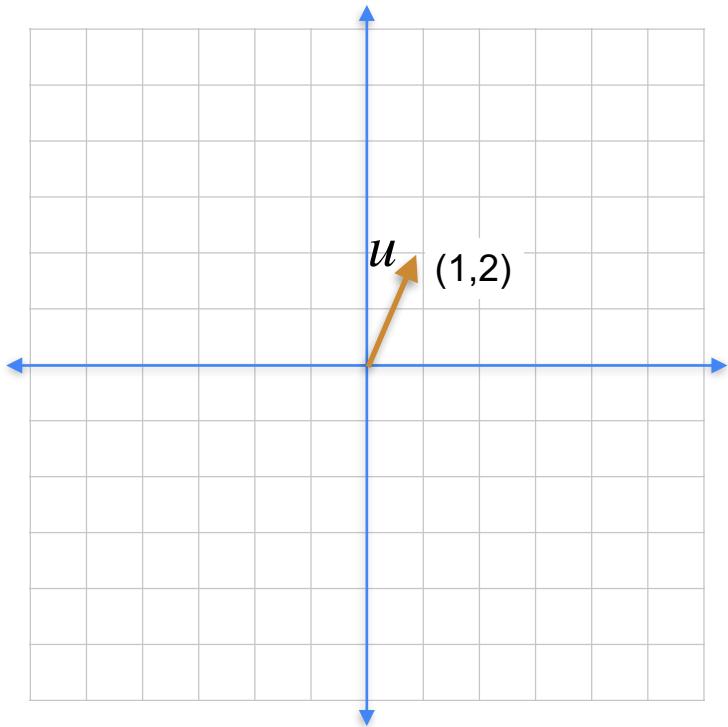


Multiplying a vector by a scalar



$$u = (1,2)$$

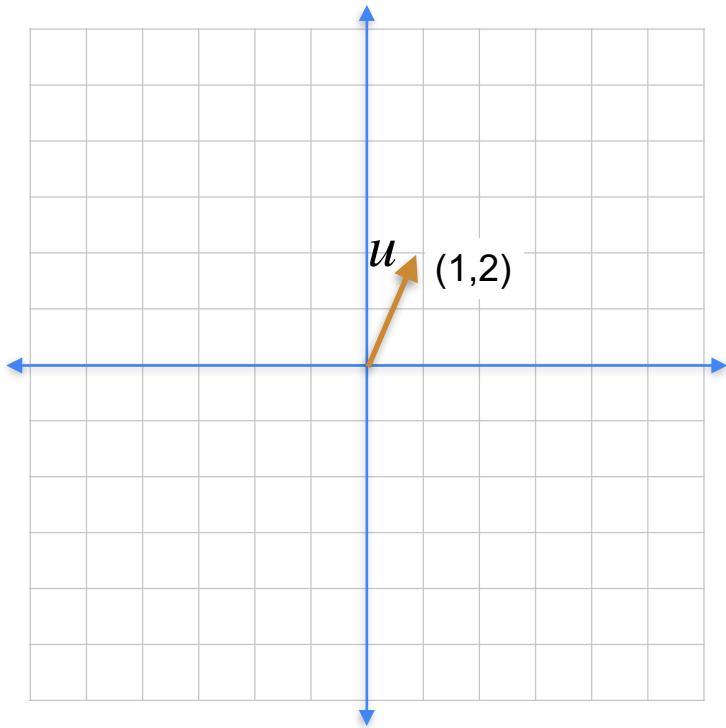
Multiplying a vector by a scalar



$$u = (1,2)$$

$$\lambda = 3$$

Multiplying a vector by a scalar

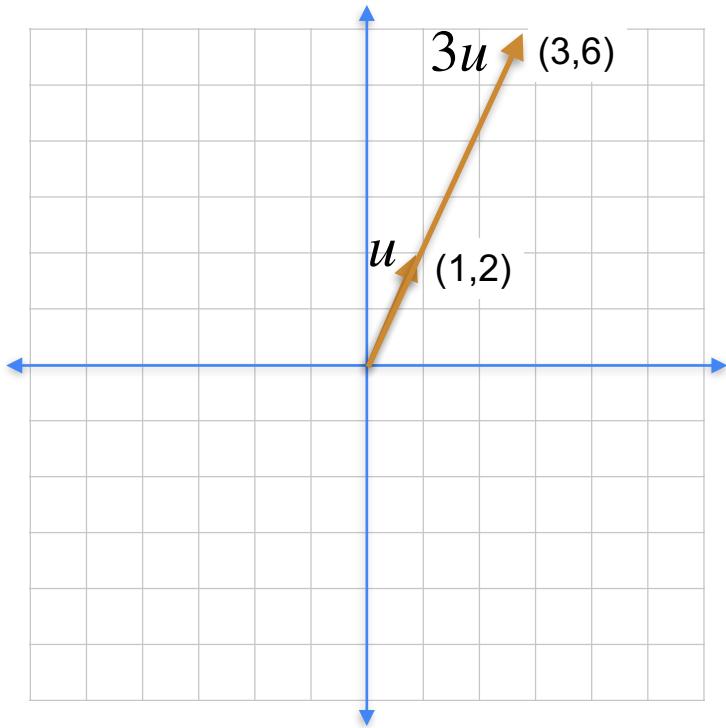


$$u = (1, 2)$$

$$\lambda = 3$$

$$\lambda u = (3, 6)$$

Multiplying a vector by a scalar

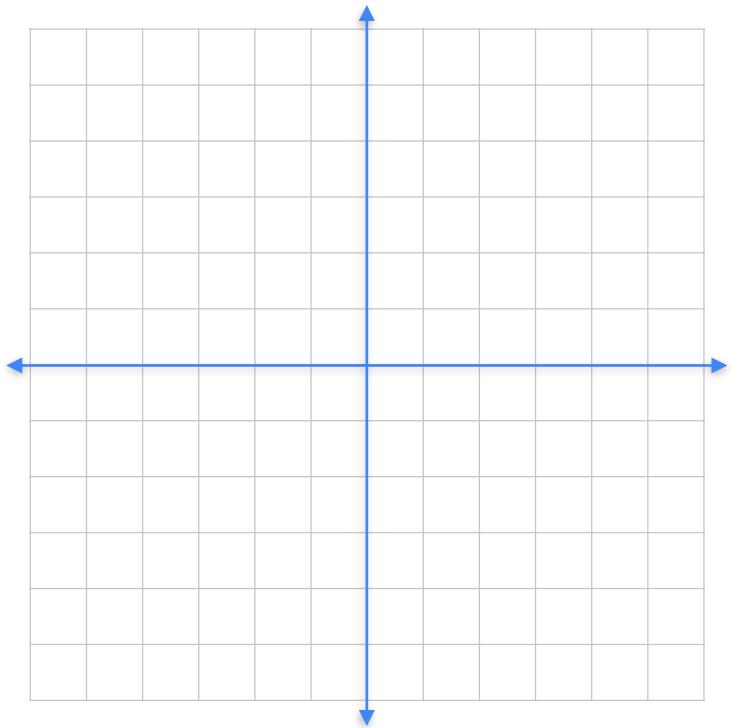


$$u = (1, 2)$$

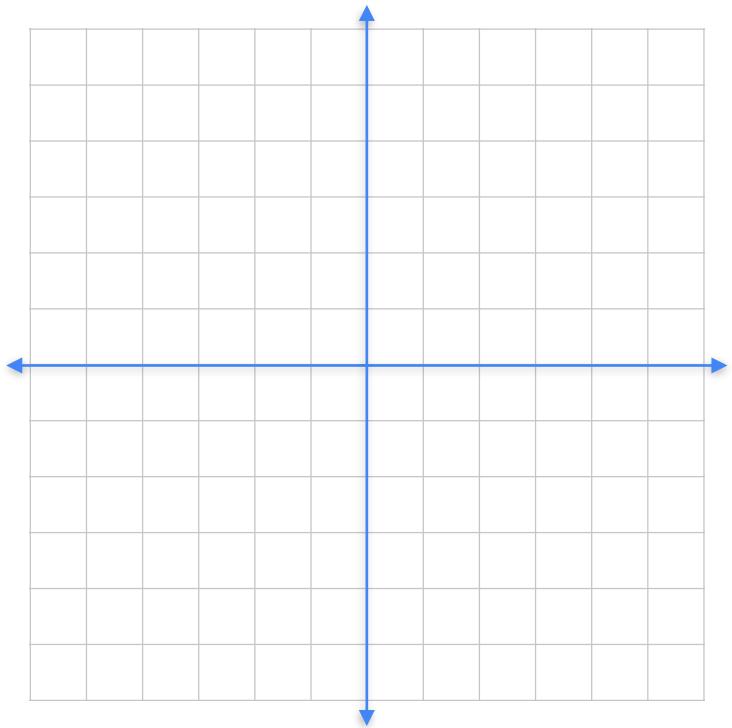
$$\lambda = 3$$

$$\lambda u = (3, 6)$$

If the scalar is negative

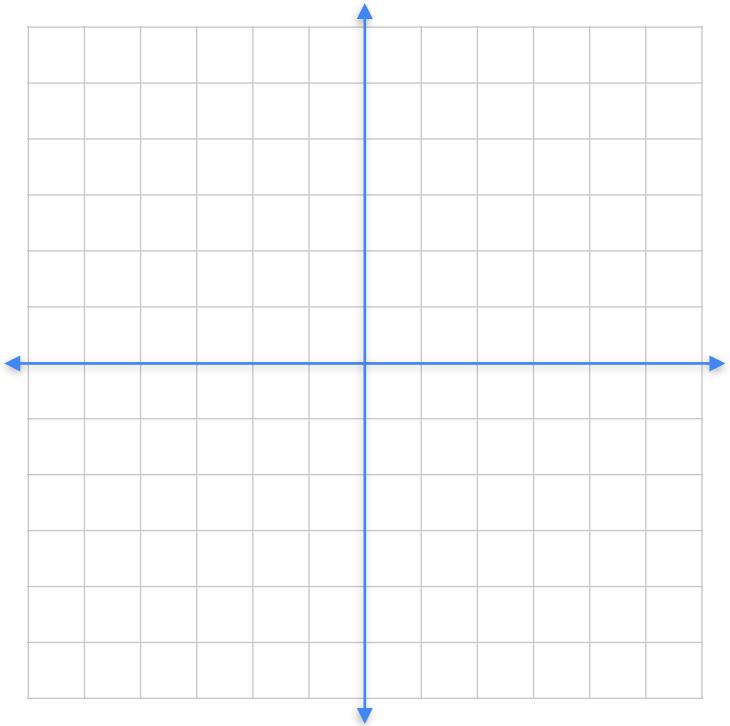


If the scalar is negative



$$u = (1, 2)$$

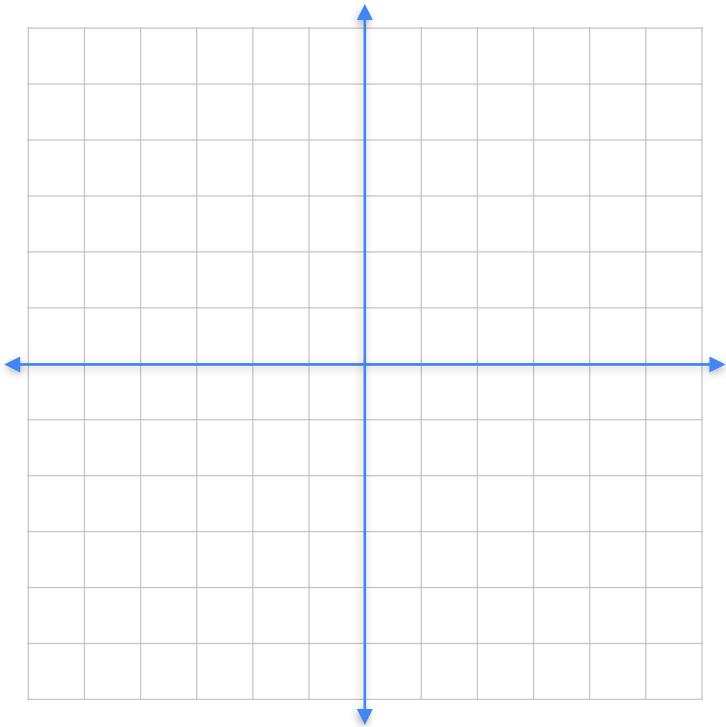
If the scalar is negative



$$u = (1, 2)$$

$$\lambda = -2$$

If the scalar is negative

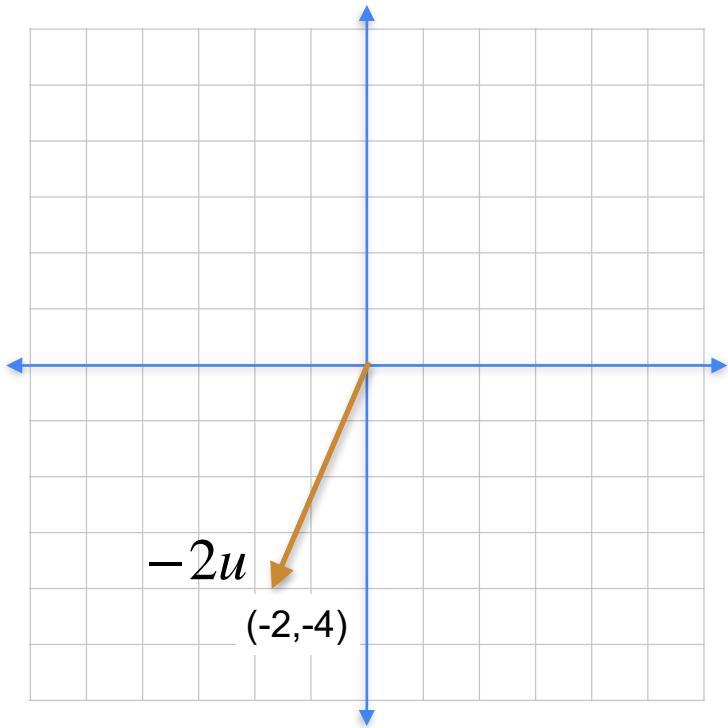


$$u = (1, 2)$$

$$\lambda = -2$$

$$\lambda u = (-2, -4)$$

If the scalar is negative

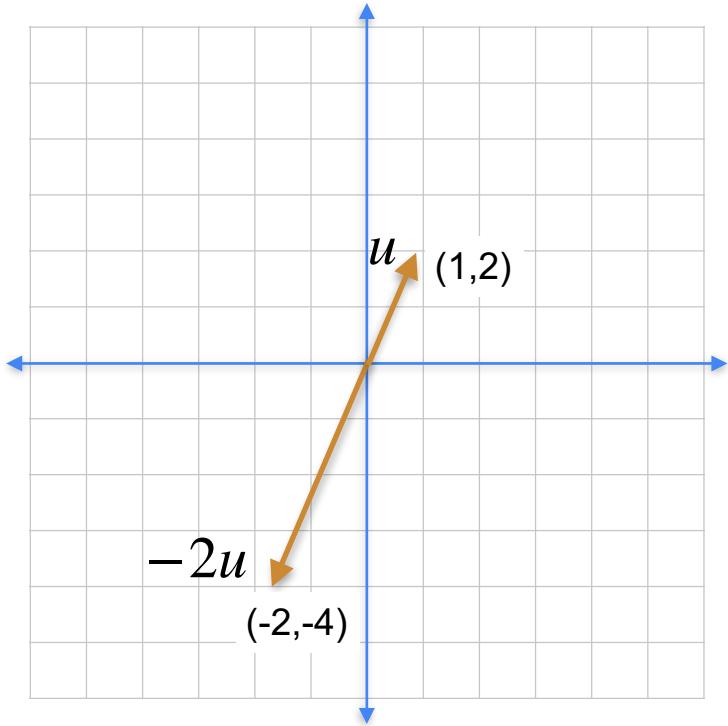


$$u = (1, 2)$$

$$\lambda = -2$$

$$\lambda u = (-2, -4)$$

If the scalar is negative



$$u = (1, 2)$$

$$\lambda = -2$$

$$\lambda u = (-2, -4)$$



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Vectors and Linear Transformations

The dot product

A shortcut for linear operations

A shortcut for linear operations

Quantities

2 apples

4 bananas

1 cherry

A shortcut for linear operations

Quantities

2 apples
4 bananas
1 cherry

Prices

apples: \$3
bananas: \$5
cherries: \$2

A shortcut for linear operations

Quantities

2 apples
4 bananas
1 cherry

Prices

apples: \$3
bananas: \$5
cherries: \$2

Total price

A shortcut for linear operations

Quantities

2 apples
4 bananas
1 cherry

	2
	4
	1

Prices

apples: \$3
bananas: \$5
cherries: \$2

Total price

A shortcut for linear operations

Quantities

2 apples

4 bananas

1 cherry

	2
	4
	1

Prices

apples: \$3

bananas: \$5

cherries: \$2

\$ 	3
\$ 	5
\$ 	2

Total price

A shortcut for linear operations

Quantities

2 apples

4 bananas

1 cherry

	2
	4
	1

Prices

apples: \$3

bananas: \$5

cherries: \$2

Total price

$$2 \times 3 = 6$$



\$	3
\$	5
\$	2

A shortcut for linear operations

Quantities

2 apples

4 bananas

1 cherry

	2
	4
	1

Prices

apples: \$3

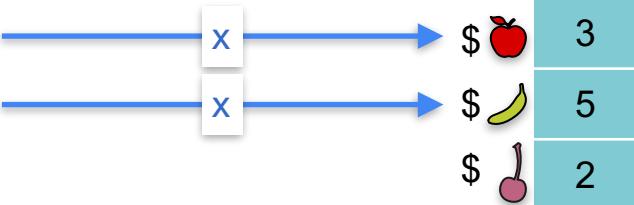
bananas: \$5

cherries: \$2

Total price

$$2 \times 3 = 6$$

$$4 \times 5 = 20$$



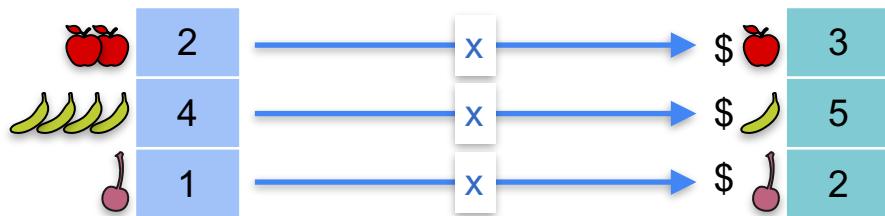
A shortcut for linear operations

Quantities

2 apples

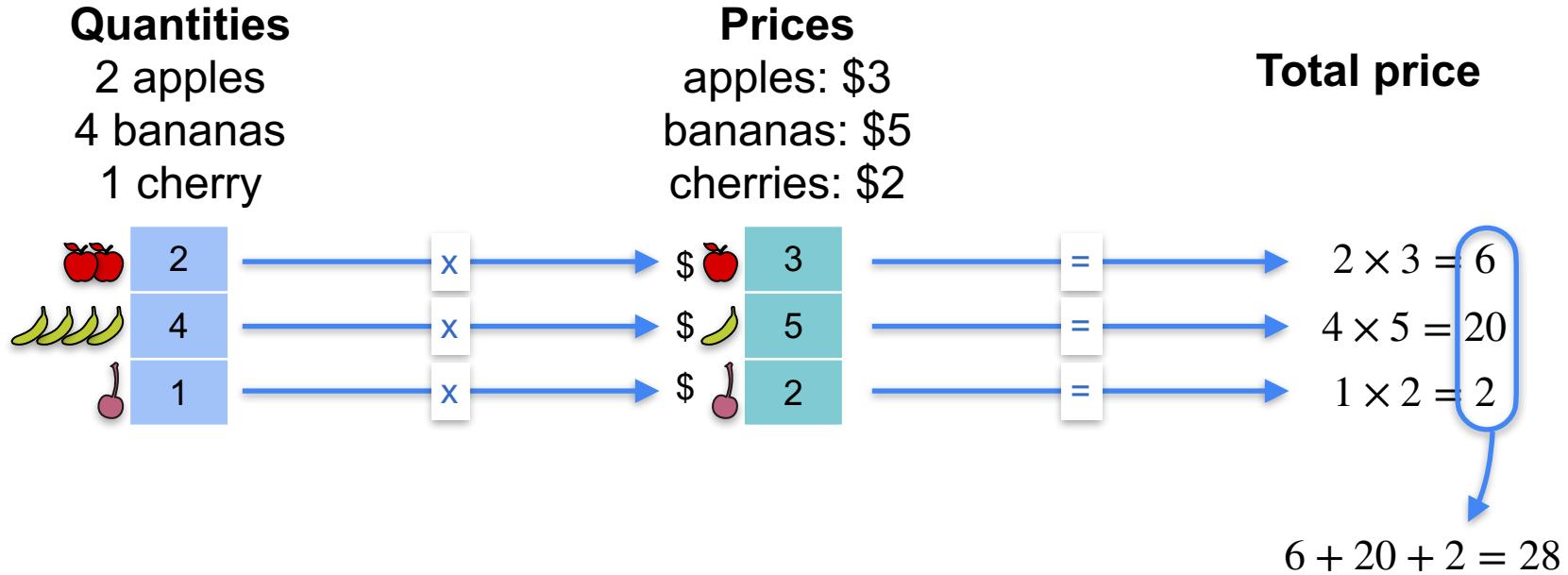
4 bananas

1 cherry



Total price

A shortcut for linear operations



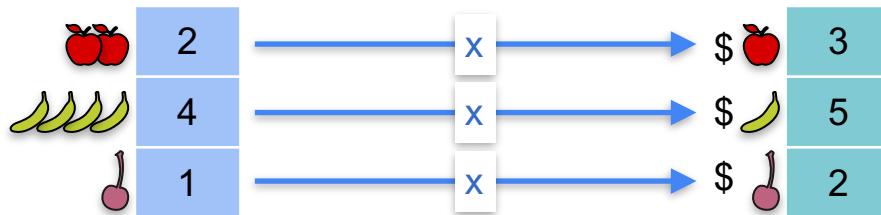
A shortcut for linear operations

Quantities

2 apples

4 bananas

1 cherry



Prices

apples: \$3

bananas: \$5

cherries: \$2

Total price

\$28

$$2 \times 3 = 6$$

$$4 \times 5 = 20$$

$$1 \times 2 = 2$$

$$6 + 20 + 2 = 28$$

The dot product

$$\begin{array}{c} \text{apple} \\ \text{banana} \\ \text{cherry} \end{array} \begin{array}{c|c} & 2 \\ \hline 4 & \\ \hline 1 & \end{array} \cdot \begin{array}{c} \$\text{apple} \\ \$\text{banana} \\ \$\text{cherry} \end{array} \begin{array}{c|c} 3 \\ \hline 5 \\ \hline 2 \end{array} = \$28$$

The dot product

The diagram shows two vectors being multiplied. The first vector, on the left, is represented by a blue grid with three rows. The top row contains two red apples. The middle row contains four yellow bananas. The bottom row contains one red cherry. To the right of this grid are its corresponding numerical values: 2, 4, and 1, stacked vertically. A multiplication sign follows this vector. To the right of the multiplication sign is a second vector, represented by a teal grid with three rows. The top row contains one red apple with a dollar sign. The middle row contains one yellow banana with a dollar sign. The bottom row contains one red cherry with a dollar sign. To the right of this grid are its corresponding numerical values: 3, 5, and 2, stacked vertically. An equals sign follows this vector, and to its right is the result '\$28'.

$$\begin{matrix} \text{apple} \\ \text{banana} \\ \text{cherry} \end{matrix} \begin{matrix} 2 \\ 4 \\ 1 \end{matrix} \cdot \begin{matrix} \$ \text{apple} \\ \$ \text{banana} \\ \$ \text{cherry} \end{matrix} \begin{matrix} 3 \\ 5 \\ 2 \end{matrix} = \$28$$

$$2 \cdot 3 + 4 \cdot 5 + 1 \cdot 2 = 28$$

The dot product

The diagram shows two vectors of fruits being multiplied. The first vector (bottom) has three elements: an apple (value 2), four bananas (value 4), and one cherry (value 1). The second vector (top) has three elements: a dollar sign and an apple (value 3), a dollar sign and a banana (value 5), and a dollar sign and a cherry (value 2). The result of the multiplication is \$28.

2	4	1
\$	apple	3
\$	banana	5
\$	cherry	2

= \$28

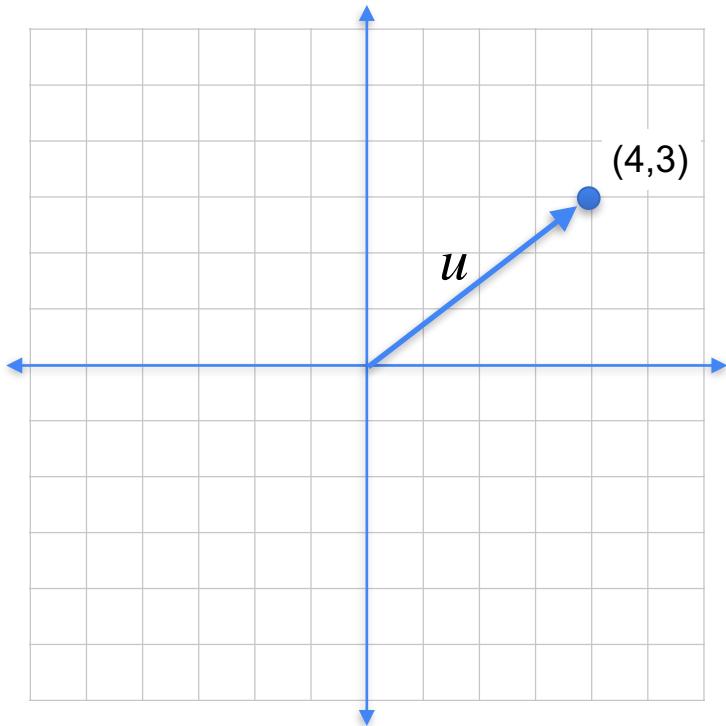
$$2 \cdot 3 + 4 \cdot 5 + 1 \cdot 2 = 28$$

The dot product

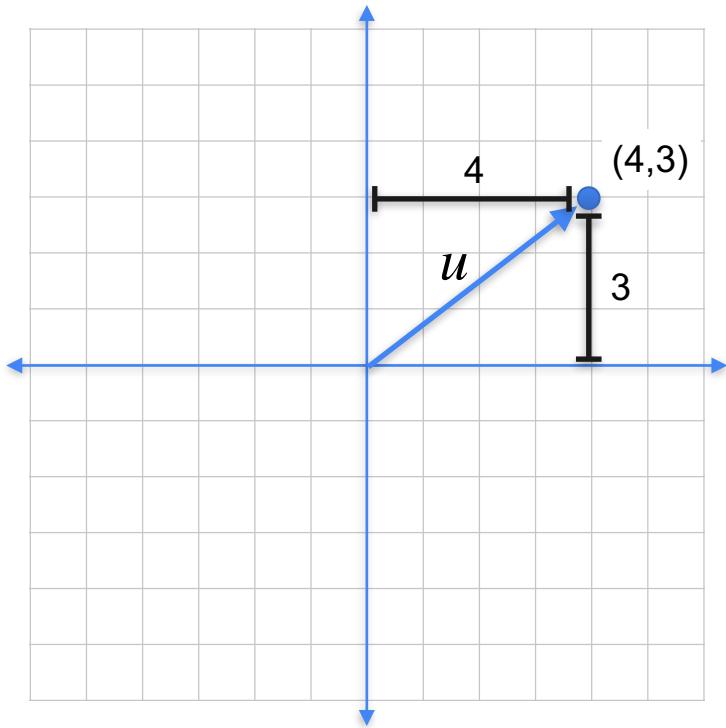
$$\begin{matrix} 2 & 4 & 1 \end{matrix} \cdot \begin{matrix} 3 \\ 5 \\ 2 \end{matrix} = 28$$

$$2 \cdot 3 + 4 \cdot 5 + 1 \cdot 2 = 28$$

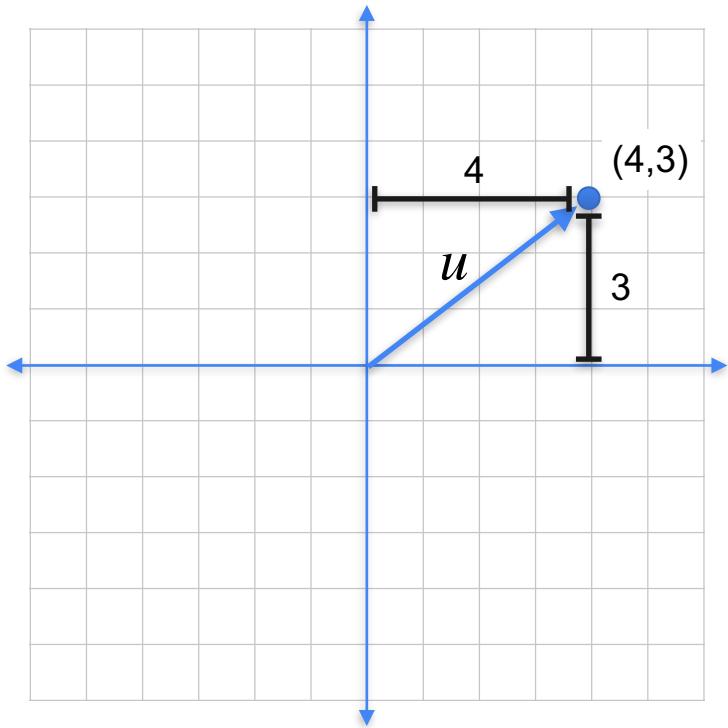
Norm of a vector using dot product



Norm of a vector using dot product

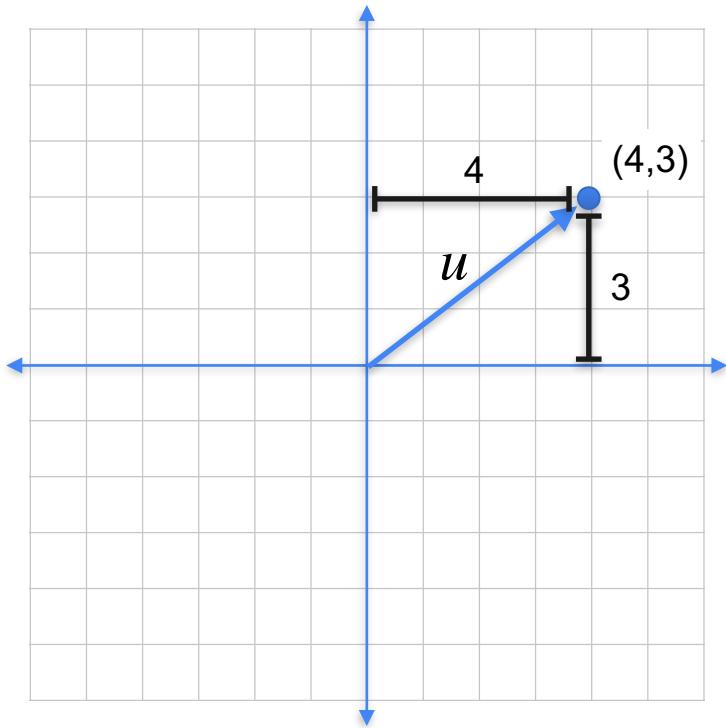


Norm of a vector using dot product



$$\sqrt{4^2 + 3^2} = \sqrt{25} = 5$$

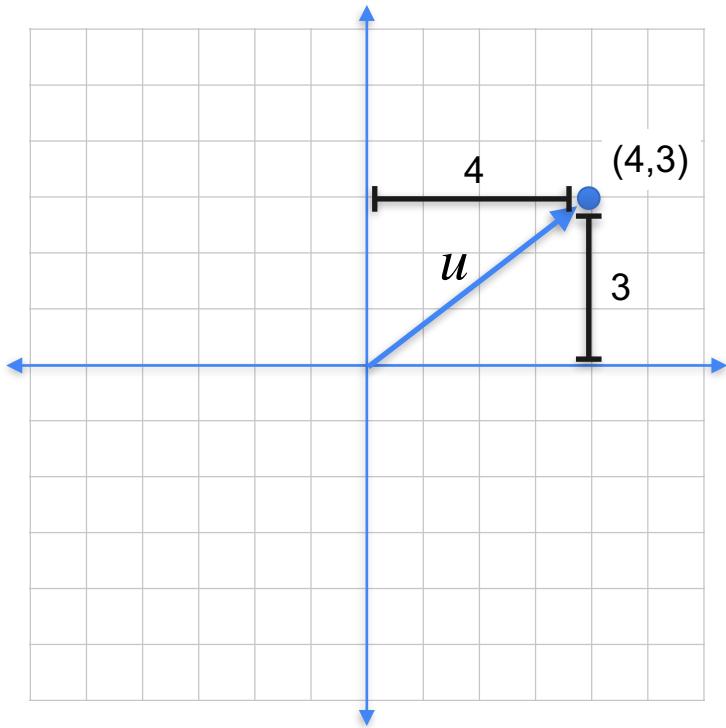
Norm of a vector using dot product



$$\sqrt{4^2 + 3^2} = \sqrt{25} = 5$$

$$\begin{array}{|c|c|} \hline 4 & 3 \\ \hline \end{array} = \begin{array}{|c|c|} \hline 4 & \\ \hline 3 & \\ \hline \end{array} = 25$$

Norm of a vector using dot product

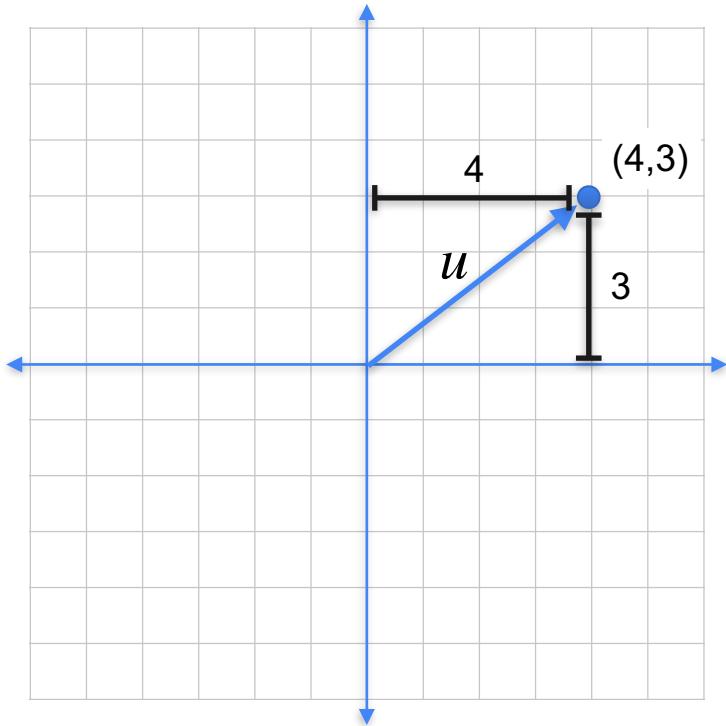


$$\sqrt{4^2 + 3^2} = \sqrt{25} = 5$$

$$\begin{array}{|c|c|} \hline 4 & 3 \\ \hline \end{array} = \begin{array}{|c|c|} \hline 4 \\ \hline 3 \\ \hline \end{array} = 25$$

$$L2 - norm = \sqrt{\text{dot product}(u, u)}$$

Norm of a vector using dot product



$$\sqrt{4^2 + 3^2} = \sqrt{25} = 5$$

$$\begin{array}{|c|c|} \hline 4 & 3 \\ \hline \end{array} = \begin{array}{|c|c|} \hline 4 \\ \hline 3 \\ \hline \end{array} = 25$$

$$L2 - norm = \sqrt{\text{dot product}(u, u)}$$

$$\| u \|_2 = \sqrt{\langle u, u \rangle}$$

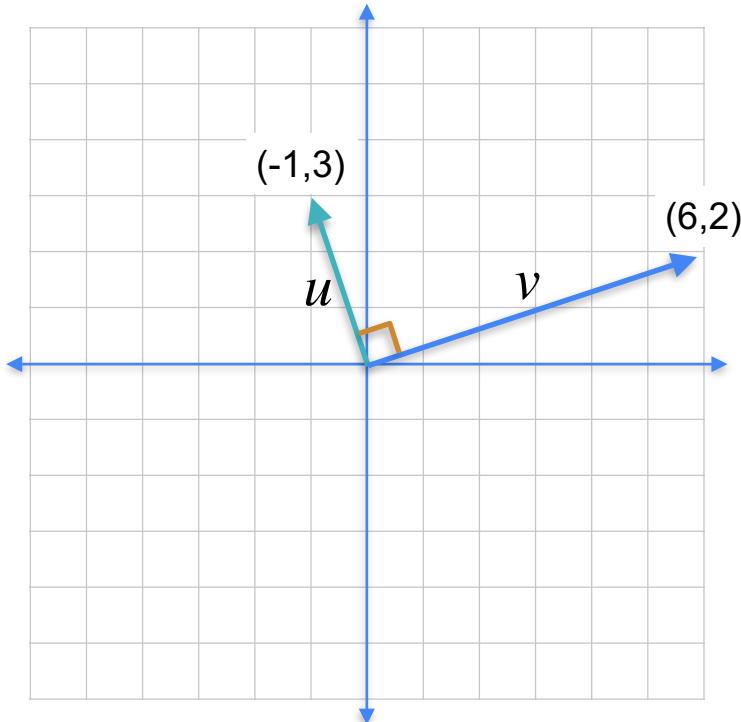


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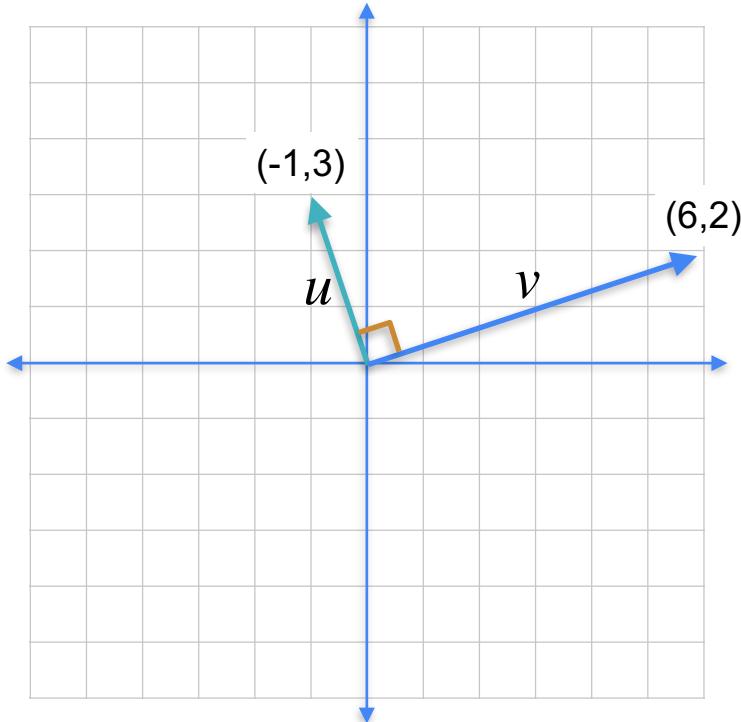
Vectors and Linear Transformations

Geometric dot product

Orthogonal vectors have dot product 0

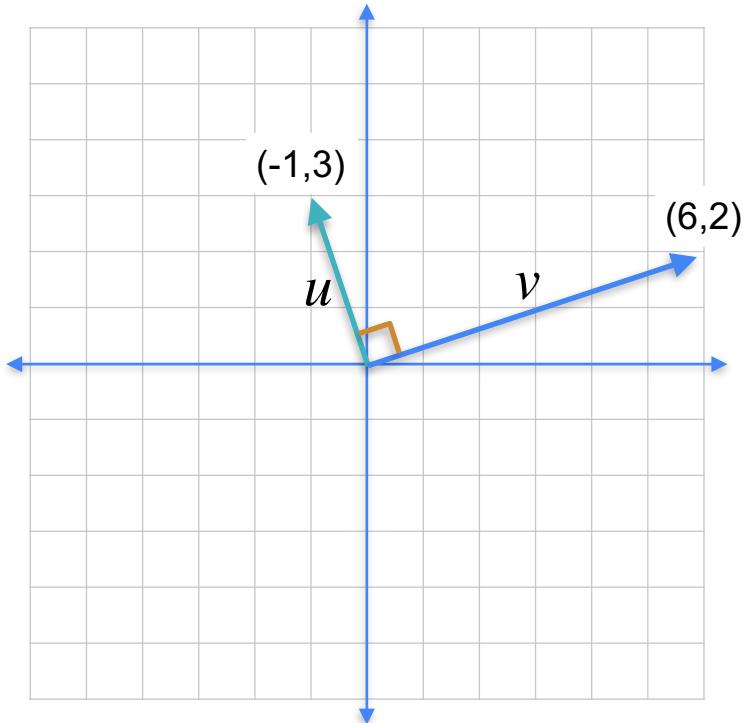


Orthogonal vectors have dot product 0



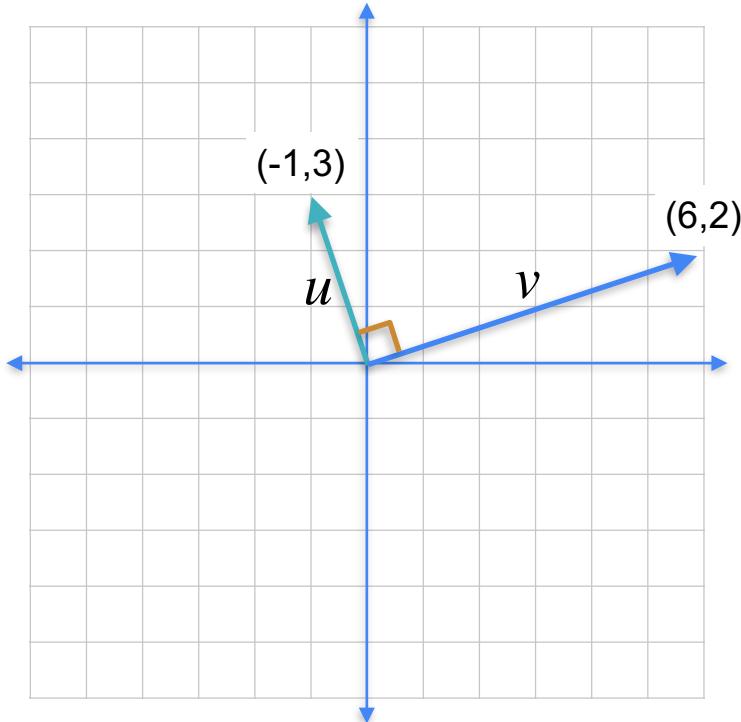
6	2
---	---

Orthogonal vectors have dot product 0



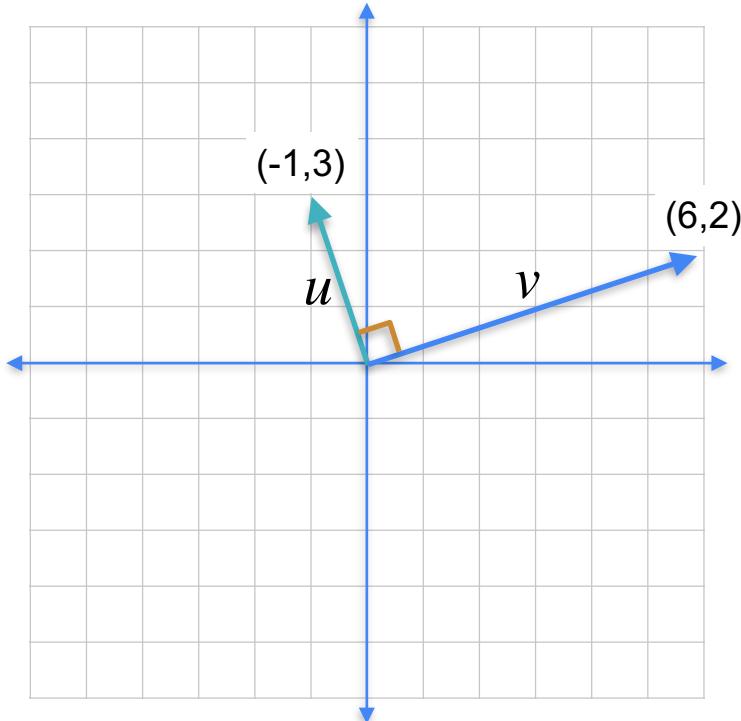
6	2	-1
3		

Orthogonal vectors have dot product 0



$$\begin{matrix} 6 & 2 \end{matrix} \cdot \begin{pmatrix} -1 \\ 3 \end{pmatrix} = 0$$

Orthogonal vectors have dot product 0



$$\begin{matrix} 6 & 2 \end{matrix} \times \begin{pmatrix} -1 \\ 3 \end{pmatrix} = 0$$

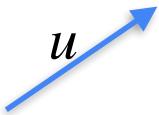
$$\langle u, v \rangle = 0$$

The dot product

The dot product

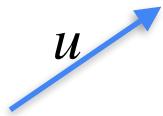


The dot product

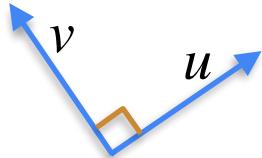


$$\langle u, u \rangle = |u|^2$$

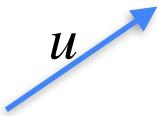
The dot product



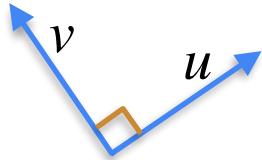
$$\langle u, u \rangle = |u|^2$$



The dot product

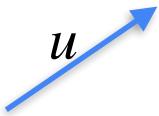


$$\langle u, u \rangle = |u|^2$$

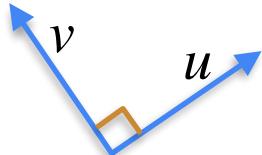


$$\langle u, v \rangle = 0$$

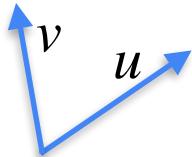
The dot product



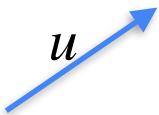
$$\langle u, u \rangle = |u|^2$$



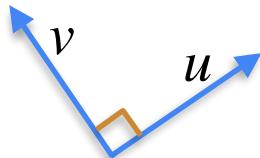
$$\langle u, v \rangle = 0$$



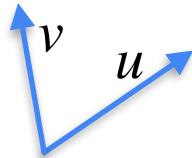
The dot product



$$\langle u, u \rangle = |u|^2$$



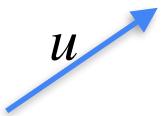
$$\langle u, v \rangle = 0$$



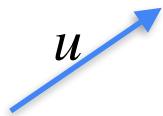
$$\langle u, v \rangle = ?$$

The dot product

The dot product

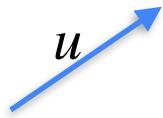


The dot product

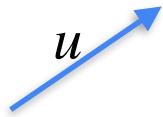


$$\langle u, u \rangle = |u|^2 = |u| \cdot |u|$$

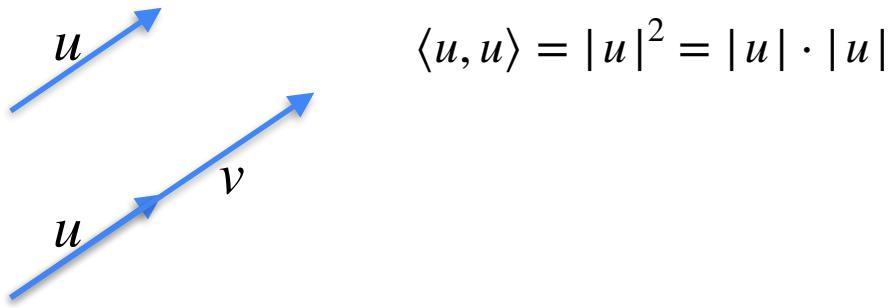
The dot product



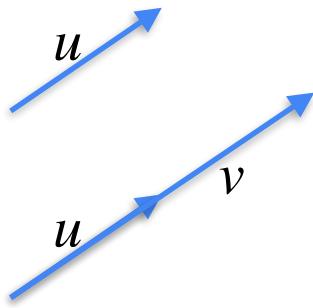
$$\langle u, u \rangle = |u|^2 = |u| \cdot |u|$$



The dot product



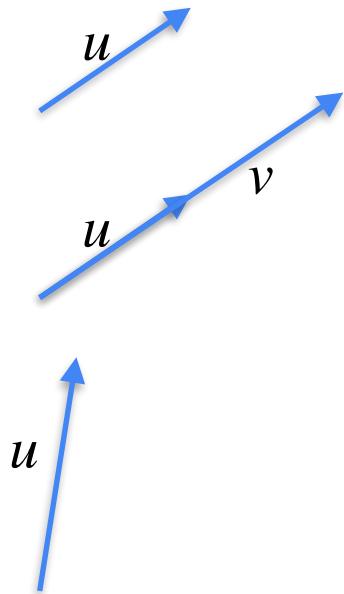
The dot product



$$\langle u, u \rangle = |u|^2 = |u| \cdot |u|$$

$$\langle u, v \rangle = |u| \cdot |v|$$

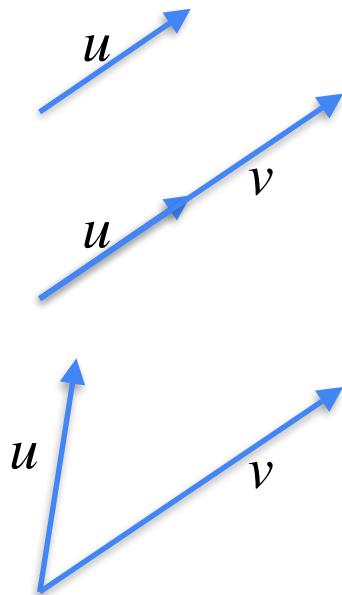
The dot product



$$\langle u, u \rangle = |u|^2 = |u| \cdot |u|$$

$$\langle u, v \rangle = |u| \cdot |v|$$

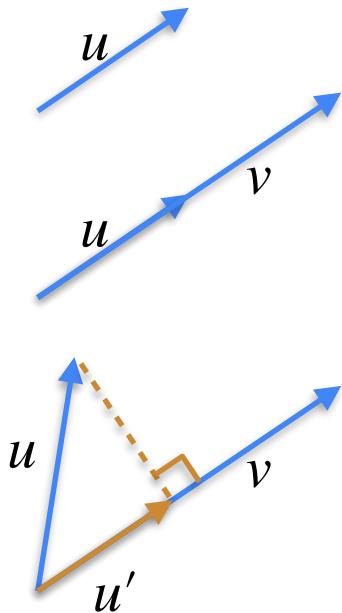
The dot product



$$\langle u, u \rangle = |u|^2 = |u| \cdot |u|$$

$$\langle u, v \rangle = |u| \cdot |v|$$

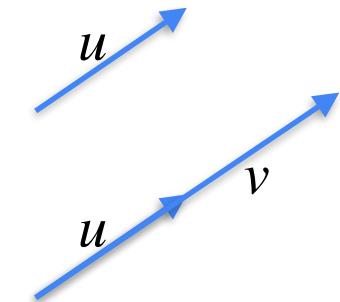
The dot product



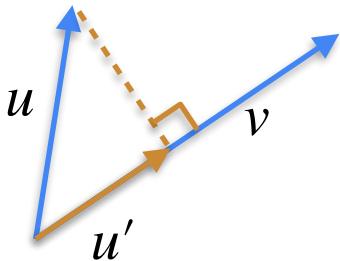
$$\langle u, u \rangle = |u|^2 = |u| \cdot |u|$$

$$\langle u, v \rangle = |u| \cdot |v|$$

The dot product



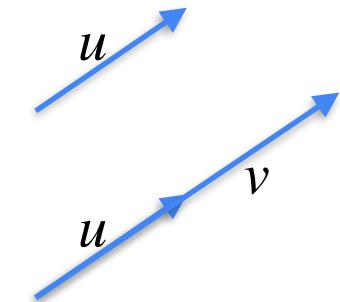
$$\langle u, u \rangle = |u|^2 = |u| \cdot |u|$$



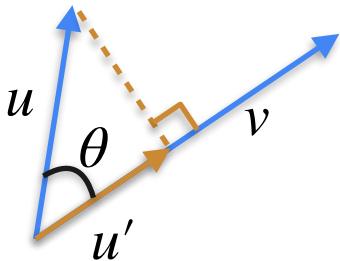
$$\langle u, v \rangle = |u| \cdot |v|$$

$$\langle u, v \rangle = |u'| \cdot |v|$$

The dot product



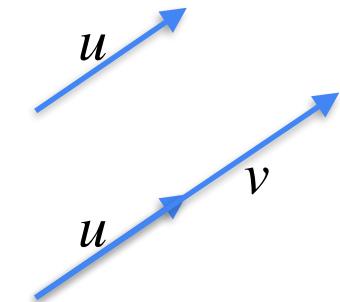
$$\langle u, u \rangle = |u|^2 = |u| \cdot |u|$$



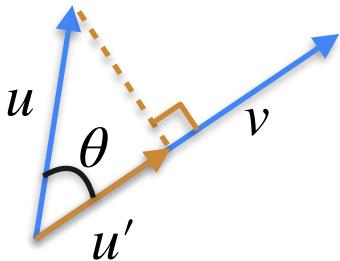
$$\langle u, v \rangle = |u| \cdot |v|$$

$$\langle u, v \rangle = |u'| \cdot |v|$$

The dot product



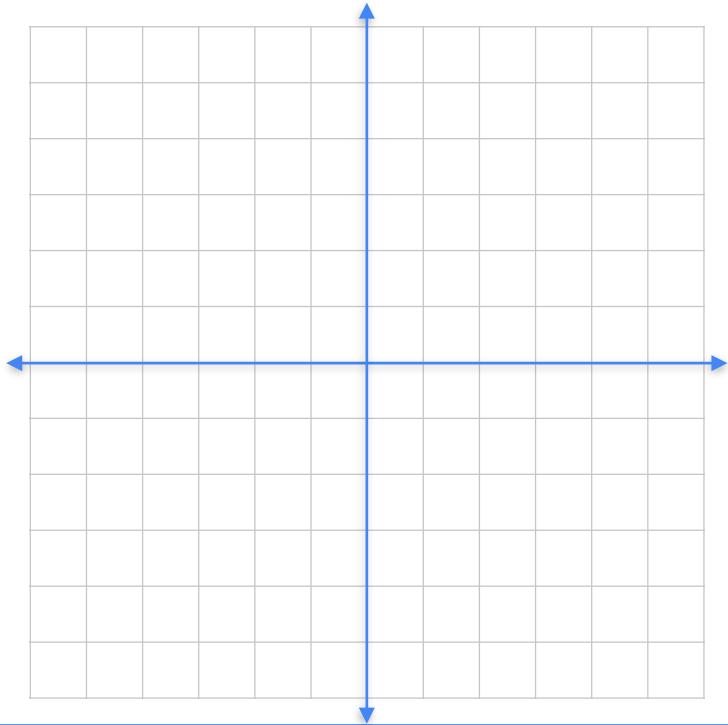
$$\langle u, u \rangle = |u|^2 = |u| \cdot |u|$$



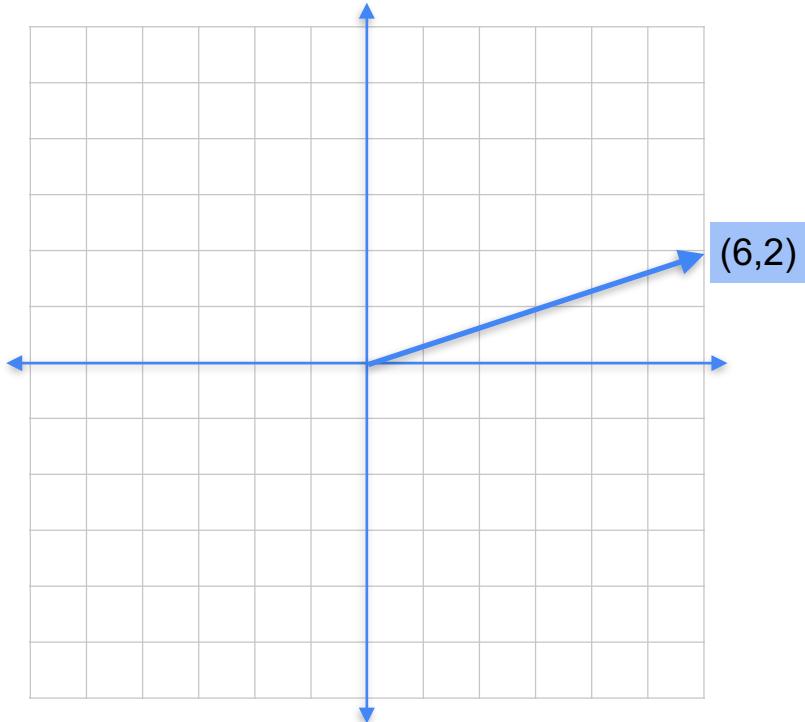
$$\langle u, v \rangle = |u| \cdot |v|$$

$$\begin{aligned}\langle u, v \rangle &= |u'| \cdot |v| \\ &= |u| |v| \cos(\theta)\end{aligned}$$

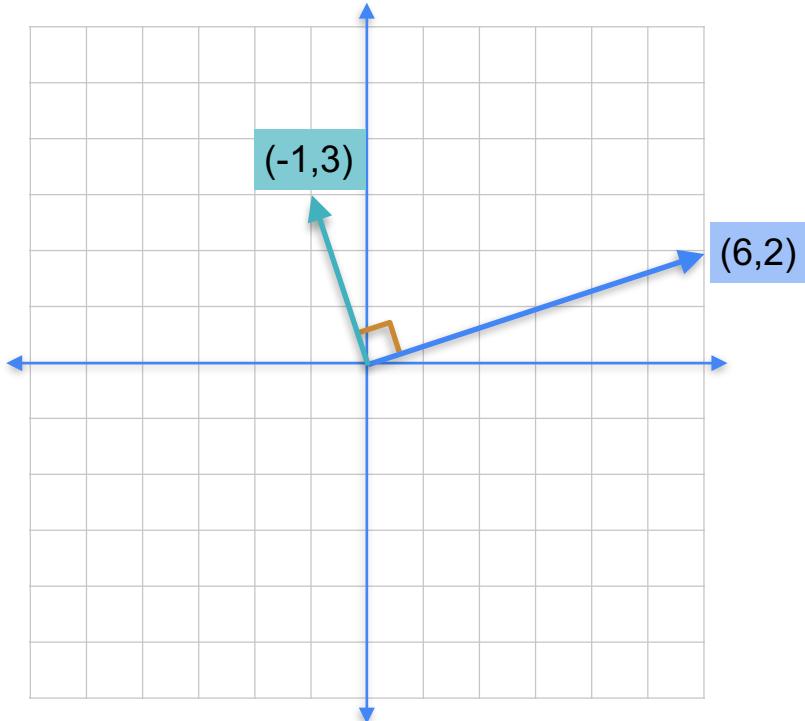
Geometric dot product



Geometric dot product

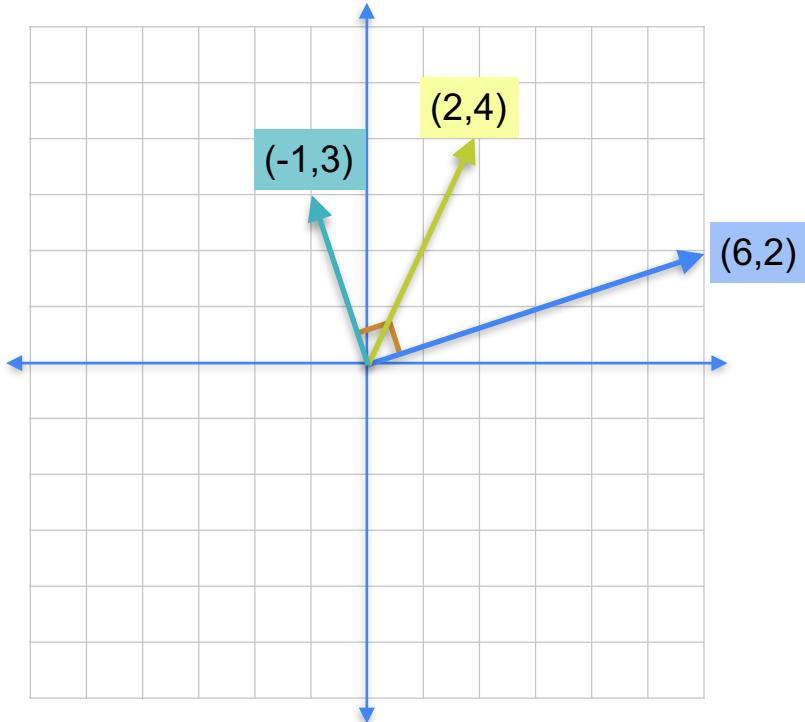


Geometric dot product



$$\begin{array}{|c|c|} \hline 6 & 2 \\ \hline \end{array} \cdot \begin{array}{|c|c|} \hline -1 \\ \hline 3 \\ \hline \end{array} = 0$$

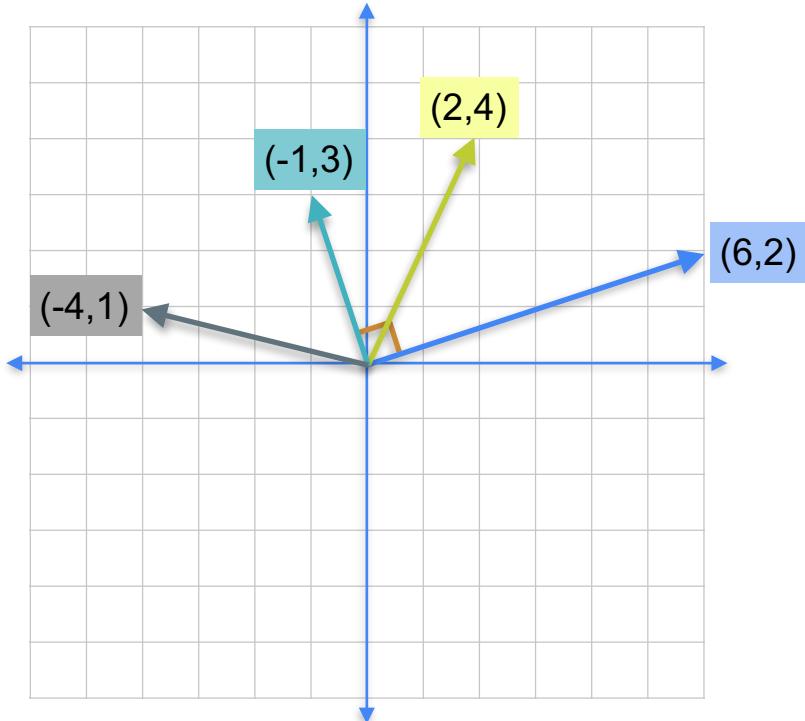
Geometric dot product



$$\begin{array}{|c|c|} \hline 6 & 2 \\ \hline \end{array} \begin{array}{|c|c|} \hline 2 \\ \hline 4 \\ \hline \end{array} = \begin{array}{|c|} \hline 20 \\ \hline \end{array}$$

$$\begin{array}{|c|c|} \hline 6 & 2 \\ \hline \end{array} \begin{array}{|c|c|} \hline -1 \\ \hline 3 \\ \hline \end{array} = \begin{array}{|c|} \hline 0 \\ \hline \end{array}$$

Geometric dot product

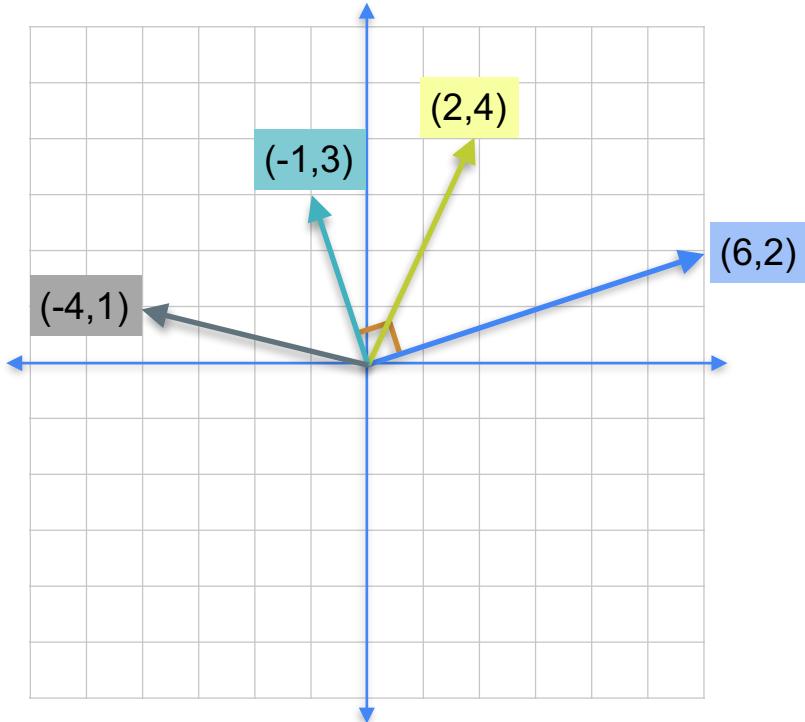


$$\begin{array}{|c|c|} \hline 6 & 2 \\ \hline \end{array} \begin{array}{|c|c|} \hline 2 \\ \hline 4 \\ \hline \end{array} = \begin{array}{|c|} \hline 20 \\ \hline \end{array}$$

$$\begin{array}{|c|c|} \hline 6 & 2 \\ \hline \end{array} \begin{array}{|c|c|} \hline -1 \\ \hline 3 \\ \hline \end{array} = \begin{array}{|c|} \hline 0 \\ \hline \end{array}$$

$$\begin{array}{|c|c|} \hline 6 & 2 \\ \hline \end{array} \begin{array}{|c|c|} \hline -4 \\ \hline 1 \\ \hline \end{array} = \begin{array}{|c|} \hline -22 \\ \hline \end{array}$$

Geometric dot product

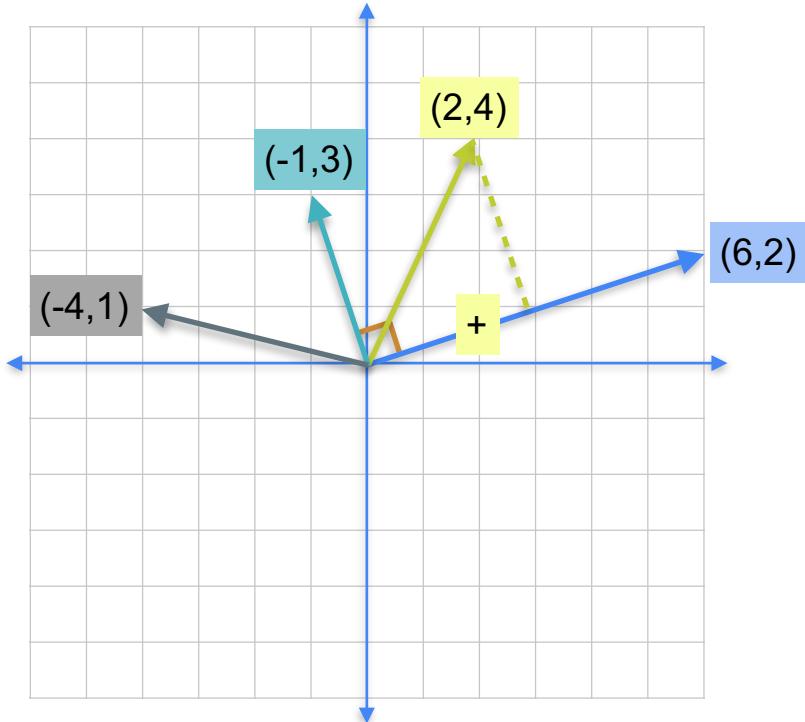


$$\begin{array}{|c|c|} \hline 6 & 2 \\ \hline \end{array} \begin{array}{|c|c|} \hline 2 \\ \hline 4 \\ \hline \end{array} = \begin{array}{|c|} \hline 20 \\ \hline \end{array} \text{ Positive}$$

$$\begin{array}{|c|c|} \hline 6 & 2 \\ \hline \end{array} \begin{array}{|c|c|} \hline -1 \\ \hline 3 \\ \hline \end{array} = \begin{array}{|c|} \hline 0 \\ \hline \end{array}$$

$$\begin{array}{|c|c|} \hline 6 & 2 \\ \hline \end{array} \begin{array}{|c|c|} \hline -4 \\ \hline 1 \\ \hline \end{array} = \begin{array}{|c|} \hline -22 \\ \hline \end{array}$$

Geometric dot product

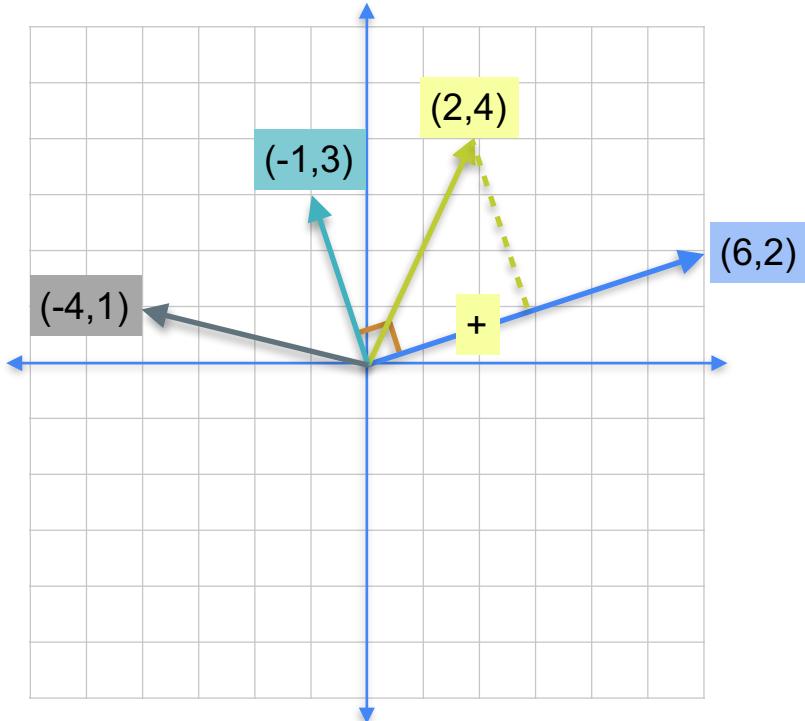


$$\begin{array}{|c|c|} \hline 6 & 2 \\ \hline \end{array} \begin{array}{|c|c|} \hline 2 \\ \hline 4 \\ \hline \end{array} = \begin{array}{|c|} \hline 20 \\ \hline \end{array} \text{ Positive}$$

$$\begin{array}{|c|c|} \hline 6 & 2 \\ \hline \end{array} \begin{array}{|c|c|} \hline -1 \\ \hline 3 \\ \hline \end{array} = \begin{array}{|c|} \hline 0 \\ \hline \end{array}$$

$$\begin{array}{|c|c|} \hline 6 & 2 \\ \hline \end{array} \begin{array}{|c|c|} \hline -4 \\ \hline 1 \\ \hline \end{array} = \begin{array}{|c|} \hline -22 \\ \hline \end{array}$$

Geometric dot product

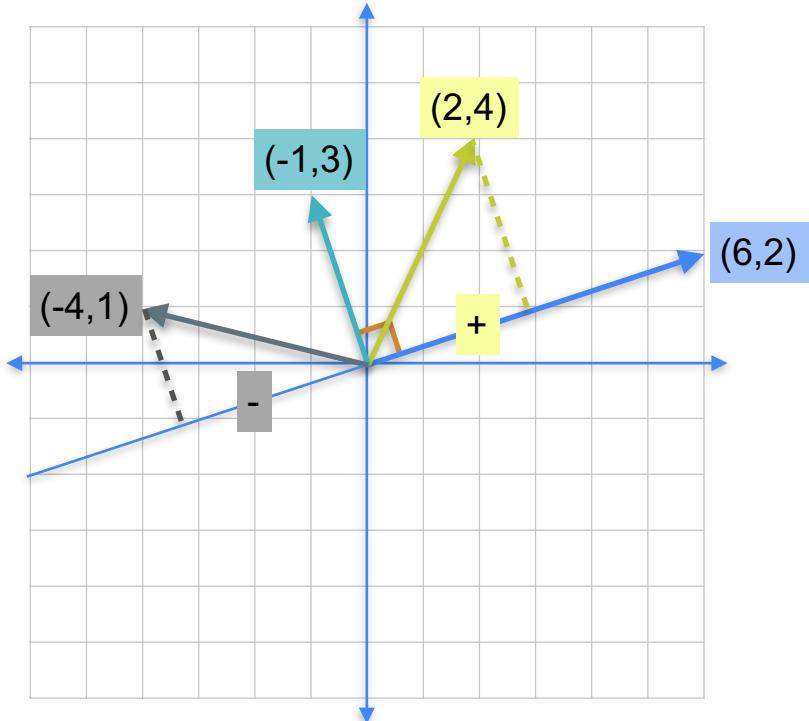


$$\begin{array}{|c|c|} \hline 6 & 2 \\ \hline \end{array} \begin{array}{|c|c|} \hline 2 \\ \hline 4 \\ \hline \end{array} = \begin{array}{|c|} \hline 20 \\ \hline \end{array} \text{ Positive}$$

$$\begin{array}{|c|c|} \hline 6 & 2 \\ \hline \end{array} \begin{array}{|c|c|} \hline -1 \\ \hline 3 \\ \hline \end{array} = \begin{array}{|c|} \hline 0 \\ \hline \end{array}$$

$$\begin{array}{|c|c|} \hline 6 & 2 \\ \hline \end{array} \begin{array}{|c|c|} \hline -4 \\ \hline 1 \\ \hline \end{array} = \begin{array}{|c|} \hline -22 \\ \hline \end{array} \text{ Negative}$$

Geometric dot product

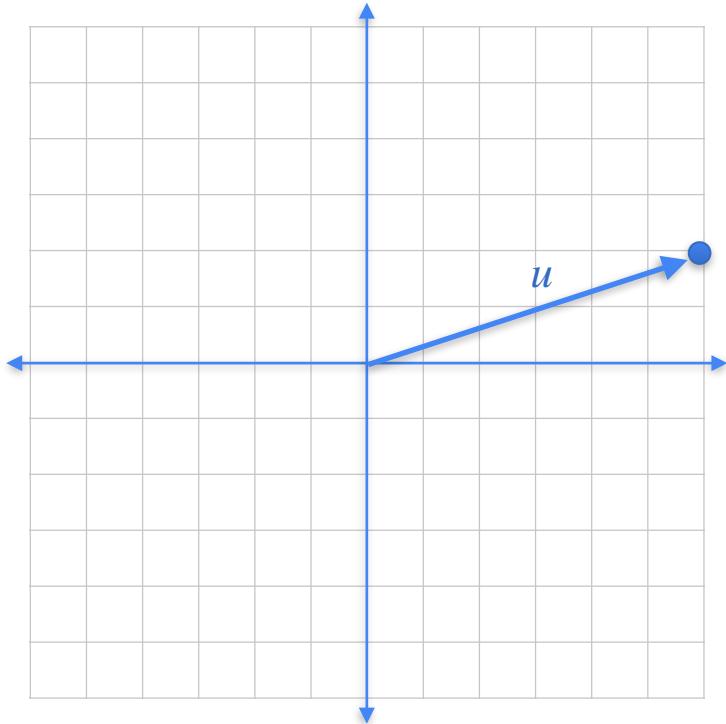


$$\begin{array}{|c|c|} \hline 6 & 2 \\ \hline \end{array} \begin{array}{|c|c|} \hline 2 \\ \hline 4 \\ \hline \end{array} = \begin{array}{|c|} \hline 20 \\ \hline \end{array} \text{ Positive}$$

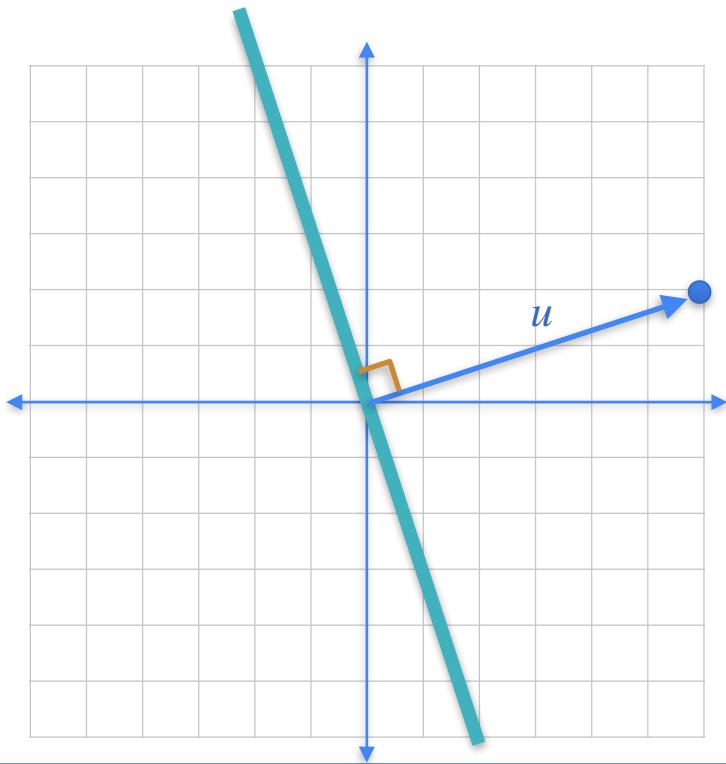
$$\begin{array}{|c|c|} \hline 6 & 2 \\ \hline \end{array} \begin{array}{|c|c|} \hline -1 \\ \hline 3 \\ \hline \end{array} = \begin{array}{|c|} \hline 0 \\ \hline \end{array}$$

$$\begin{array}{|c|c|} \hline 6 & 2 \\ \hline \end{array} \begin{array}{|c|c|} \hline -4 \\ \hline 1 \\ \hline \end{array} = \begin{array}{|c|} \hline -22 \\ \hline \end{array} \text{ Negative}$$

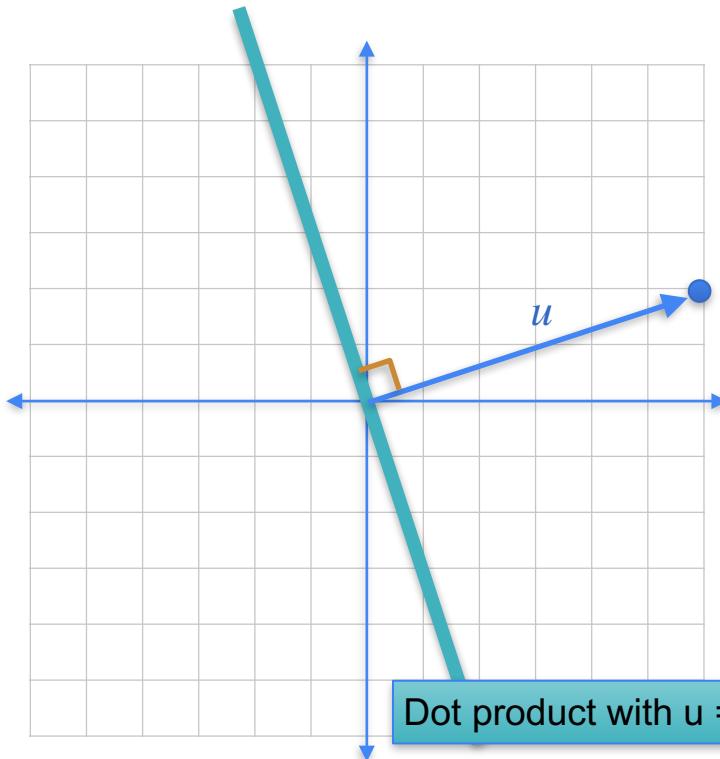
Geometric dot product



Geometric dot product

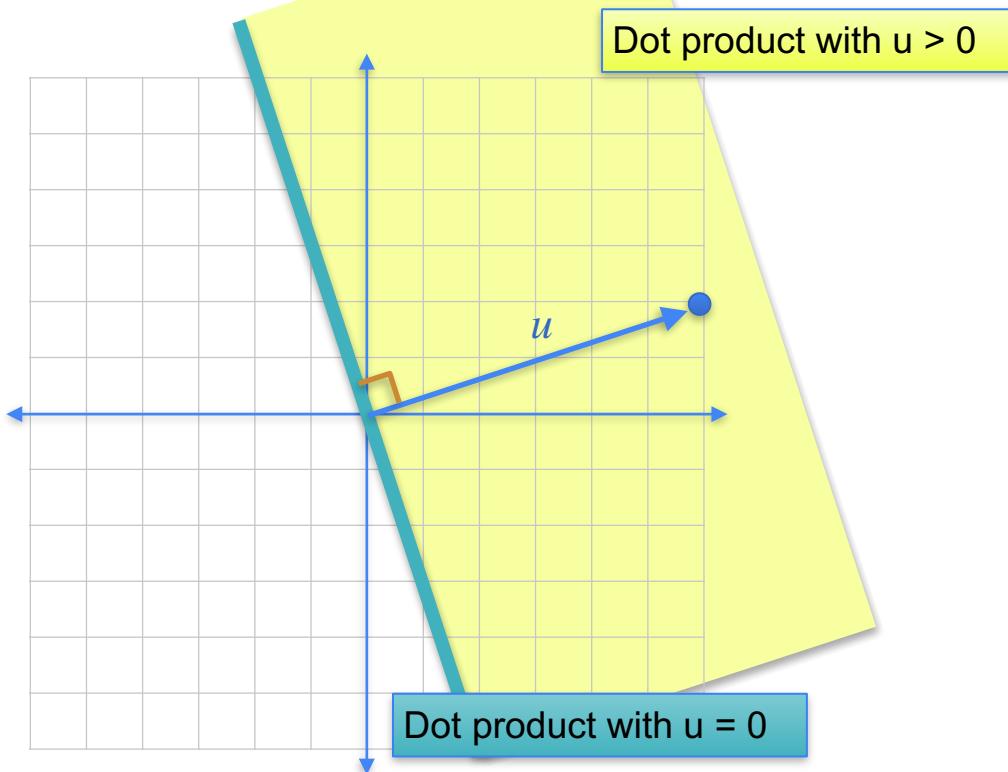


Geometric dot product



$$\langle u, v \rangle = 0$$

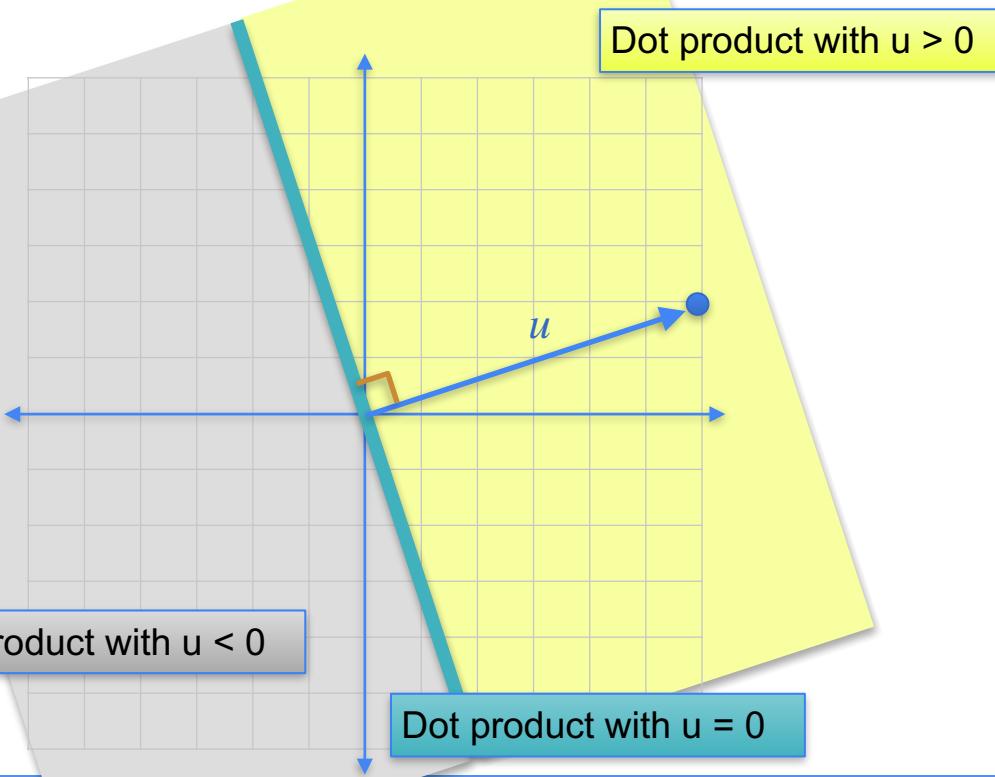
Geometric dot product



$$\langle u, v \rangle > 0$$

$$\langle u, v \rangle = 0$$

Geometric dot product



$$\langle u, v \rangle > 0$$

$$\langle u, v \rangle = 0$$

$$\langle u, v \rangle < 0$$



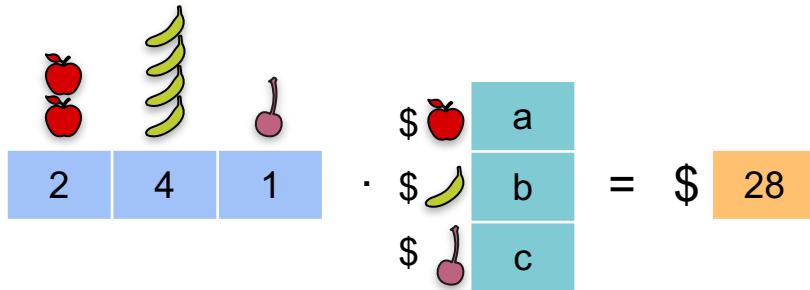
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Vectors and Linear Transformations

**Multiplying a matrix by a
vector**

Equations as dot product

$$2a + 4b + c = 28$$



Equations as dot product

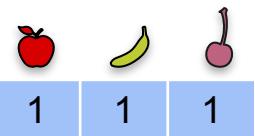
$$a + b + c = 10$$

$$a + 2b + c = 15$$

$$a + b + 2c = 12$$

Equations as dot product

$$a + b + c = 10$$



$$a + 2b + c = 15$$

$$a + b + 2c = 12$$

Equations as dot product

$$a + b + c = 10$$

$$a + 2b + c = 15$$

$$a + b + 2c = 12$$

					a
1	1	1			b
					c

Equations as dot product

$$a + b + c = 10$$

$$a + 2b + c = 15$$

$$a + b + 2c = 12$$

The diagram illustrates the first equation $a + b + c = 10$ as a dot product. It shows a row vector of fruit counts (apple, banana, cherry) with values 1, 1, 1 respectively, multiplied by a column vector of fruit prices (\$apple, \$banana, \$cherry) with values a , b , c respectively, resulting in a scalar value of \$10.

1	1	1
---	---	---

·

\$	apple	a
\$	banana	b
\$	cherry	c

= \$ 10

Equations as dot product

$$a + b + c = 10$$

$$a + 2b + c = 15$$

$$a + b + 2c = 12$$

The diagram illustrates the equation $a + b + c = 10$ using vectors of fruit icons. On the left, there are three blue boxes containing the numbers 1, 1, and 1, each accompanied by an icon of an apple, banana, and cherry respectively. To the right of these boxes is a multiplication sign followed by another set of boxes: a top row with \$ and fruit icons (apple, banana, cherry) and a bottom row with \$ and the letters a, b, and c. To the right of this is a yellow box containing the number 10, preceded by a dollar sign. An equals sign follows the multiplication, and to its right is a vertical orange box containing a dollar sign and the number 10.

The diagram illustrates the equation $a + 2b + c = 15$ using vectors of fruit icons. It features three blue boxes with the numbers 1, 2, and 1, each with an apple, banana, and cherry icon above them. To the right is a vertical orange box containing a dollar sign and the number 15.



Equations as dot product

$$a + b + c = 10$$

The diagram illustrates the equation $a + b + c = 10$ using vectors. On the left, there is a vector of fruits: an apple, a banana, and a cherry, each represented by a red icon. Below this vector is a row of three blue boxes, each containing the number 1. To the right of the fruit vector is a dot product expression: the fruit vector multiplied by a column vector of prices (\$). The column vector has three rows labeled 'a', 'b', and 'c' respectively, with dollar signs (\$) preceding them. To the right of the dot product is an equals sign followed by a yellow box containing the number 10.

$$a + 2b + c = 15$$

The diagram illustrates the equation $a + 2b + c = 15$ using vectors. It follows a similar structure to the first diagram, but the fruit vector now contains two apples and one banana. The row of three blue boxes below it also contains two 1s and one 2. The column vector of prices is identical to the first diagram, with rows labeled 'a', 'b', and 'c'. The result is an equals sign followed by a yellow box containing the number 15.

$$a + b + 2c = 12$$

(vertical line)

Equations as dot product

$$a + b + c = 10$$

The diagram illustrates the equation $a + b + c = 10$ using vectors. On the left, there is a vector of fruits: an apple, a banana, and a cherry, each represented by a red icon. Below this vector is a blue horizontal bar divided into three equal segments, each labeled with the number 1. To the right of the fruit vector is a dot product expression: $\begin{matrix} \text{apple} \\ 1 \\ \text{banana} \\ 1 \\ \text{cherry} \\ 1 \end{matrix} \cdot \begin{matrix} \$ \text{apple} \\ a \\ \$ \text{banana} \\ b \\ \$ \text{cherry} \\ c \end{matrix} = \$ 10$. The fruit vector is multiplied by a vector where each element is labeled with a dollar sign and the name of the fruit, followed by a variable (a, b, or c). The result of the dot product is \$10, shown in an orange box.

$$a + 2b + c = 15$$

The diagram illustrates the equation $a + 2b + c = 15$ using vectors. It follows a similar structure to the first diagram, but the fruit vector has two bananas instead of one. The fruit vector is: an apple, two bananas, and a cherry. Below it is a blue horizontal bar divided into four equal segments, with the first three labeled 1 and the fourth labeled 2. To the right is the dot product expression: $\begin{matrix} \text{apple} \\ 1 \\ \text{banana} \\ 2 \\ \text{banana} \\ 1 \\ \text{cherry} \\ 1 \end{matrix} \cdot \begin{matrix} \$ \text{apple} \\ a \\ \$ \text{banana} \\ b \\ \$ \text{banana} \\ b \\ \$ \text{cherry} \\ c \end{matrix} = \$ 15$. The result of the dot product is \$15, shown in an orange box.

$$a + b + 2c = 12$$

Equations as dot product

$$a + b + c = 10$$

The diagram illustrates the equation $a + b + c = 10$ using vectors. On the left, there is a vector of fruits: an apple, a banana, and a cherry, each represented by a red icon. Below this vector is a blue horizontal bar divided into three equal segments, each labeled with the number 1. To the right of the fruit vector is a multiplication sign (\cdot). Next is a dollar sign ($\$$) followed by a teal 3x3 matrix. The matrix has columns for the fruit icons and rows for the numbers 1, 2, and 3. The matrix is labeled with variables: a in the top-right cell, b in the middle-right cell, and c in the bottom-right cell. To the right of the matrix is another dollar sign ($\$$) followed by a yellow box containing the number 10.

$$a + 2b + c = 15$$

The diagram illustrates the equation $a + 2b + c = 15$ using vectors. It follows a similar structure to the first diagram. On the left, there is a vector of fruits: an apple, a banana, and a cherry. Below it is a blue horizontal bar divided into three segments, with the middle segment labeled 2 and the right segment labeled 1. To the right of the fruit vector is a multiplication sign (\cdot). Next is a dollar sign ($\$$) followed by a teal 3x3 matrix. The matrix has columns for the fruit icons and rows for the numbers 1, 2, and 3. The matrix is labeled with variables: a in the top-right cell, b in the middle-right cell, and c in the bottom-right cell. To the right of the matrix is another dollar sign ($\$$) followed by a yellow box containing the number 15.

$$a + b + 2c = 12$$

The diagram illustrates the equation $a + b + 2c = 12$ using vectors. It follows a similar structure to the first two diagrams. On the left, there is a vector of fruits: an apple, a banana, and a cherry. Below it is a blue horizontal bar divided into three segments, with the rightmost segment labeled 2. To the right of the fruit vector is a multiplication sign (\cdot). There is no dollar sign or matrix present in this part of the diagram.

Equations as dot product

$$a + b + c = 10$$

The diagram illustrates the equation $a + b + c = 10$ using vectors. On the left, there is a vector of fruits: one apple, one banana, and one cherry. This vector is multiplied by a scalar vector $\begin{bmatrix} a \\ b \\ c \end{bmatrix}$, resulting in a total value of \$10.

1	1	1
\$	apple	a
\$	banana	b
\$	cherry	c

$$\begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \$10$$

$$a + 2b + c = 15$$

The diagram illustrates the equation $a + 2b + c = 15$ using vectors. It shows a vector of fruits (one apple, two bananas, one cherry) multiplied by a scalar vector $\begin{bmatrix} a \\ b \\ c \end{bmatrix}$, resulting in a total value of \$15.

1	2	1
\$	apple	a
\$	banana	b
\$	cherry	c

$$\begin{bmatrix} 1 & 2 & 1 \end{bmatrix} \cdot \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \$15$$

$$a + b + 2c = 12$$

The diagram illustrates the equation $a + b + 2c = 12$ using vectors. It shows a vector of fruits (one apple, one banana, two cherries) multiplied by a scalar vector $\begin{bmatrix} a \\ b \\ c \end{bmatrix}$, resulting in a total value of \$12.

1	1	2
\$	apple	a
\$	banana	b
\$	cherry	c

$$\begin{bmatrix} 1 & 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \$12$$

Equations as dot product

$$a + b + c = 10$$

A diagram illustrating the equation $a + b + c = 10$. It shows three vectors on the left: a red apple icon with a value of 1 below it, a yellow banana icon with a value of 1 below it, and a pink cherry icon with a value of 1 below it. To the right of these vectors is a multiplication sign followed by a matrix multiplication symbol (\cdot). The matrix has three columns, each corresponding to one of the three icons above. The first column is labeled 'a' at the top and has a dollar sign (\$) before the bottom cell. The second column is labeled 'b' at the top and has a dollar sign (\$) before the bottom cell. The third column is labeled 'c' at the top and has a dollar sign (\$) before the bottom cell. To the right of the matrix is an equals sign followed by a yellow box containing the number 10.

$$a + 2b + c = 15$$

A diagram illustrating the equation $a + 2b + c = 15$. It shows three vectors on the left: a red apple icon with a value of 1 below it, a yellow banana icon with a value of 2 below it, and a pink cherry icon with a value of 1 below it. To the right of these vectors is a multiplication sign followed by a matrix multiplication symbol (\cdot). The matrix has three columns, each corresponding to one of the three icons above. The first column is labeled 'a' at the top and has a dollar sign (\$) before the bottom cell. The second column is labeled 'b' at the top and has a dollar sign (\$) before the bottom cell. The third column is labeled 'c' at the top and has a dollar sign (\$) before the bottom cell. To the right of the matrix is an equals sign followed by a yellow box containing the number 15.

$$a + b + 2c = 12$$

A diagram illustrating the equation $a + b + 2c = 12$. It shows three vectors on the left: a red apple icon with a value of 1 below it, a yellow banana icon with a value of 1 below it, and a pink cherry icon with a value of 2 below it. To the right of these vectors is a multiplication sign followed by a matrix multiplication symbol (\cdot). The matrix has three columns, each corresponding to one of the three icons above. The first column is labeled 'a' at the top and has a dollar sign (\$) before the bottom cell. The second column is labeled 'b' at the top and has a dollar sign (\$) before the bottom cell. The third column is labeled 'c' at the top and has a dollar sign (\$) before the bottom cell. To the right of the matrix is an equals sign followed by a yellow box containing the number 12.

Equations as dot product

$$a + b + c = 10$$

The diagram illustrates the equation $a + b + c = 10$ using vectors. On the left, there is a vector of fruits: one apple, one banana, and one cherry. This vector is multiplied by a scalar vector $\begin{bmatrix} a \\ b \\ c \end{bmatrix}$, resulting in a total value of \$10.

1	1	1
\$	apple	a
\$	banana	b
\$	cherry	c

$$\begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \$10$$

$$a + 2b + c = 15$$

The diagram illustrates the equation $a + 2b + c = 15$ using vectors. It shows a vector of fruits (one apple, two bananas, one cherry) multiplied by a scalar vector $\begin{bmatrix} a \\ b \\ c \end{bmatrix}$, resulting in a total value of \$15.

1	2	1
\$	apple	a
\$	banana	b
\$	cherry	c

$$\begin{bmatrix} 1 & 2 & 1 \end{bmatrix} \cdot \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \$15$$

$$a + b + 2c = 12$$

The diagram illustrates the equation $a + b + 2c = 12$ using vectors. It shows a vector of fruits (one apple, one banana, two cherries) multiplied by a scalar vector $\begin{bmatrix} a \\ b \\ c \end{bmatrix}$, resulting in a total value of \$12.

1	1	2
\$	apple	a
\$	banana	b
\$	cherry	c

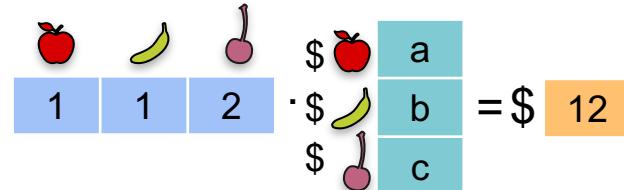
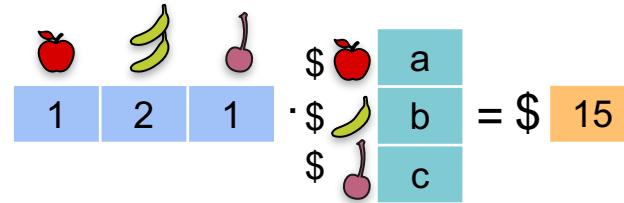
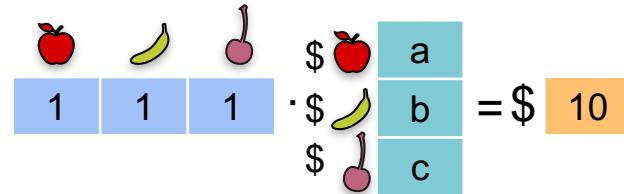
$$\begin{bmatrix} 1 & 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \$12$$

Equations as dot product

$$a + b + c = 10$$

$$a + 2b + c = 15$$

$$a + b + 2c = 12$$

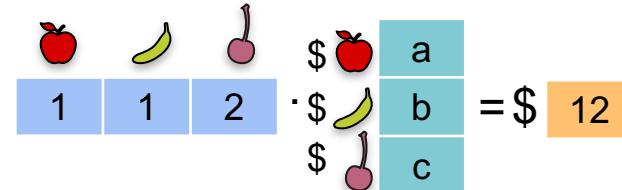
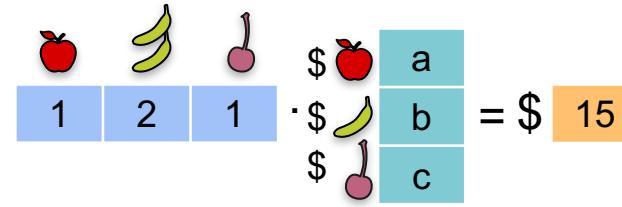
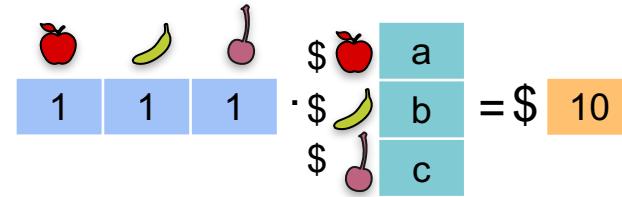


Equations as dot product

$$a + b + c = 10$$

$$a + 2b + c = 15$$

$$a + b + 2c = 12$$



Equations as dot product

System of equations

$$a + b + c = 10$$

$$a + 2b + c = 15$$

$$a + b + 2c = 12$$

Matrix product

The diagram illustrates the conversion of a system of linear equations into a matrix equation. On the left, three fruit icons (apple, banana, cherry) are positioned above a 3x3 matrix of coefficients. To the right of the multiplication sign is another 3x3 matrix where the columns represent the variables a , b , and c . The resulting matrix on the far right contains the constants 10, 15, and 12.

1	1	1	\$	apple	a	= \$	10
1	2	1	\$	banana	b	= \$	15
1	1	2	\$	cherry	c	= \$	12

Equations as dot product

System of equations

$$a + b + c = 10$$

$$a + 2b + c = 15$$

$$a + b + 2c = 12$$

Matrix product

$$\begin{matrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{matrix} \begin{matrix} a \\ b \\ c \end{matrix} = \begin{matrix} 10 \\ 15 \\ 12 \end{matrix}$$



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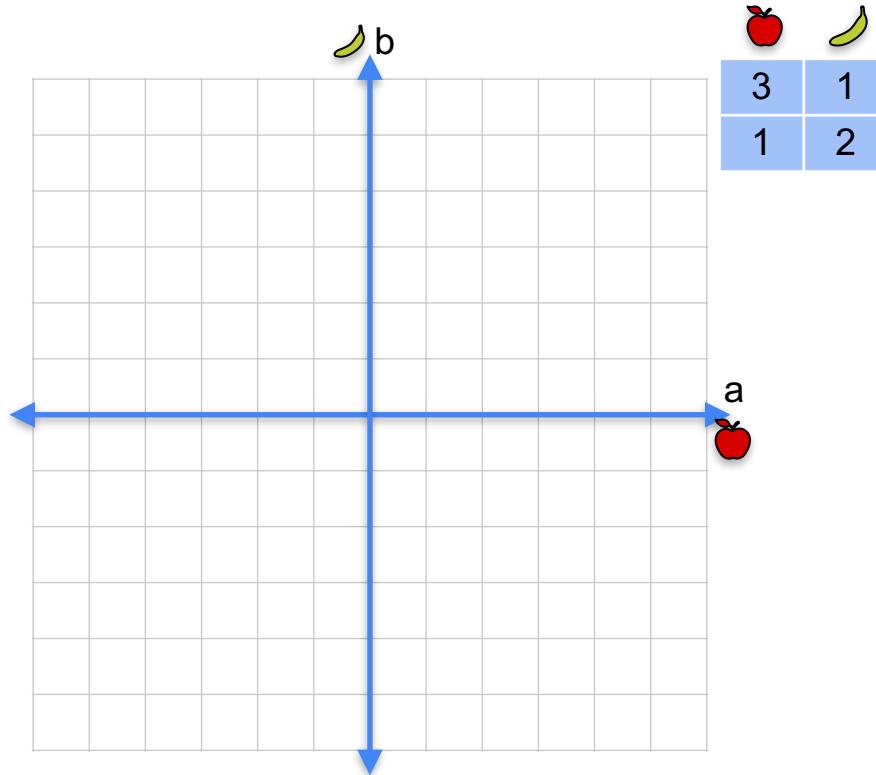
Vectors and Linear Transformations

**Matrices as linear
transformations**

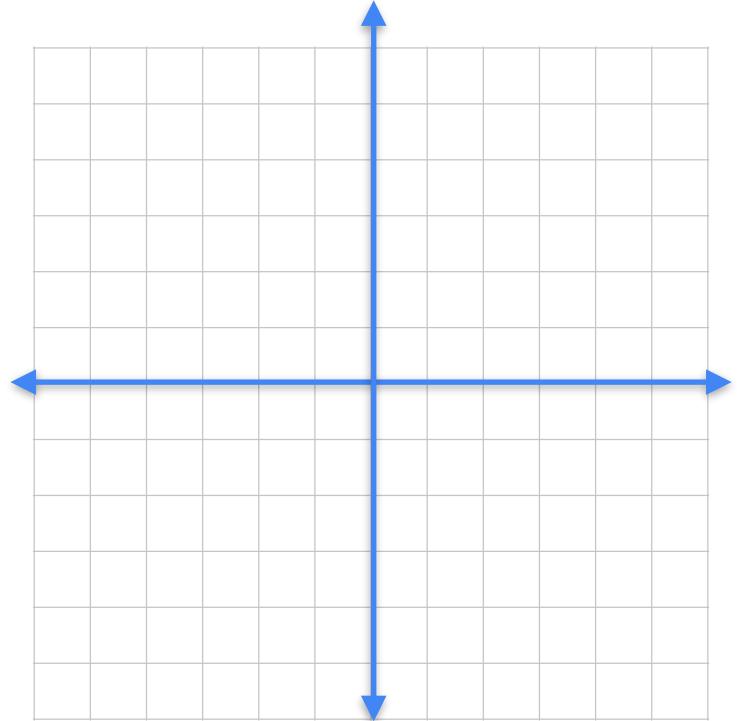
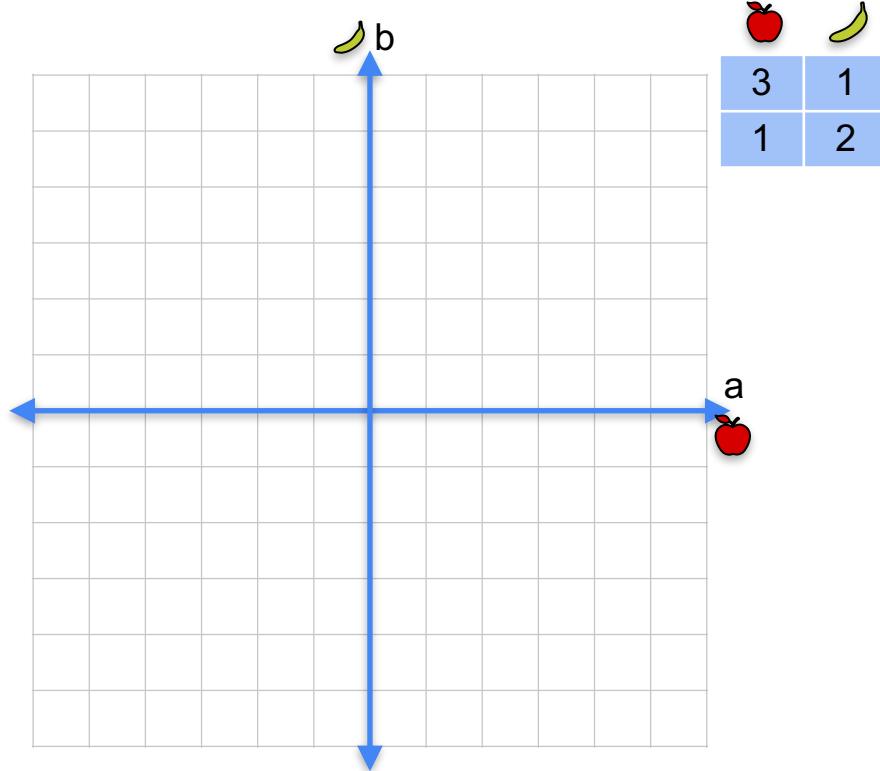
Matrices as linear transformations

	
3	1
1	2

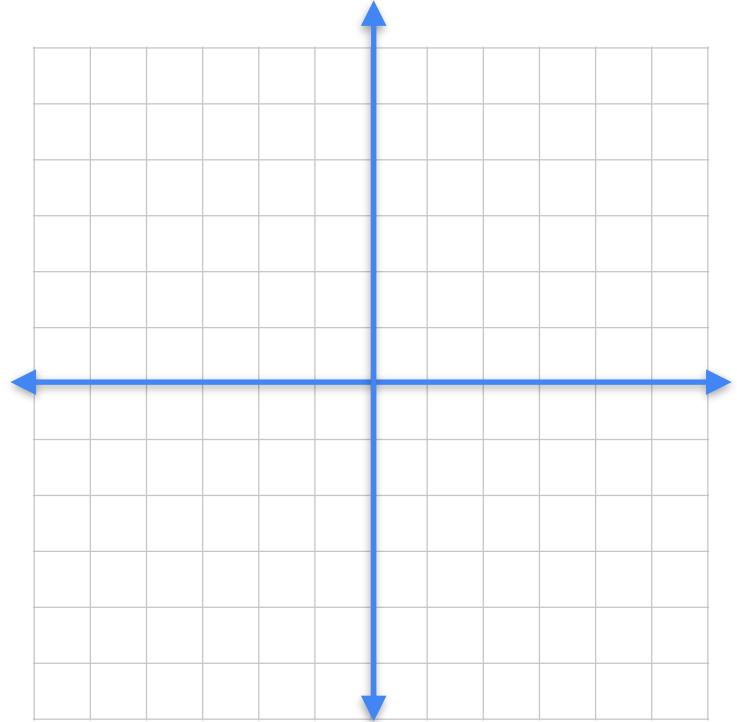
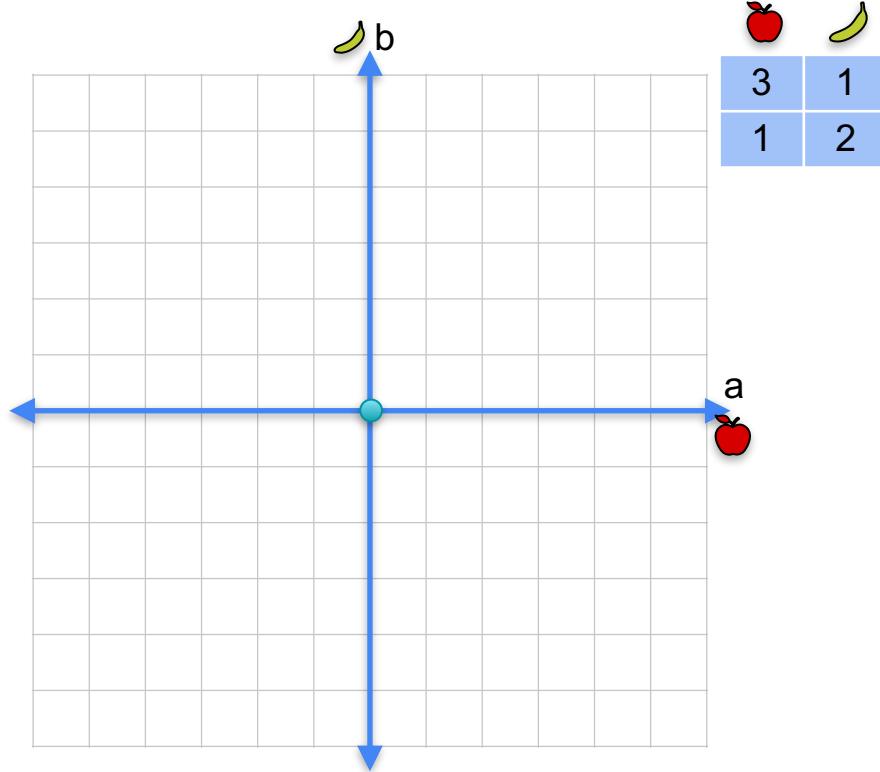
Matrices as linear transformations



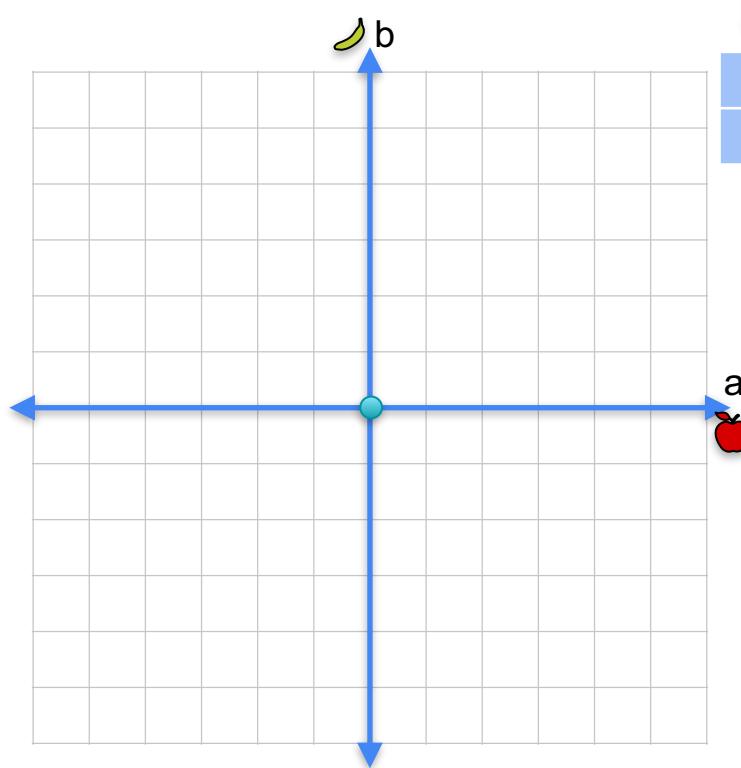
Matrices as linear transformations



Matrices as linear transformations

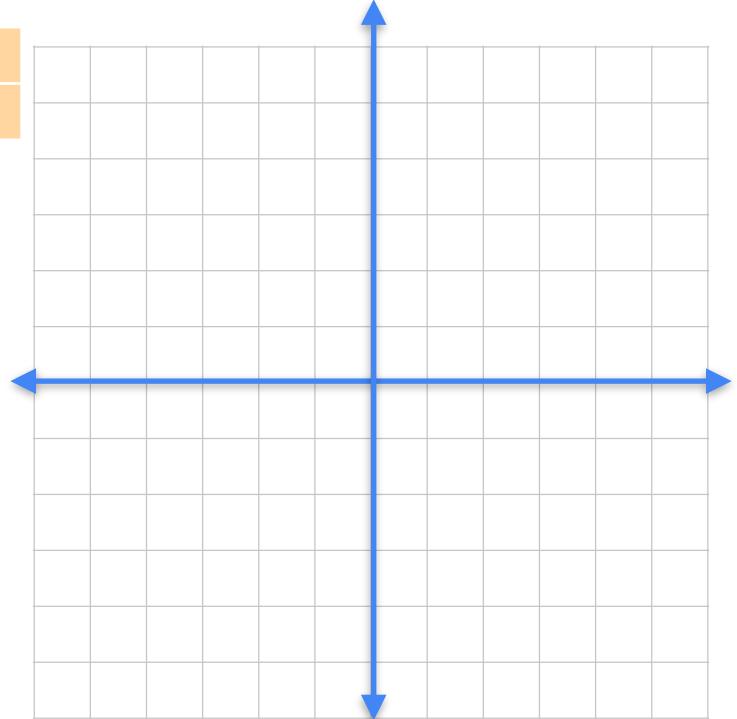


Matrices as linear transformations

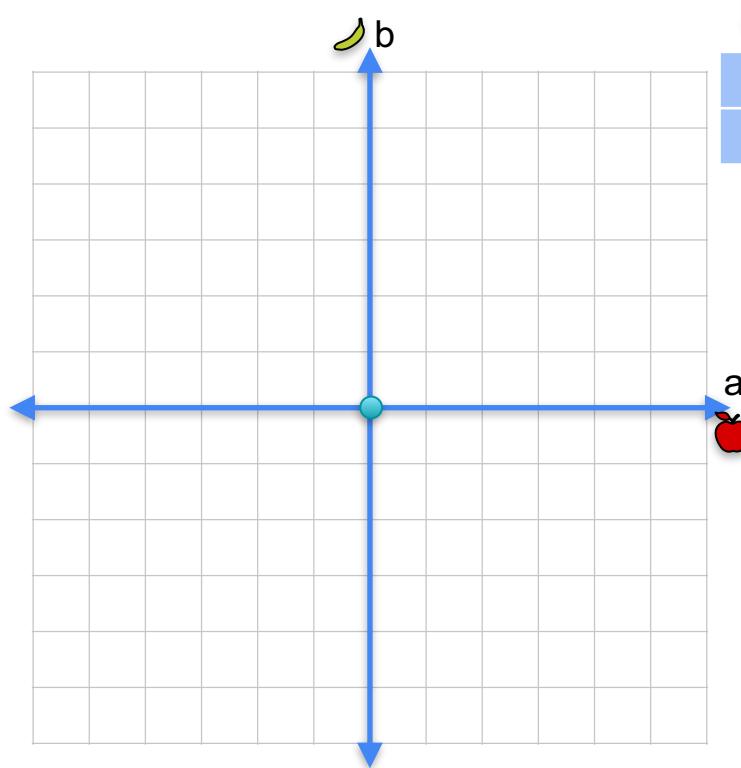


$$\begin{matrix} \text{apple} & \text{banana} \\ \begin{matrix} 3 & 1 \\ 1 & 2 \end{matrix} & \begin{matrix} 0 \\ 0 \end{matrix} \\ = & \begin{matrix} 0 \\ 0 \end{matrix} \end{matrix}$$

$(0,0) \rightarrow (0,0)$

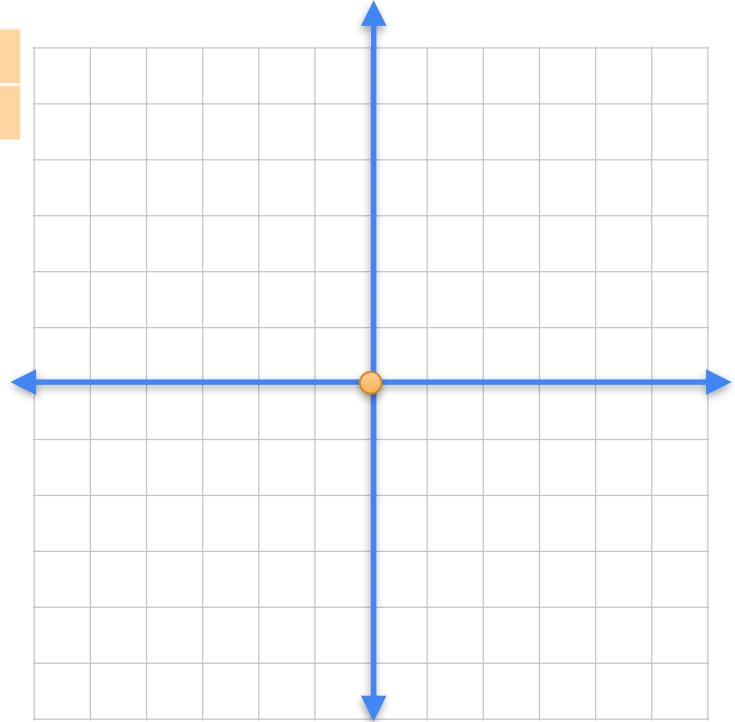


Matrices as linear transformations

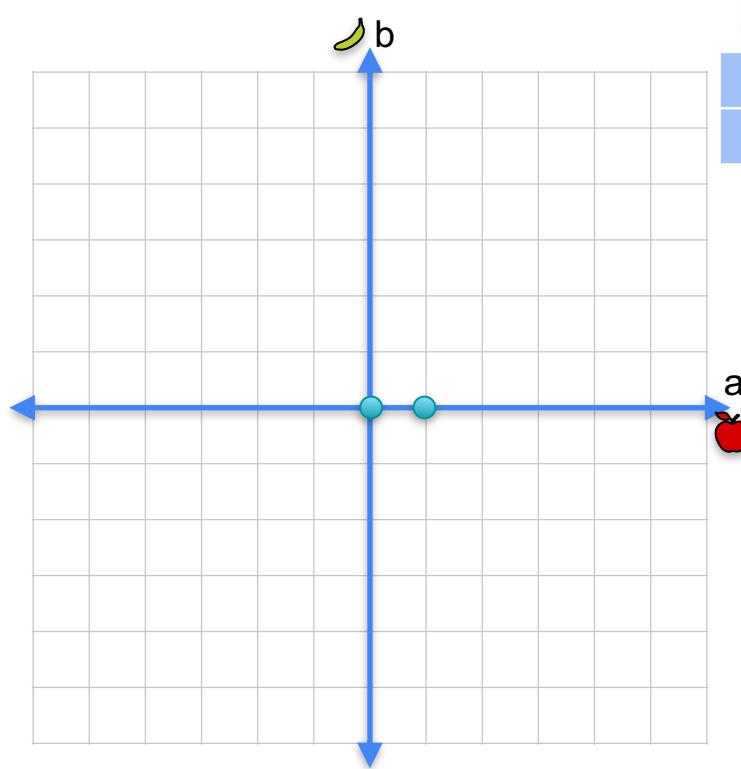


$$\begin{matrix} \text{apple} & \text{banana} \\ \begin{matrix} 3 & 1 \\ 1 & 2 \end{matrix} & \begin{matrix} 0 \\ 0 \end{matrix} \\ = & \begin{matrix} 0 \\ 0 \end{matrix} \end{matrix}$$

$(0,0) \rightarrow (0,0)$

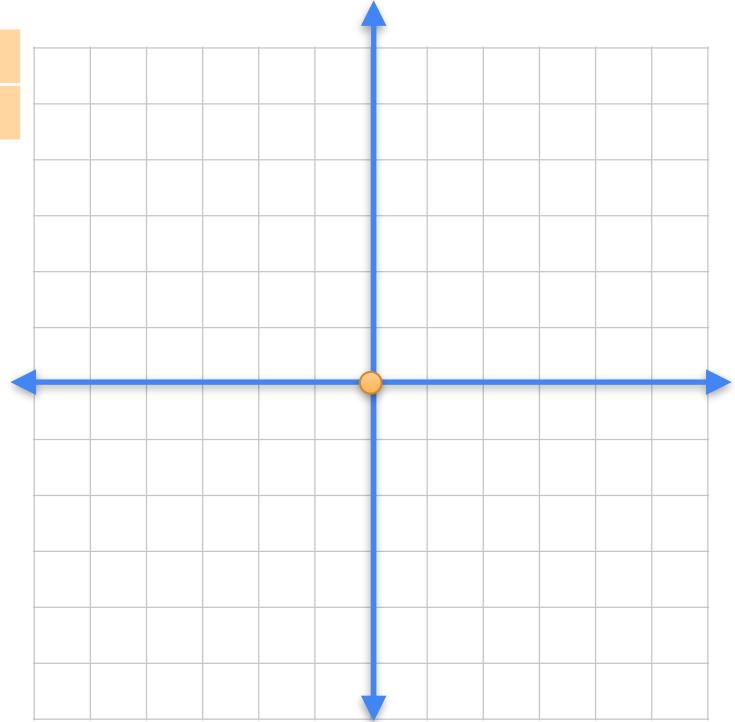


Matrices as linear transformations

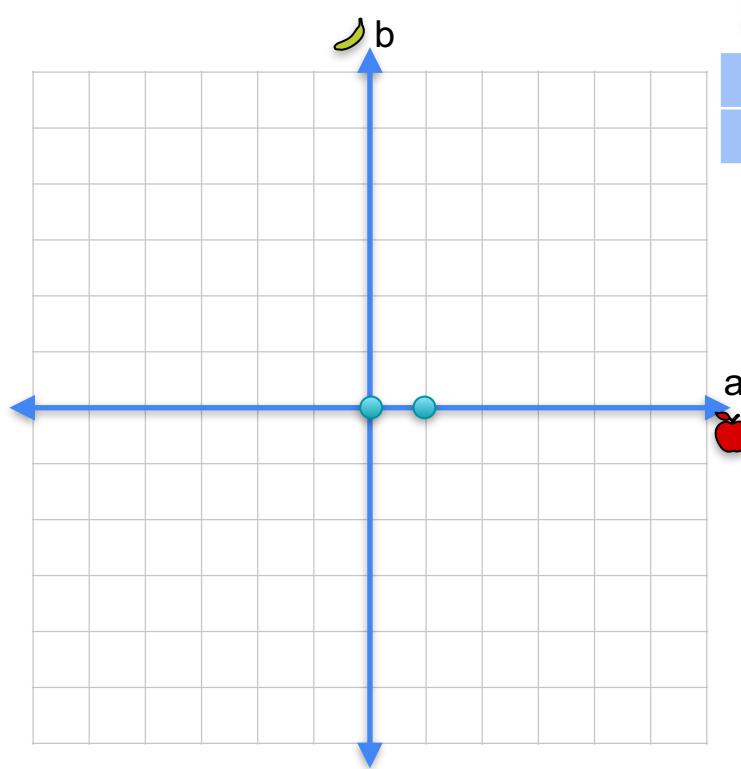


$$\begin{matrix} \text{apple} & \text{banana} \\ \begin{matrix} 3 & 1 \\ 1 & 2 \end{matrix} & \begin{matrix} 0 \\ 0 \end{matrix} \\ = & \begin{matrix} 0 \\ 0 \end{matrix} \end{matrix}$$

$(0,0) \rightarrow (0,0)$

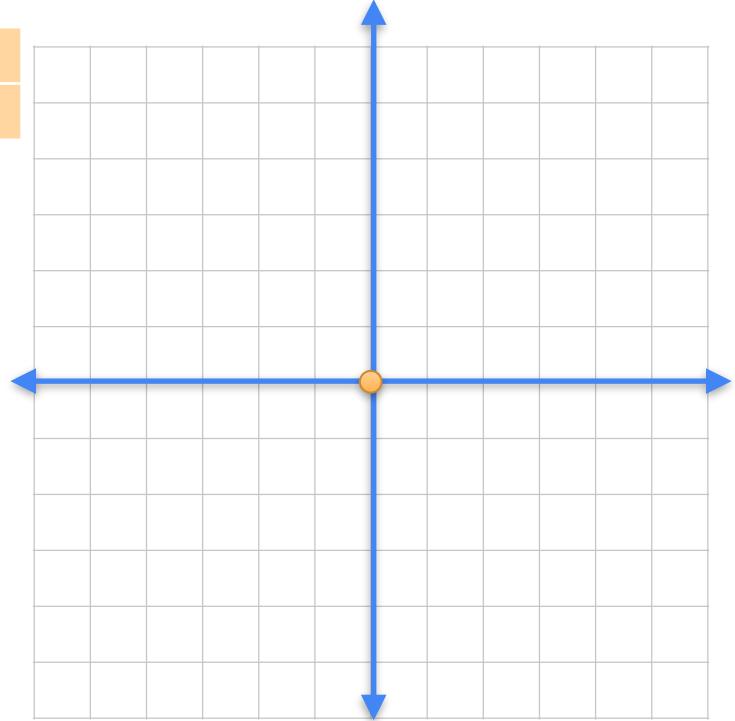


Matrices as linear transformations

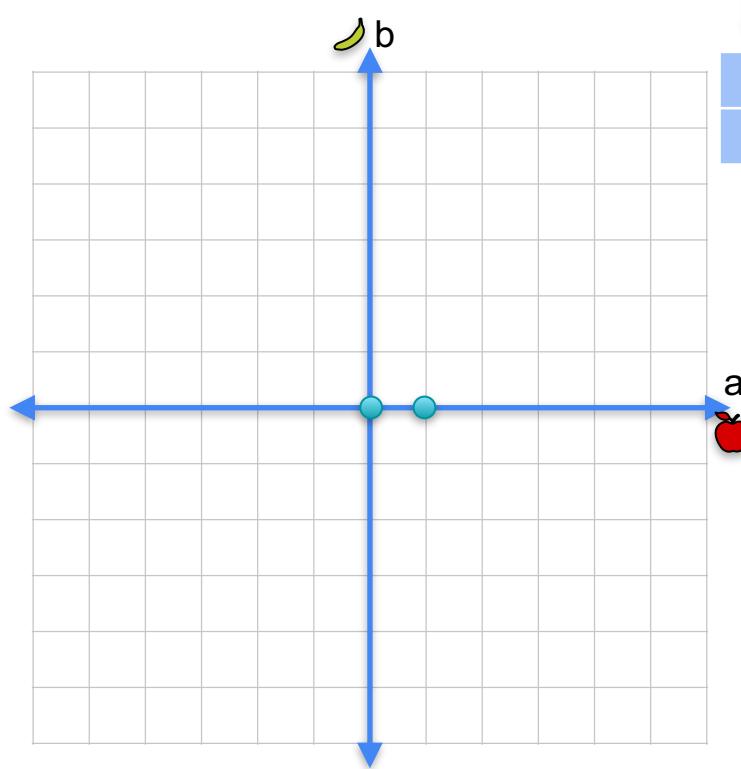


$$\begin{matrix} \text{apple} & \text{banana} \\ \begin{matrix} 3 & 1 \\ 1 & 2 \end{matrix} & \begin{matrix} 1 \\ 0 \end{matrix} \end{matrix} = \begin{matrix} 3 \\ 1 \end{matrix}$$

$$\begin{aligned} (0,0) &\rightarrow (0,0) \\ (1,0) &\rightarrow (3,1) \end{aligned}$$

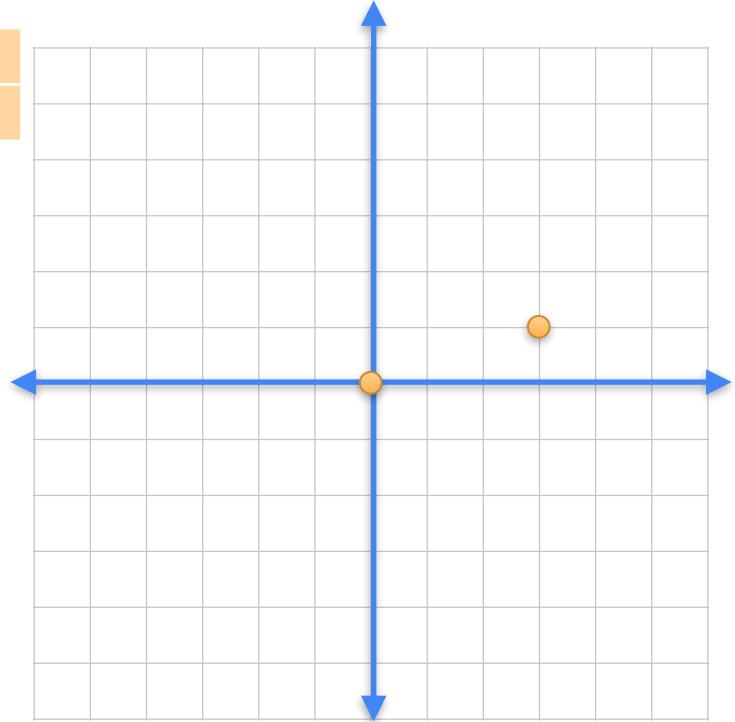


Matrices as linear transformations

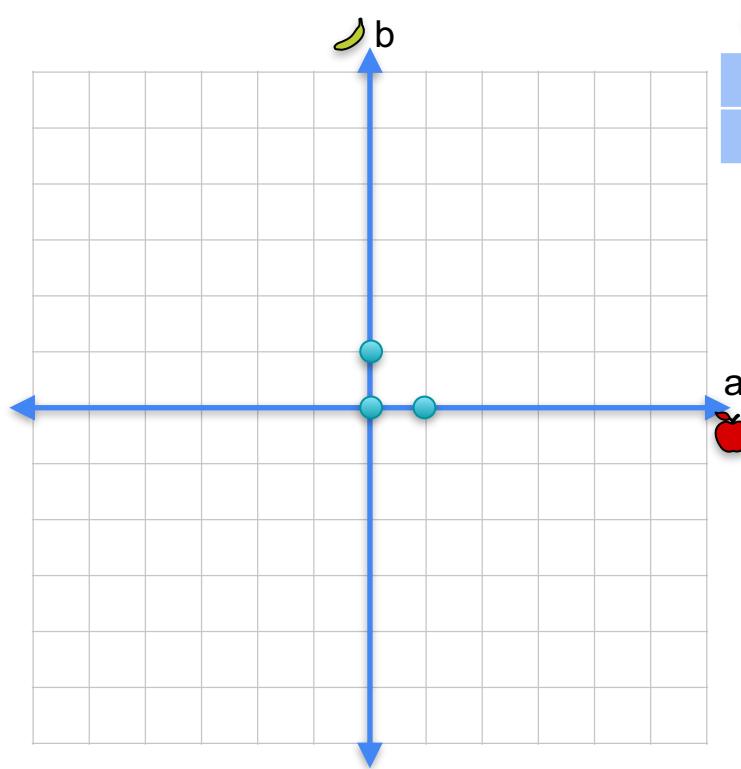


$$\begin{matrix} \text{apple} & \text{banana} \\ \begin{matrix} 3 & 1 \\ 1 & 2 \end{matrix} & \begin{matrix} 1 \\ 0 \end{matrix} \end{matrix} = \begin{matrix} 3 \\ 1 \end{matrix}$$

$$\begin{aligned} (0,0) &\rightarrow (0,0) \\ (1,0) &\rightarrow (3,1) \end{aligned}$$

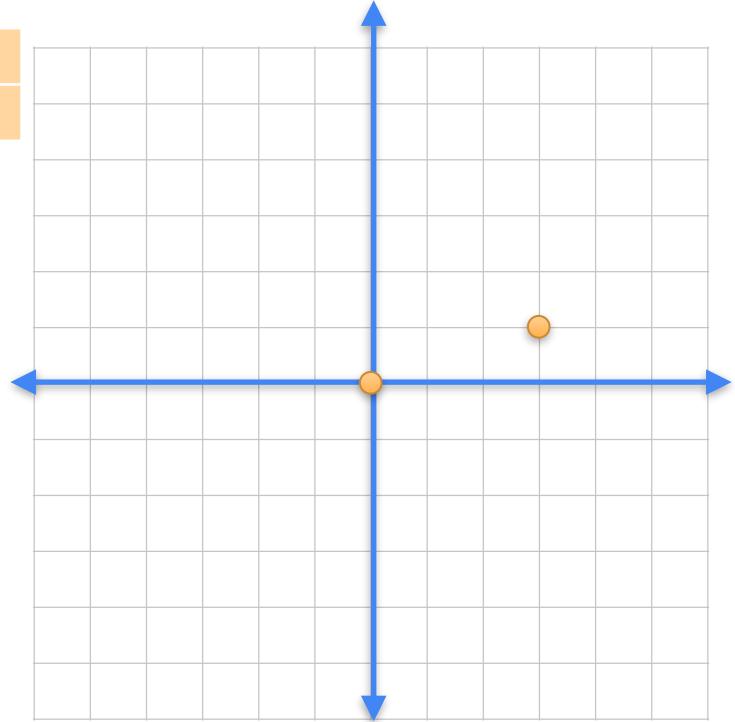


Matrices as linear transformations

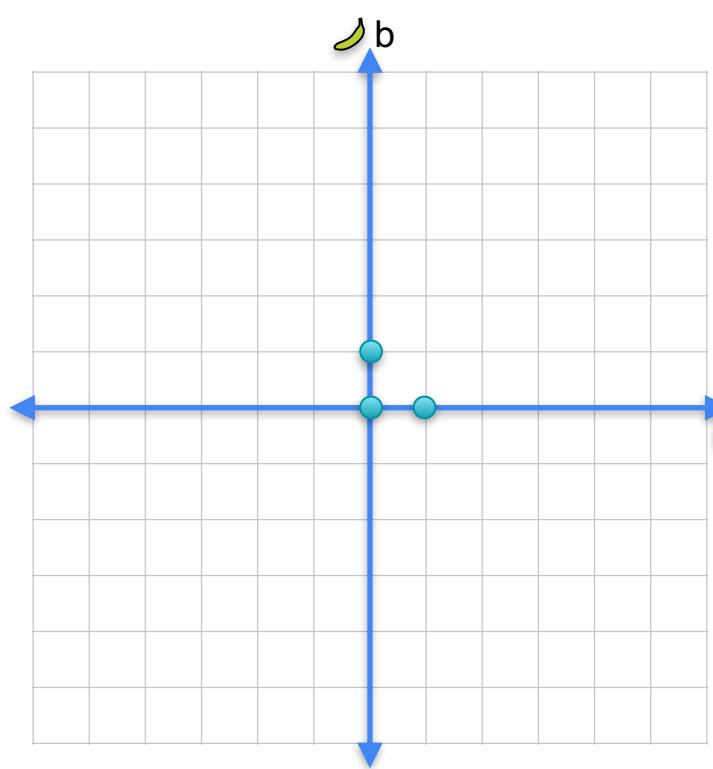


$$\begin{matrix} \text{apple} & \text{banana} \\ \begin{matrix} 3 & 1 \\ 1 & 2 \end{matrix} & \begin{matrix} 1 \\ 0 \end{matrix} \end{matrix} = \begin{matrix} 3 \\ 1 \end{matrix}$$

$$\begin{aligned} (0,0) &\rightarrow (0,0) \\ (1,0) &\rightarrow (3,1) \end{aligned}$$

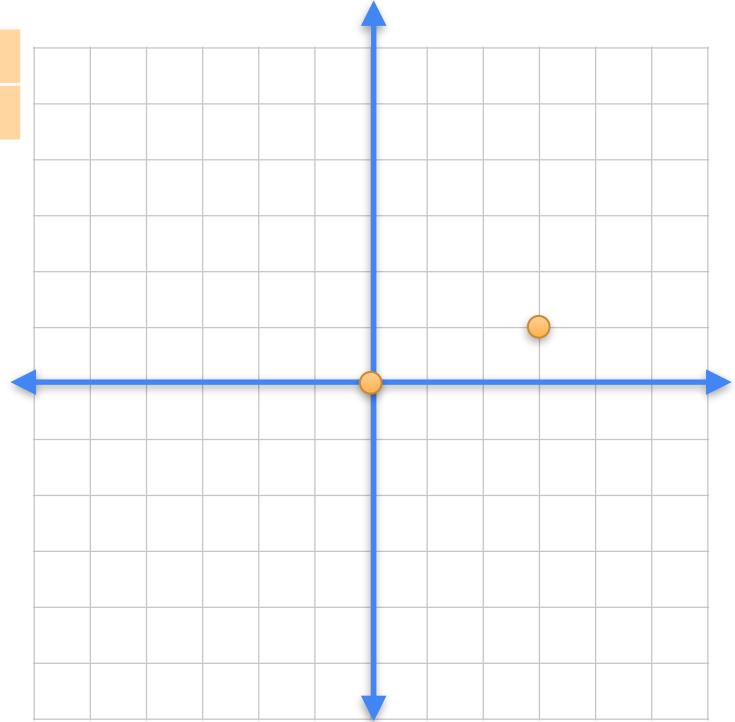


Matrices as linear transformations

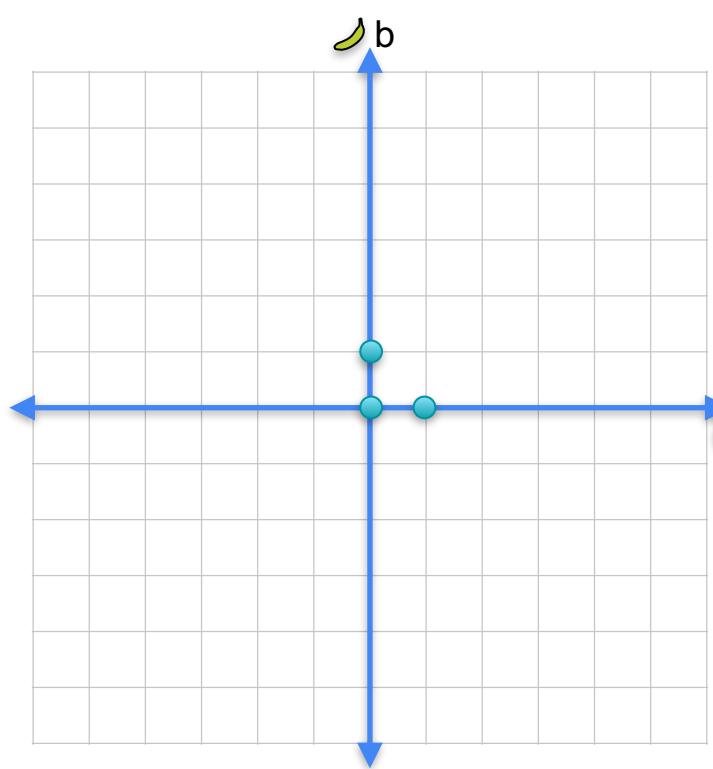


$$\begin{matrix} \text{apple} & \text{banana} \\ \begin{matrix} 3 & 1 \\ 1 & 2 \end{matrix} & \begin{matrix} 0 \\ 1 \end{matrix} \\ = & \begin{matrix} 1 \\ 2 \end{matrix} \end{matrix}$$

$(0,0) \rightarrow (0,0)$
 $(1,0) \rightarrow (3,1)$
 $(0,1) \rightarrow (1,2)$

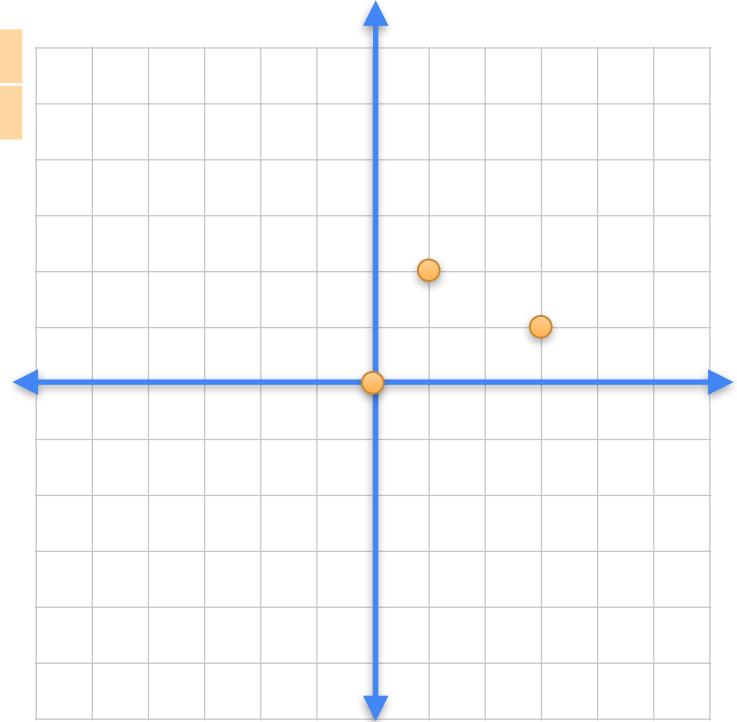


Matrices as linear transformations

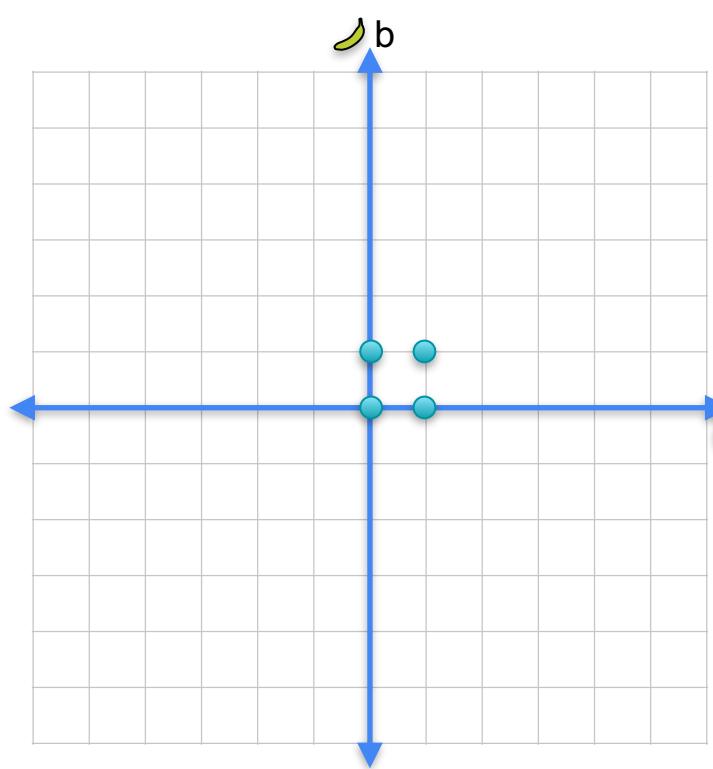


$$\begin{matrix} \text{apple} & \text{banana} \\ \begin{matrix} 3 & 1 \\ 1 & 2 \end{matrix} & \begin{matrix} 0 \\ 1 \end{matrix} \end{matrix} = \begin{matrix} 1 \\ 2 \end{matrix}$$

$$\begin{aligned} (0,0) &\rightarrow (0,0) \\ (1,0) &\rightarrow (3,1) \\ (0,1) &\rightarrow (1,2) \end{aligned}$$

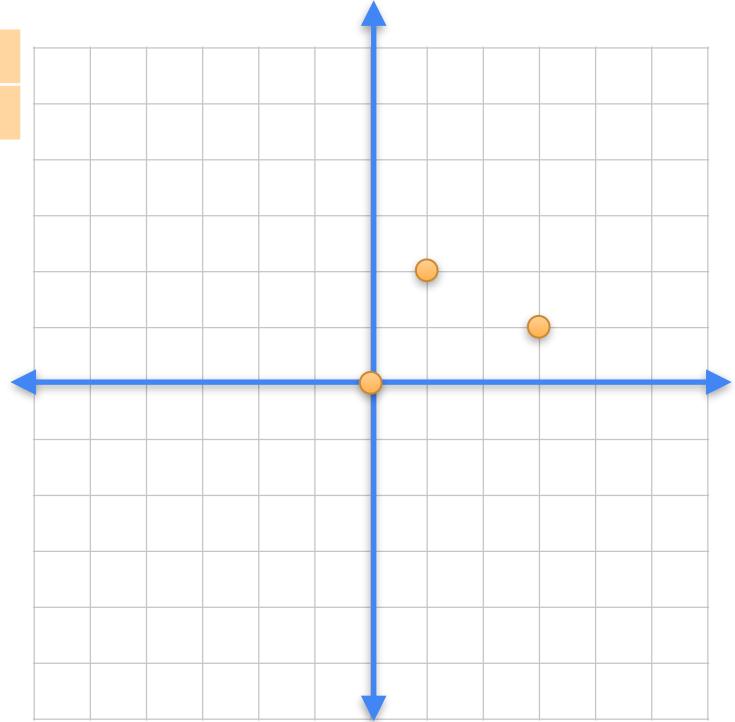


Matrices as linear transformations

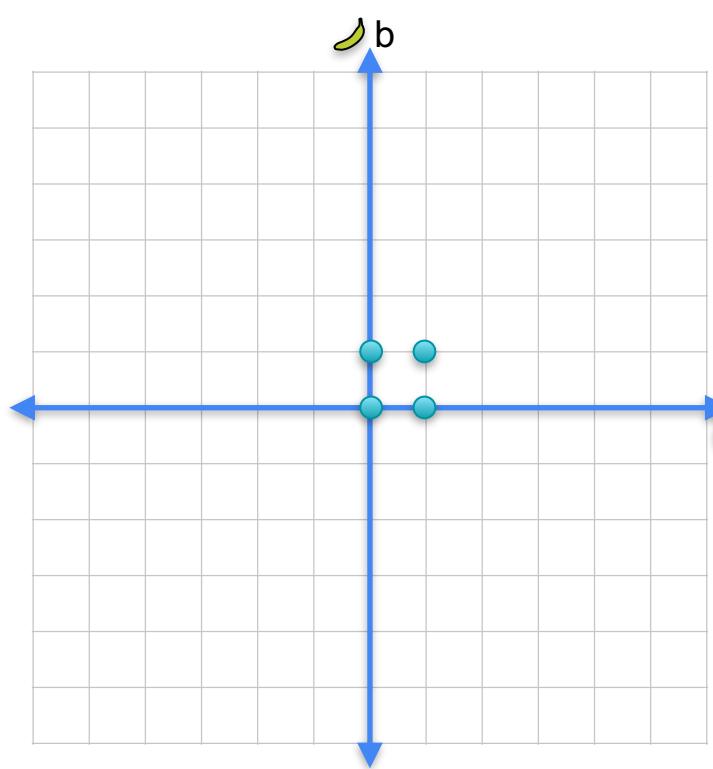


$$\begin{matrix} \text{apple} & \text{banana} \\ \begin{matrix} 3 & 1 \\ 1 & 2 \end{matrix} & \begin{matrix} 0 \\ 1 \end{matrix} \\ = & \begin{matrix} 1 \\ 2 \end{matrix} \end{matrix}$$

$(0,0) \rightarrow (0,0)$
 $(1,0) \rightarrow (3,1)$
 $(0,1) \rightarrow (1,2)$

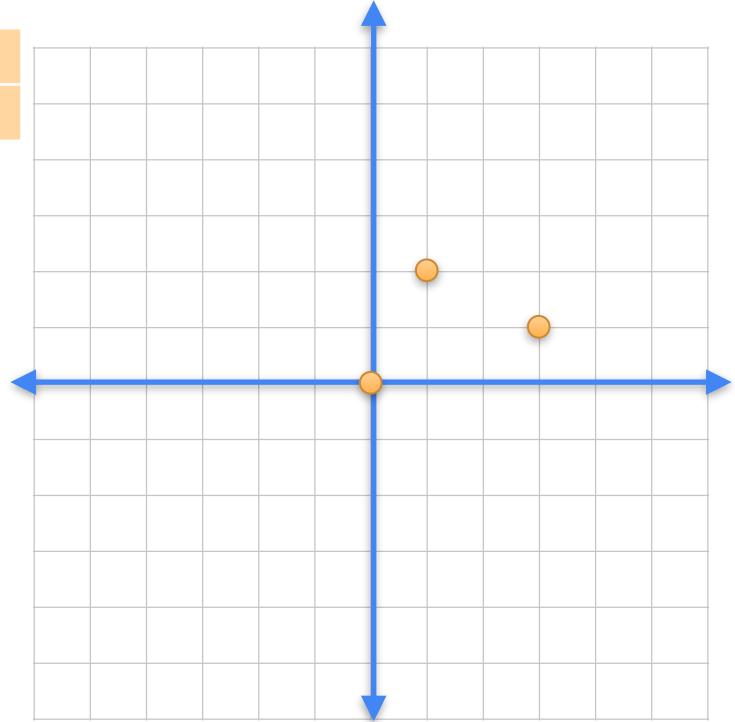


Matrices as linear transformations

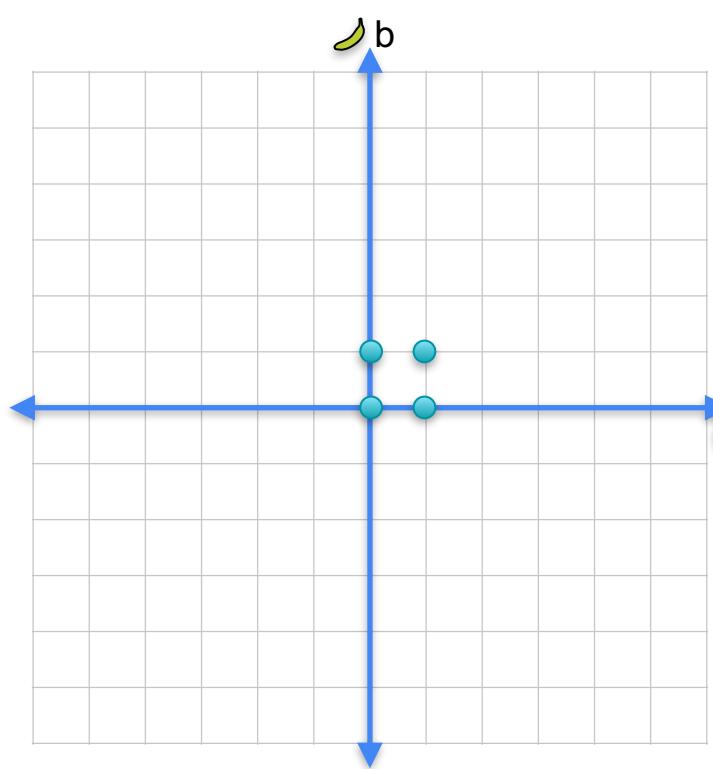


$$\begin{matrix} \text{apple} & \text{banana} \\ \begin{matrix} 3 & 1 \\ 1 & 2 \end{matrix} & \begin{matrix} 1 \\ 1 \end{matrix} \end{matrix} = \begin{matrix} 4 \\ 3 \end{matrix}$$

- $(0,0) \rightarrow (0,0)$
- $(1,0) \rightarrow (3,1)$
- $(0,1) \rightarrow (1,2)$
- $(1,1) \rightarrow (4,3)$

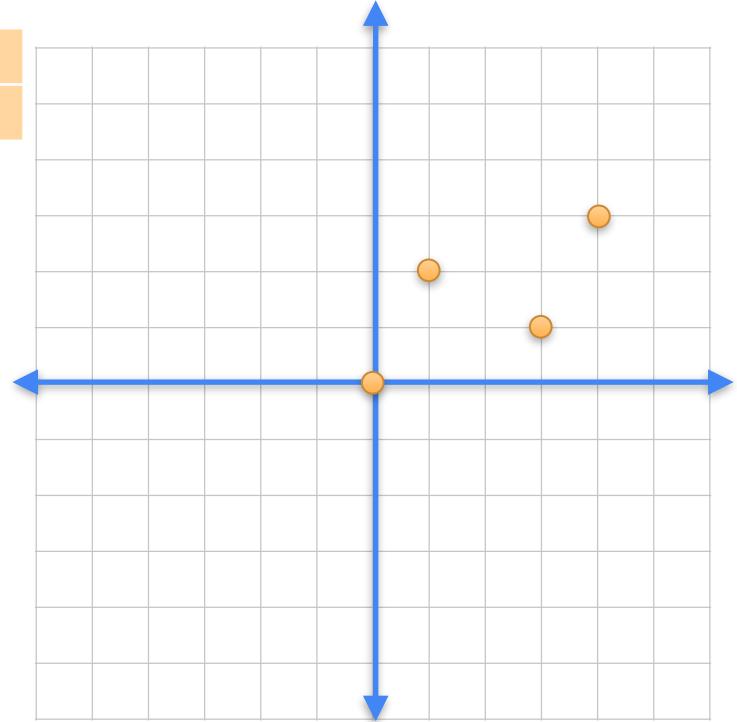


Matrices as linear transformations

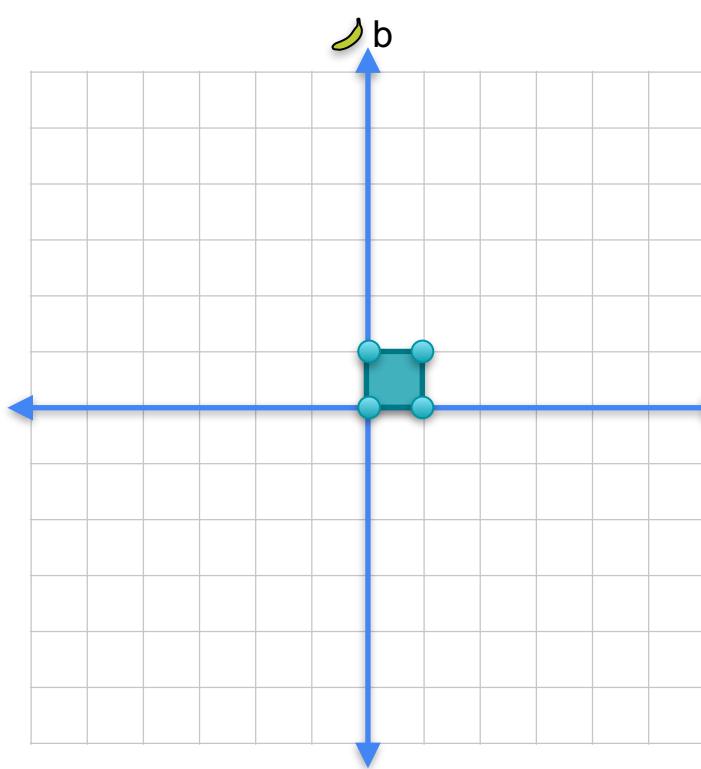


$$\begin{matrix} \text{apple} & \text{banana} \\ \begin{matrix} 3 & 1 \\ 1 & 2 \end{matrix} & \begin{matrix} 1 \\ 1 \end{matrix} \end{matrix} = \begin{matrix} 4 \\ 3 \end{matrix}$$

$$\begin{aligned} (0,0) &\rightarrow (0,0) \\ (1,0) &\rightarrow (3,1) \\ (0,1) &\rightarrow (1,2) \\ (1,1) &\rightarrow (4,3) \end{aligned}$$

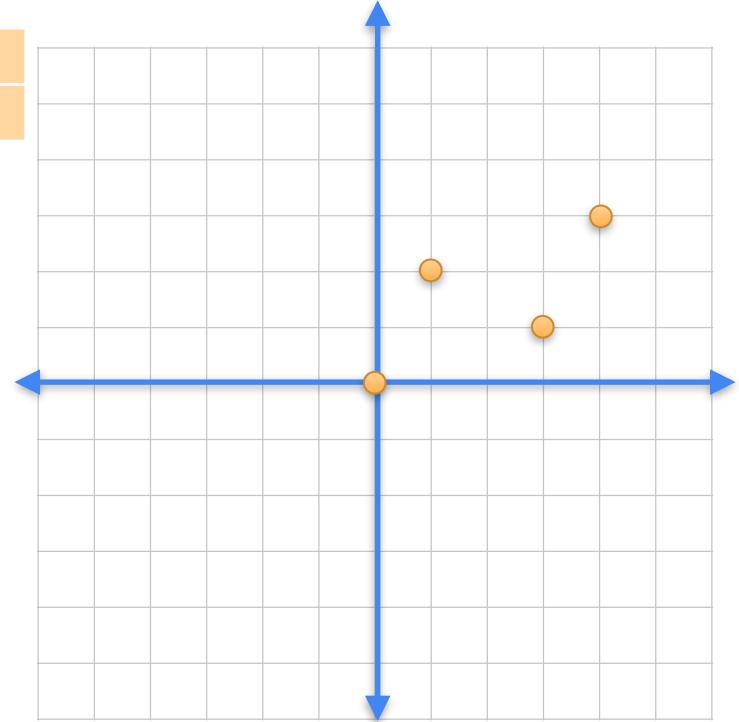


Matrices as linear transformations

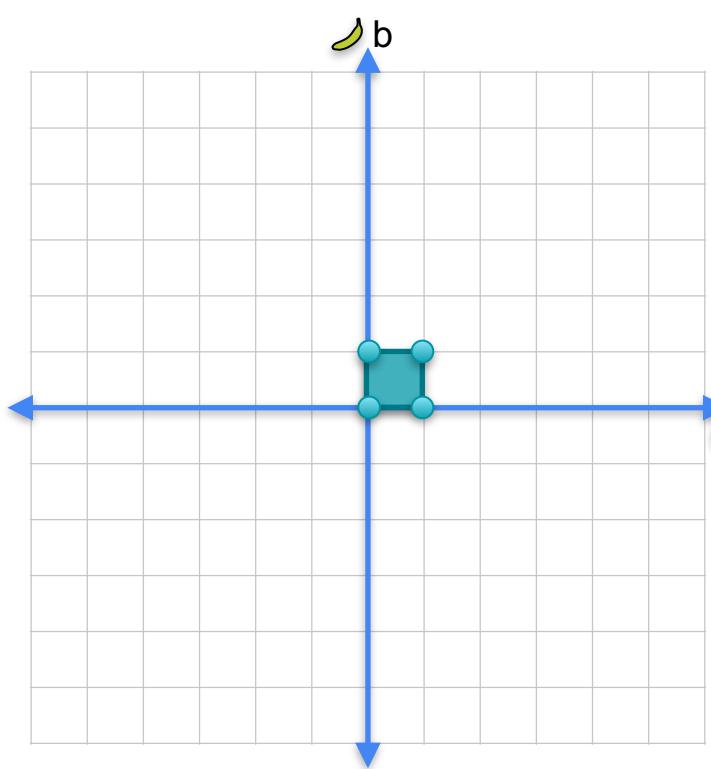


$$\begin{matrix} \text{apple} & \text{banana} \\ \begin{matrix} 3 & 1 \\ 1 & 2 \end{matrix} & \begin{matrix} 1 \\ 1 \end{matrix} \end{matrix} = \begin{matrix} 4 \\ 3 \end{matrix}$$

- $(0,0) \rightarrow (0,0)$
- $(1,0) \rightarrow (3,1)$
- $(0,1) \rightarrow (1,2)$
- $(1,1) \rightarrow (4,3)$

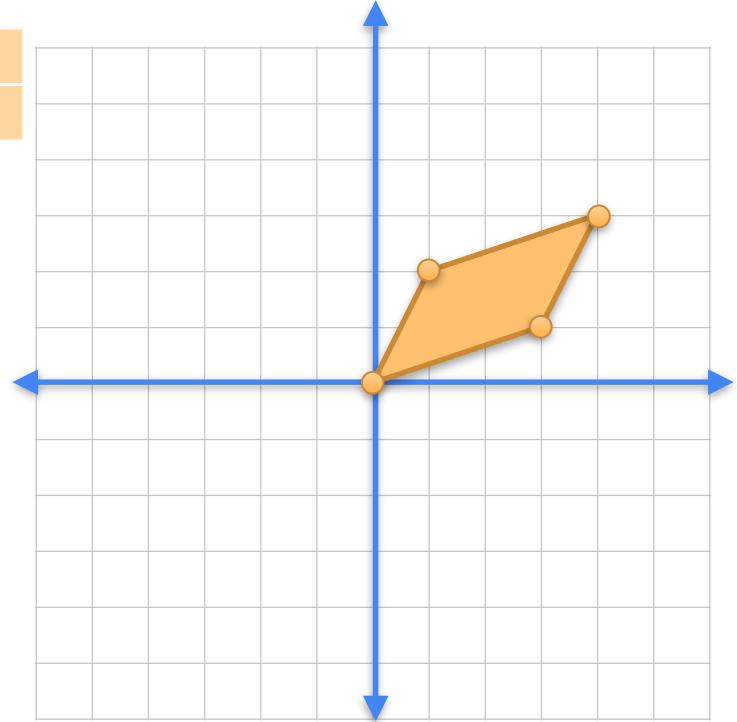


Matrices as linear transformations

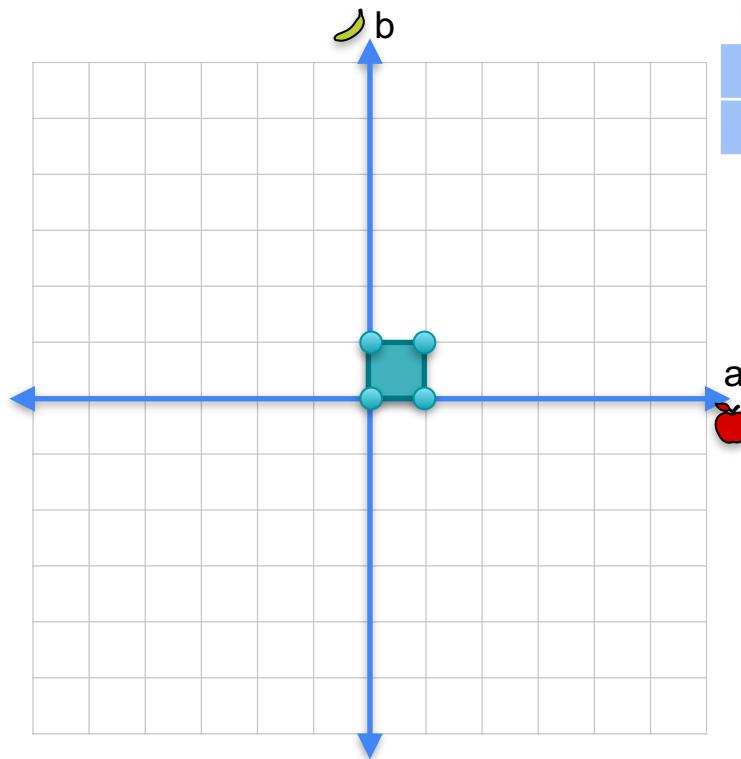


$$\begin{matrix} \text{apple} & \text{banana} \\ \begin{matrix} 3 & 1 \\ 1 & 2 \end{matrix} & \begin{matrix} 1 \\ 1 \end{matrix} \end{matrix} = \begin{matrix} 4 \\ 3 \end{matrix}$$

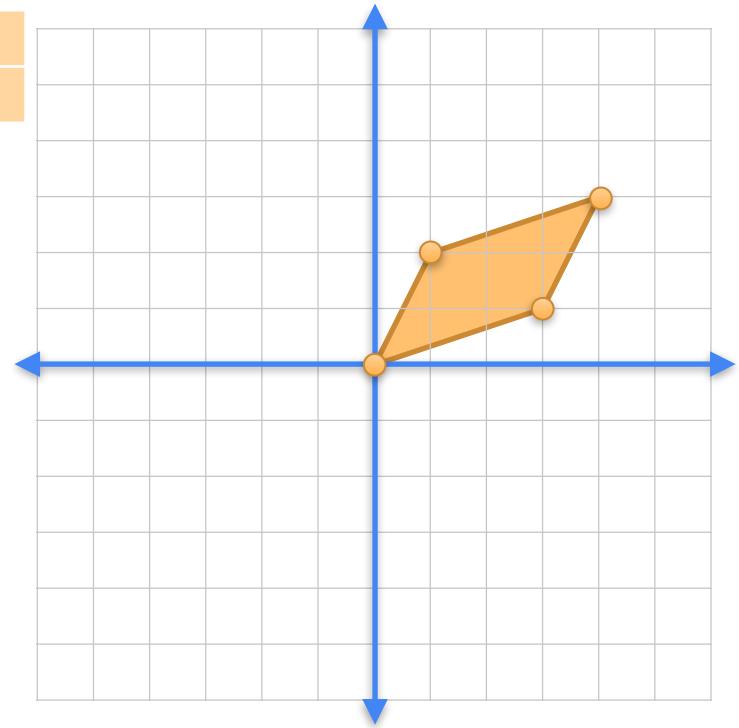
$$\begin{aligned} (0,0) &\rightarrow (0,0) \\ (1,0) &\rightarrow (3,1) \\ (0,1) &\rightarrow (1,2) \\ (1,1) &\rightarrow (4,3) \end{aligned}$$



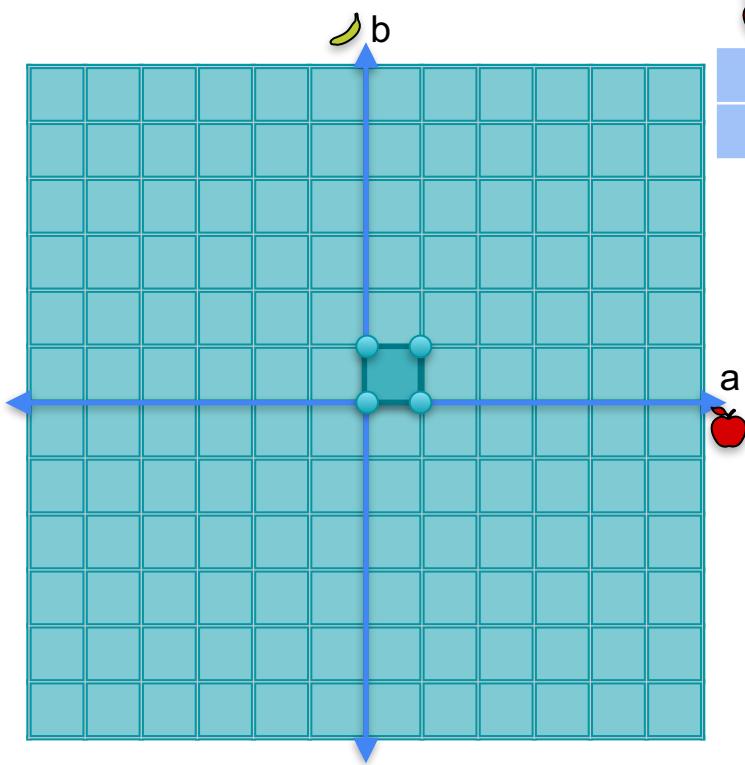
Matrices as linear transformations



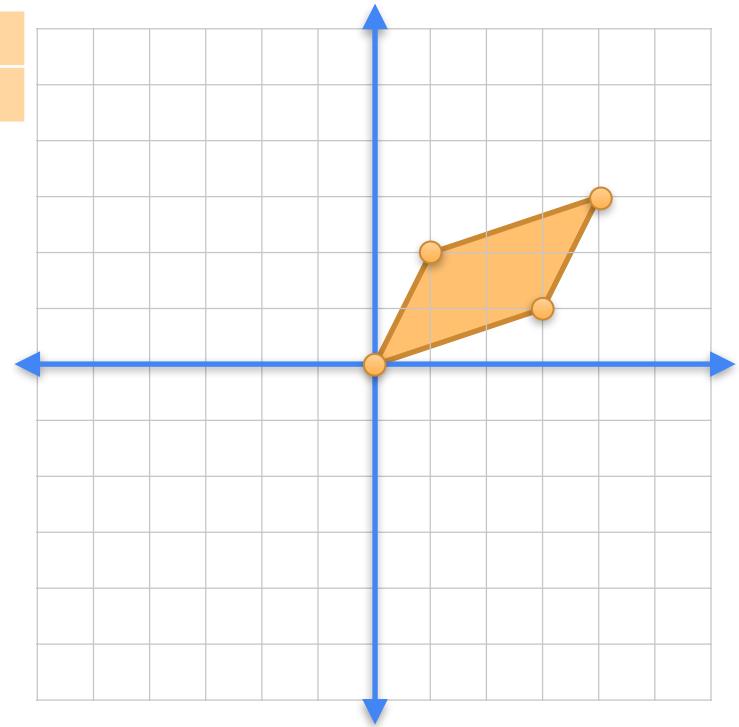
$$\begin{matrix} \text{apple} & \text{banana} \\ \begin{matrix} 3 & 1 & -2 \\ 1 & 2 & 3 \end{matrix} & = \begin{matrix} -3 \\ 4 \end{matrix} \end{matrix}$$



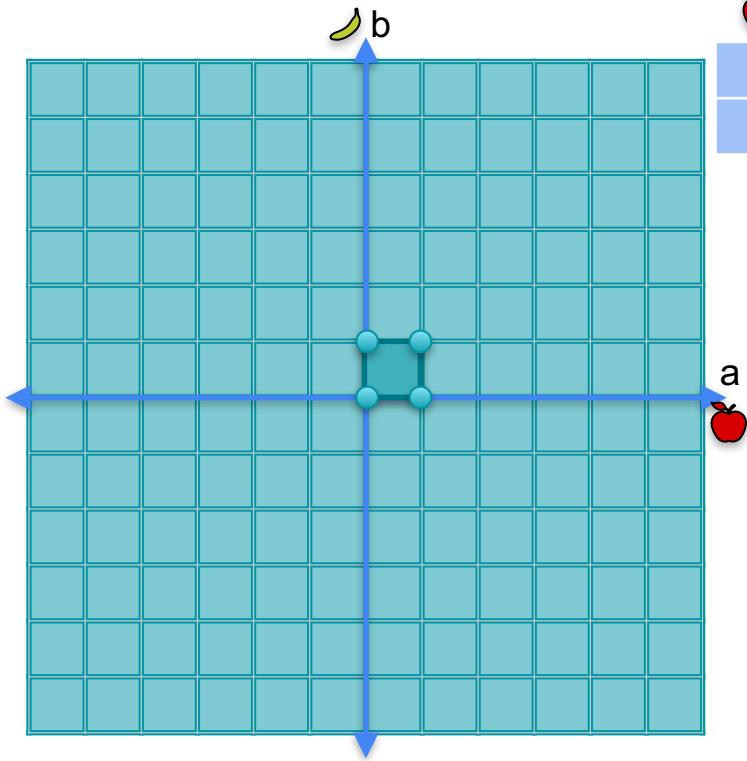
Matrices as linear transformations



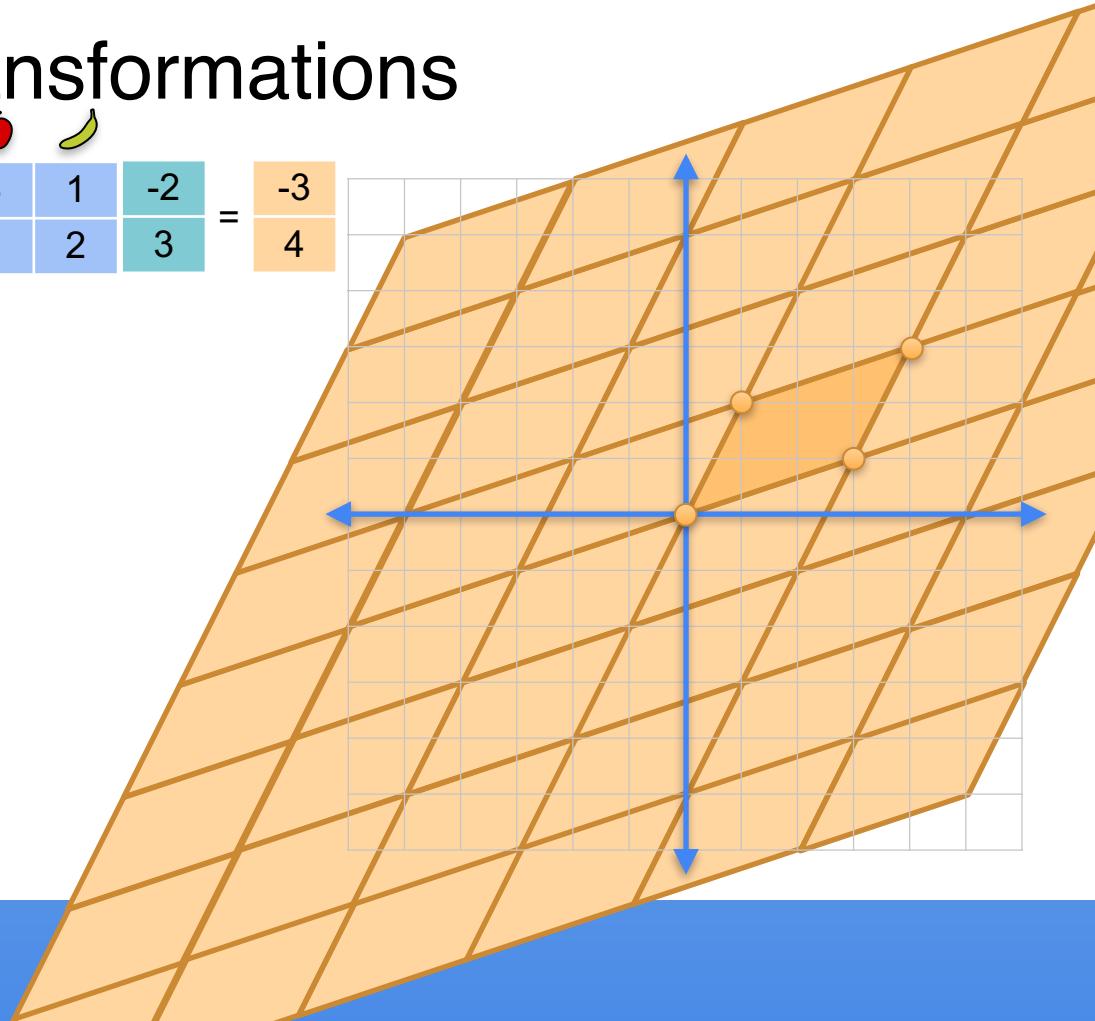
$$\begin{matrix} \text{apple} & \text{banana} \\ \begin{matrix} 3 & 1 \\ 1 & 2 \end{matrix} & \begin{matrix} -2 \\ 3 \end{matrix} = \begin{matrix} -3 \\ 4 \end{matrix} \end{matrix}$$



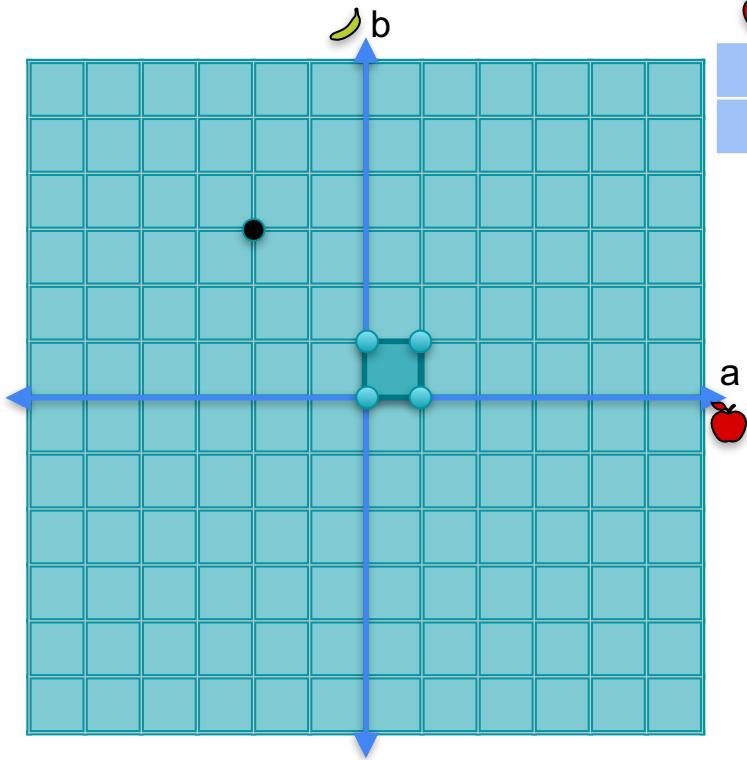
Matrices as linear transformations



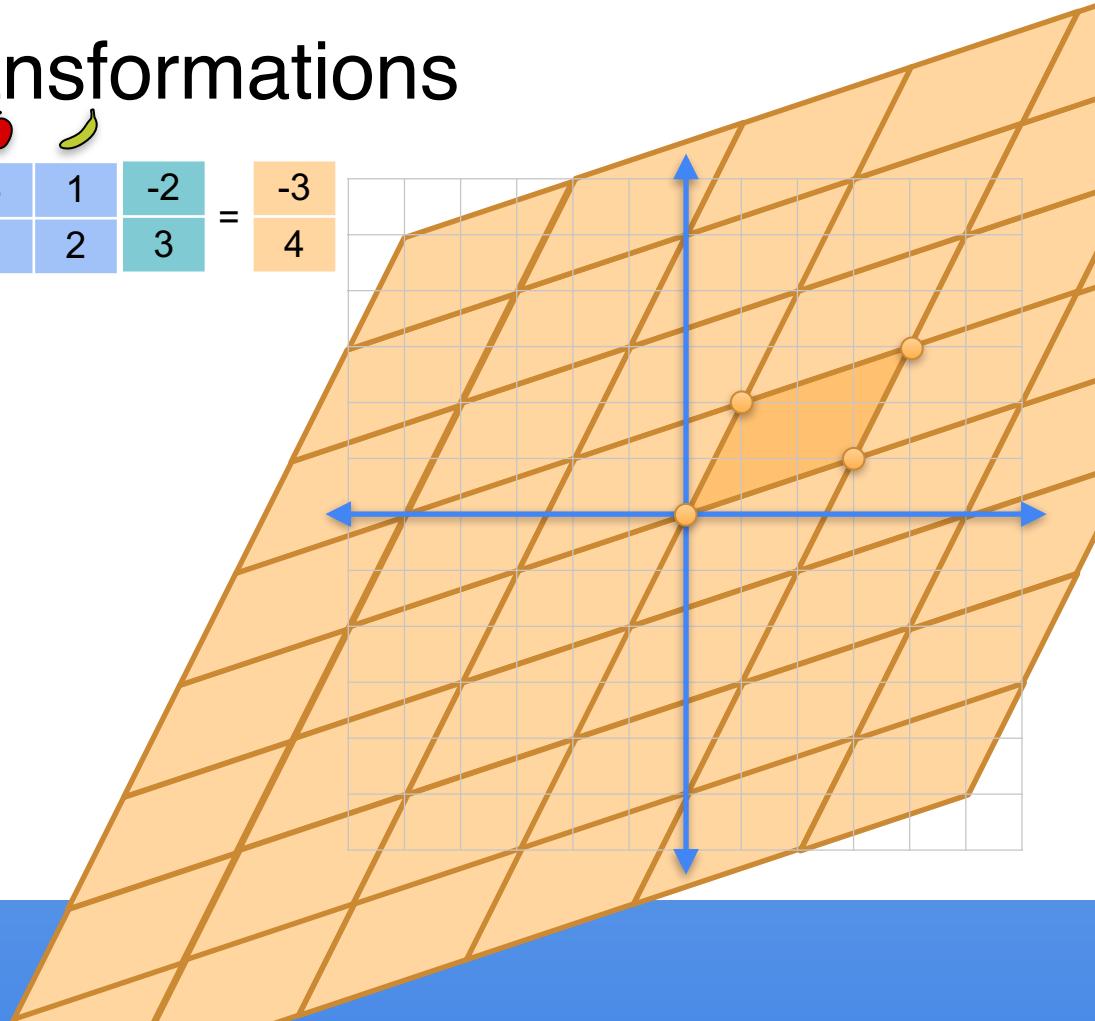
$$\begin{array}{|c c|} \hline 3 & 1 \\ \hline 1 & 2 \\ \hline \end{array} \begin{array}{|c c|} \hline -2 \\ \hline 3 \\ \hline \end{array} = \begin{array}{|c|} \hline -3 \\ \hline 4 \\ \hline \end{array}$$



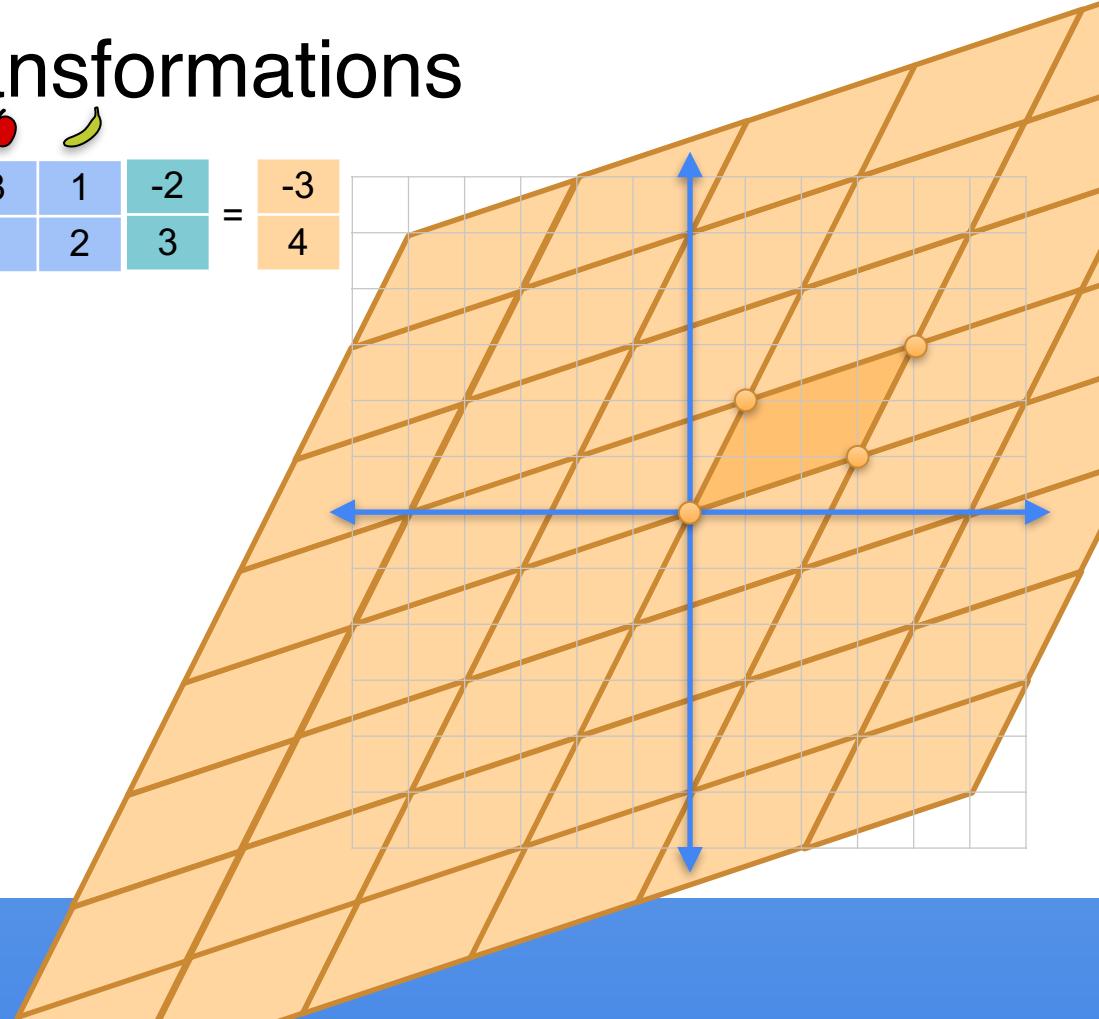
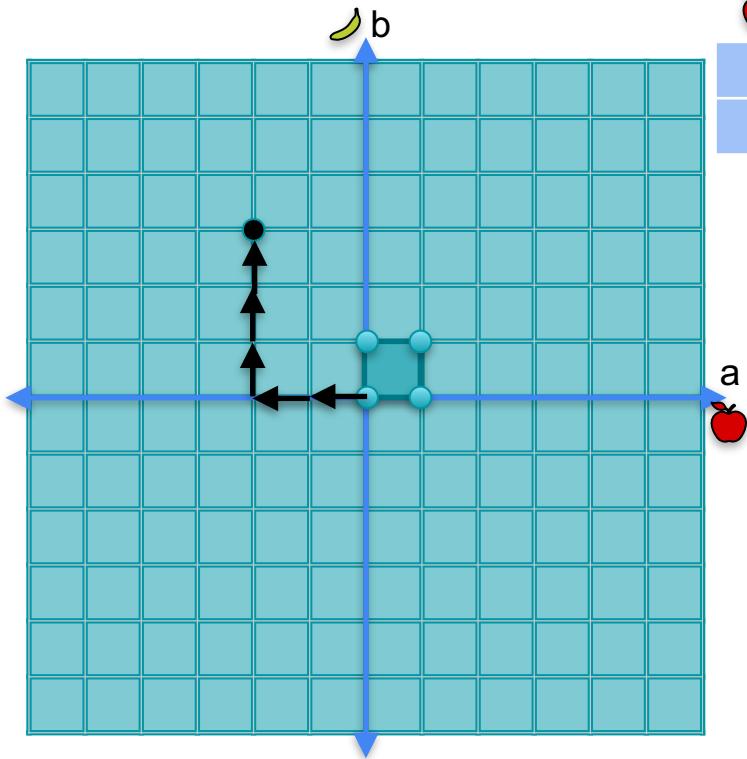
Matrices as linear transformations



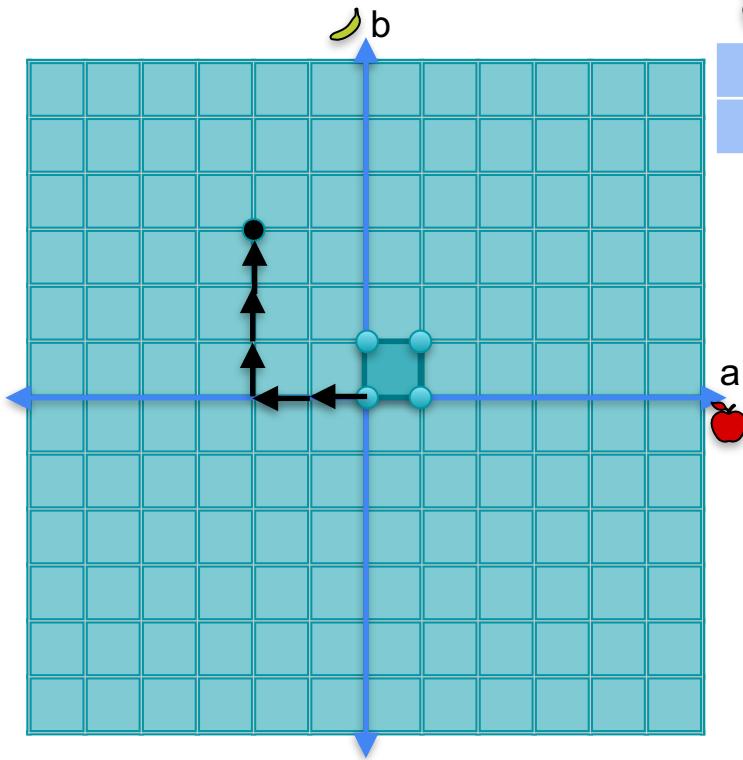
$$\begin{matrix} \text{apple} & \text{banana} \end{matrix} \begin{pmatrix} 3 & 1 \\ 1 & 2 \end{pmatrix} \begin{pmatrix} -2 \\ 3 \end{pmatrix} = \begin{pmatrix} -3 \\ 4 \end{pmatrix}$$



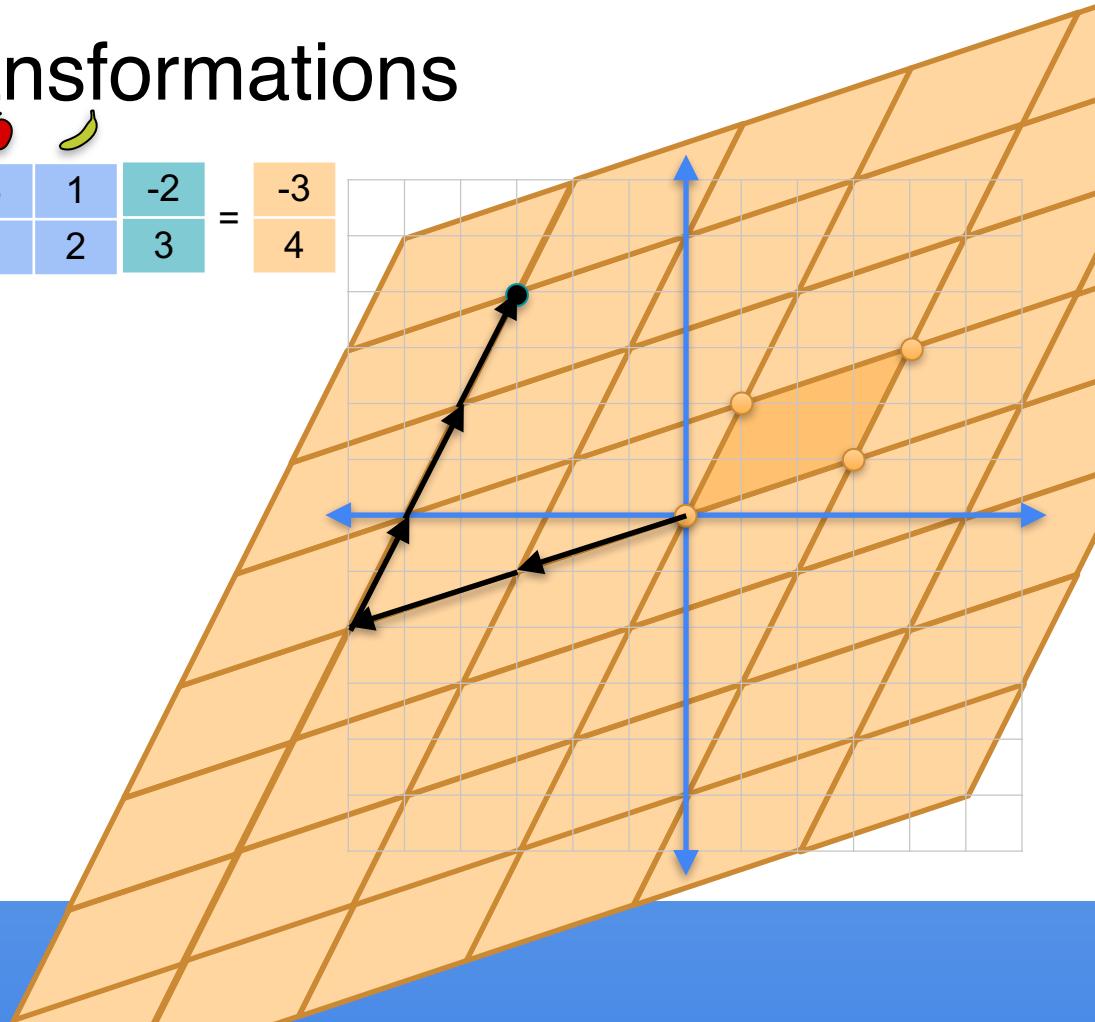
Matrices as linear transformations



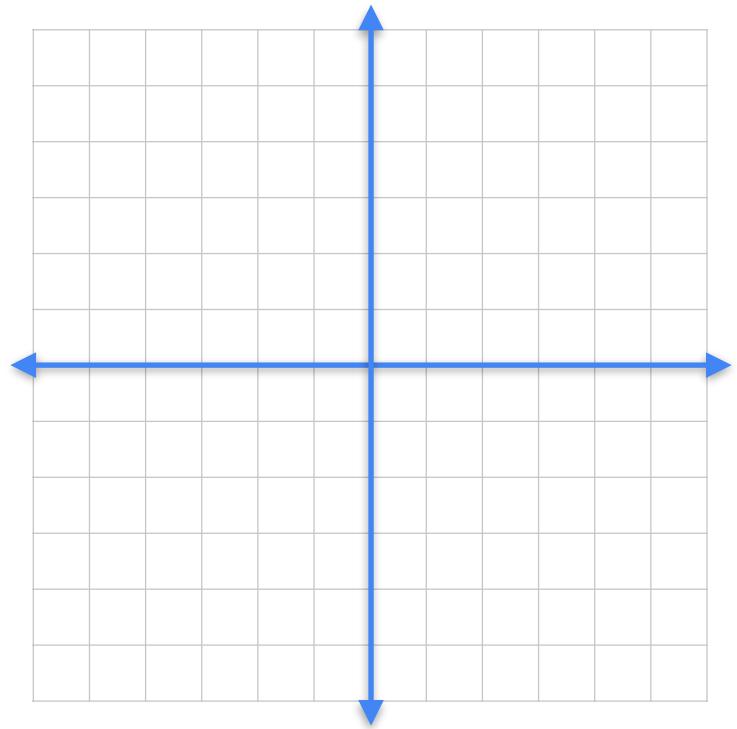
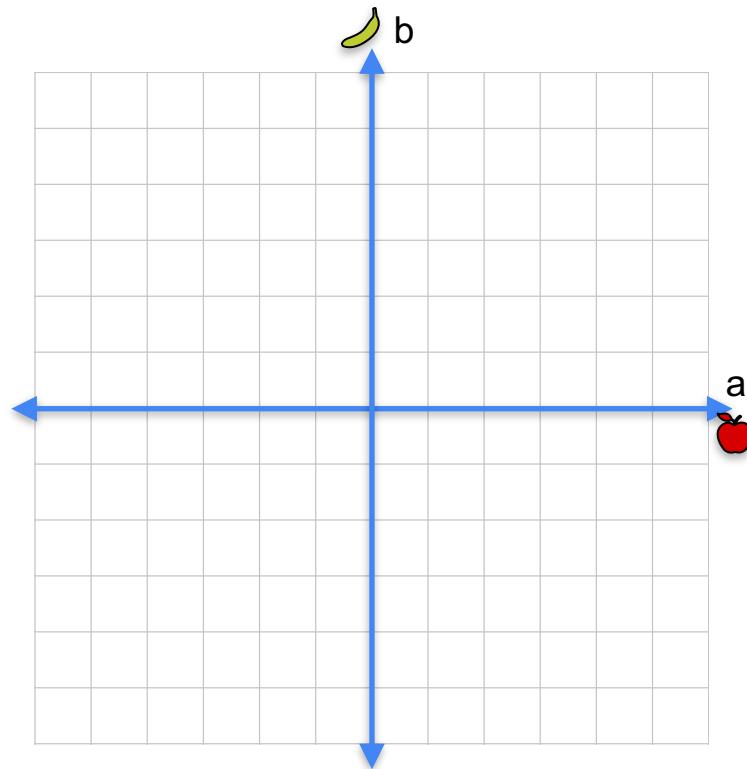
Matrices as linear transformations



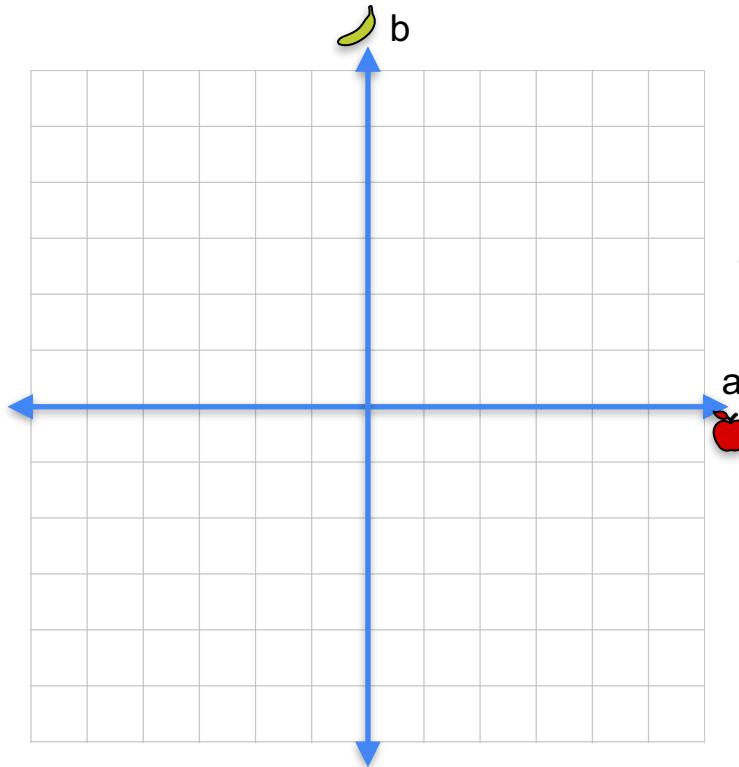
$$\begin{matrix} \text{apple} & \text{banana} \end{matrix} \quad \begin{matrix} 3 & 1 & -2 \\ 1 & 2 & 3 \end{matrix} = \begin{matrix} -3 \\ 4 \end{matrix}$$



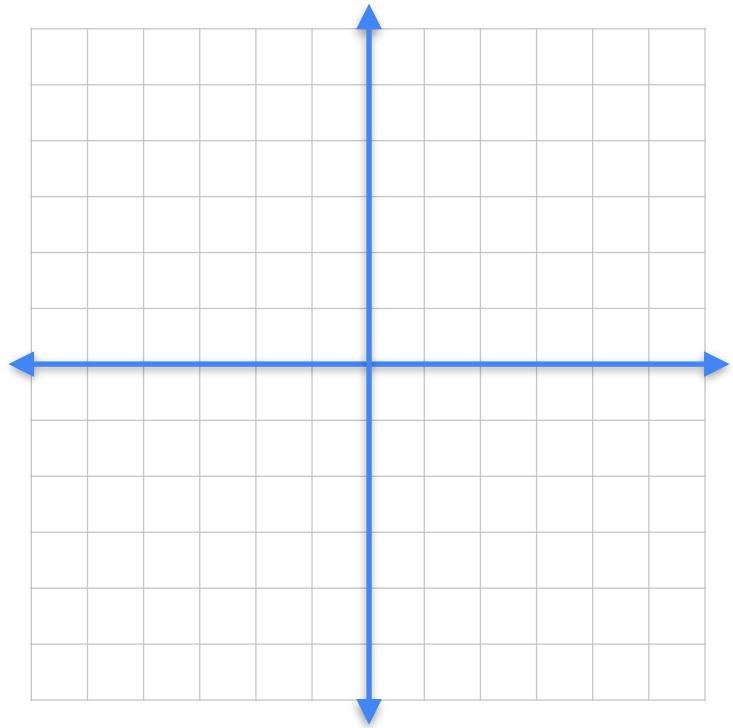
Systems of equations as linear transformations



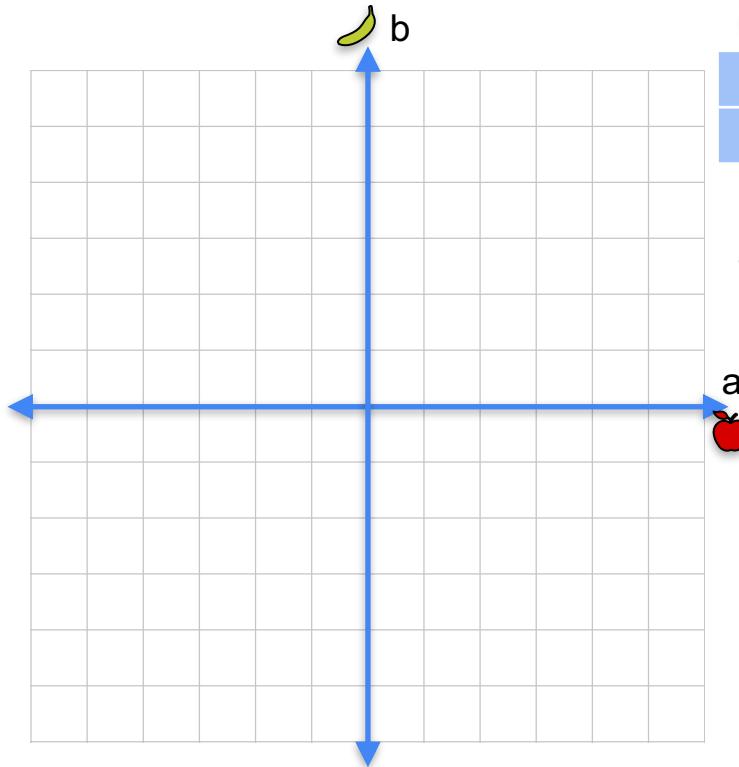
Systems of equations as linear transformations



First day: $3a + b$
Second day: $a + 2b$

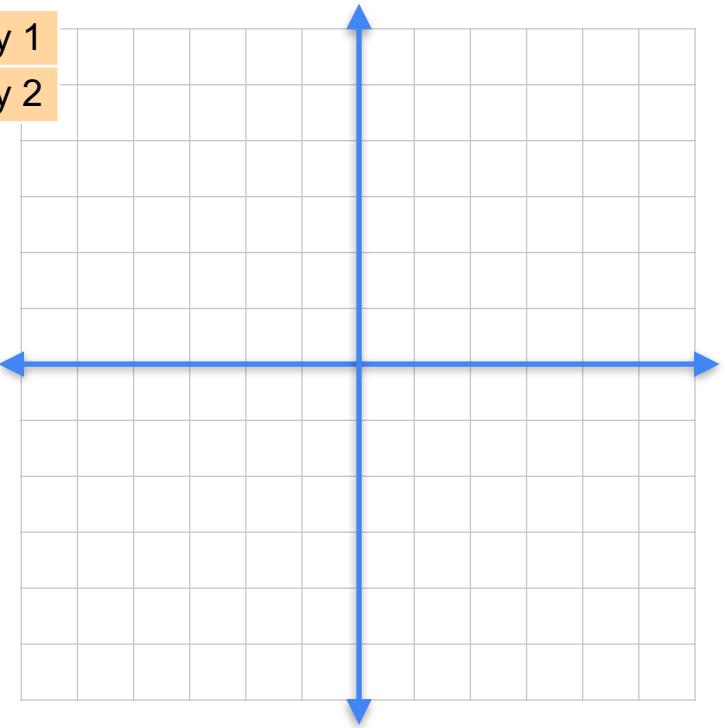


Systems of equations as linear transformations

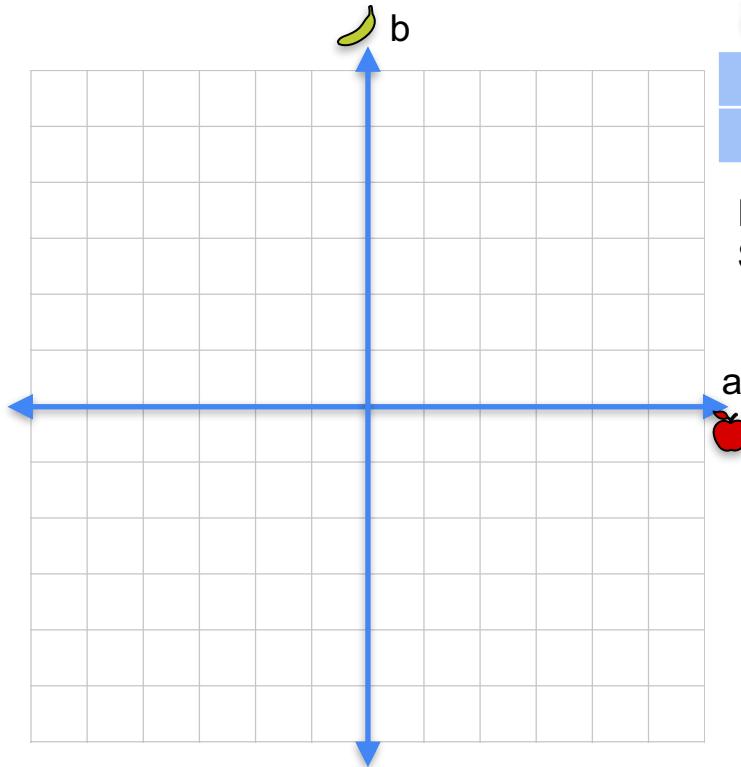


$$\begin{matrix} \text{apple} & \text{banana} \\ 3 & 1 \\ 1 & 2 \end{matrix} \begin{matrix} a \\ b \end{matrix} = \begin{matrix} \text{Day 1} \\ \text{Day 2} \end{matrix}$$

First day: $3a + b$
Second day: $a + 2b$

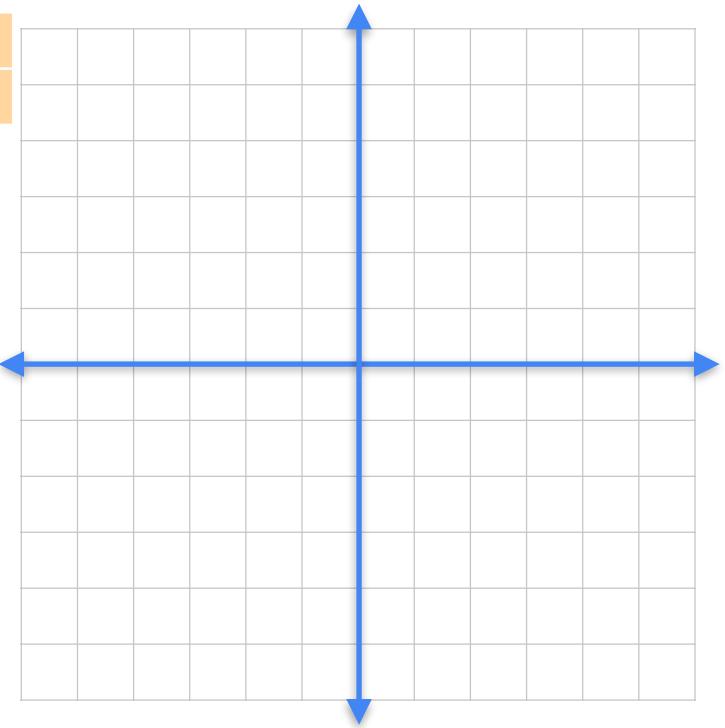


Systems of equations as linear transformations

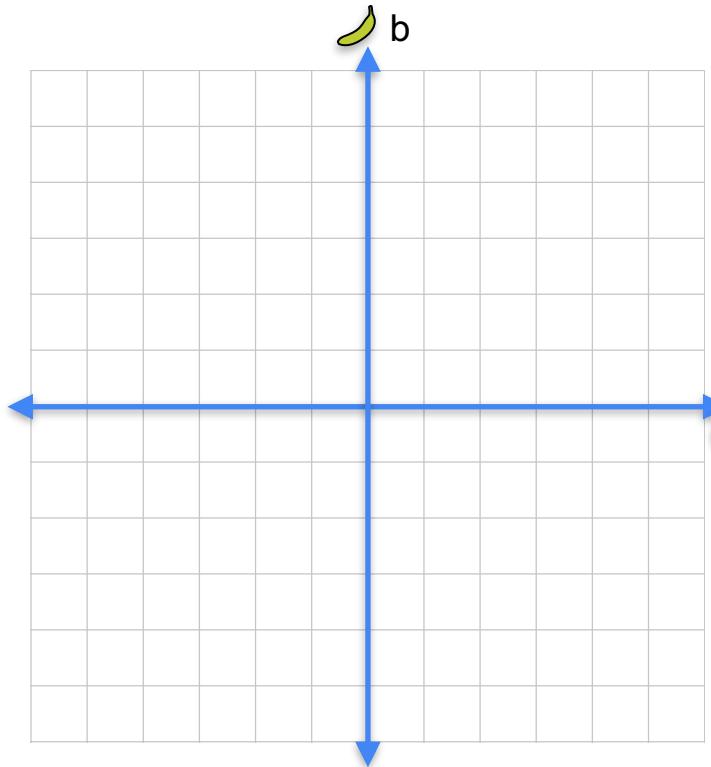


$$\begin{matrix} \text{apple} & \text{banana} \\ \begin{matrix} 3 & 1 \\ 1 & 2 \end{matrix} & \begin{matrix} 1 \\ 1 \end{matrix} \end{matrix} = \begin{matrix} 4 \\ 3 \end{matrix}$$

First day: $3a + b$
Second day: $a + 2b$



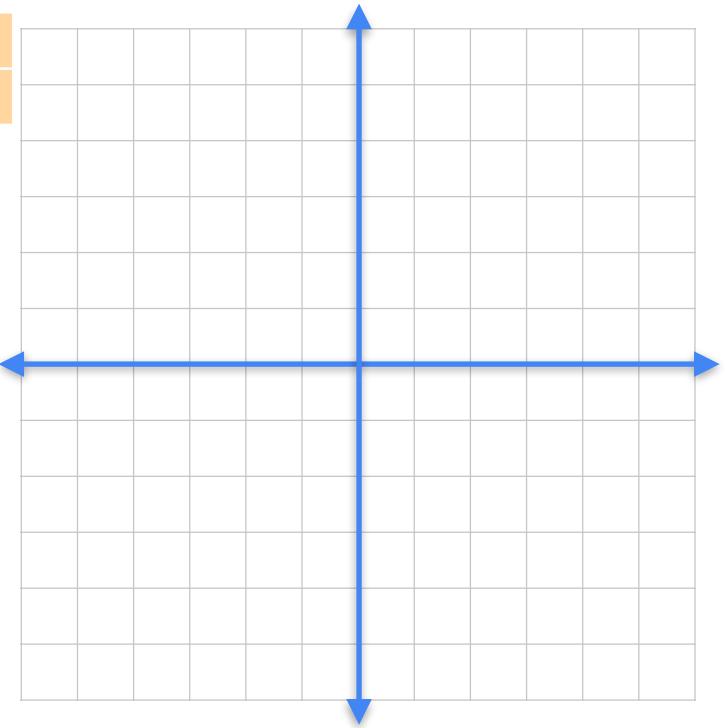
Systems of equations as linear transformations



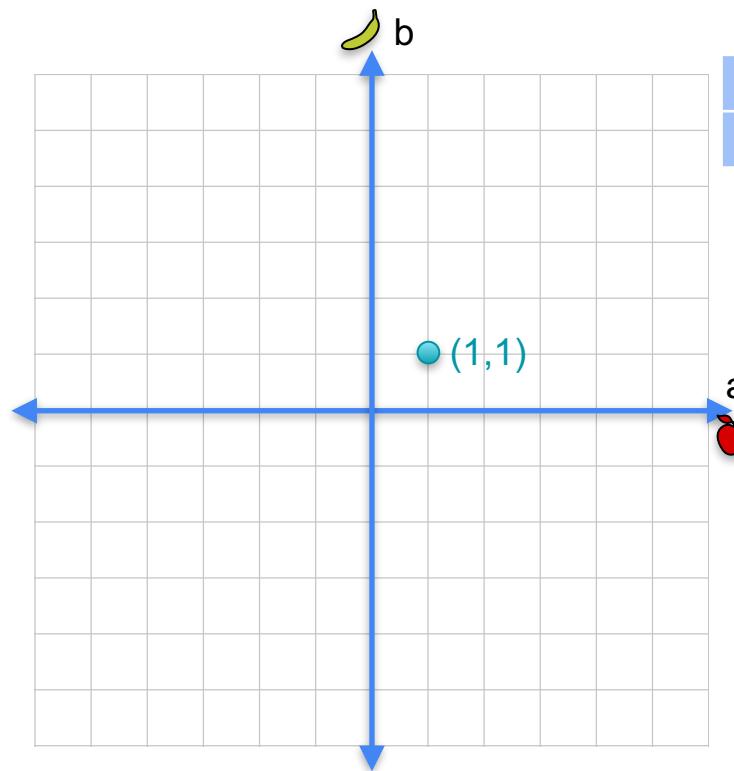
$$\begin{matrix} \text{apple} & \text{banana} \\ \begin{matrix} 3 & 1 \\ 1 & 2 \end{matrix} & \begin{matrix} 1 \\ 1 \end{matrix} \end{matrix} = \begin{matrix} \text{apple} \\ \text{banana} \\ \begin{matrix} 4 \\ 3 \end{matrix} \end{matrix}$$

First day: $3a + b$
Second day: $a + 2b$

$$(1,1) \rightarrow (4,3)$$



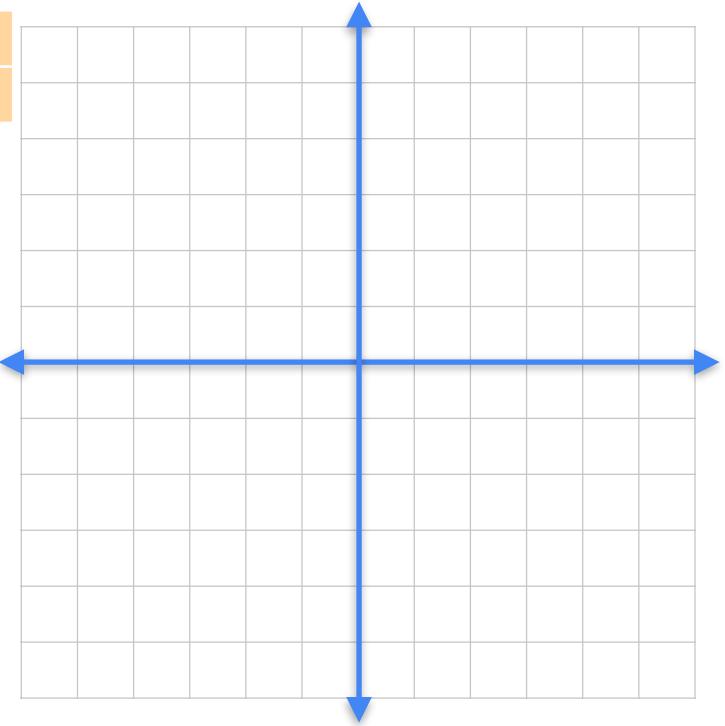
Systems of equations as linear transformations



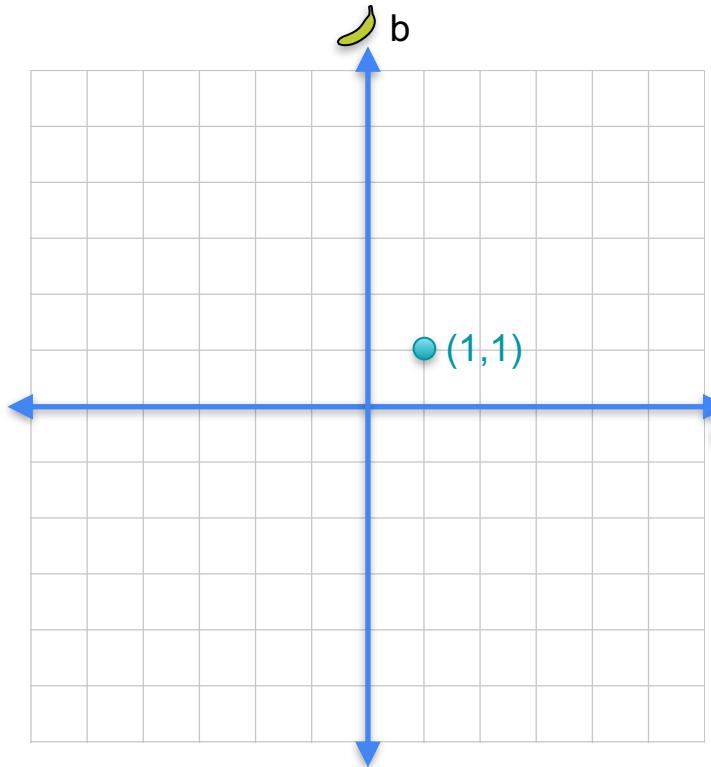
$$\begin{matrix} \text{apple} & \text{banana} \\ 3 & 1 \\ 1 & 2 \end{matrix} = \begin{matrix} \text{banana} \\ 4 \end{matrix}$$

First day: $3a + b$

Second day: $a + 2b$

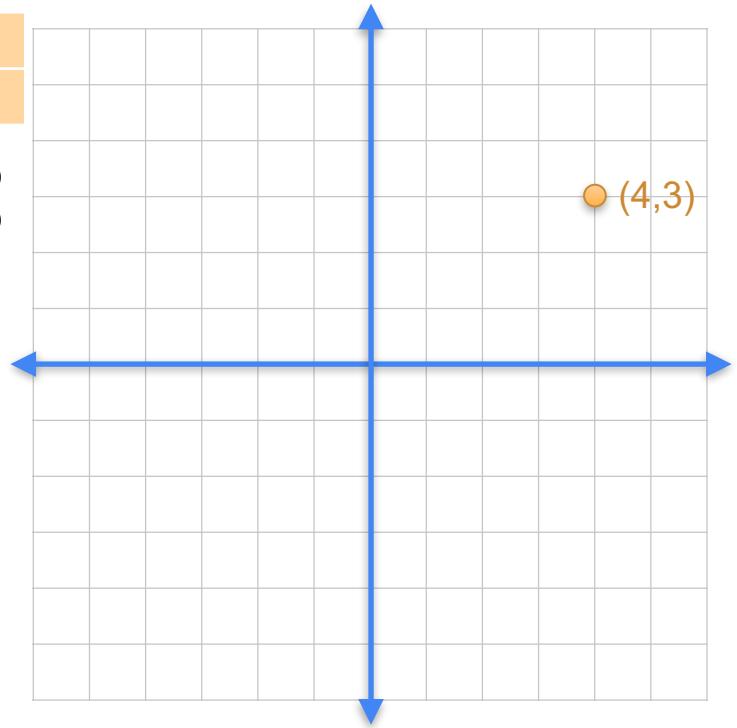


Systems of equations as linear transformations

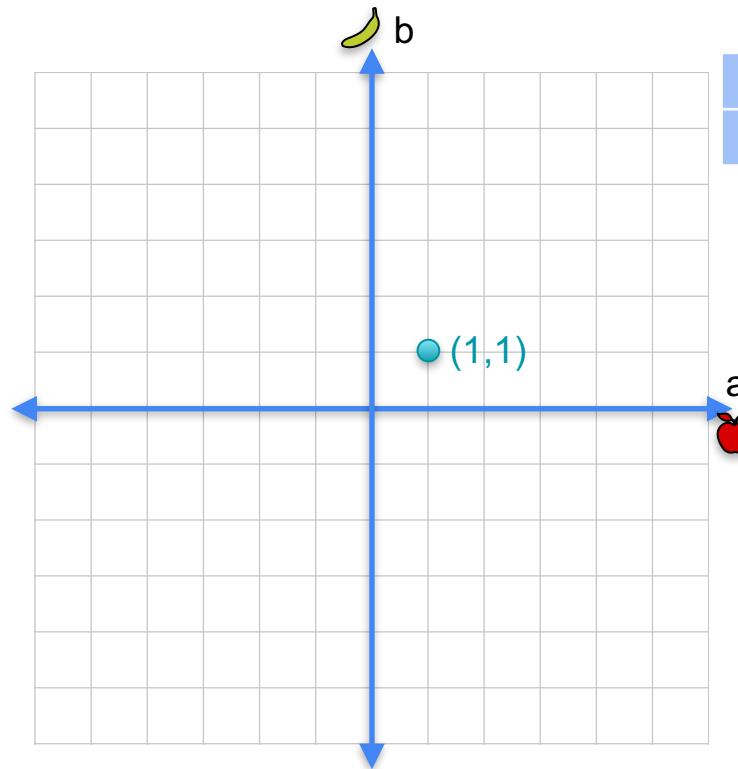


3	1	1	=	4
1	2	1		3

First day: $3a + b$
Second day: $a + 2b$

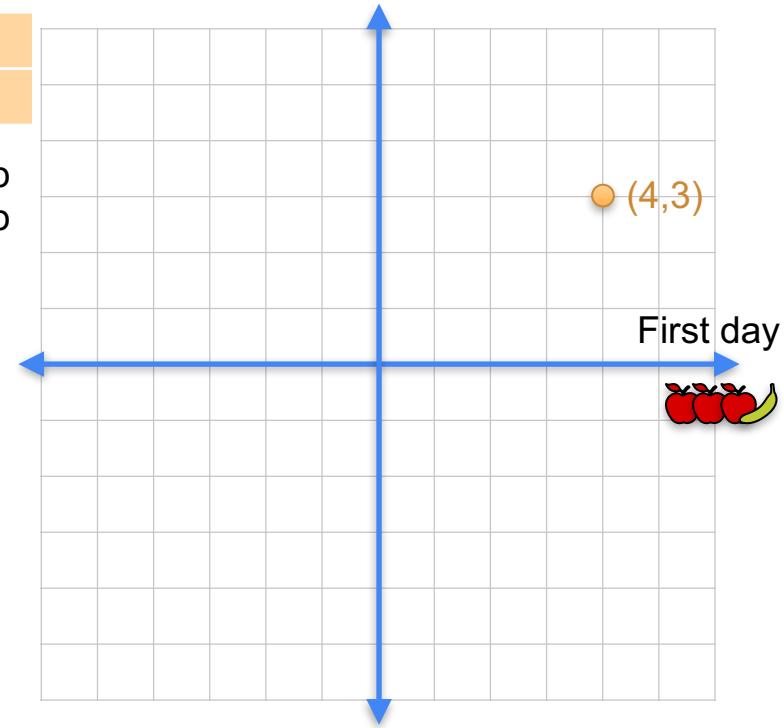


Systems of equations as linear transformations

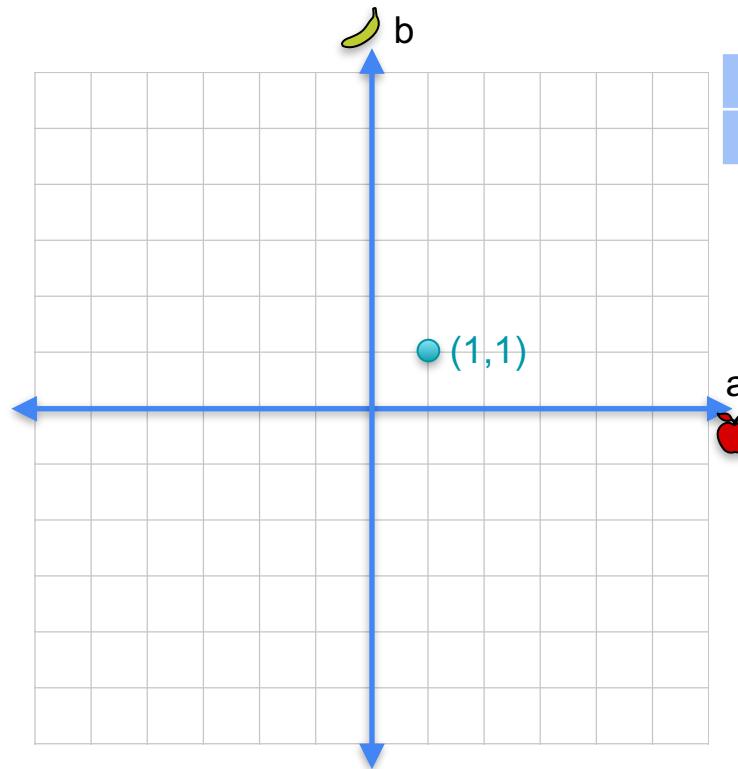


$$\begin{matrix} \text{apple} & \text{banana} \\ \begin{matrix} 3 & 1 \\ 1 & 2 \end{matrix} & \begin{matrix} 1 \\ 1 \end{matrix} \end{matrix} = \begin{matrix} 4 \\ 3 \end{matrix}$$

First day: $3a + b$
Second day: $a + 2b$

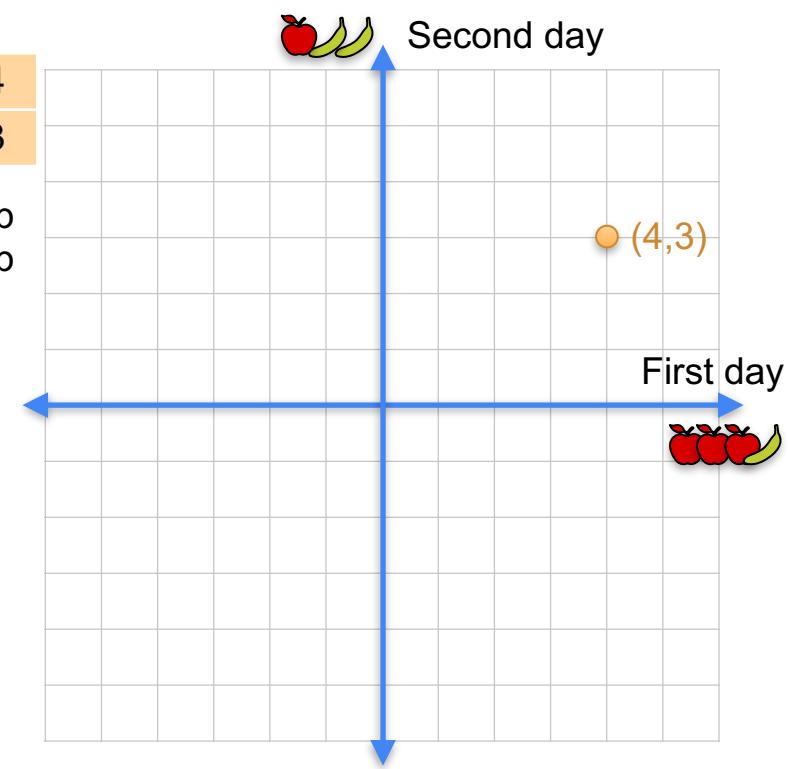


Systems of equations as linear transformations



$$\begin{matrix} \text{apple} & \text{banana} \\ 3 & 1 & 1 \\ 1 & 2 & 1 \end{matrix} = \begin{matrix} \text{yellow} & \text{blue} \\ 4 & 3 \end{matrix}$$

First day: $3a + b$
Second day: $a + 2b$



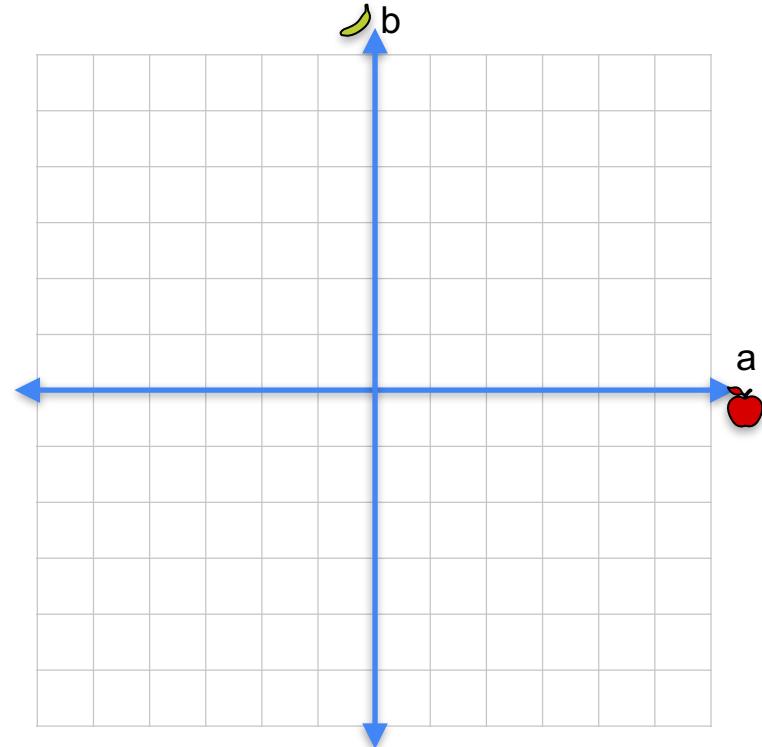
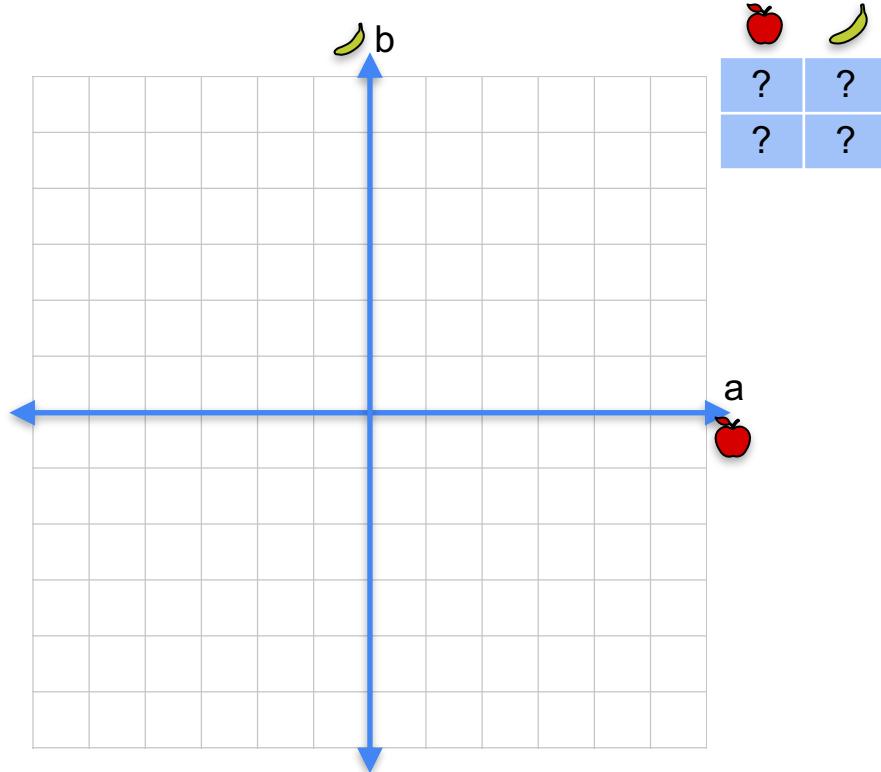


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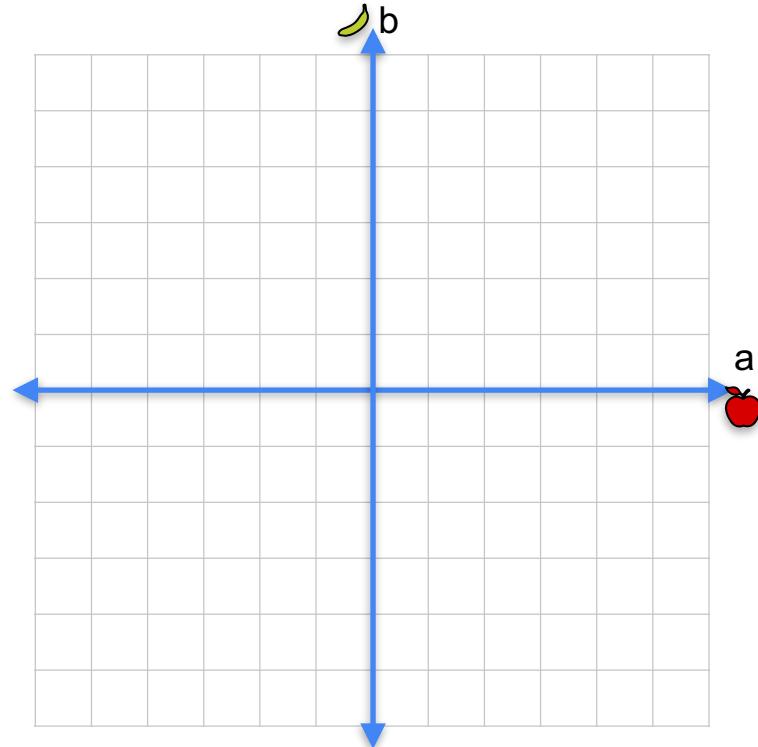
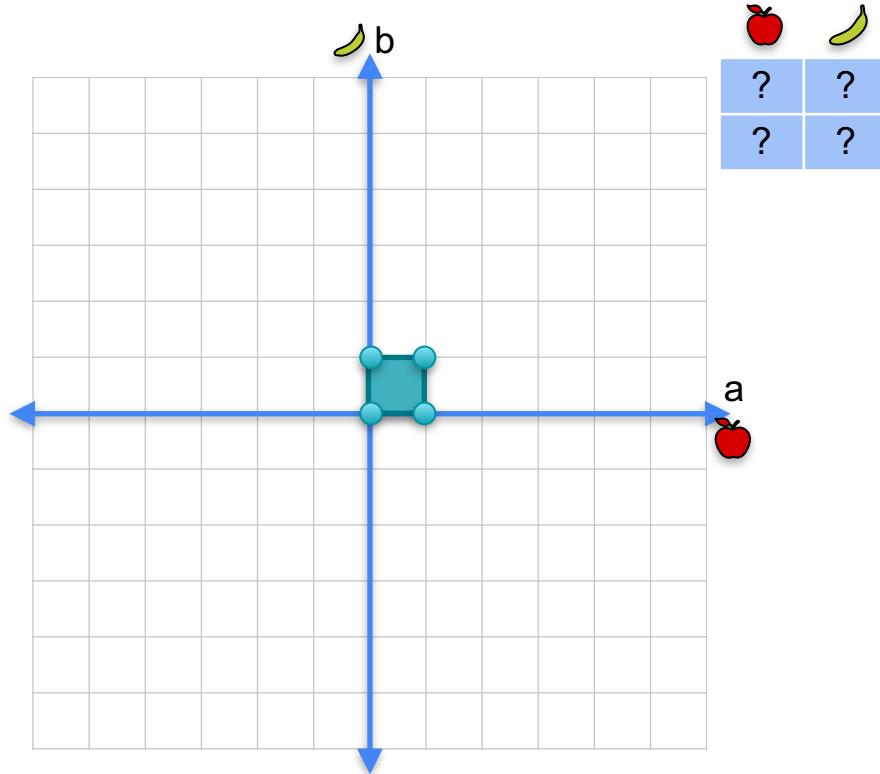
Vectors and Linear Transformations

**Linear transformations as
matrices**

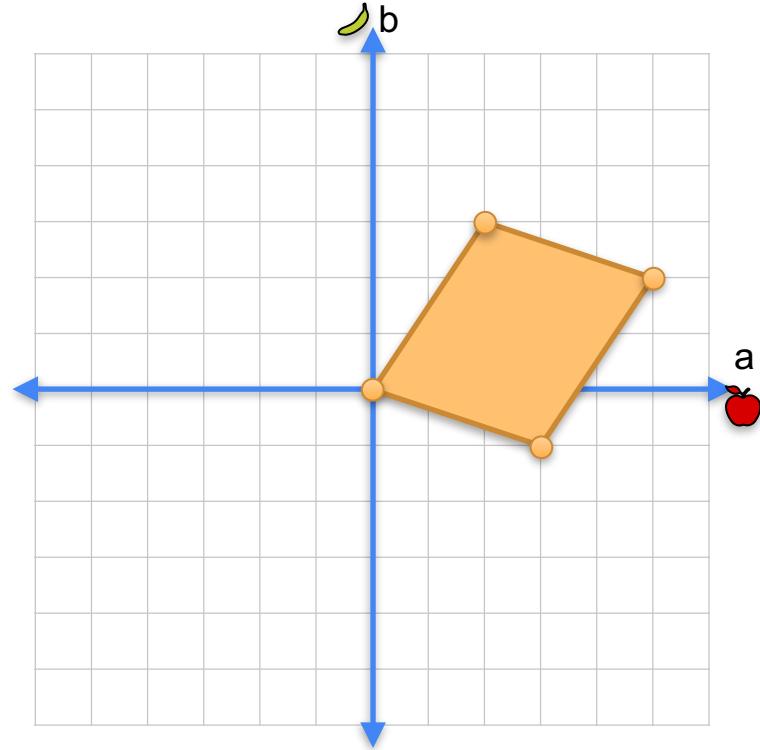
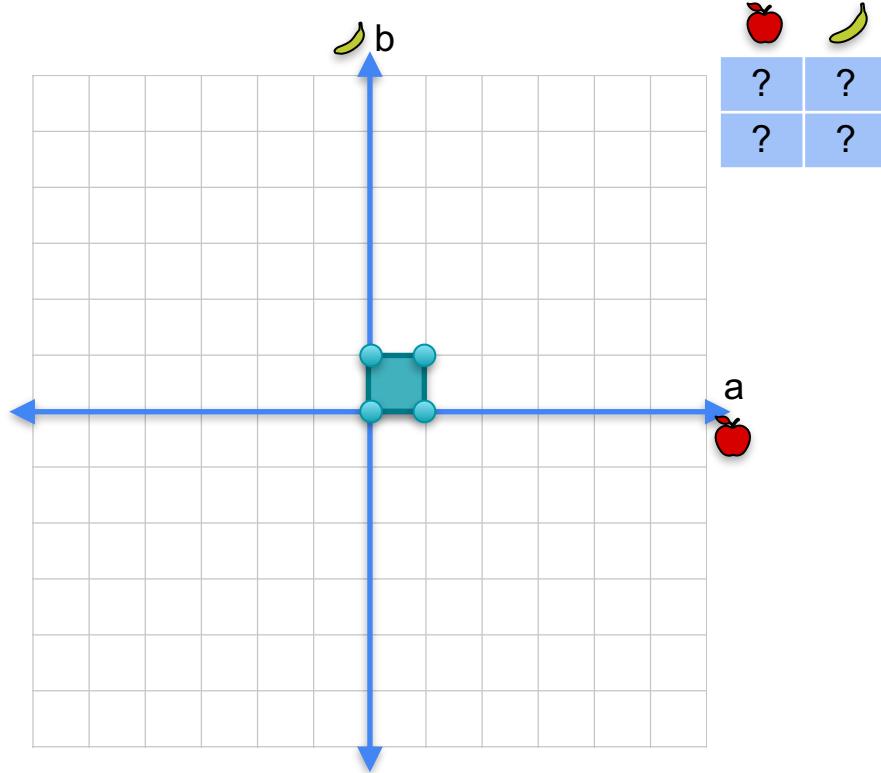
Linear transformations as matrices



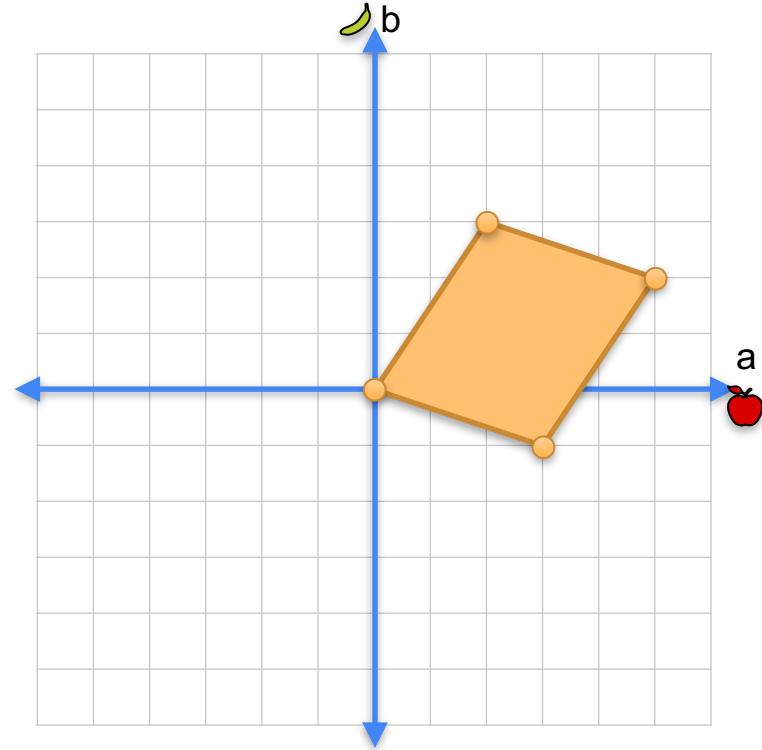
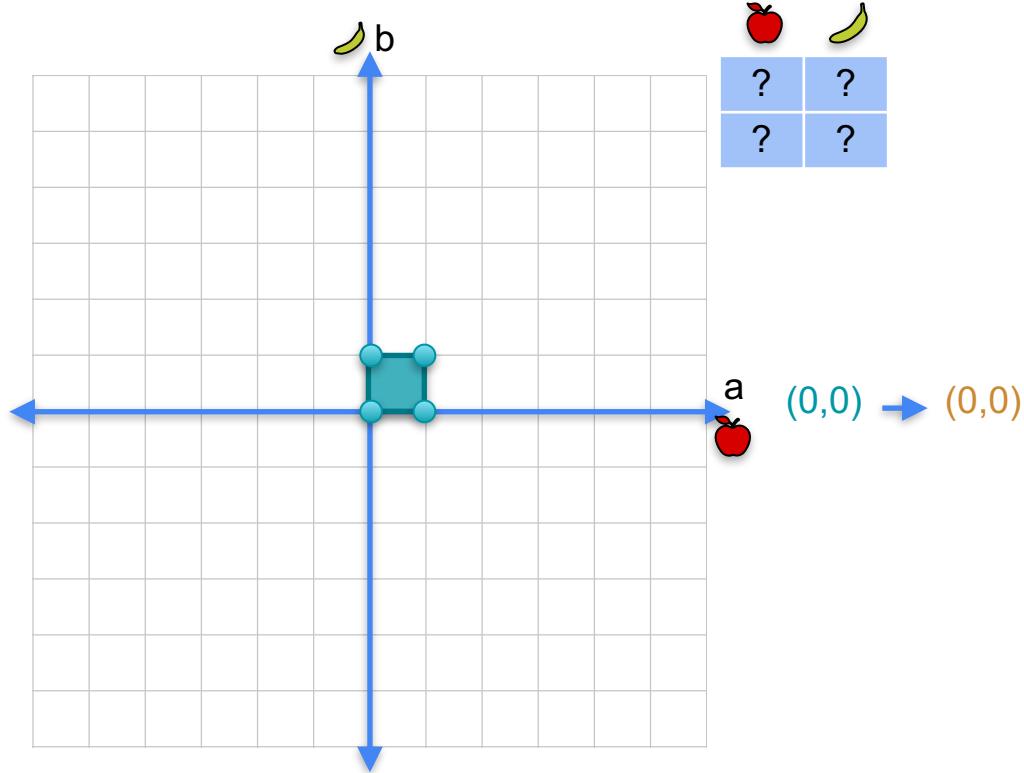
Linear transformations as matrices



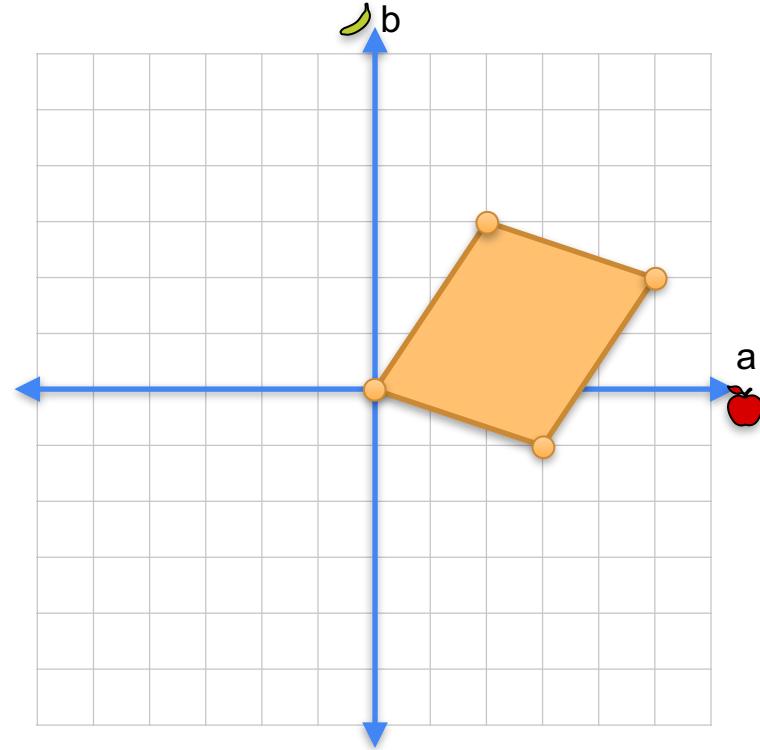
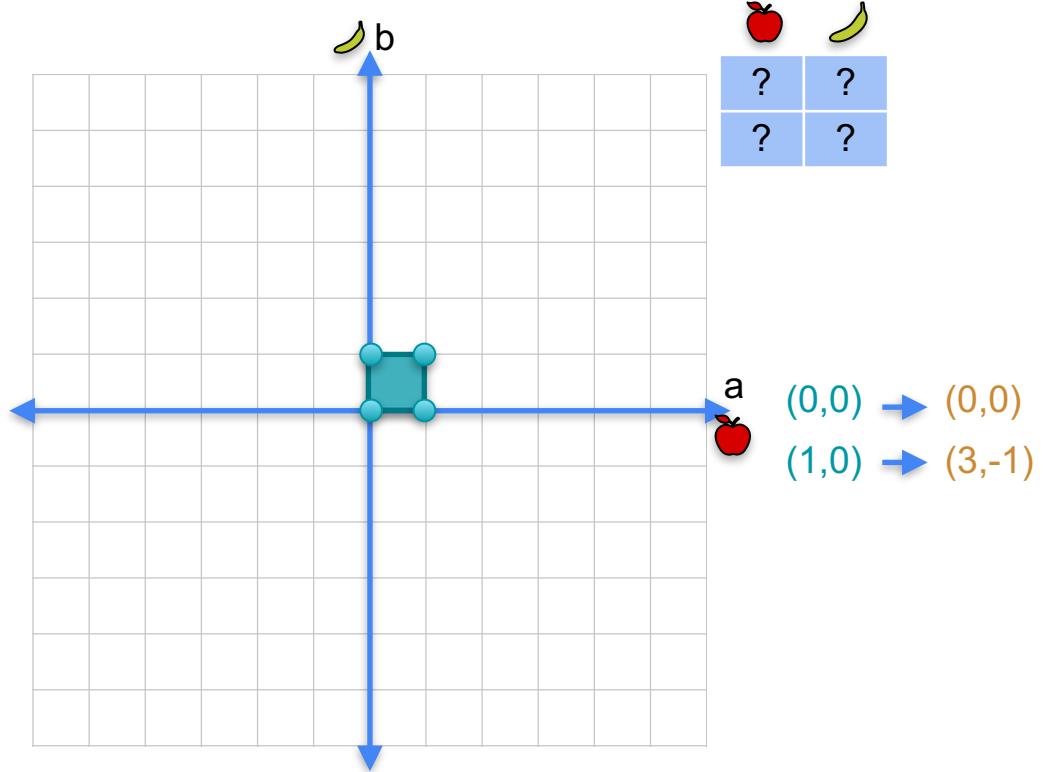
Linear transformations as matrices



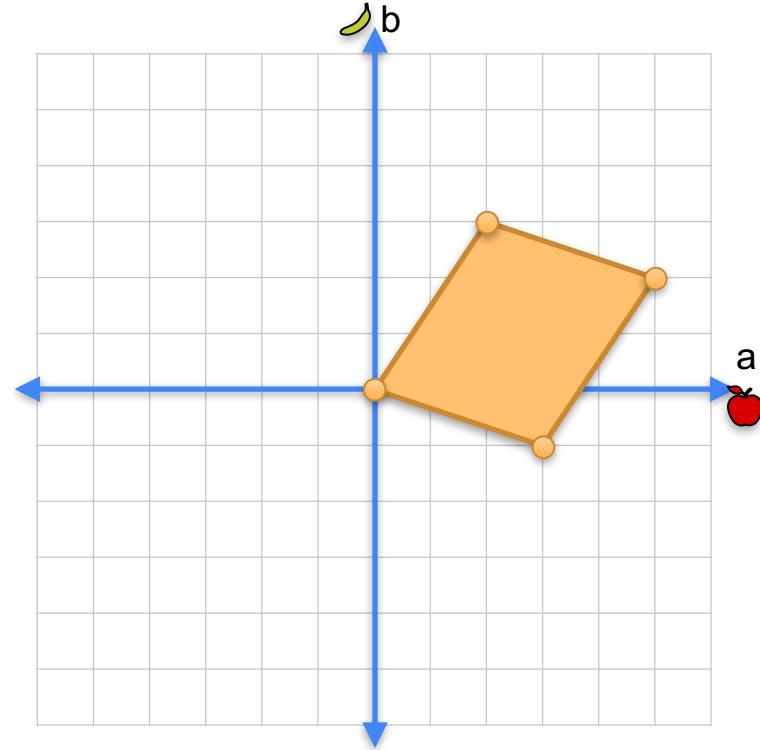
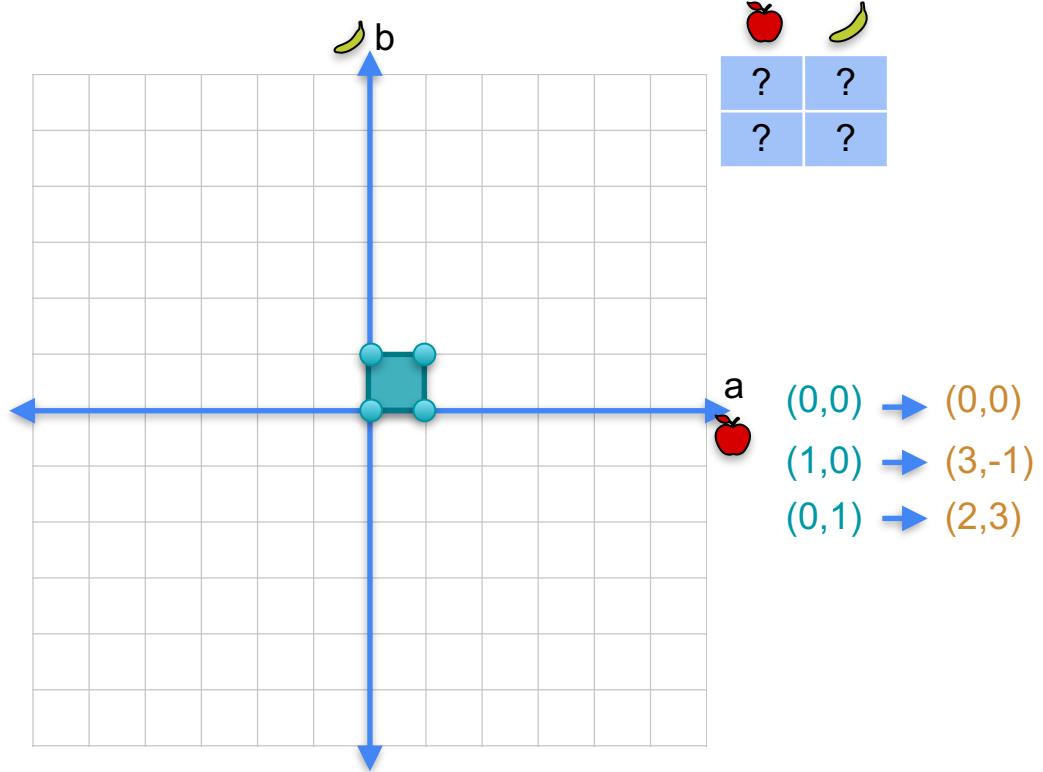
Linear transformations as matrices



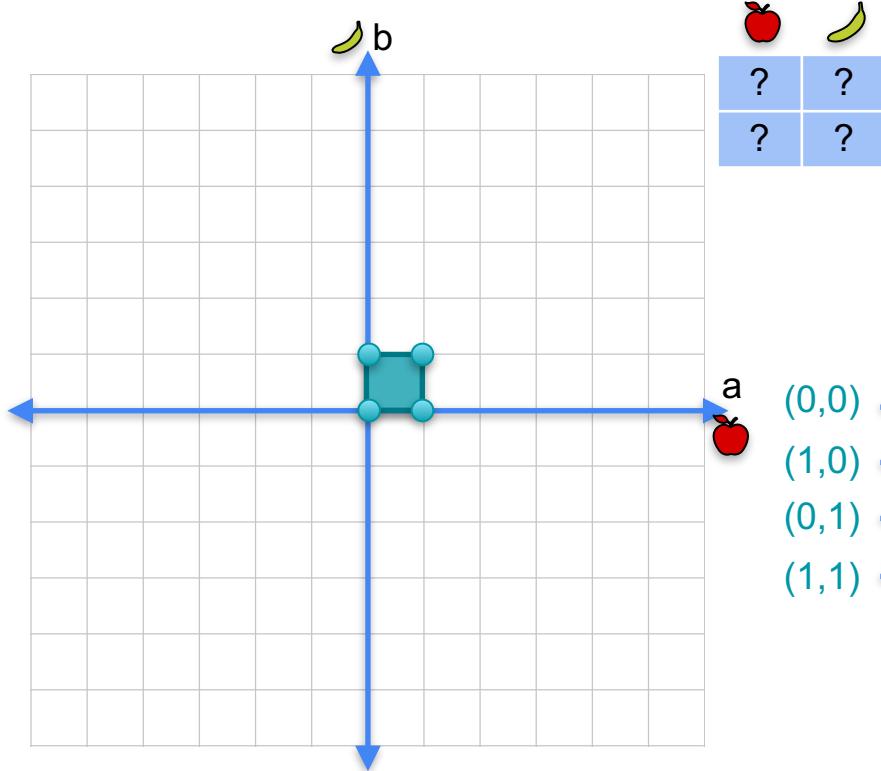
Linear transformations as matrices



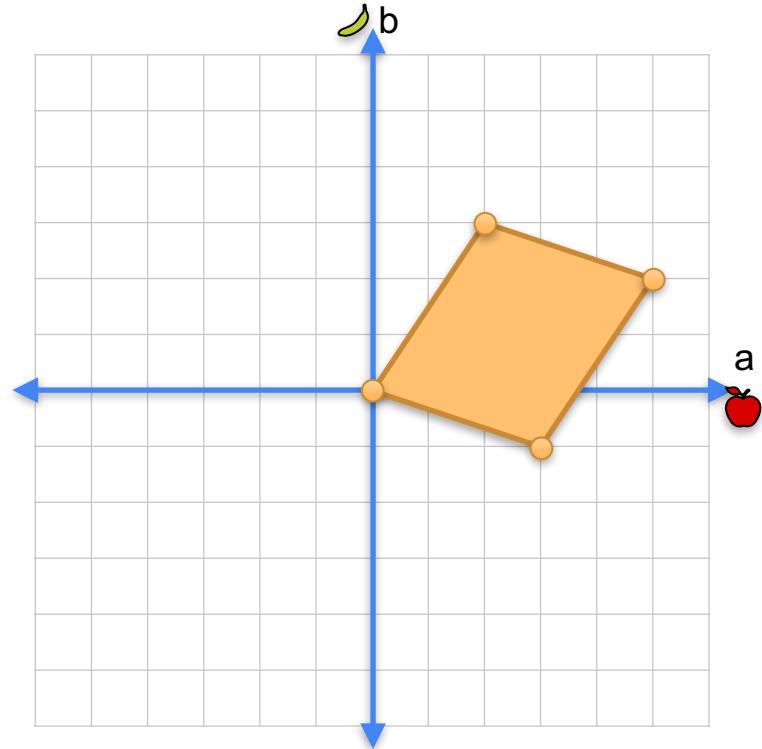
Linear transformations as matrices



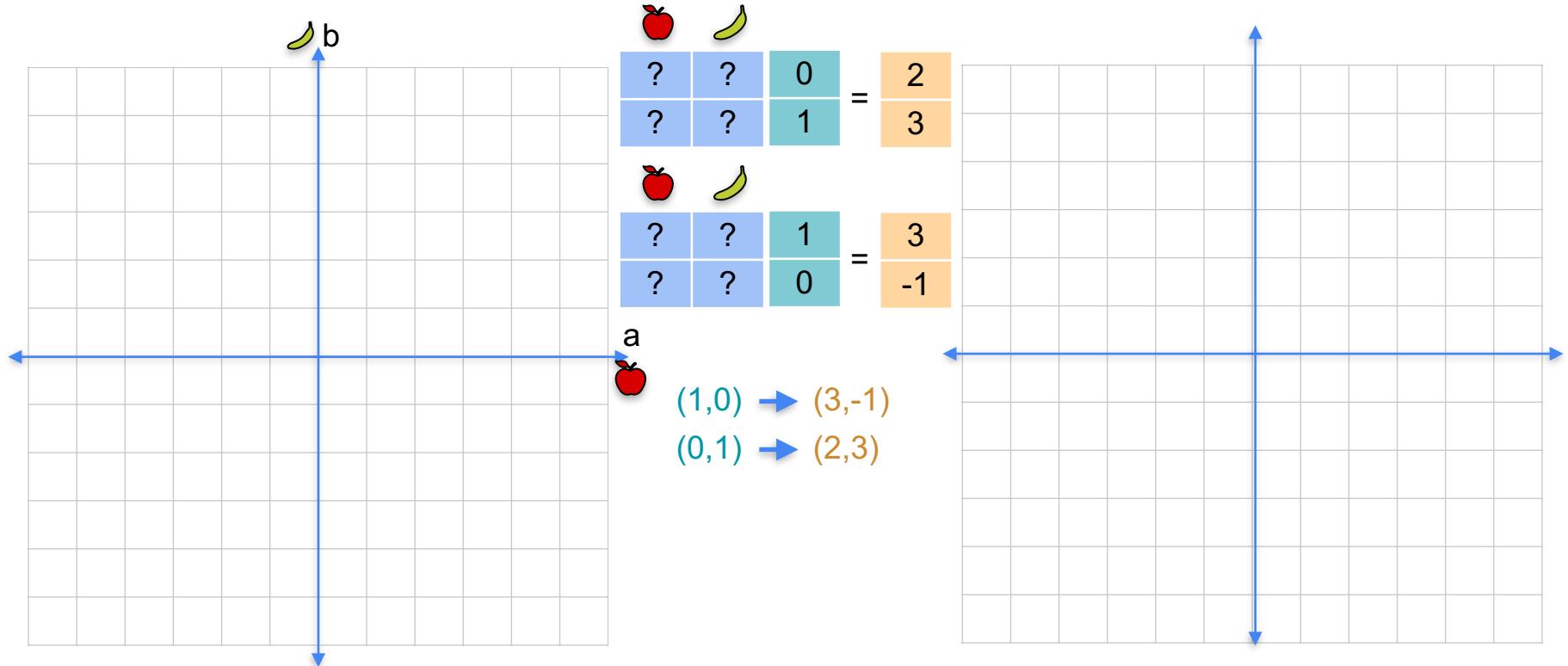
Linear transformations as matrices



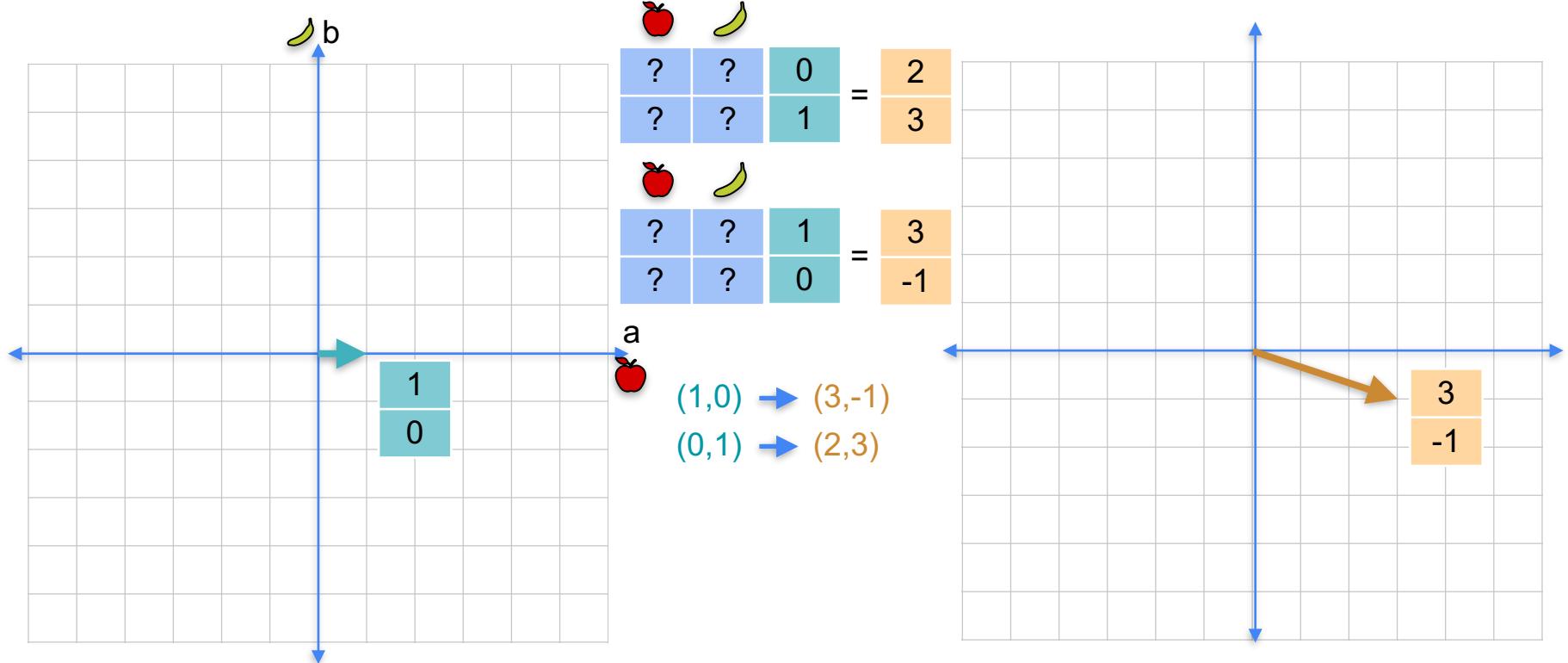
$$\begin{aligned}(0,0) &\rightarrow (0,0) \\(1,0) &\rightarrow (3,-1) \\(0,1) &\rightarrow (2,3) \\(1,1) &\rightarrow (5,2)\end{aligned}$$



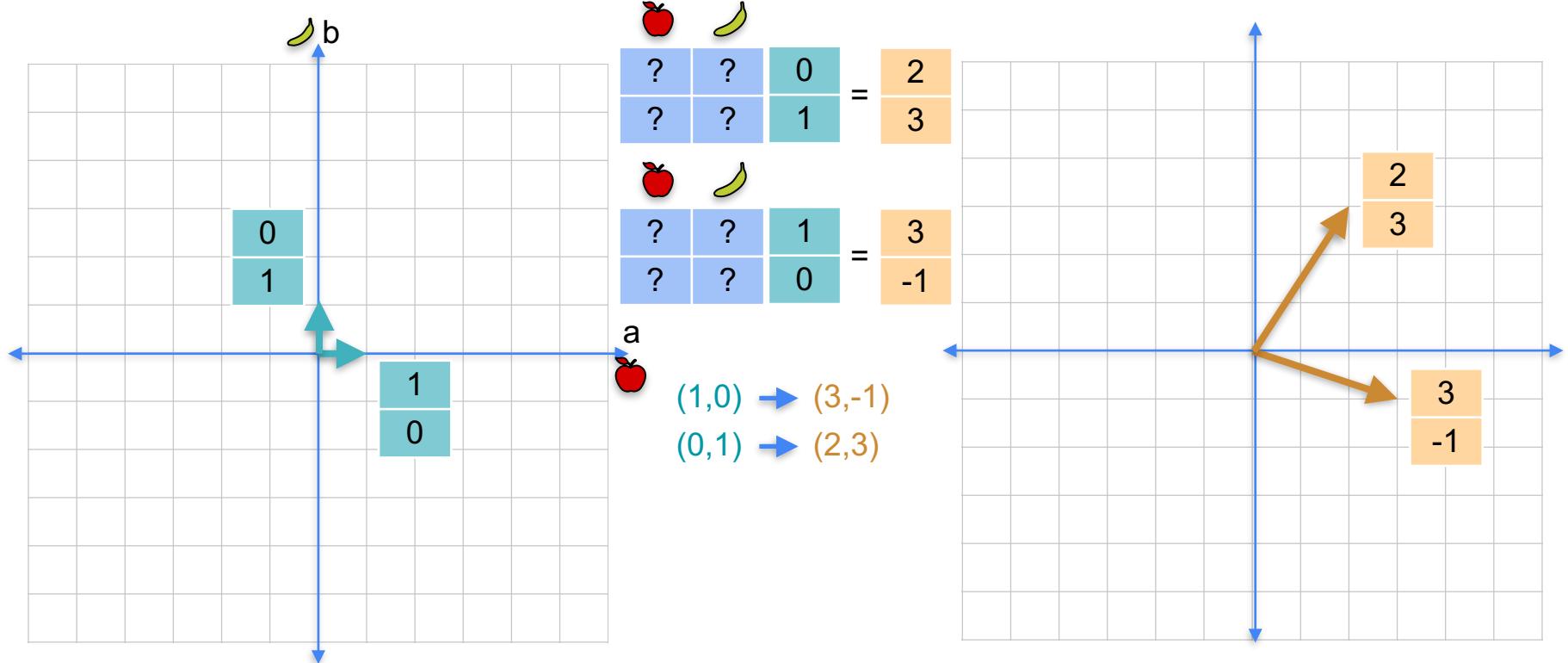
Linear transformations as matrices



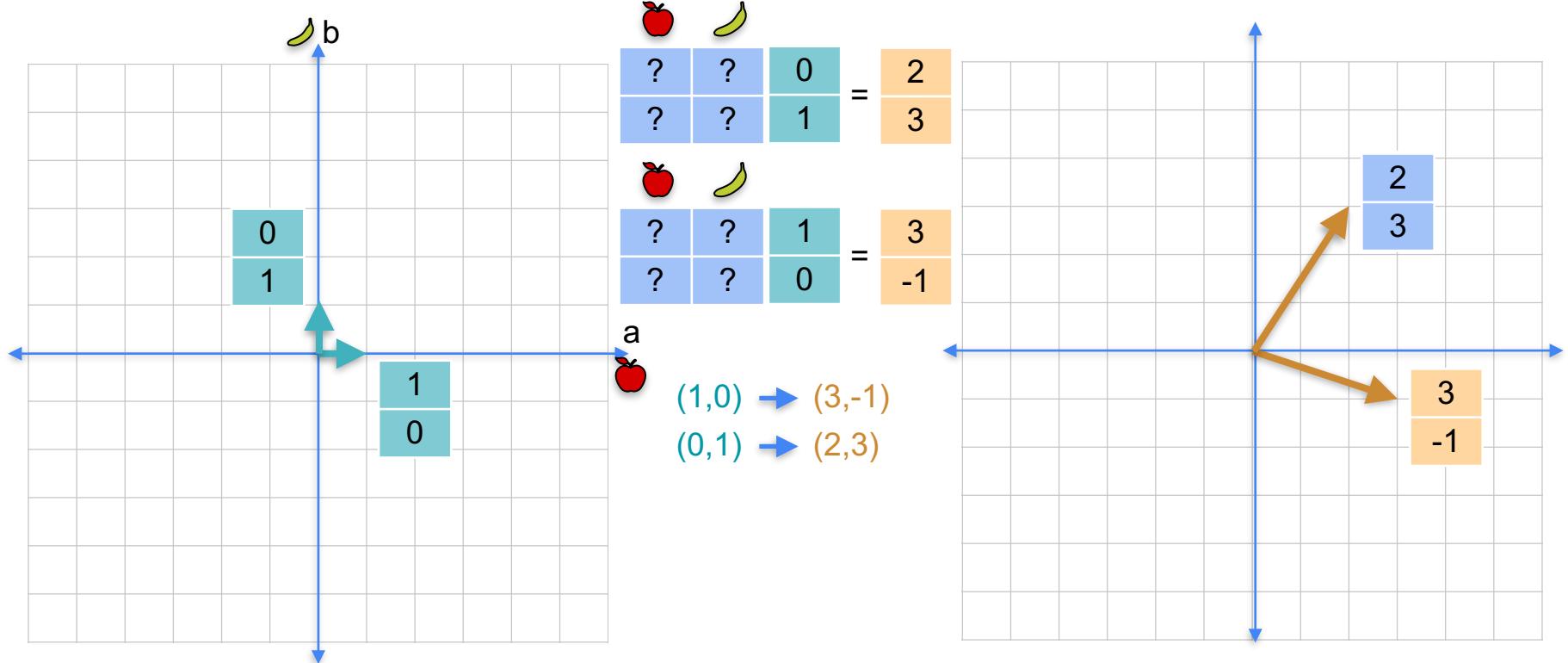
Linear transformations as matrices



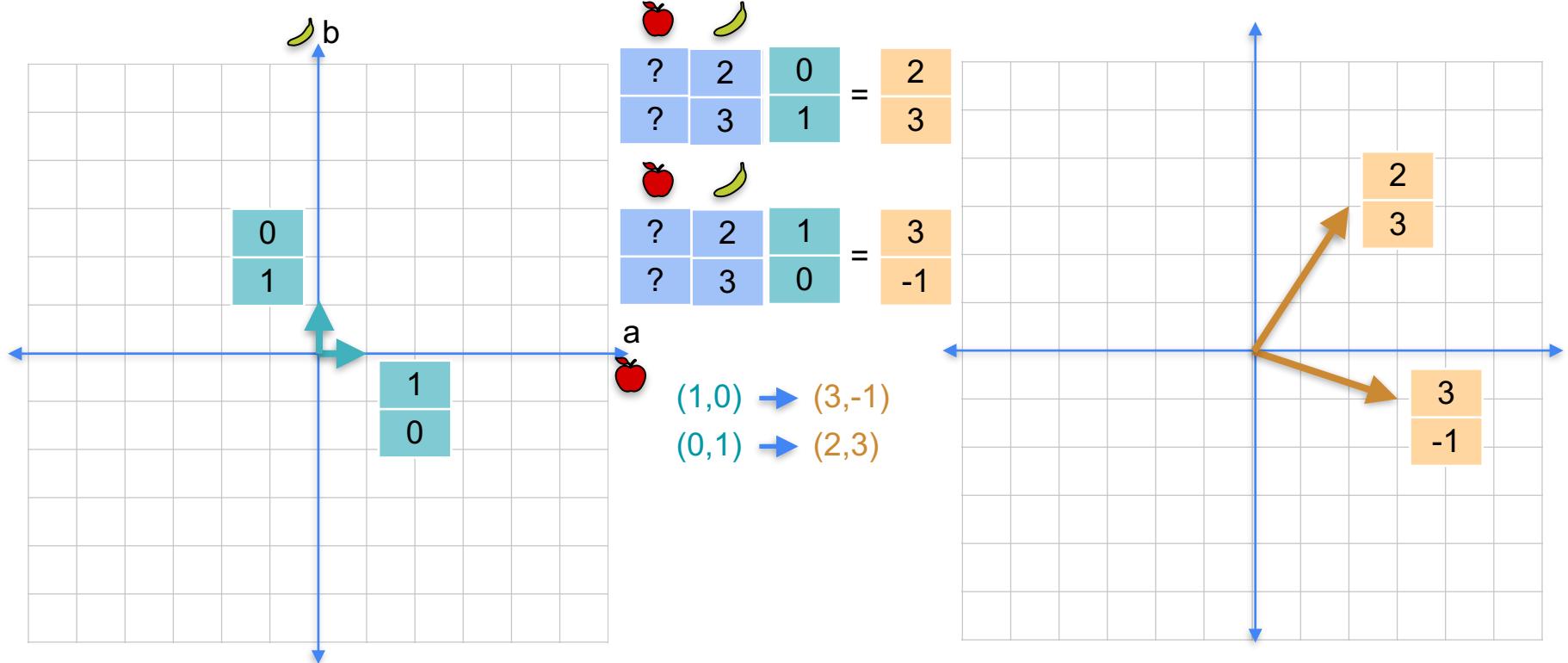
Linear transformations as matrices



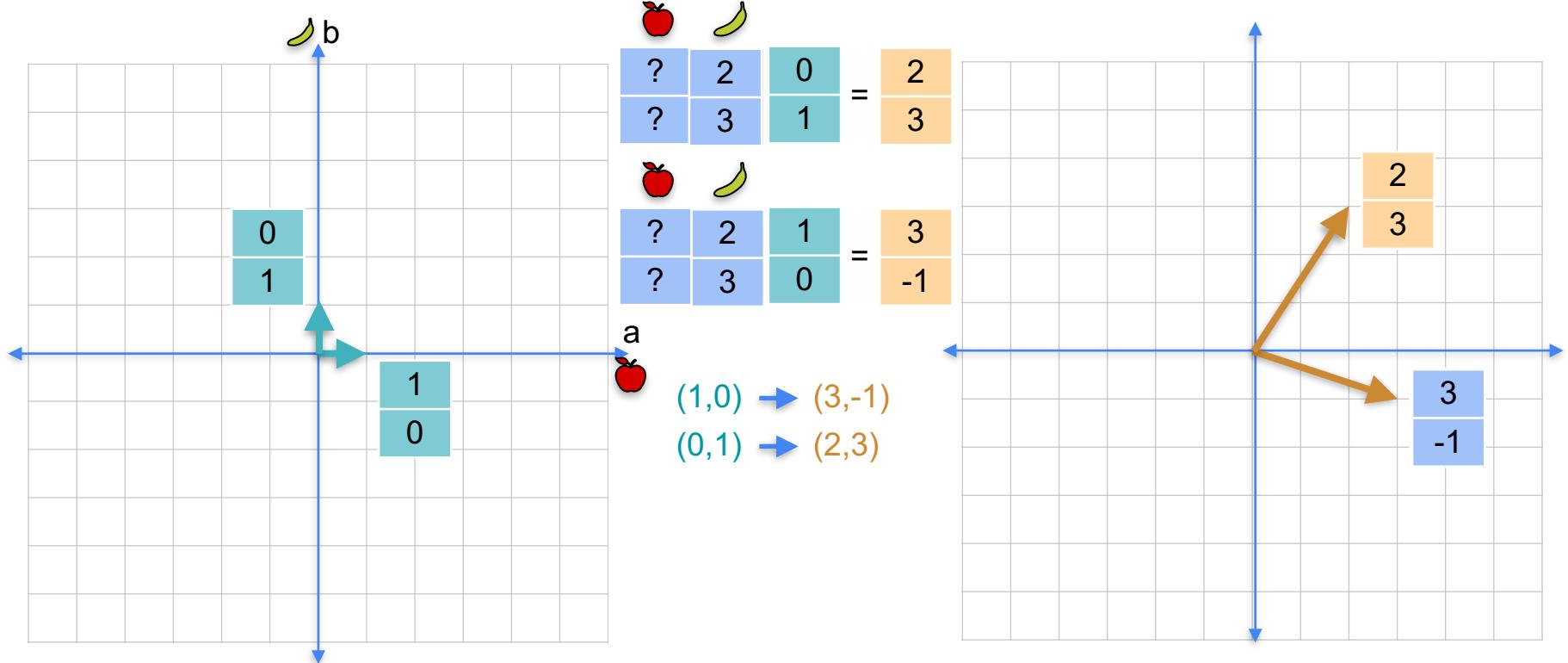
Linear transformations as matrices



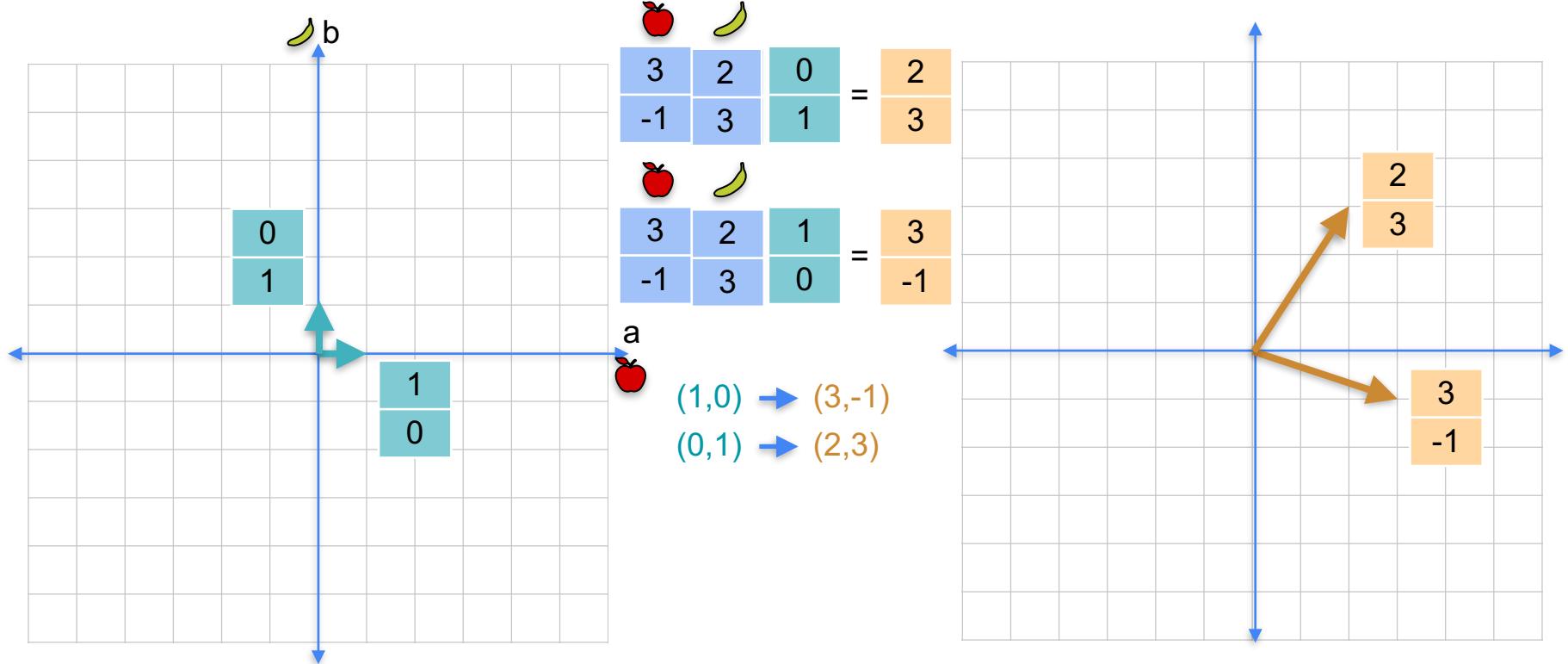
Linear transformations as matrices



Linear transformations as matrices



Linear transformations as matrices



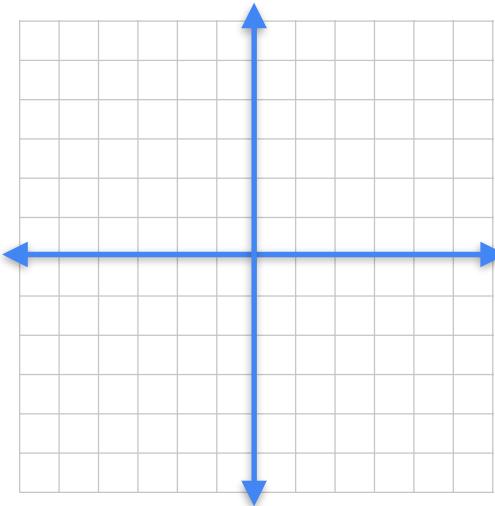
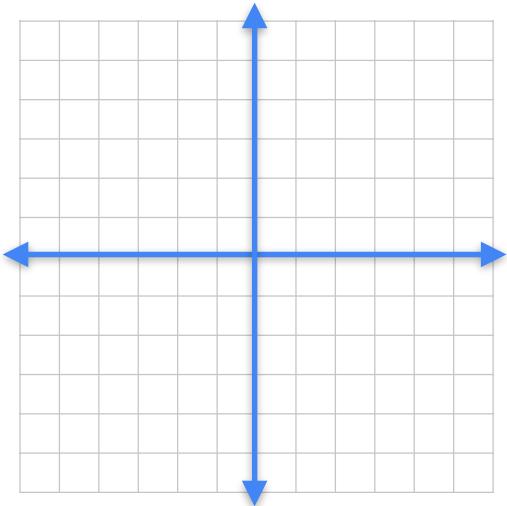


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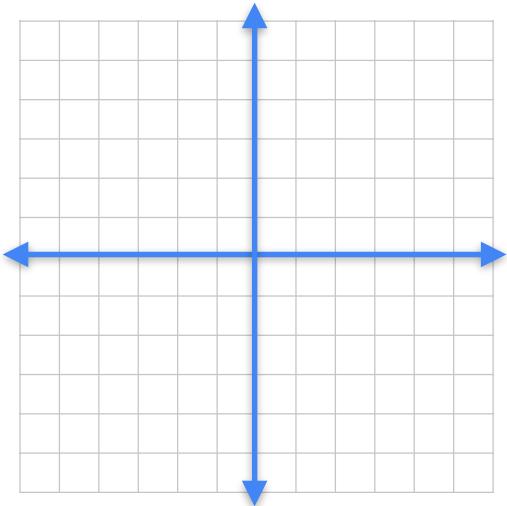
Vectors and Linear Transformations

Matrix multiplication

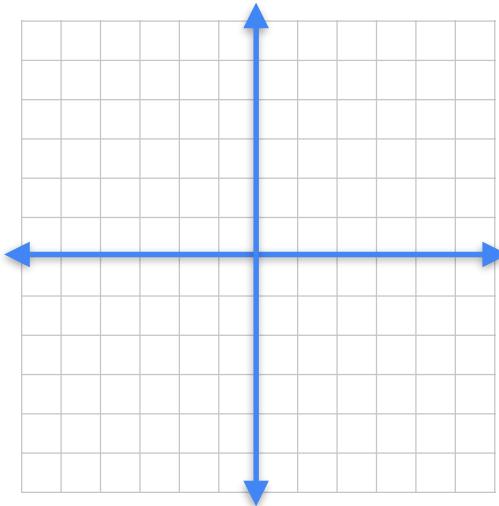
Combining linear transformations



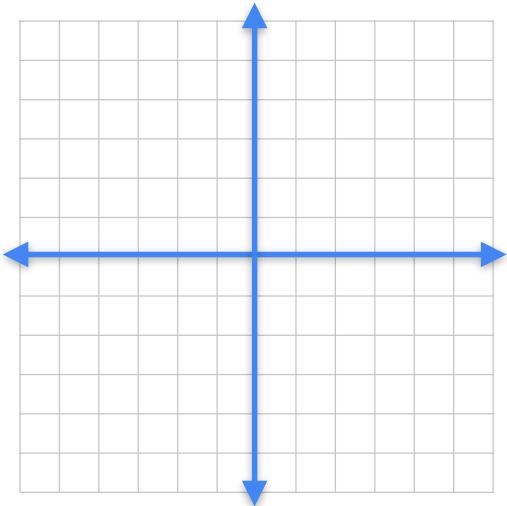
Combining linear transformations



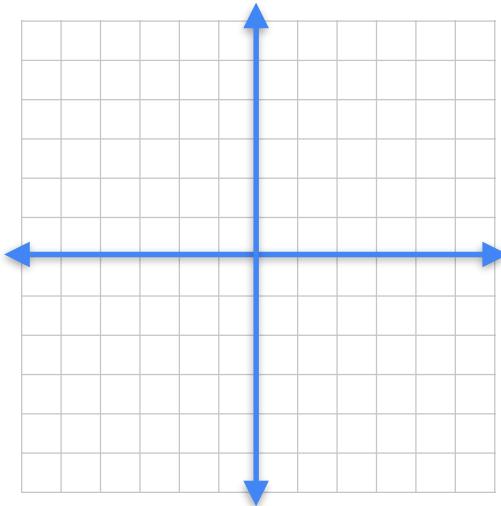
3	1
1	2



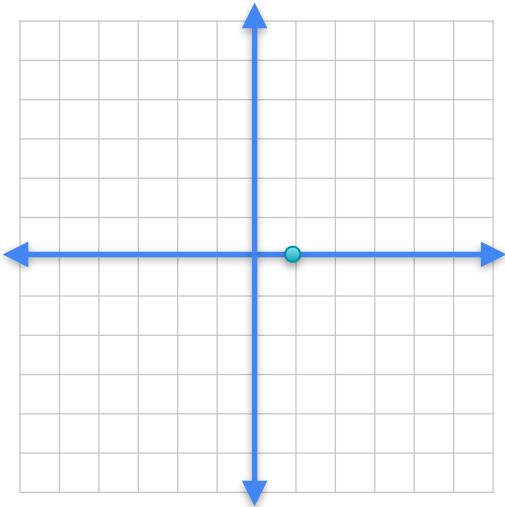
Combining linear transformations



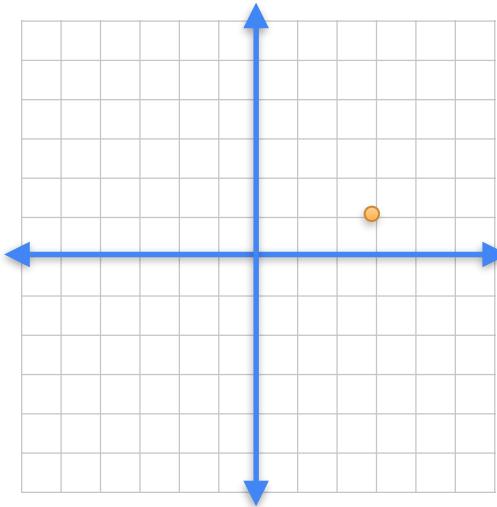
$$\begin{bmatrix} 3 & 1 & 1 \\ 1 & 2 & 0 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$



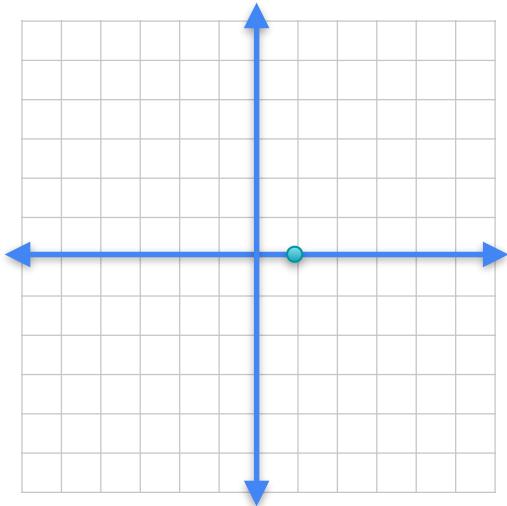
Combining linear transformations



$$\begin{bmatrix} 3 & 1 & 1 \\ 1 & 2 & 0 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

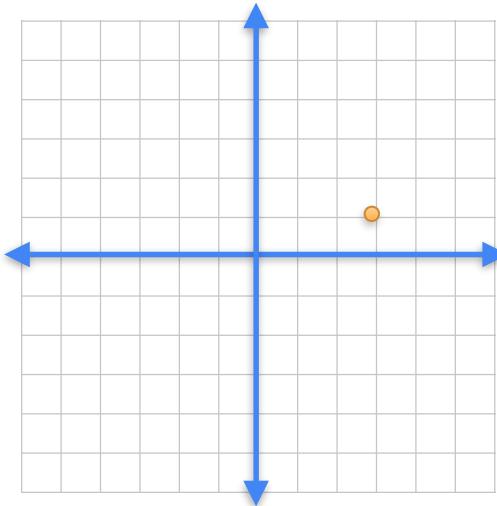


Combining linear transformations

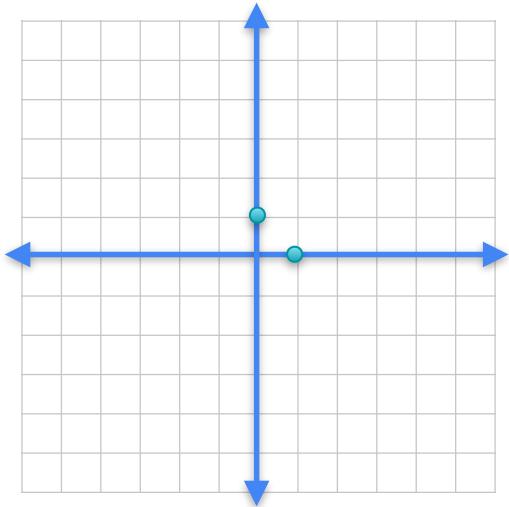


$$\begin{bmatrix} 3 & 1 & 1 \\ 1 & 2 & 0 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 1 & 0 \\ 1 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

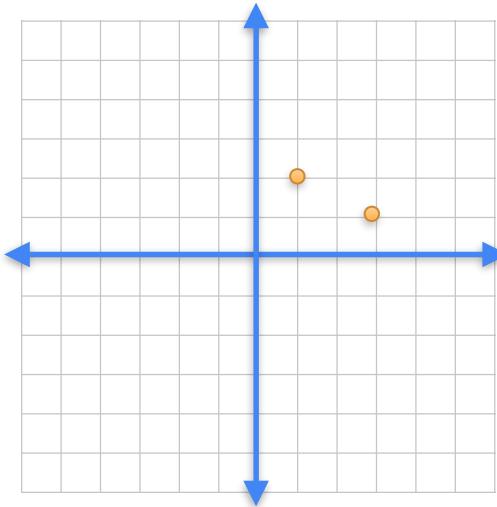


Combining linear transformations

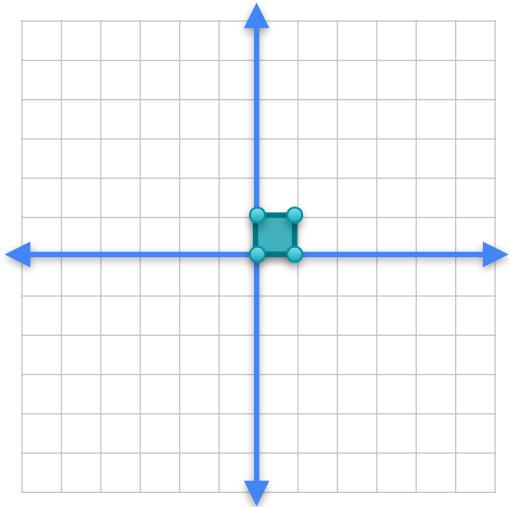


$$\begin{bmatrix} 3 & 1 & 1 \\ 1 & 2 & 0 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 1 & 0 \\ 1 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

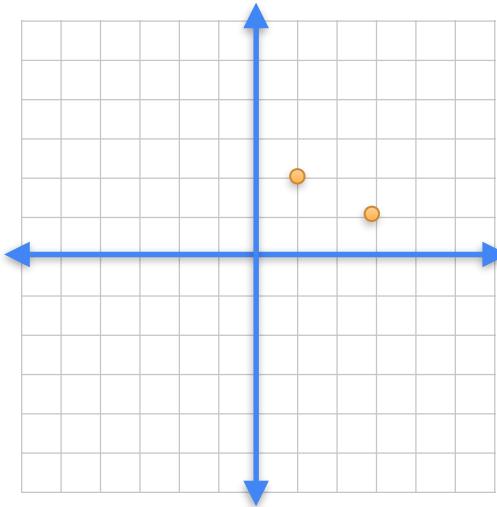


Combining linear transformations

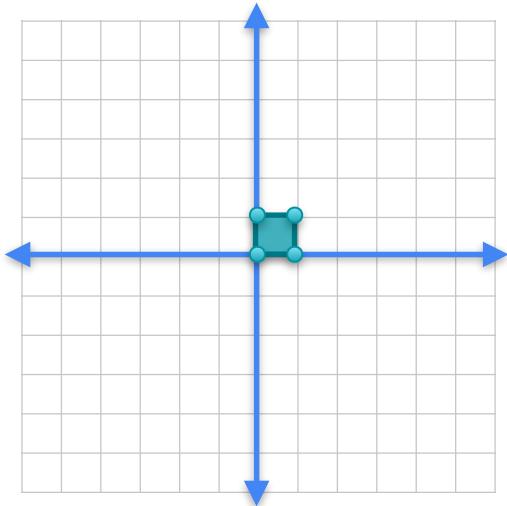


$$\begin{bmatrix} 3 & 1 & 1 \\ 1 & 2 & 0 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 1 & 0 \\ 1 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

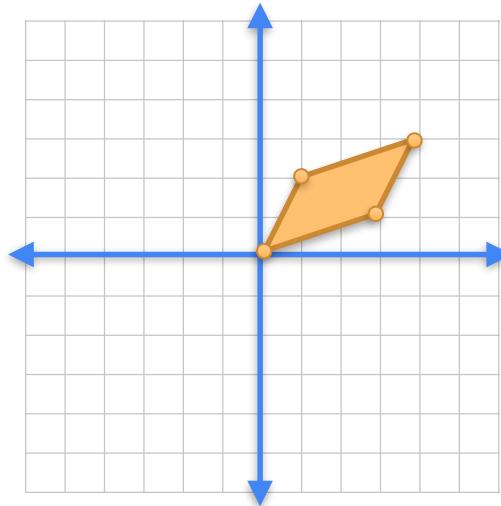


Combining linear transformations

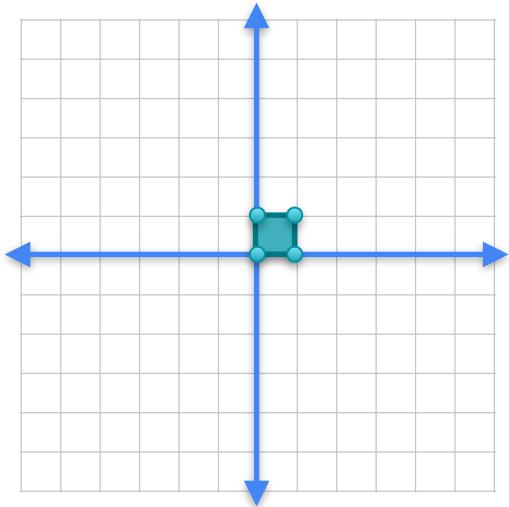


$$\begin{bmatrix} 3 & 1 & 1 \\ 1 & 2 & 0 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 1 & 0 \\ 1 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

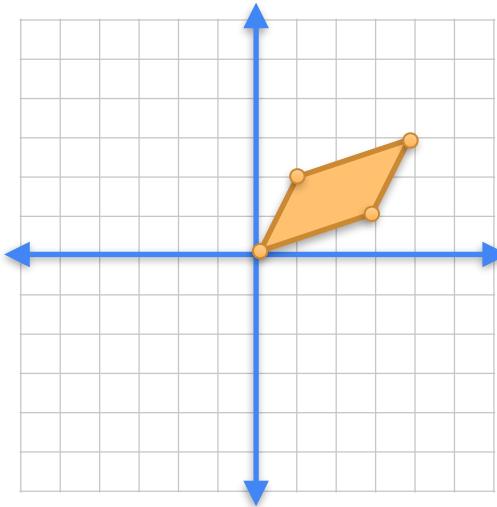


Combining linear transformations

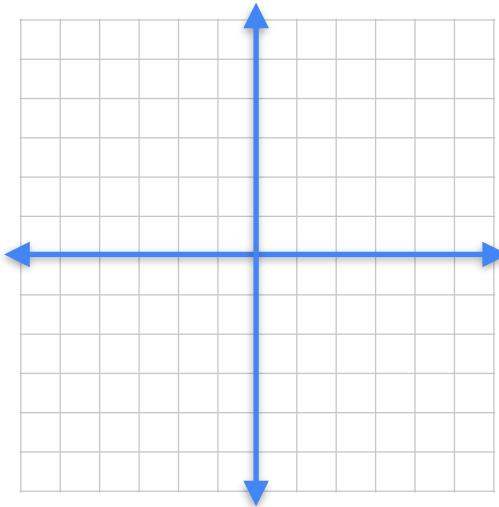
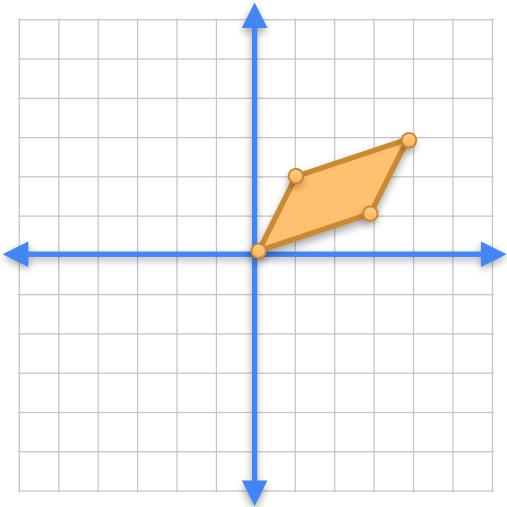


$$\begin{bmatrix} 3 & 1 & 1 \\ 1 & 2 & 0 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

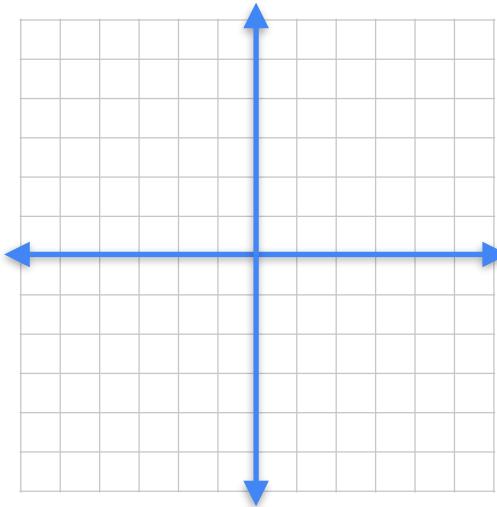
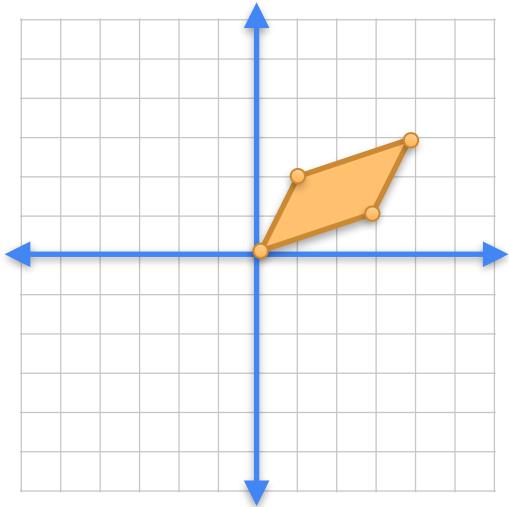
$$\begin{bmatrix} 3 & 1 & 0 \\ 1 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$



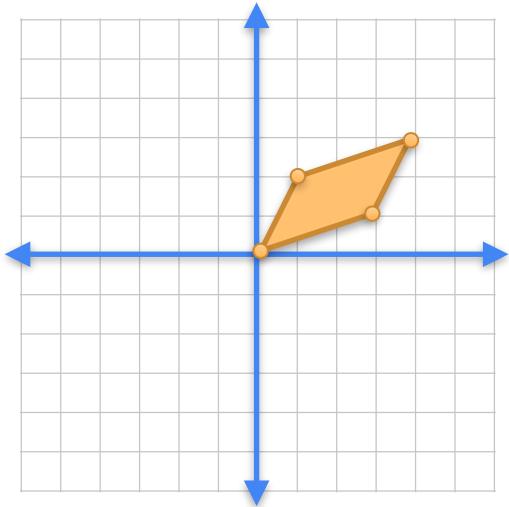
Combining linear transformations



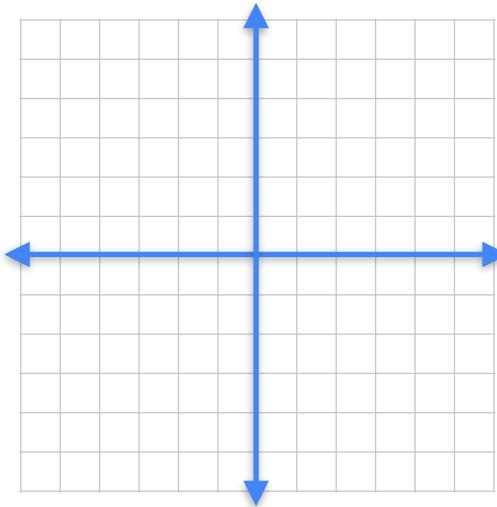
Combining linear transformations



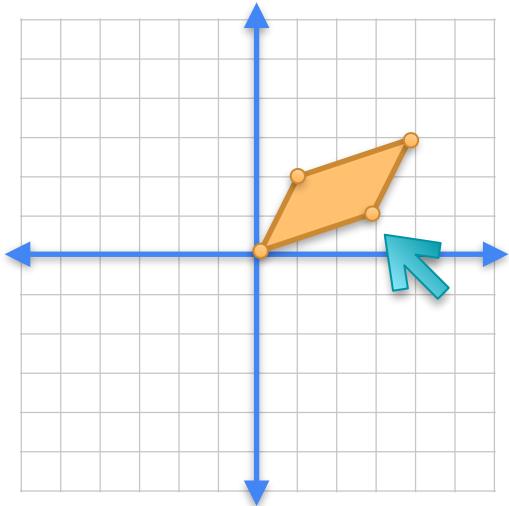
Combining linear transformations



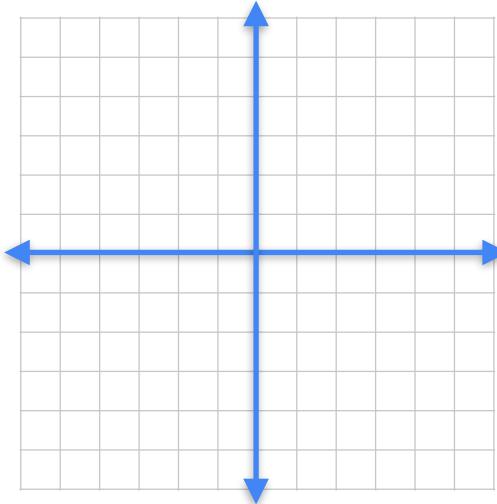
$$\begin{matrix} 2 & -1 & 3 \\ 0 & 2 & 1 \end{matrix} = \begin{matrix} 5 \\ 2 \end{matrix}$$



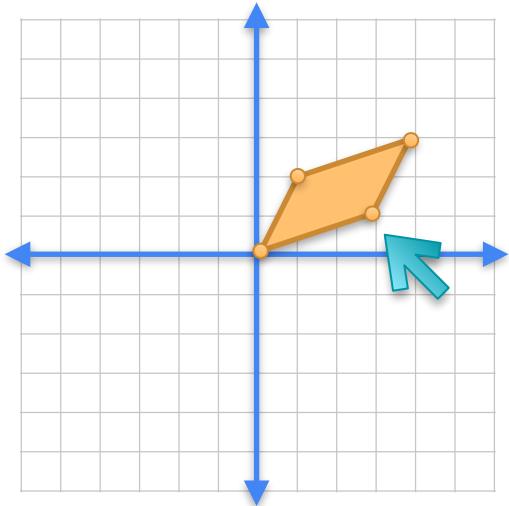
Combining linear transformations



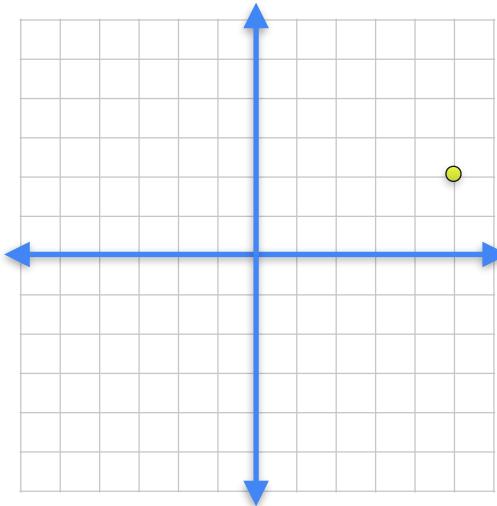
$$\begin{bmatrix} 2 & -1 & 3 \\ 0 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 5 \\ 2 \end{bmatrix}$$



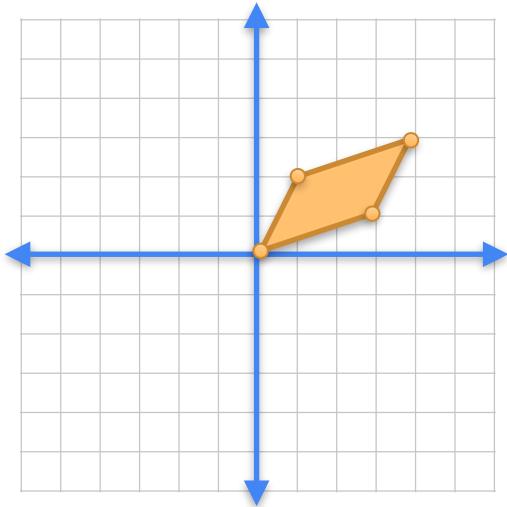
Combining linear transformations



$$\begin{matrix} 2 & -1 & 3 \\ 0 & 2 & 1 \end{matrix} = \begin{matrix} 5 \\ 2 \end{matrix}$$

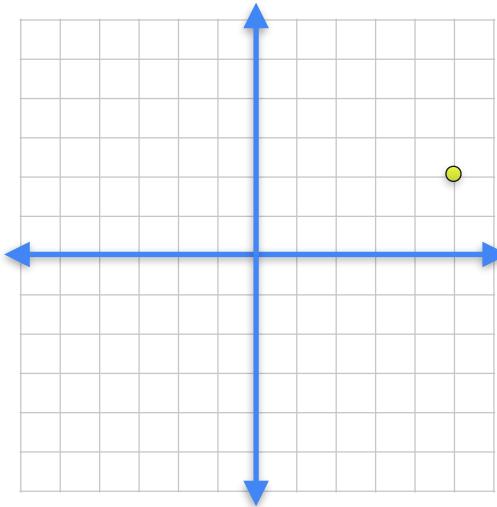


Combining linear transformations

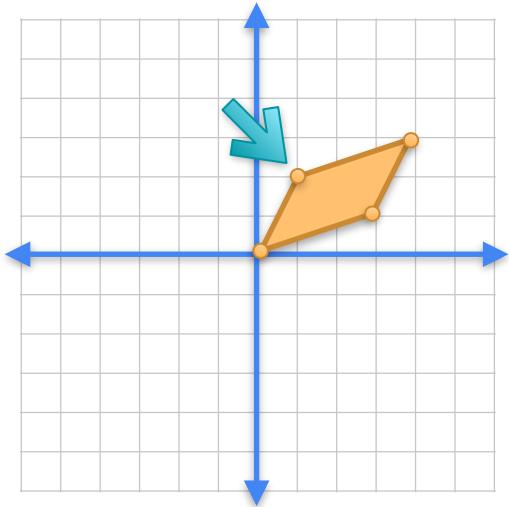


$$\begin{bmatrix} 2 & -1 & 3 \\ 0 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 5 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} 2 & -1 & 1 \\ 0 & 2 & 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 4 \end{bmatrix}$$

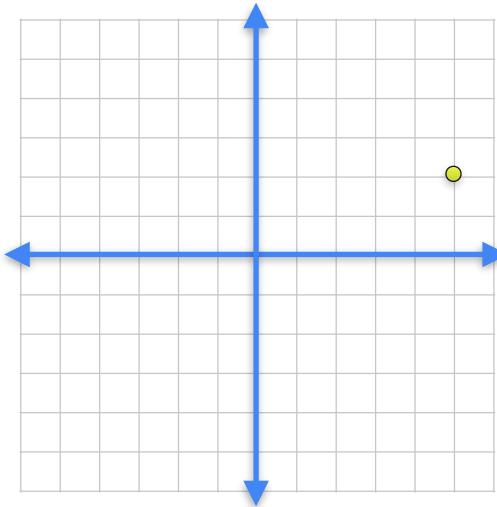


Combining linear transformations

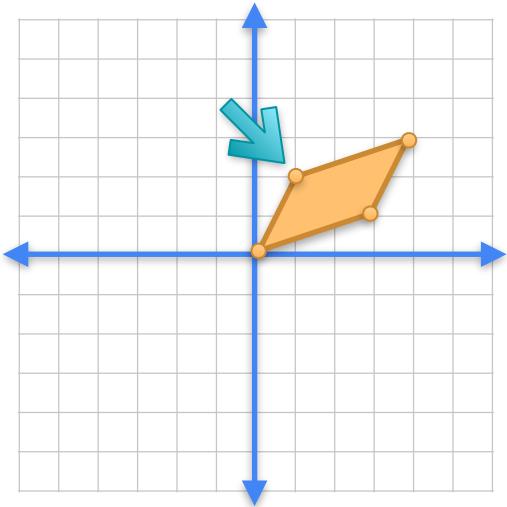


$$\begin{bmatrix} 2 & -1 & 3 \\ 0 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 5 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} 2 & -1 & 1 \\ 0 & 2 & 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 4 \end{bmatrix}$$

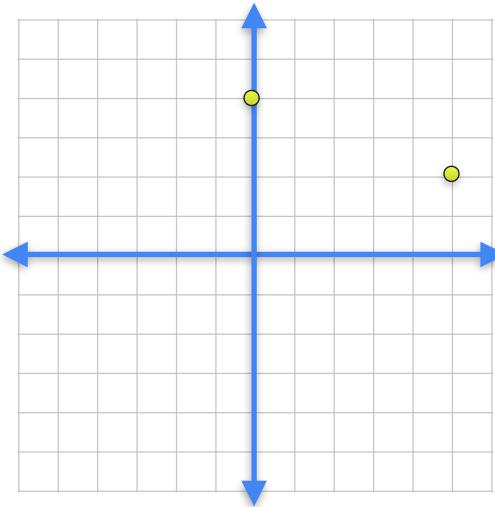


Combining linear transformations

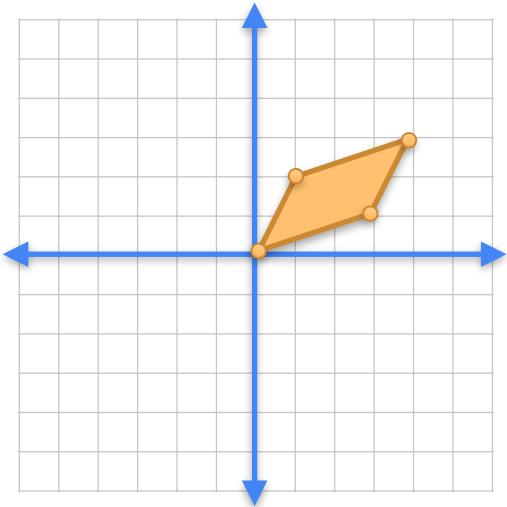


$$\begin{matrix} 2 & -1 & 3 \\ 0 & 2 & 1 \end{matrix} = \begin{matrix} 5 \\ 2 \end{matrix}$$

$$\begin{matrix} 2 & -1 & 1 \\ 0 & 2 & 2 \end{matrix} = \begin{matrix} 0 \\ 4 \end{matrix}$$

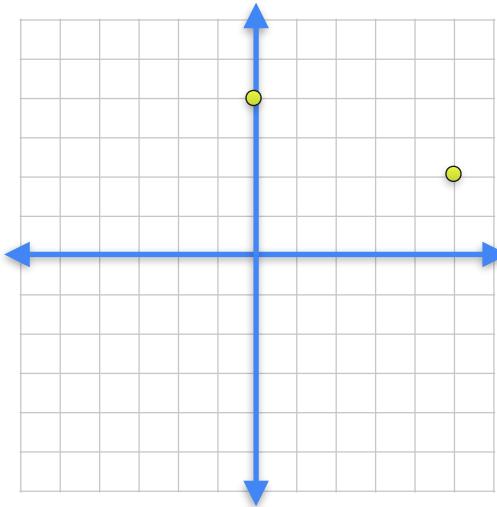


Combining linear transformations

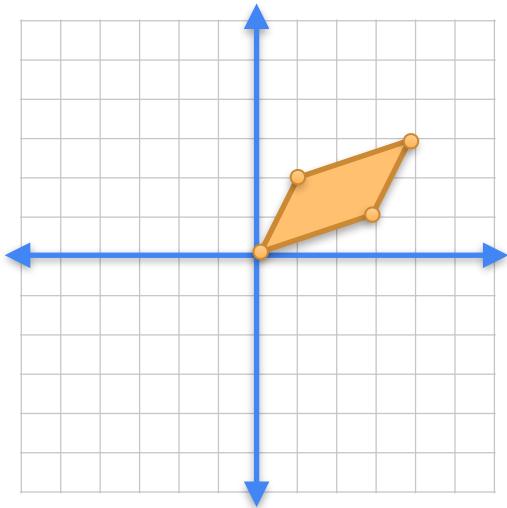


$$\begin{matrix} 2 & -1 & 3 \\ 0 & 2 & 1 \end{matrix} = \begin{matrix} 5 \\ 2 \end{matrix}$$

$$\begin{matrix} 2 & -1 & 1 \\ 0 & 2 & 2 \end{matrix} = \begin{matrix} 0 \\ 4 \end{matrix}$$

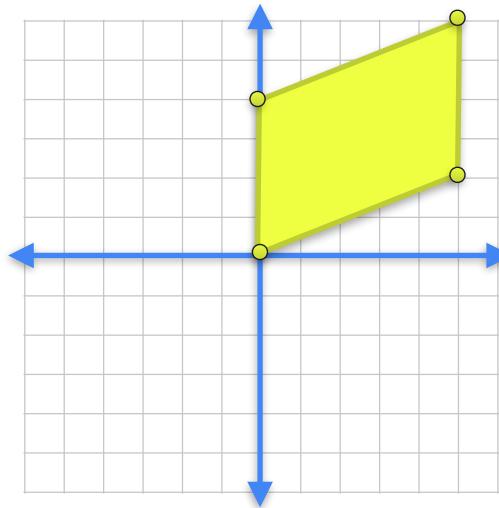


Combining linear transformations

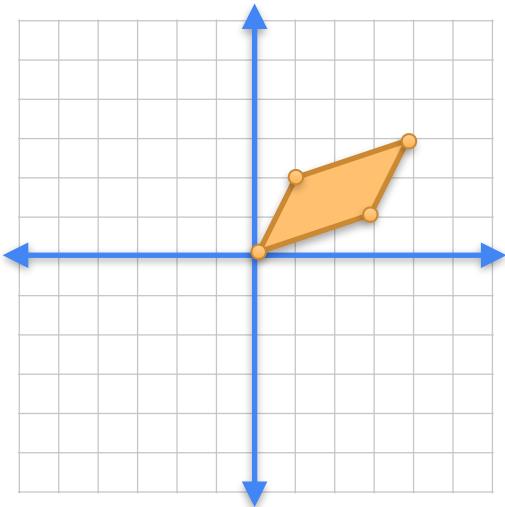


$$\begin{matrix} 2 & -1 & 3 \\ 0 & 2 & 1 \end{matrix} = \begin{matrix} 5 \\ 2 \end{matrix}$$

$$\begin{matrix} 2 & -1 & 1 \\ 0 & 2 & 2 \end{matrix} = \begin{matrix} 0 \\ 4 \end{matrix}$$

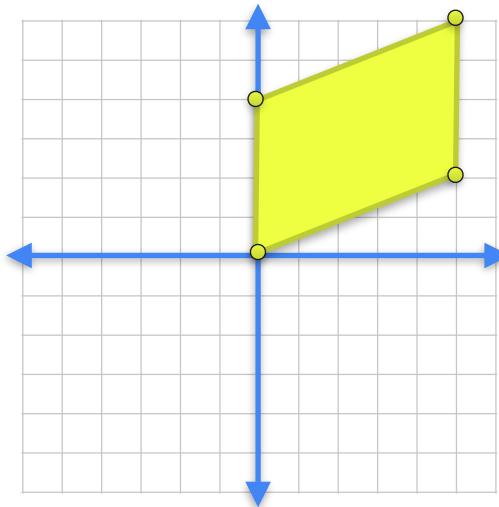


Combining linear transformations

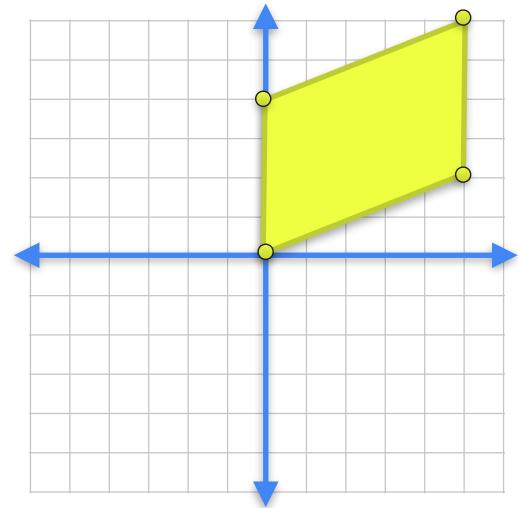
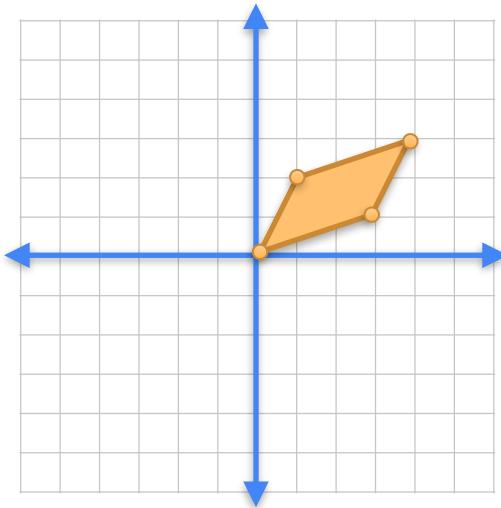
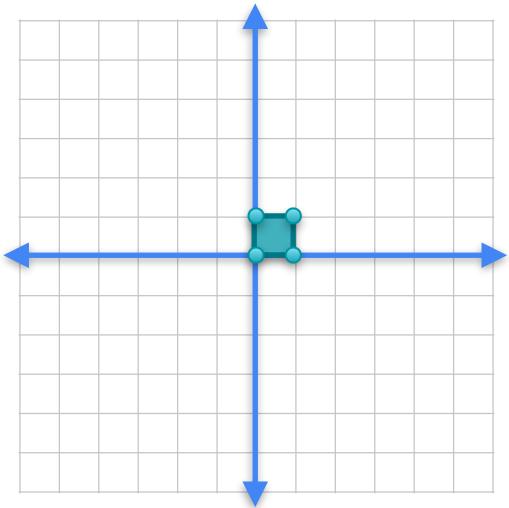


$$\begin{matrix} 2 & -1 & 3 \\ 0 & 2 & 1 \end{matrix} = \begin{matrix} 5 \\ 2 \end{matrix}$$

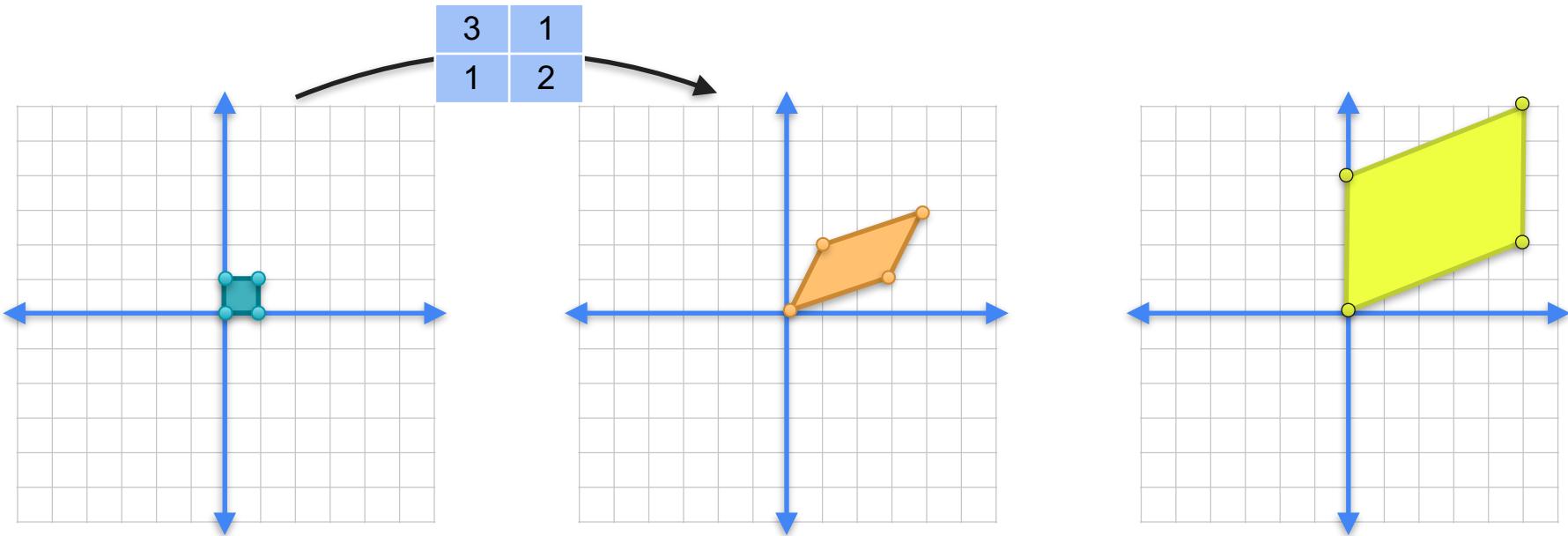
$$\begin{matrix} 2 & -1 & 1 \\ 0 & 2 & 2 \end{matrix} = \begin{matrix} 0 \\ 4 \end{matrix}$$



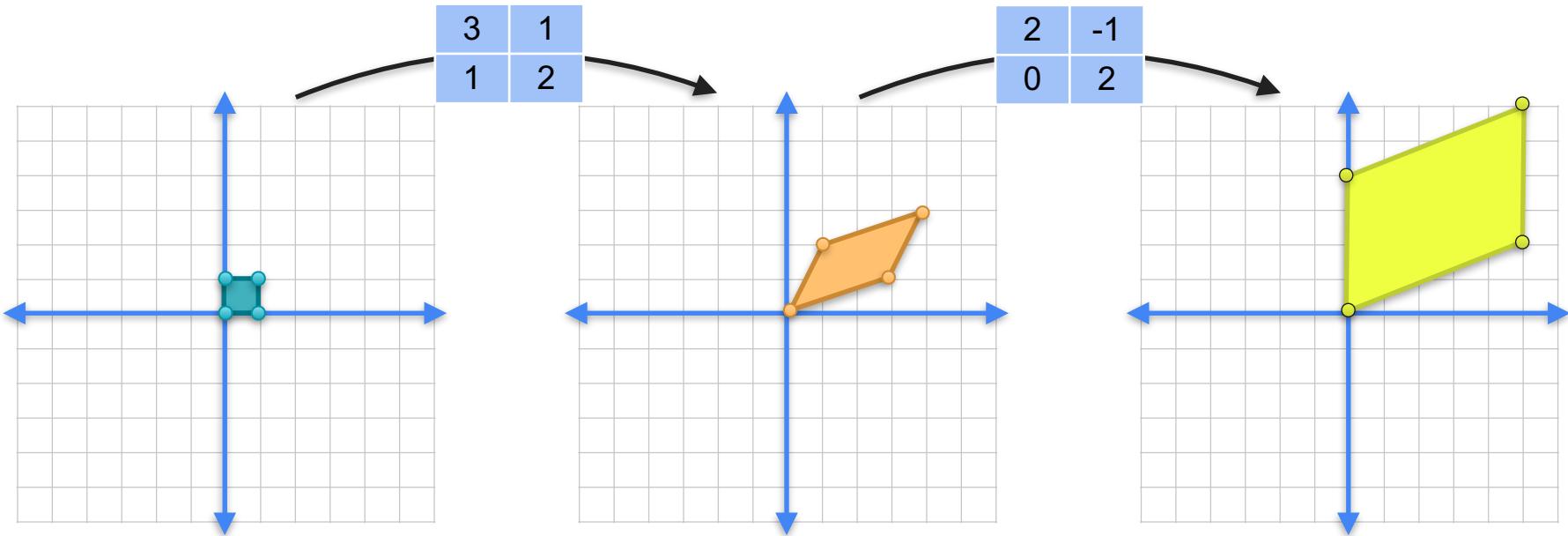
Combining linear transformations



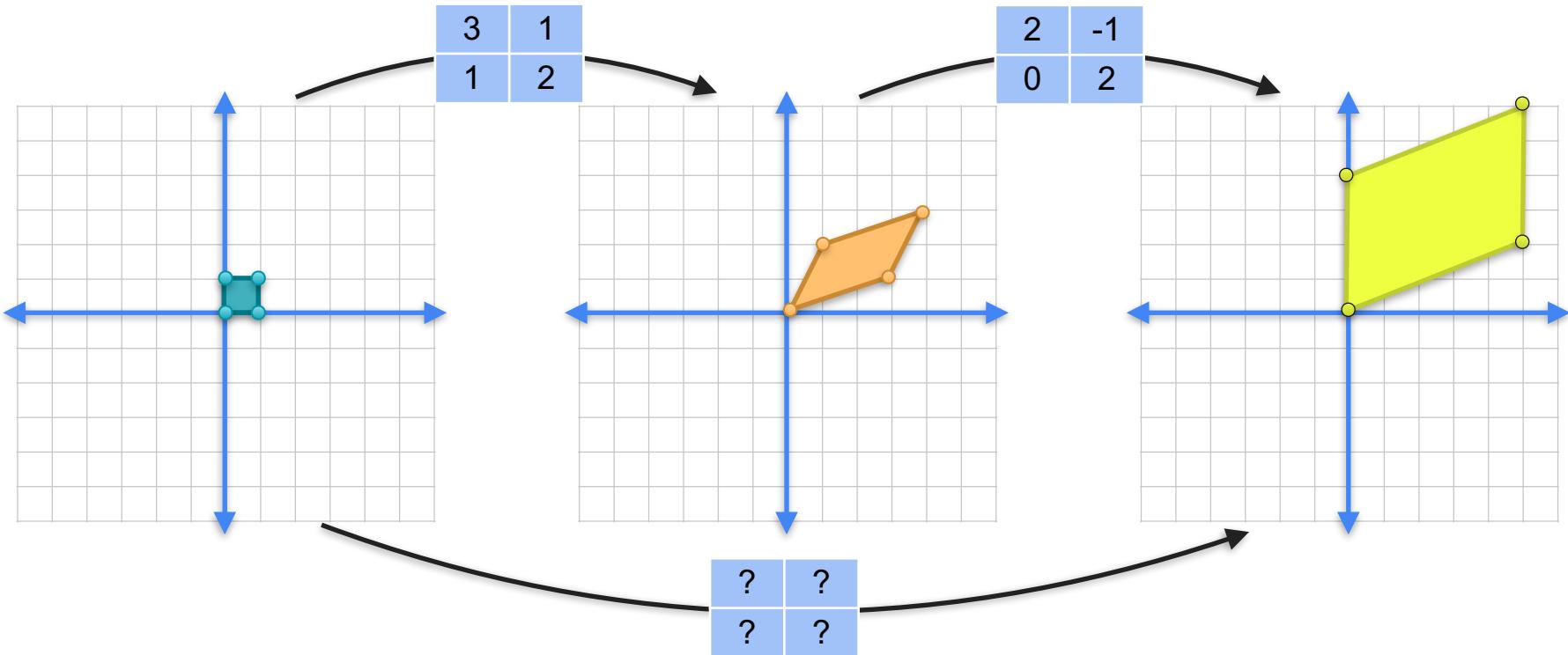
Combining linear transformations



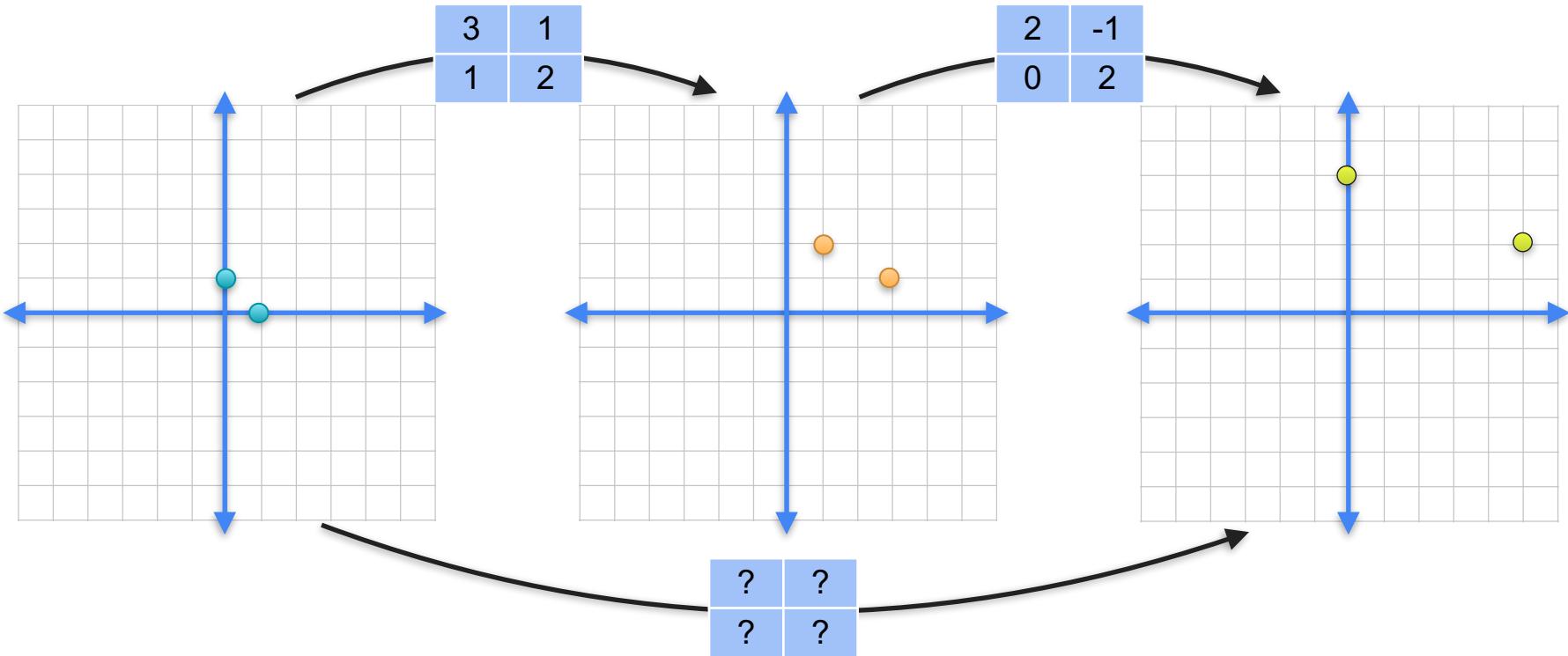
Combining linear transformations



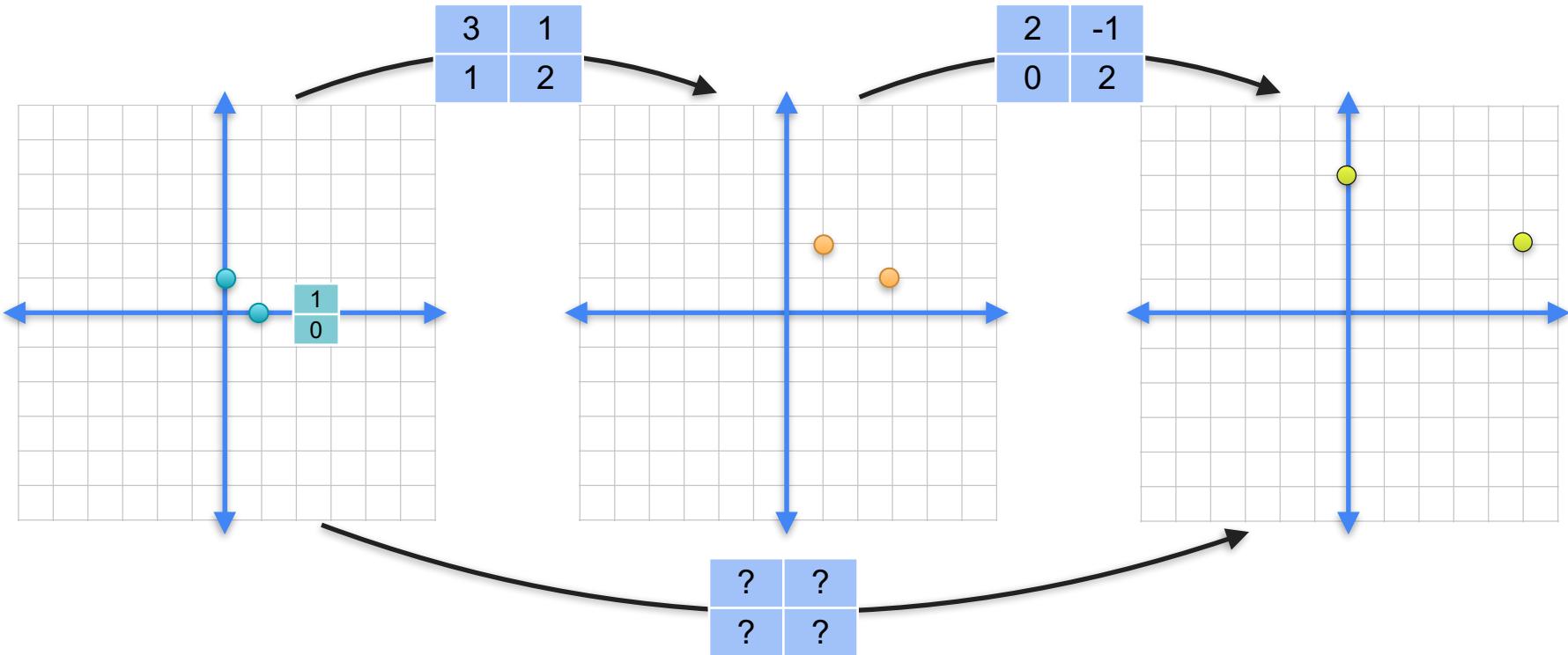
Combining linear transformations



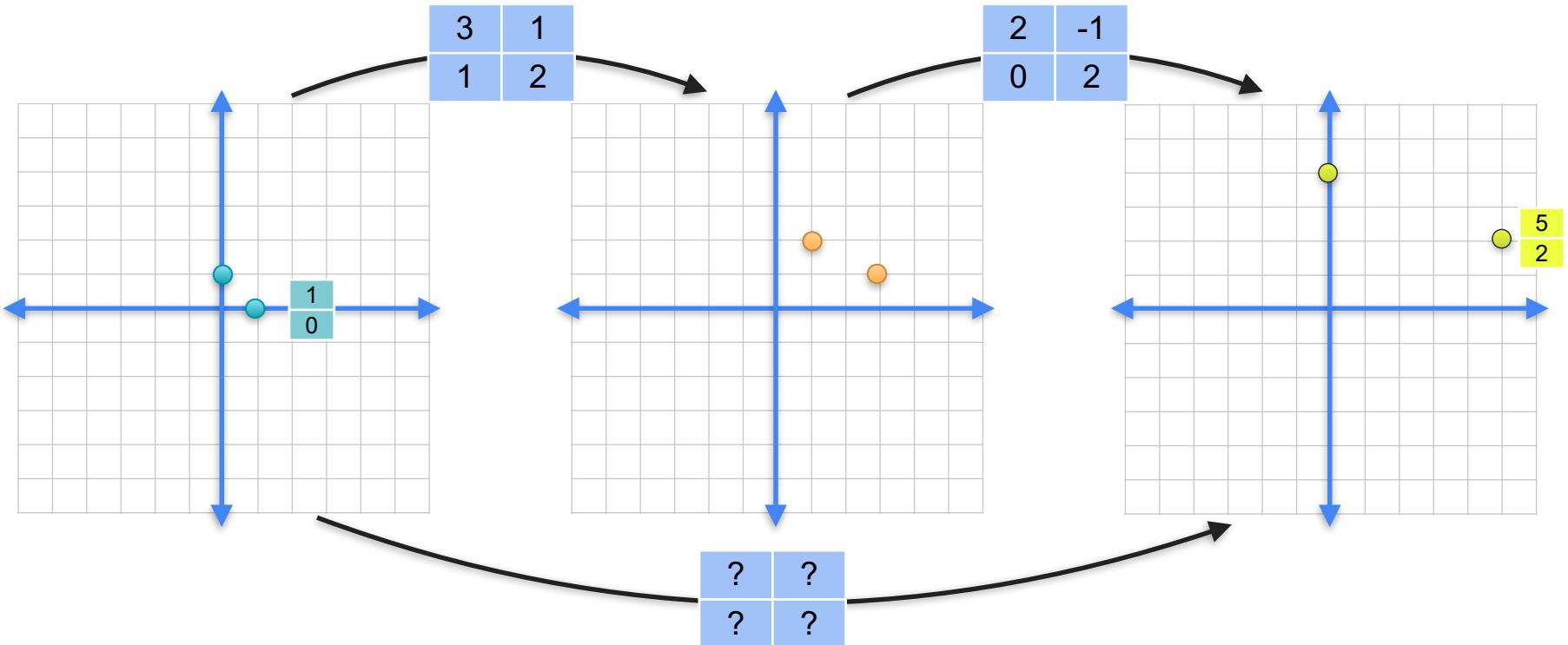
Combining linear transformations



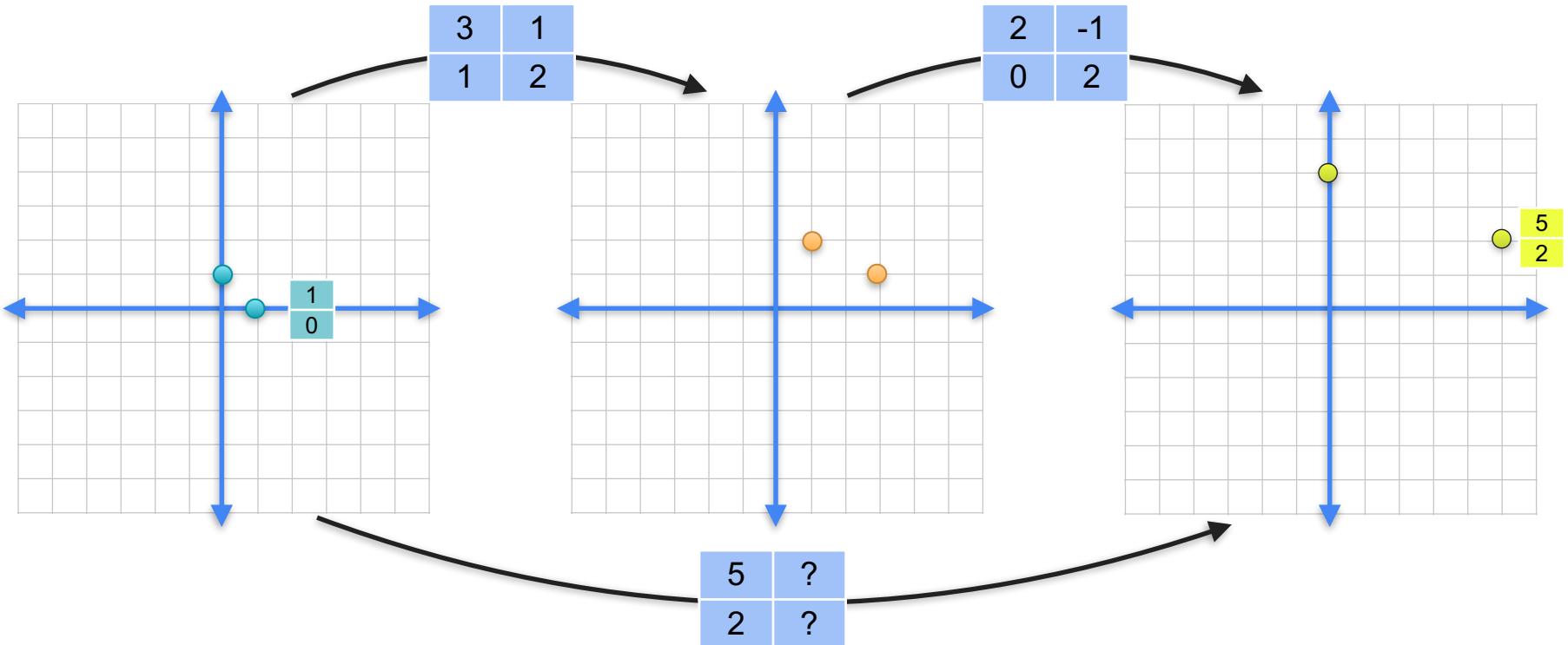
Combining linear transformations



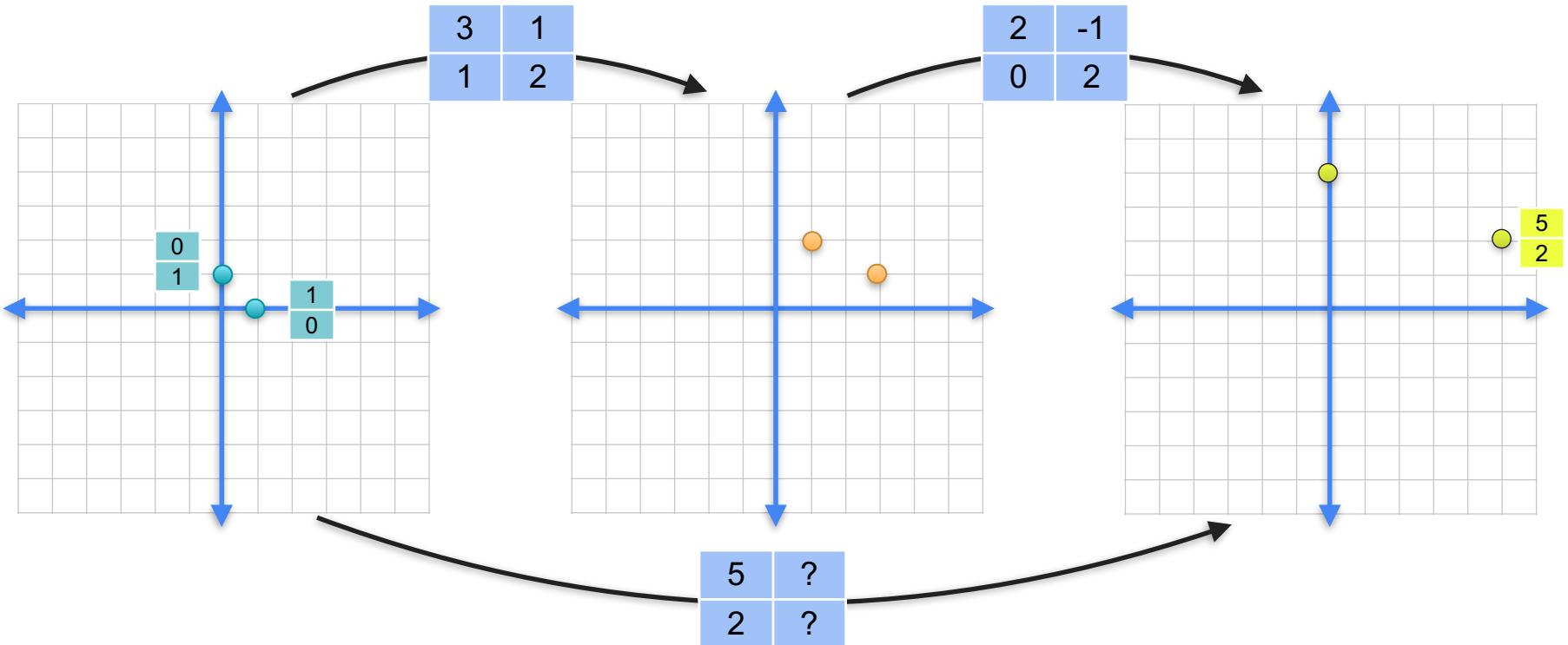
Combining linear transformations



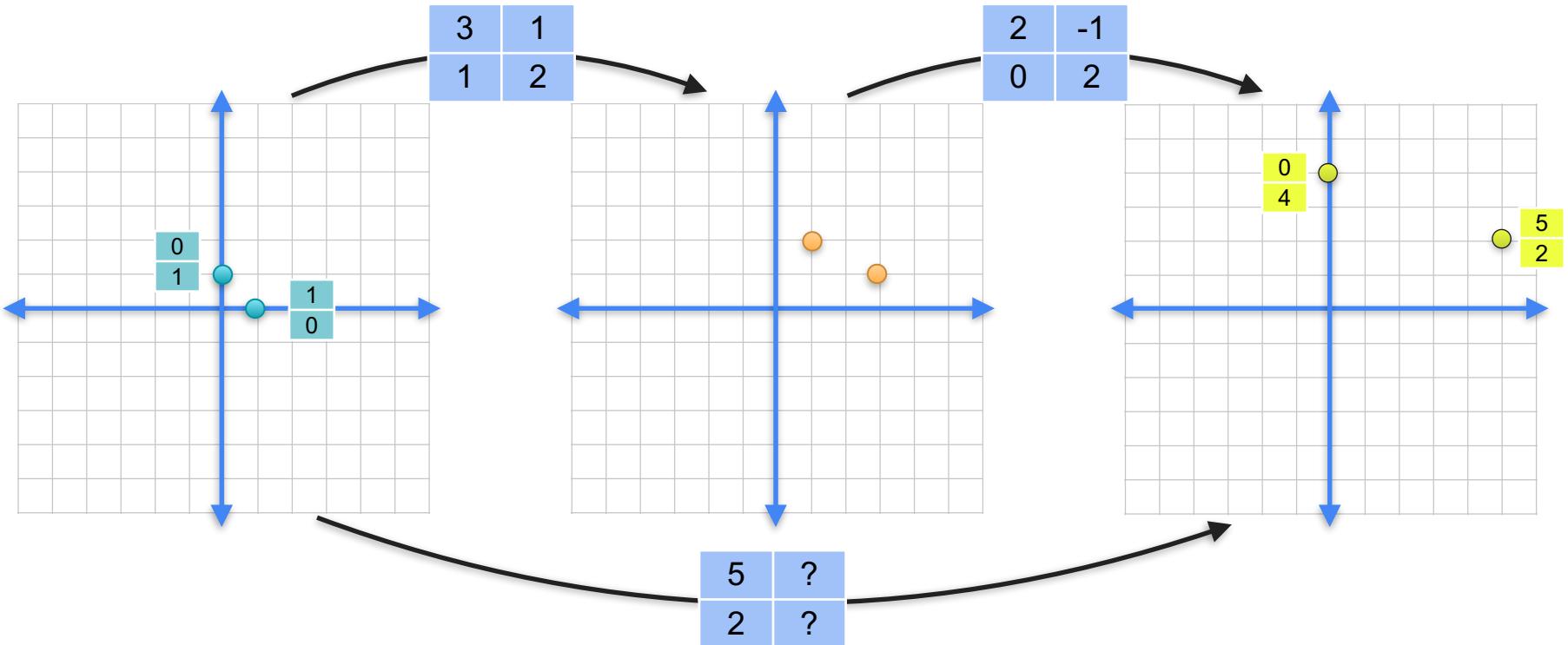
Combining linear transformations



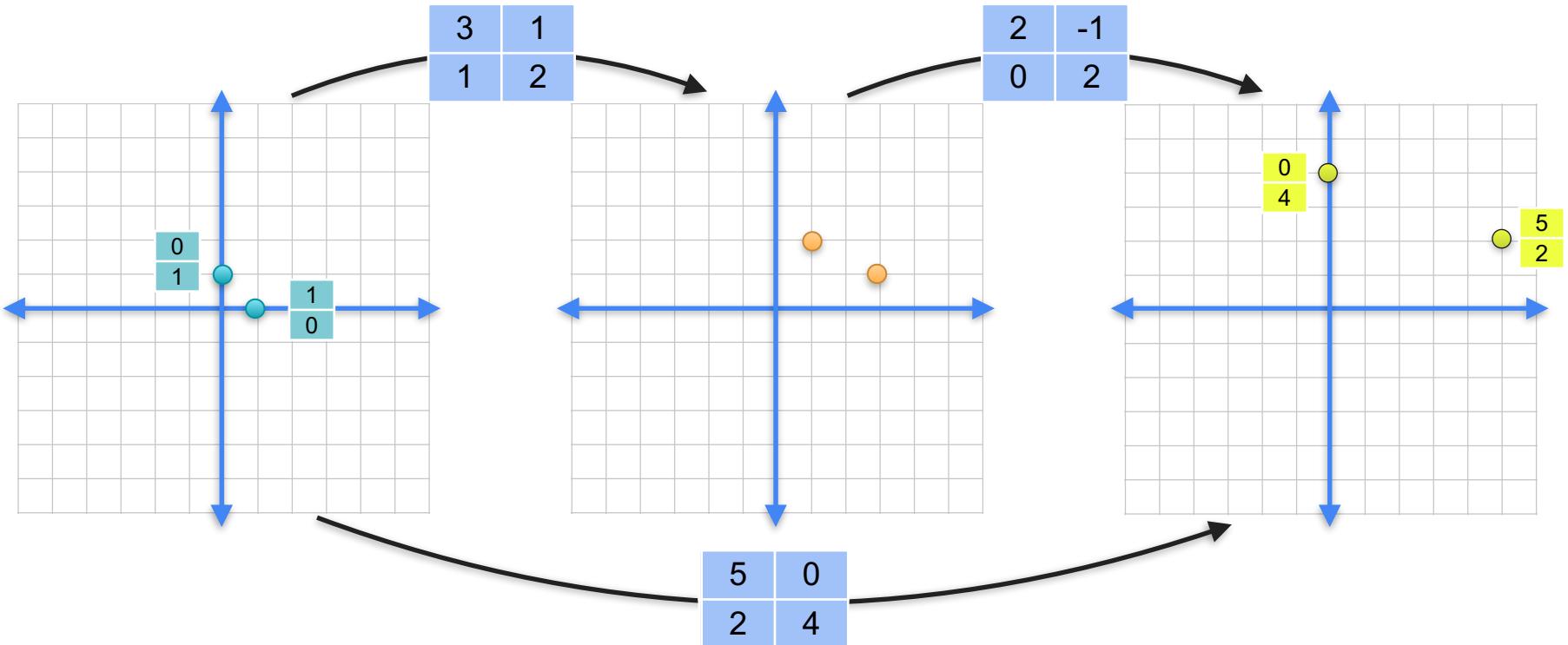
Combining linear transformations



Combining linear transformations



Combining linear transformations



Combining linear transformations

$$\begin{bmatrix} 2 & -1 \\ 0 & 2 \end{bmatrix} \cdot \begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ 2 & 4 \end{bmatrix}$$

Combining linear transformations

First
↓

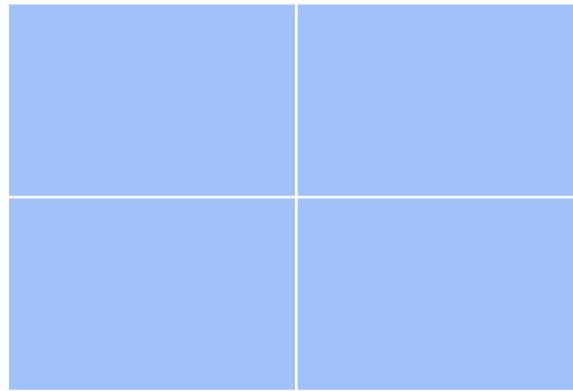
$$\begin{matrix} 2 & -1 \\ 0 & 2 \end{matrix} \cdot \begin{matrix} 3 & 1 \\ 1 & 2 \end{matrix} = \begin{matrix} 5 & 0 \\ 2 & 4 \end{matrix}$$

Combining linear transformations

$$\begin{array}{c} \text{Second} \\ \downarrow \\ \begin{array}{|c|c|} \hline 2 & -1 \\ \hline 0 & 2 \\ \hline \end{array} \end{array} \cdot \begin{array}{c} \text{First} \\ \downarrow \\ \begin{array}{|c|c|} \hline 3 & 1 \\ \hline 1 & 2 \\ \hline \end{array} \end{array} = \begin{array}{|c|c|} \hline 5 & 0 \\ \hline 2 & 4 \\ \hline \end{array}$$

Multiplying matrices

$$\begin{matrix} 2 & -1 \\ 0 & 2 \end{matrix} \cdot \begin{matrix} 5 & 0 \\ 0 & 4 \end{matrix} =$$



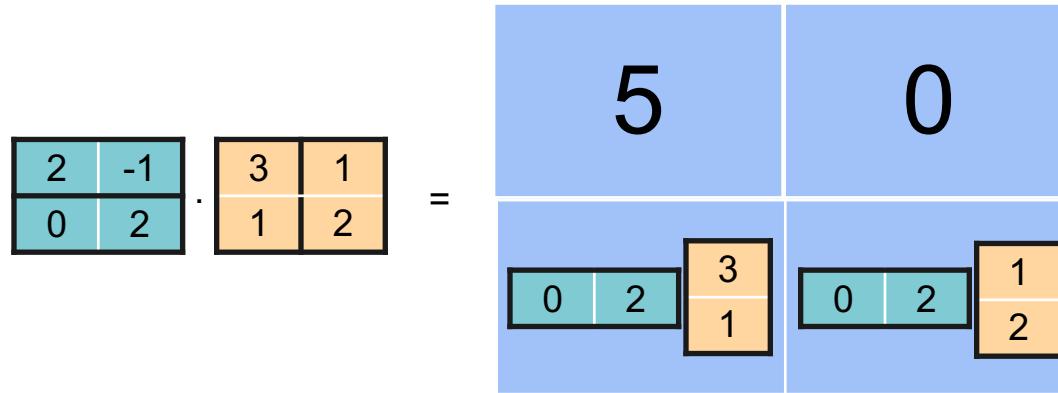
Multiplying matrices

$$\begin{matrix} 2 & -1 \\ 0 & 2 \end{matrix} \cdot \begin{matrix} 3 & 1 \\ 1 & 2 \end{matrix} = \begin{matrix} & & 3 & 1 \\ 2 & -1 & | & 3 & 1 \\ & & 1 & 2 \\ \hline & & 0 & 2 & | & 3 & 1 \\ & & 0 & 2 & | & 1 & 2 \end{matrix}$$

Multiplying matrices

$$\begin{matrix} 2 & -1 \\ 0 & 2 \end{matrix} \cdot \begin{matrix} 3 & 1 \\ 1 & 2 \end{matrix} = \begin{matrix} 5 & & & \\ & \begin{matrix} 2 & -1 \\ 1 & 2 \end{matrix} & \begin{matrix} 1 & \\ 2 & \end{matrix} & \\ & \begin{matrix} 0 & 2 \\ 1 & 2 \end{matrix} & \begin{matrix} 3 & \\ 1 & \end{matrix} & \begin{matrix} 1 & \\ 2 & \end{matrix} \end{matrix}$$

Multiplying matrices

$$\begin{matrix} 2 & -1 \\ 0 & 2 \end{matrix} \cdot \begin{matrix} 3 & 1 \\ 1 & 2 \end{matrix} = \begin{matrix} 5 & 0 \\ 0 & 2 \end{matrix}$$


The diagram illustrates the multiplication of two 2x2 matrices. The first matrix, on the left, has cyan cells containing 2 and -1 in the top row, and 0 and 2 in the bottom row. The second matrix, also on the left, has orange cells containing 3 and 1 in the top row, and 1 and 2 in the bottom row. A large blue square on the right represents the result of their multiplication. Inside this blue square, there is a cyan cell containing 5 and an orange cell containing 1. To the left of the blue square, the two original matrices are shown side-by-side with a multiplication dot between them. To the right of the blue square, there is an equals sign. Below the blue square, there are two smaller blocks: one with a cyan cell (0, 2) and an orange cell (3, 1), and another with a cyan cell (0, 2) and an orange cell (1, 2).

Multiplying matrices

$$\begin{matrix} 2 & -1 \\ 0 & 2 \end{matrix} \cdot \begin{matrix} 3 & 1 \\ 1 & 2 \end{matrix} = \begin{matrix} 5 & 0 \\ 2 & \begin{matrix} 0 & 2 \\ 1 & 2 \end{matrix} \end{matrix}$$

Multiplying matrices

$$\begin{matrix} 2 & -1 \\ 0 & 2 \end{matrix} \cdot \begin{matrix} 3 & 1 \\ 1 & 2 \end{matrix} = \begin{matrix} 5 & 0 \\ 2 & 4 \end{matrix}$$



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Vectors and Linear Transformations

The identity matrix

The identity matrix

1	0	0	0	0
0	1	0	0	0
0	0	1	0	0
0	0	0	1	0
0	0	0	0	1

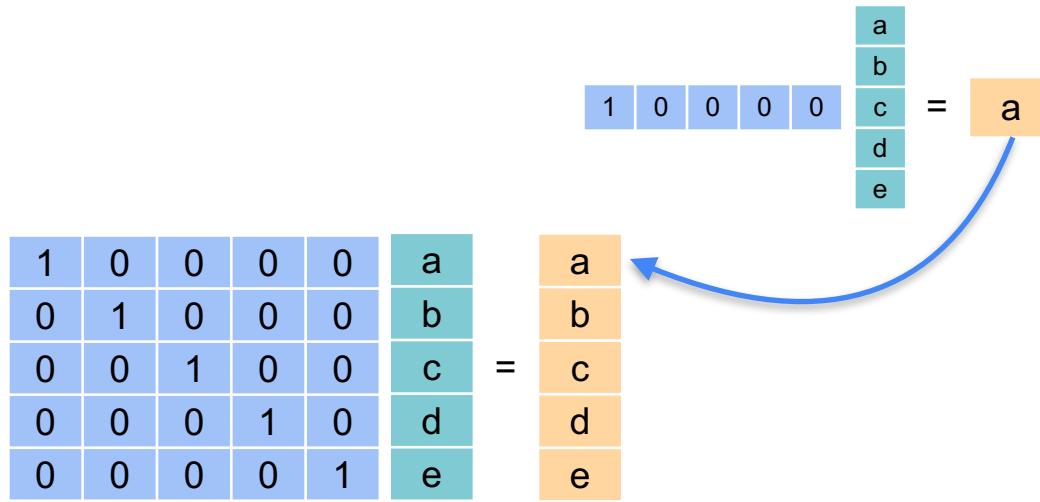
The identity matrix

1	0	0	0	0	a
0	1	0	0	0	b
0	0	1	0	0	c
0	0	0	1	0	d
0	0	0	0	1	e

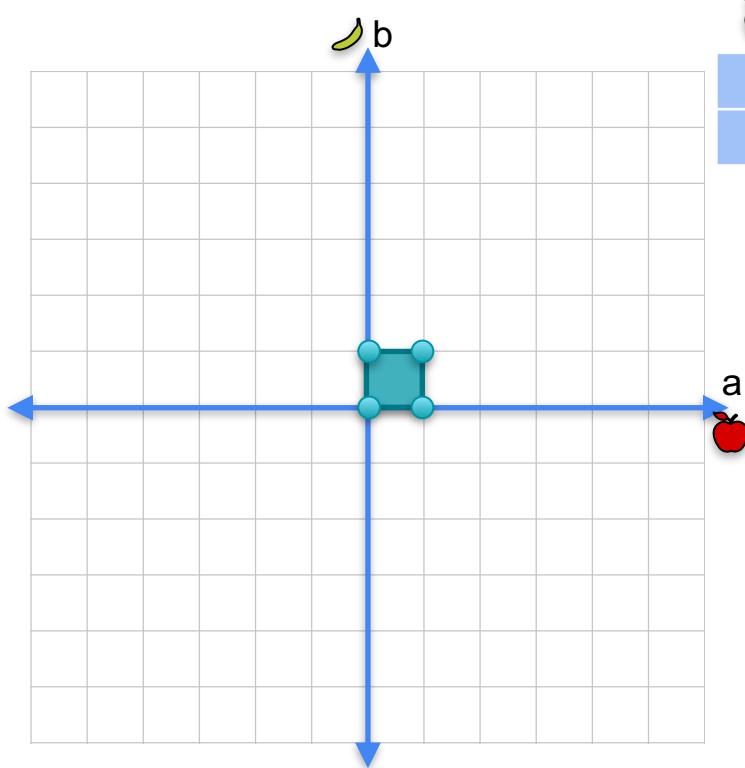
The identity matrix

$$\begin{array}{|c|c|c|c|c|c|} \hline 1 & 0 & 0 & 0 & 0 & \text{a} \\ \hline 0 & 1 & 0 & 0 & 0 & \text{b} \\ \hline 0 & 0 & 1 & 0 & 0 & \text{c} \\ \hline 0 & 0 & 0 & 1 & 0 & \text{d} \\ \hline 0 & 0 & 0 & 0 & 1 & \text{e} \\ \hline \end{array} = \begin{array}{|c|} \hline \text{a} \\ \hline \text{b} \\ \hline \text{c} \\ \hline \text{d} \\ \hline \text{e} \\ \hline \end{array}$$

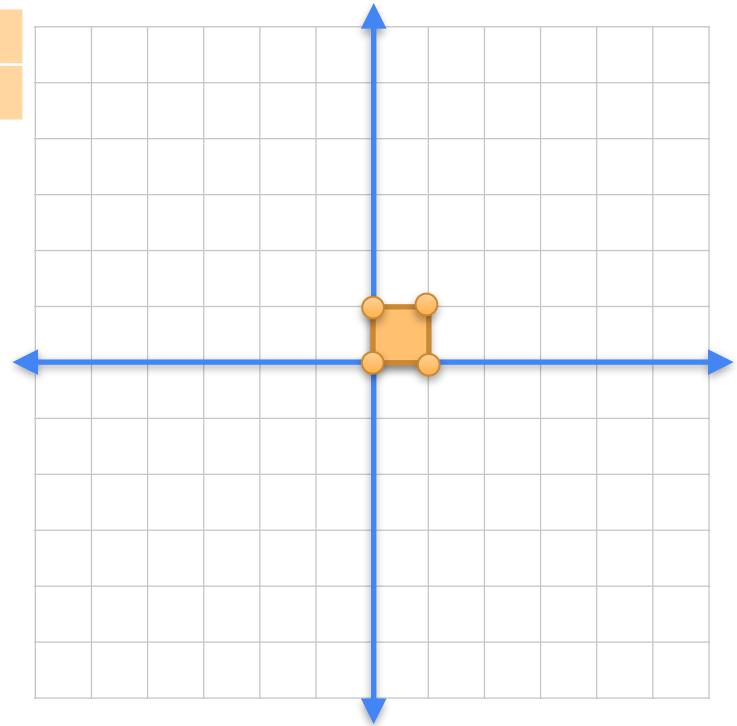
The identity matrix



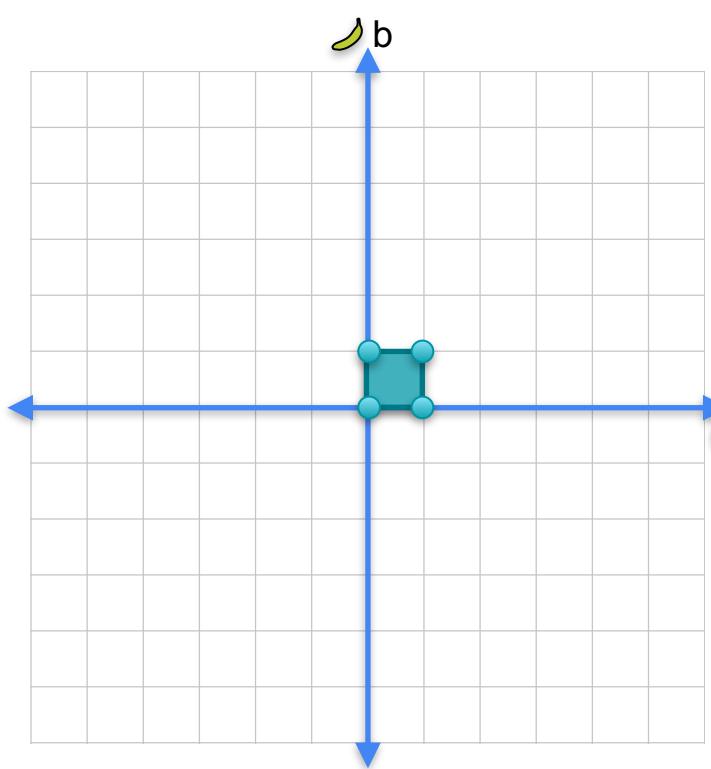
The identity matrix



$$\begin{matrix} \text{apple} & \text{banana} \\ 1 & 0 \\ 0 & 1 \end{matrix} \times \begin{matrix} x \\ y \end{matrix} = \begin{matrix} x \\ y \end{matrix}$$

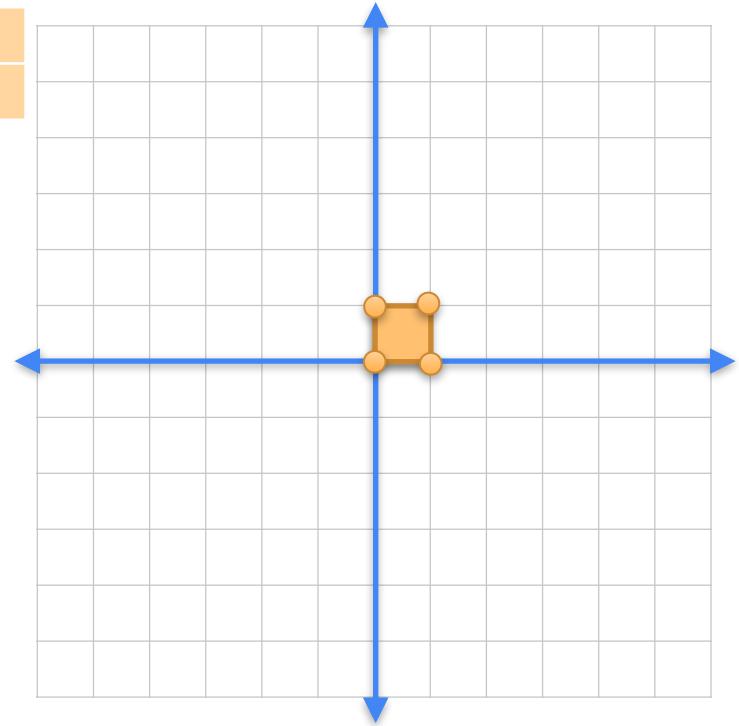


The identity matrix



$$\begin{array}{c} \text{apple} \quad \text{banana} \\ \begin{matrix} 1 & 0 \\ 0 & 1 \end{matrix} \end{array} = \begin{array}{c} x \\ y \end{array}$$

$$\begin{aligned} (0,0) &\rightarrow (0,0) \\ (1,0) &\rightarrow (1,0) \\ (0,1) &\rightarrow (0,1) \\ (1,1) &\rightarrow (1,1) \end{aligned}$$





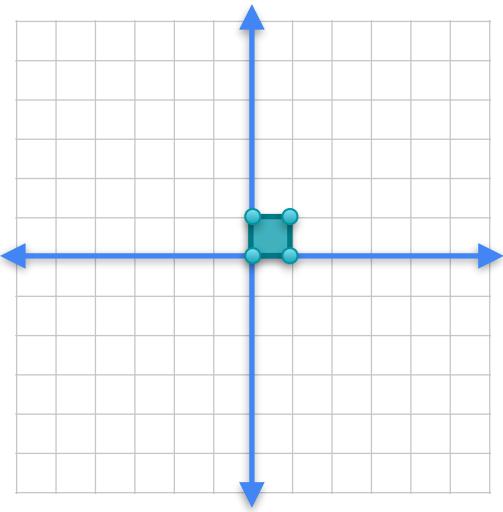
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Vectors and Linear Transformations

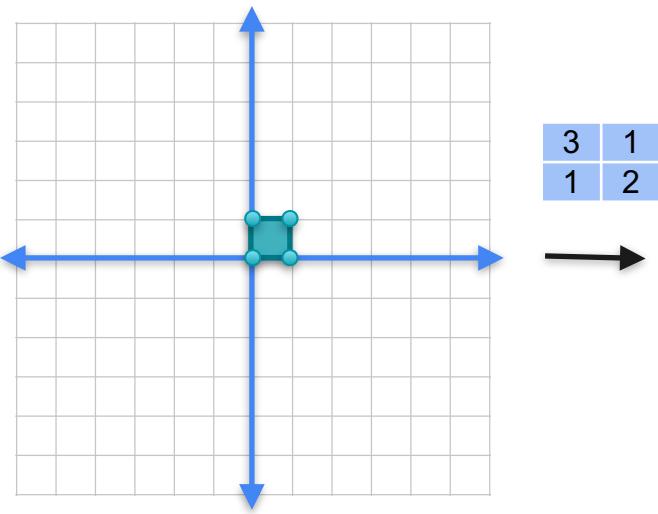
Matrix inverse

Matrix inverses

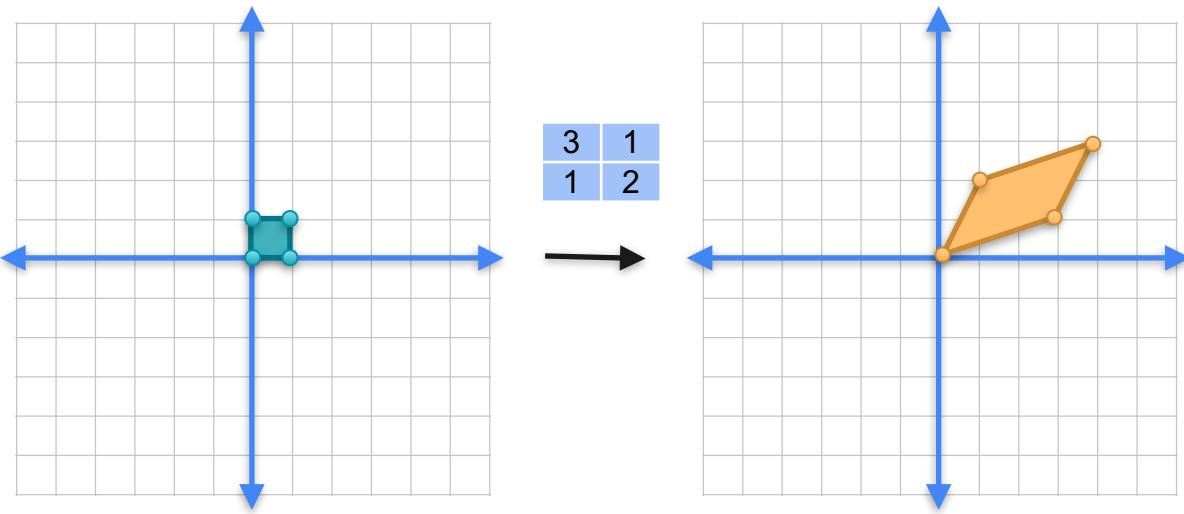
Matrix inverses



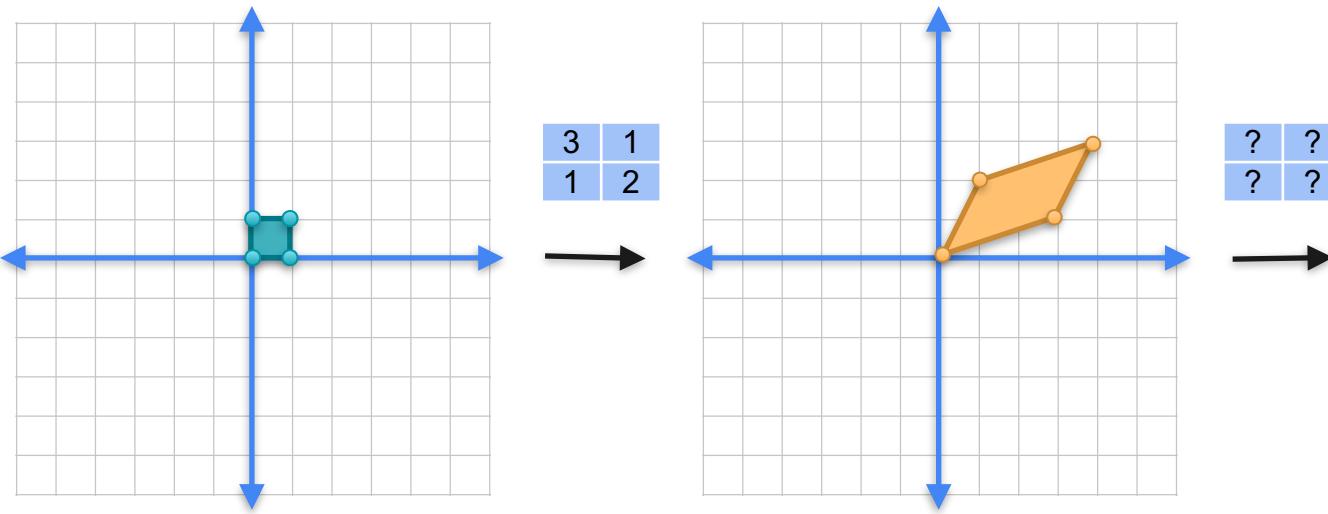
Matrix inverses



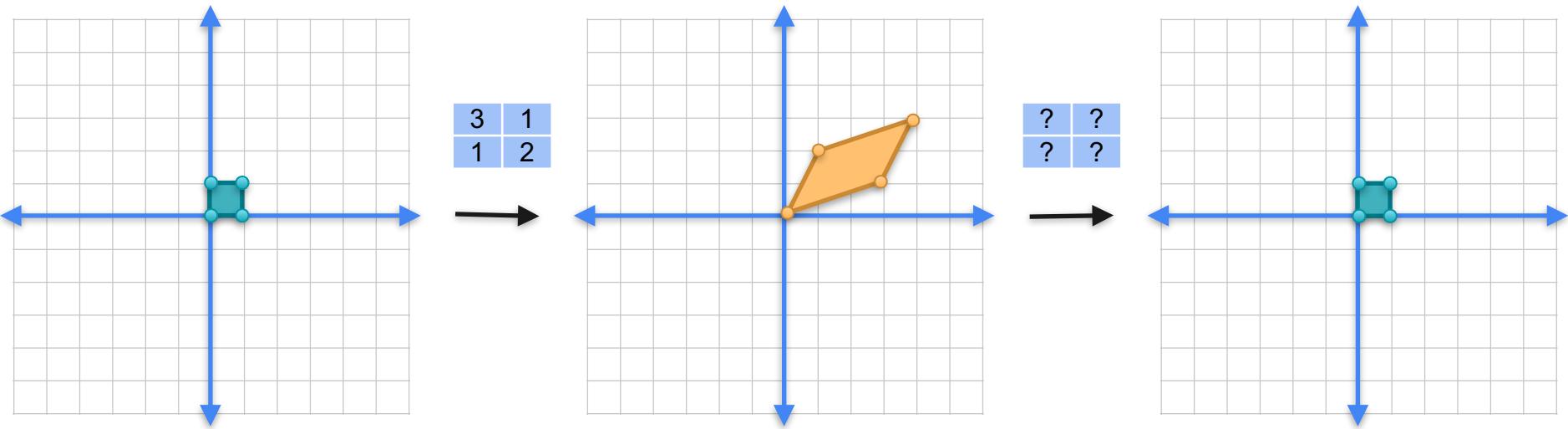
Matrix inverses



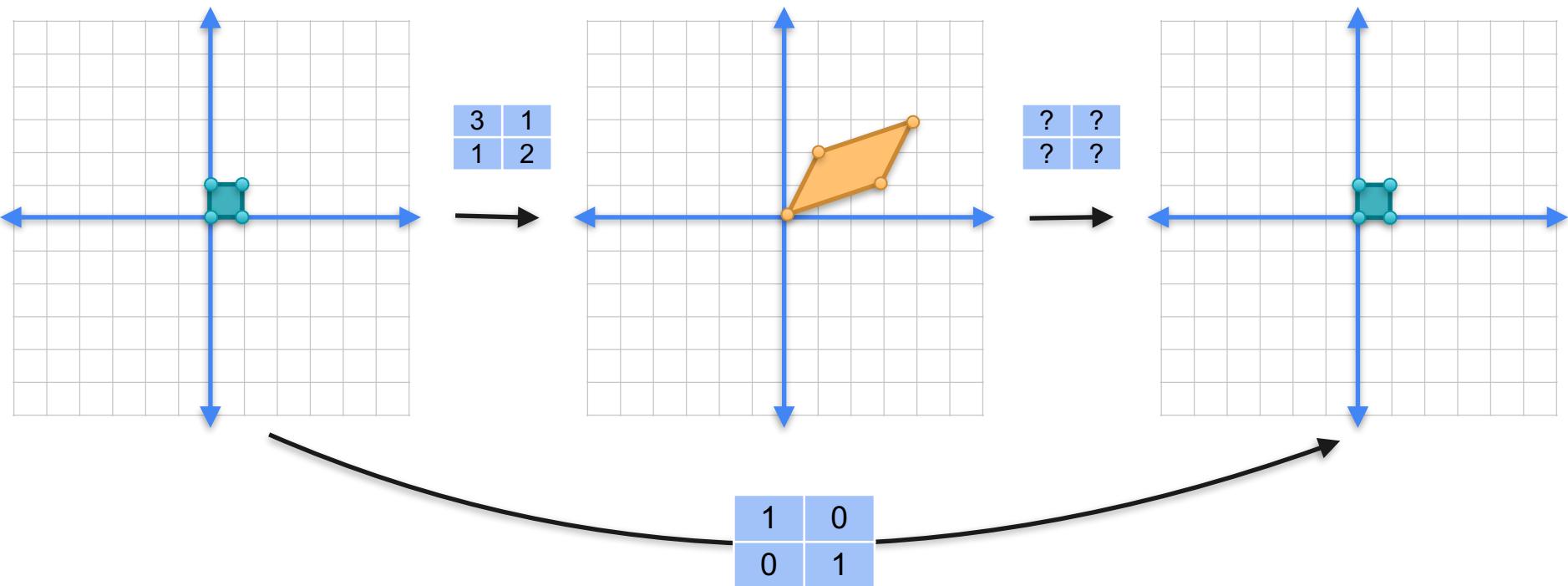
Matrix inverses



Matrix inverses



Matrix inverses



Multiplying matrices

Multiplying matrices

a	b
c	d

Multiplying matrices

$$\begin{matrix} a & b \\ c & d \end{matrix} \cdot \begin{matrix} 3 & 1 \\ 1 & 2 \end{matrix}$$

Multiplying matrices

$$\begin{matrix} a & b \\ c & d \end{matrix} \cdot \begin{matrix} 3 & 1 \\ 1 & 2 \end{matrix} = \begin{matrix} 1 & 0 \\ 0 & 1 \end{matrix}$$

Multiplying matrices

$$\begin{matrix} a & b \\ c & d \end{matrix} \cdot \begin{matrix} 3 & 1 \\ 1 & 2 \end{matrix} = \begin{matrix} 1 & 0 \\ 0 & 1 \end{matrix}$$

The diagram illustrates the multiplication of two 2x2 matrices. The first matrix is labeled with variables a, b, c, d . The second matrix has specific values: 3 in the top-left, 1 in the top-right, 1 in the bottom-left, and 2 in the bottom-right. The result of the multiplication is the identity matrix. A blue arrow points from the bottom-left cell of the first matrix (which contains the value -1) up to the top-left cell of the second matrix (which contains the value a).

Multiplying matrices

$$\begin{matrix} a & b \\ c & d \end{matrix} \cdot \begin{matrix} 3 & 1 \\ 1 & 2 \end{matrix} = \begin{matrix} 1 & 0 \\ 0 & 1 \end{matrix}$$
$$\begin{matrix} 3 & 1 \\ 1 & 2 \end{matrix}^{-1} = \begin{matrix} 2/5 & -1/5 \\ -1/5 & 3/5 \end{matrix}$$


How to find an inverse?

$$\begin{array}{|c|c|} \hline a & b \\ \hline c & d \\ \hline \end{array} \cdot \begin{array}{|c|c|} \hline 3 & 1 \\ \hline 1 & 2 \\ \hline \end{array} = \begin{array}{|c|c|} \hline 1 & 0 \\ \hline 0 & 1 \\ \hline \end{array}$$

How to find an inverse?

$$\begin{array}{|c|c|} \hline a & b \\ \hline c & d \\ \hline \end{array} \cdot \begin{array}{|c|c|} \hline 3 & 1 \\ \hline 1 & 2 \\ \hline \end{array} = \begin{array}{|c|c|} \hline 1 & 0 \\ \hline 0 & 1 \\ \hline \end{array}$$

$$\begin{array}{|c|c|} \hline a & b \\ \hline \end{array} \left| \begin{array}{c} 3 \\ 1 \end{array} \right. = \begin{array}{|c|} \hline 1 \\ \hline \end{array}$$

$$\begin{array}{|c|c|} \hline a & b \\ \hline \end{array} \left| \begin{array}{c} 1 \\ 2 \end{array} \right. = \begin{array}{|c|} \hline 0 \\ \hline \end{array}$$

$$\begin{array}{|c|c|} \hline c & d \\ \hline \end{array} \left| \begin{array}{c} 3 \\ 1 \end{array} \right. = \begin{array}{|c|} \hline 0 \\ \hline \end{array}$$

$$\begin{array}{|c|c|} \hline c & d \\ \hline \end{array} \left| \begin{array}{c} 1 \\ 2 \end{array} \right. = \begin{array}{|c|} \hline 1 \\ \hline \end{array}$$

How to find an inverse?

$$\begin{array}{|c|c|} \hline a & b \\ \hline c & d \\ \hline \end{array} \cdot \begin{array}{|c|c|} \hline 3 & 1 \\ \hline 1 & 2 \\ \hline \end{array} = \begin{array}{|c|c|} \hline 1 & 0 \\ \hline 0 & 1 \\ \hline \end{array}$$

$$\begin{array}{|c|c|} \hline a & b \\ \hline \end{array} \left[\begin{array}{|c|c|} \hline 3 & 1 \\ \hline \end{array} \right] = \begin{array}{|c|} \hline 1 \\ \hline \end{array} \quad 3a + 1b = 1$$

$$\begin{array}{|c|c|} \hline a & b \\ \hline \end{array} \left[\begin{array}{|c|c|} \hline 1 & 2 \\ \hline \end{array} \right] = \begin{array}{|c|} \hline 0 \\ \hline \end{array} \quad 1a + 2b = 0$$

$$\begin{array}{|c|c|} \hline c & d \\ \hline \end{array} \left[\begin{array}{|c|c|} \hline 3 & 1 \\ \hline \end{array} \right] = \begin{array}{|c|} \hline 0 \\ \hline \end{array} \quad 3c + 1d = 0$$

$$\begin{array}{|c|c|} \hline c & d \\ \hline \end{array} \left[\begin{array}{|c|c|} \hline 1 & 2 \\ \hline \end{array} \right] = \begin{array}{|c|} \hline 1 \\ \hline \end{array} \quad 1c + 2d = 1$$

How to find an inverse?

$$\begin{array}{|c|c|} \hline a & b \\ \hline c & d \\ \hline \end{array} \cdot \begin{array}{|c|c|} \hline 3 & 1 \\ \hline 1 & 2 \\ \hline \end{array} = \begin{array}{|c|c|} \hline 1 & 0 \\ \hline 0 & 1 \\ \hline \end{array}$$

$$\begin{array}{|c|c|} \hline a & b \\ \hline \end{array} \left[\begin{array}{|c|c|} \hline 3 & 1 \\ \hline \end{array} \right] = \begin{array}{|c|} \hline 1 \\ \hline \end{array} \quad 3a + 1b = 1 \quad a = \frac{2}{5}$$

$$\begin{array}{|c|c|} \hline a & b \\ \hline \end{array} \left[\begin{array}{|c|c|} \hline 1 & 2 \\ \hline \end{array} \right] = \begin{array}{|c|} \hline 0 \\ \hline \end{array} \quad 1a + 2b = 0 \quad b = -\frac{1}{5}$$

$$\begin{array}{|c|c|} \hline c & d \\ \hline \end{array} \left[\begin{array}{|c|c|} \hline 3 & 1 \\ \hline \end{array} \right] = \begin{array}{|c|} \hline 0 \\ \hline \end{array} \quad 3c + 1d = 0 \quad c = -\frac{1}{5}$$

$$\begin{array}{|c|c|} \hline c & d \\ \hline \end{array} \left[\begin{array}{|c|c|} \hline 1 & 2 \\ \hline \end{array} \right] = \begin{array}{|c|} \hline 1 \\ \hline \end{array} \quad 1c + 2d = 1 \quad d = \frac{3}{5}$$

Quiz

- Find the inverse of the following matrix. If you find that the task is impossible, feel free to click on “I couldn’t find it”

5	2
1	2

Solution

- By solving the corresponding system of linear equations, we get the following.

$$\begin{array}{|c|c|} \hline 5 & 2 \\ \hline 1 & 2 \\ \hline \end{array} \cdot \begin{array}{|c|c|} \hline a & b \\ \hline c & d \\ \hline \end{array} = \begin{array}{|c|c|} \hline 1 & 0 \\ \hline 0 & 1 \\ \hline \end{array}$$

$$\begin{array}{|c|c|} \hline 5 & 2 \\ \hline \end{array} \begin{array}{|c|} \hline a \\ \hline c \\ \hline \end{array} = \begin{array}{|c|} \hline 1 \\ \hline \end{array}$$

$$\begin{array}{|c|c|} \hline 5 & 2 \\ \hline \end{array} \begin{array}{|c|} \hline b \\ \hline d \\ \hline \end{array} = \begin{array}{|c|} \hline 0 \\ \hline \end{array}$$

Solution

- By solving the corresponding system of linear equations, we get the following.

$$\begin{matrix} 5 & 2 \\ 1 & 2 \end{matrix} \cdot \begin{matrix} a & b \\ c & d \end{matrix} = \begin{matrix} 1 & 0 \\ 0 & 1 \end{matrix}$$

$$\begin{matrix} 5 & 2 \\ 1 & 2 \end{matrix} \begin{matrix} a \\ c \end{matrix} = \begin{matrix} 1 \end{matrix}$$

$$\begin{matrix} 5 & 2 \\ 1 & 2 \end{matrix} \begin{matrix} b \\ d \end{matrix} = \begin{matrix} 0 \end{matrix}$$

$$\begin{matrix} 1 & 2 \\ 1 & 2 \end{matrix} \begin{matrix} a \\ c \end{matrix} = \begin{matrix} 0 \end{matrix}$$

$$\begin{matrix} 1 & 2 \\ 1 & 2 \end{matrix} \begin{matrix} b \\ d \end{matrix} = \begin{matrix} 1 \end{matrix}$$

Solution

- By solving the corresponding system of linear equations, we get the following.

$$\begin{array}{|c|c|} \hline 5 & 2 \\ \hline 1 & 2 \\ \hline \end{array} \cdot \begin{array}{|c|c|} \hline a & b \\ \hline c & d \\ \hline \end{array} = \begin{array}{|c|c|} \hline 1 & 0 \\ \hline 0 & 1 \\ \hline \end{array}$$

$$\begin{array}{|c|c|} \hline 5 & 2 \\ \hline \end{array} \begin{array}{|c|} \hline a \\ \hline c \\ \hline \end{array} = \begin{array}{|c|} \hline 1 \\ \hline \end{array} \quad \bullet \quad 5a + 2c = 1$$

$$\begin{array}{|c|c|} \hline 5 & 2 \\ \hline \end{array} \begin{array}{|c|} \hline b \\ \hline d \\ \hline \end{array} = \begin{array}{|c|} \hline 0 \\ \hline \end{array} \quad \bullet \quad 5b + 2d = 0$$

$$\begin{array}{|c|c|} \hline 1 & 2 \\ \hline \end{array} \begin{array}{|c|} \hline a \\ \hline c \\ \hline \end{array} = \begin{array}{|c|} \hline 0 \\ \hline \end{array} \quad \bullet \quad a + 2c = 0$$

$$\begin{array}{|c|c|} \hline 1 & 2 \\ \hline \end{array} \begin{array}{|c|} \hline b \\ \hline d \\ \hline \end{array} = \begin{array}{|c|} \hline 1 \\ \hline \end{array} \quad \bullet \quad b + 2d = 1$$

Solution

- By solving the corresponding system of linear equations, we get the following.

$$\begin{matrix} 5 & 2 \\ 1 & 2 \end{matrix} \cdot \begin{matrix} a & b \\ c & d \end{matrix} = \begin{matrix} 1 & 0 \\ 0 & 1 \end{matrix}$$

$$\begin{matrix} 5 & 2 \\ 1 & 2 \end{matrix} \begin{matrix} a \\ c \end{matrix} = \begin{matrix} 1 \end{matrix}$$

$$\bullet 5a + 2c = 1$$

$$\bullet a = 1/4$$

$$\begin{matrix} 5 & 2 \\ 1 & 2 \end{matrix} \begin{matrix} b \\ d \end{matrix} = \begin{matrix} 0 \end{matrix}$$

$$\bullet 5b + 2d = 0$$

$$\begin{matrix} 1 & 2 \\ 1 & 2 \end{matrix} \begin{matrix} a \\ c \end{matrix} = \begin{matrix} 0 \end{matrix}$$

$$\bullet a + 2c = 0$$

$$\begin{matrix} 1 & 2 \\ 1 & 2 \end{matrix} \begin{matrix} b \\ d \end{matrix} = \begin{matrix} 1 \end{matrix}$$

$$\bullet b + 2d = 1$$

Solution

- By solving the corresponding system of linear equations, we get the following.

$$\begin{array}{|c|c|} \hline 5 & 2 \\ \hline 1 & 2 \\ \hline \end{array} \cdot \begin{array}{|c|c|} \hline a & b \\ \hline c & d \\ \hline \end{array} = \begin{array}{|c|c|} \hline 1 & 0 \\ \hline 0 & 1 \\ \hline \end{array}$$

$$\begin{array}{|c|c|} \hline 5 & 2 \\ \hline \end{array} \left| \begin{array}{|c|c|} \hline a \\ \hline c \\ \hline \end{array} \right. = \begin{array}{|c|} \hline 1 \\ \hline \end{array}$$

$$\bullet 5a + 2c = 1$$

$$\bullet a = 1/4$$

$$\begin{array}{|c|c|} \hline 5 & 2 \\ \hline \end{array} \left| \begin{array}{|c|c|} \hline b \\ \hline d \\ \hline \end{array} \right. = \begin{array}{|c|} \hline 0 \\ \hline \end{array}$$

$$\bullet 5b + 2d = 0$$

$$\bullet b = -1/4$$

$$\begin{array}{|c|c|} \hline 1 & 2 \\ \hline \end{array} \left| \begin{array}{|c|c|} \hline a \\ \hline c \\ \hline \end{array} \right. = \begin{array}{|c|} \hline 0 \\ \hline \end{array}$$

$$\bullet a + 2c = 0$$

$$\begin{array}{|c|c|} \hline 1 & 2 \\ \hline \end{array} \left| \begin{array}{|c|c|} \hline b \\ \hline d \\ \hline \end{array} \right. = \begin{array}{|c|} \hline 1 \\ \hline \end{array}$$

$$\bullet b + 2d = 1$$

Solution

- By solving the corresponding system of linear equations, we get the following.

$$\begin{array}{|c|c|} \hline 5 & 2 \\ \hline 1 & 2 \\ \hline \end{array} \cdot \begin{array}{|c|c|} \hline a & b \\ \hline c & d \\ \hline \end{array} = \begin{array}{|c|c|} \hline 1 & 0 \\ \hline 0 & 1 \\ \hline \end{array}$$

$$\begin{array}{|c|c|} \hline 5 & 2 \\ \hline \end{array} \left| \begin{array}{|c|} \hline a \\ \hline c \\ \hline \end{array} \right. = \begin{array}{|c|} \hline 1 \\ \hline \end{array}$$

$$\bullet 5a + 2c = 1$$

$$\bullet a = 1/4$$

$$\begin{array}{|c|c|} \hline 5 & 2 \\ \hline \end{array} \left| \begin{array}{|c|} \hline b \\ \hline d \\ \hline \end{array} \right. = \begin{array}{|c|} \hline 0 \\ \hline \end{array}$$

$$\bullet 5b + 2d = 0$$

$$\bullet b = -1/4$$

$$\begin{array}{|c|c|} \hline 1 & 2 \\ \hline \end{array} \left| \begin{array}{|c|} \hline a \\ \hline c \\ \hline \end{array} \right. = \begin{array}{|c|} \hline 0 \\ \hline \end{array}$$

$$\bullet a + 2c = 0$$

$$\bullet c = -1/8$$

$$\begin{array}{|c|c|} \hline 1 & 2 \\ \hline \end{array} \left| \begin{array}{|c|} \hline b \\ \hline d \\ \hline \end{array} \right. = \begin{array}{|c|} \hline 1 \\ \hline \end{array}$$

$$\bullet b + 2d = 1$$

Solution

- By solving the corresponding system of linear equations, we get the following.

$$\begin{matrix} 5 & 2 \\ 1 & 2 \end{matrix} \cdot \begin{matrix} a & b \\ c & d \end{matrix} = \begin{matrix} 1 & 0 \\ 0 & 1 \end{matrix}$$

$$\begin{matrix} 5 & 2 \\ 1 & 2 \end{matrix} \left| \begin{matrix} a \\ c \end{matrix} \right. = \begin{matrix} 1 \\ 0 \end{matrix}$$

$$\bullet 5a + 2c = 1$$

$$\bullet a = 1/4$$

$$\begin{matrix} 5 & 2 \\ 1 & 2 \end{matrix} \left| \begin{matrix} b \\ d \end{matrix} \right. = \begin{matrix} 0 \\ 1 \end{matrix}$$

$$\bullet 5b + 2d = 0$$

$$\bullet b = -1/4$$

$$\begin{matrix} 1 & 2 \\ 1 & 2 \end{matrix} \left| \begin{matrix} a \\ c \end{matrix} \right. = \begin{matrix} 0 \\ 0 \end{matrix}$$

$$\bullet a + 2c = 0$$

$$\bullet c = -1/8$$

$$\begin{matrix} 1 & 2 \\ 1 & 2 \end{matrix} \left| \begin{matrix} b \\ d \end{matrix} \right. = \begin{matrix} 1 \\ 1 \end{matrix}$$

$$\bullet b + 2d = 1$$

$$\bullet d = 5/8$$

Quiz

- Find the inverse of the following matrix. If you find that the task is impossible, feel free to click on “I’m reaching a dead end”

1	1
2	2

Solutions

- The inverse doesn't exist!

We need to solve the following system of linear equations:

$$\begin{array}{|c|c|} \hline 1 & 1 \\ \hline 2 & 2 \\ \hline \end{array} \quad \begin{array}{|c|c|} \hline a & b \\ \hline c & d \\ \hline \end{array} \quad = \quad \begin{array}{|c|c|} \hline 1 & 0 \\ \hline 0 & 1 \\ \hline \end{array}$$

$$a + c = 1$$

$$2b + 2d = 1$$

$$2a + 2c = 0$$

$$b + d = 0$$

This is clearly a contradiction, since equation 1 says $a+c=1$, and equation 3 says $2a+2c=0$.



DeepLearning.AI

Vectors and Linear Transformations

Which matrices have an inverse?

Which matrices have inverses?

Which matrices have inverses?

$$5^{-1} = 0.2$$

Which matrices have inverses?

$$5^{-1} = 0.2$$

$$8^{-1} = 0.125$$

Which matrices have inverses?

$$5^{-1} = 0.2$$

$$8^{-1} = 0.125$$

$$0^{-1} = ???$$

Which matrices have inverses?

$$5^{-1} = 0.2$$

$$8^{-1} = 0.125$$

$$0^{-1} = ???$$

$$\begin{array}{|c|c|} \hline 3 & 1 \\ \hline 1 & 2 \\ \hline \end{array}^{-1} = \begin{array}{|c|c|} \hline 0.4 & -0.2 \\ \hline -0.2 & 0.6 \\ \hline \end{array}$$

Which matrices have inverses?

$$5^{-1} = 0.2$$

$$8^{-1} = 0.125$$

$$0^{-1} = ???$$

$$\begin{vmatrix} 3 & 1 \\ 1 & 2 \end{vmatrix}^{-1} = \begin{vmatrix} 0.4 & -0.2 \\ -0.2 & 0.6 \end{vmatrix}$$

$$\begin{vmatrix} 5 & 2 \\ 1 & 2 \end{vmatrix}^{-1} = \begin{vmatrix} 0.25 & -0.25 \\ -0.125 & 0.625 \end{vmatrix}$$

Which matrices have inverses?

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$$\begin{pmatrix} 5 & 2 \\ 1 & 2 \end{pmatrix}^{-1} = \begin{pmatrix} 0.25 & -0.25 \\ -0.125 & 0.625 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 1 \\ 2 & 2 \end{pmatrix} = \begin{pmatrix} ? & ? \\ ? & ? \end{pmatrix}$$

Which matrices have inverses?

$$5^{-1} = 0.2$$

$$8^{-1} = 0.125$$

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$$\begin{pmatrix} 3 & 1 \\ 1 & 2 \end{pmatrix}^{-1} = \begin{pmatrix} 0.4 & -0.2 \\ -0.2 & 0.6 \end{pmatrix}$$

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$$\begin{pmatrix} 1 & 1 \\ 2 & 2 \end{pmatrix} = \begin{pmatrix} ? & ? \\ ? & ? \end{pmatrix}$$

Non-singular matrix

Which matrices have inverses?

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Non-singular matrix

Non-singular matrix

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Non-singular matrix

Non-singular matrix

Singular matrix

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$$\begin{pmatrix} 1 & 1 \\ 2 & 2 \end{pmatrix} = \begin{pmatrix} ? & ? \\ ? & ? \end{pmatrix}$$

Non-singular matrix
Invertible

Non-singular matrix

Singular matrix

Which matrices have inverses?

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$$\begin{pmatrix} 3 & 1 \\ 1 & 2 \end{pmatrix}^{-1} = \begin{pmatrix} 0.4 & -0.2 \\ -0.2 & 0.6 \end{pmatrix}$$

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Non-singular matrix
Invertible

Non-singular matrix
Invertible

Singular matrix

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Non-singular matrix
Invertible

Non-singular matrix
Invertible

Singular matrix
Non-invertible

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$$\begin{pmatrix} 1 & 1 \\ 2 & 2 \end{pmatrix} = \begin{pmatrix} ? & ? \\ ? & ? \end{pmatrix}$$

Non-singular matrix
Invertible

Det = 5

Non-singular matrix
Invertible

Singular matrix
Non-invertible

Which matrices have inverses?

$$5^{-1} = 0.2$$

$$8^{-1} = 0.125$$

$$0^{-1} = ???$$

$$\begin{pmatrix} 3 & 1 \\ 1 & 2 \end{pmatrix}^{-1} = \begin{pmatrix} 0.4 & -0.2 \\ -0.2 & 0.6 \end{pmatrix}$$

$$\begin{pmatrix} 5 & 2 \\ 1 & 2 \end{pmatrix}^{-1} = \begin{pmatrix} 0.25 & -0.25 \\ -0.125 & 0.625 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 1 \\ 2 & 2 \end{pmatrix} = \begin{pmatrix} ? & ? \\ ? & ? \end{pmatrix}$$

Non-singular matrix
Invertible

Det = 5

Non-singular matrix
Invertible

Det = 8

Singular matrix
Non-invertible

Which matrices have inverses?

$$5^{-1} = 0.2$$

$$8^{-1} = 0.125$$

$$0^{-1} = ???$$

$$\begin{pmatrix} 3 & 1 \\ 1 & 2 \end{pmatrix}^{-1} = \begin{pmatrix} 0.4 & -0.2 \\ -0.2 & 0.6 \end{pmatrix}$$

$$\begin{pmatrix} 5 & 2 \\ 1 & 2 \end{pmatrix}^{-1} = \begin{pmatrix} 0.25 & -0.25 \\ -0.125 & 0.625 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 1 \\ 2 & 2 \end{pmatrix} = \begin{pmatrix} ? & ? \\ ? & ? \end{pmatrix}$$

Non-singular matrix
Invertible

Det = 5

Non-singular matrix
Invertible

Det = 8

Singular matrix
Non-invertible

Det = 0

Which matrices have inverses?

$$5^{-1} = 0.2$$

$$8^{-1} = 0.125$$

$$0^{-1} = ???$$

$$\begin{vmatrix} 3 & 1 \\ 1 & 2 \end{vmatrix}^{-1} = \begin{vmatrix} 0.4 & -0.2 \\ -0.2 & 0.6 \end{vmatrix}$$

$$\begin{vmatrix} 5 & 2 \\ 1 & 2 \end{vmatrix}^{-1} = \begin{vmatrix} 0.25 & -0.25 \\ -0.125 & 0.625 \end{vmatrix}$$

$$\begin{vmatrix} 1 & 1 \\ 2 & 2 \end{vmatrix} = \begin{vmatrix} ? & ? \\ ? & ? \end{vmatrix}$$

Non-singular matrix
Invertible

Non-singular matrix
Invertible

Singular matrix
Non-invertible

Det = 5 ← Det = 8 →

Non-zero determinants

Which matrices have inverses?

$$5^{-1} = 0.2$$

$$8^{-1} = 0.125$$

$$0^{-1} = ???$$

$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 0.4 & -0.2 \\ -0.2 & 0.6 \end{bmatrix}$$

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$$\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix} = \begin{bmatrix} ? & ? \\ ? & ? \end{bmatrix}$$

Non-singular matrix
Invertible

Non-singular matrix
Invertible

Singular matrix
Non-invertible

Det = 5 ← Det = 8 →

Non-zero determinants

Det = 0 ← Zero determinant →



DeepLearning.AI

Vectors and Linear Transformations

**Neural networks and
matrices**

AI , ML , DL , RL



**MATRIX
MULTIPLICATION**

Quiz: Natural language processing

Spam	Lottery	Win
Yes	1	1
Yes	2	1
No	0	0
Yes	0	2
No	0	1
No	1	0
Yes	2	2
Yes	2	0
Yes	1	2

Quiz: Natural language processing

Spam	Lottery	Win
Yes	1	1
Yes	2	1
No	0	0
Yes	0	2
No	0	1
No	1	0
Yes	2	2
Yes	2	0
Yes	1	2

Scores:

Lottery: ____ points

Win: ____ points

Quiz: Natural language processing

Spam	Lottery	Win
Yes	1	1
Yes	2	1
No	0	0
Yes	0	2
No	0	1
No	1	0
Yes	2	2
Yes	2	0
Yes	1	2

Scores:

Lottery: ____ points

Win: ____ points

Examples

Lottery: 3 point

Win: 2 points

“Win, win the lottery!” : 7points

Quiz: Natural language processing

Spam	Lottery	Win
Yes	1	1
Yes	2	1
No	0	0
Yes	0	2
No	0	1
No	1	0
Yes	2	2
Yes	2	0
Yes	1	2

Scores:

Lottery: ____ points

Win: ____ points

Examples

Lottery: 3 point

Win: 2 points

“Win, win the lottery!” : 7points

Rule:

If the number of points of the sentence is bigger than ____,
then the email is spam.

Quiz: Natural language processing

Spam	Lottery	Win
Yes	1	1
Yes	2	1
No	0	0
Yes	0	2
No	0	1
No	1	0
Yes	2	2
Yes	2	0
Yes	1	2

Scores:

Lottery: ____ points

Win: ____ points

Examples

Lottery: 3 point

Win: 2 points

“Win, win the lottery!” : 7points

Rule:

If the number of points of the sentence is bigger than ____,
then the email is spam.

Goal: Find the best points and threshold

Lottery: ____ point

Win: ____ point

Threshold: ____ points

Quiz: Natural language processing

Spam	Lottery	Win
Yes	1	1
Yes	2	1
No	0	0
Yes	0	2
No	0	1
No	1	0
Yes	2	2
Yes	2	0
Yes	1	2

Score	> 1.5?
2	Yes
3	Yes
0	No
2	Yes
1	No
1	No
4	Yes
2	Yes
3	Yes

Solution:

Lottery: 1 point

Win: 1 point

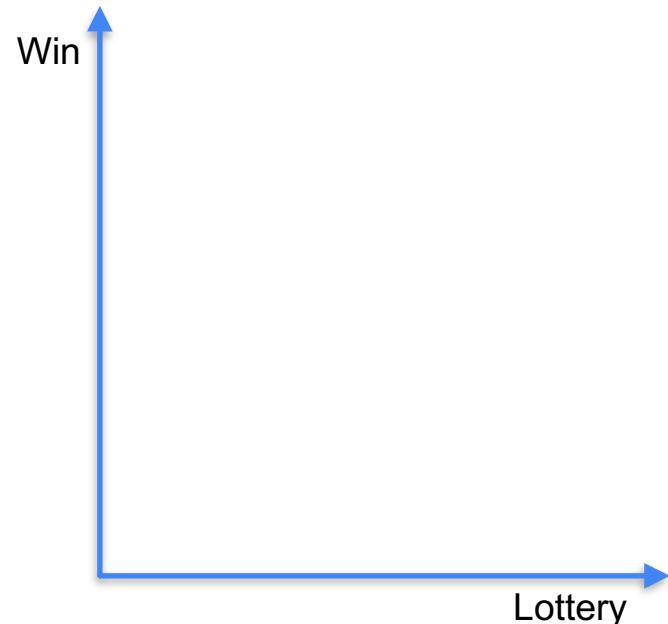
Threshold: 1.5 points

Graphical natural language processing

Spam	Lottery	Win
Yes	1	1
Yes	2	1
No	0	0
Yes	0	2
No	0	1
No	1	0
Yes	2	2
Yes	2	0
Yes	1	2

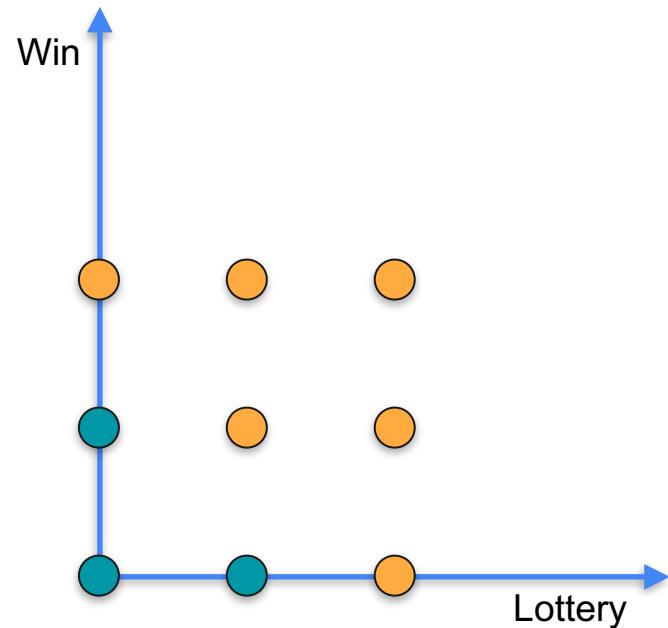
Graphical natural language processing

Spam	Lottery	Win
Yes	1	1
Yes	2	1
No	0	0
Yes	0	2
No	0	1
No	1	0
Yes	2	2
Yes	2	0
Yes	1	2



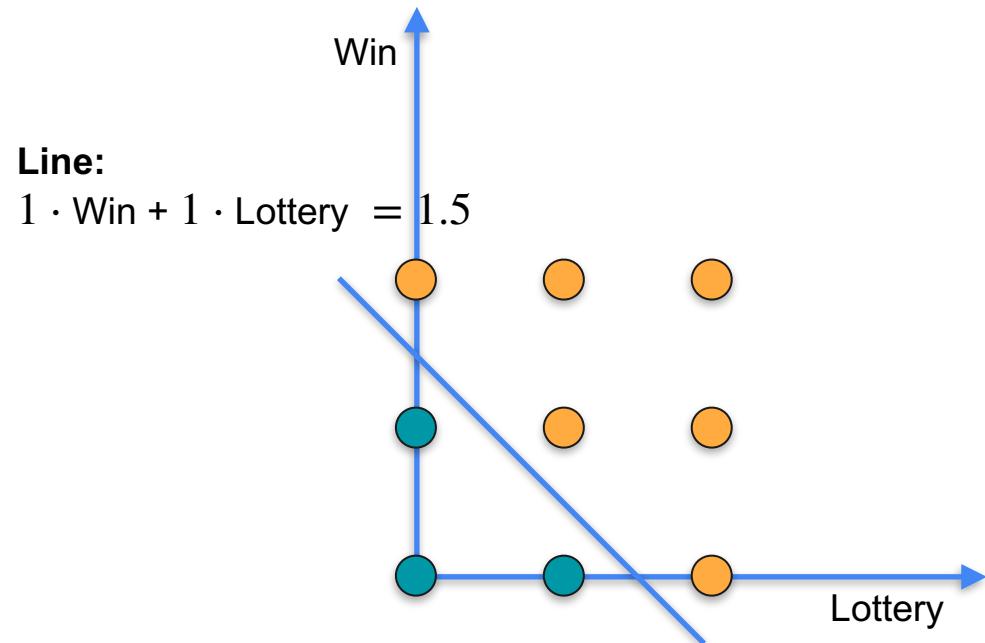
Graphical natural language processing

Spam	Lottery	Win
Yes	1	1
Yes	2	1
No	0	0
Yes	0	2
No	0	1
No	1	0
Yes	2	2
Yes	2	0
Yes	1	2



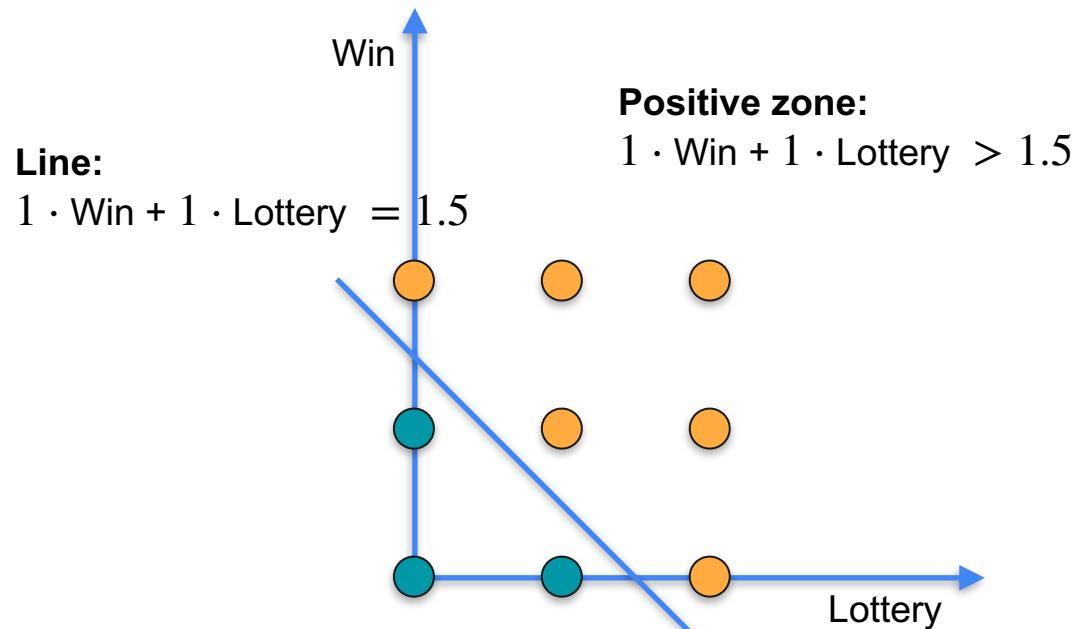
Graphical natural language processing

Spam	Lottery	Win
Yes	1	1
Yes	2	1
No	0	0
Yes	0	2
No	0	1
No	1	0
Yes	2	2
Yes	2	0
Yes	1	2



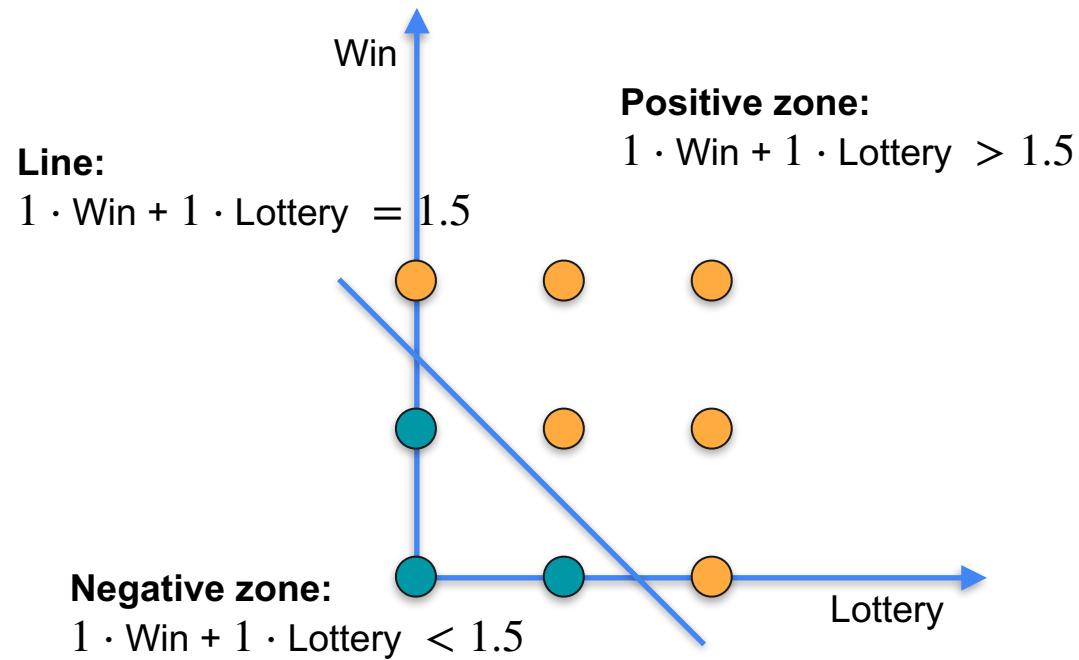
Graphical natural language processing

Spam	Lottery	Win
Yes	1	1
Yes	2	1
No	0	0
Yes	0	2
No	0	1
No	1	0
Yes	2	2
Yes	2	0
Yes	1	2



Graphical natural language processing

Spam	Lottery	Win
Yes	1	1
Yes	2	1
No	0	0
Yes	0	2
No	0	1
No	1	0
Yes	2	2
Yes	2	0
Yes	1	2



Graphical natural language processing

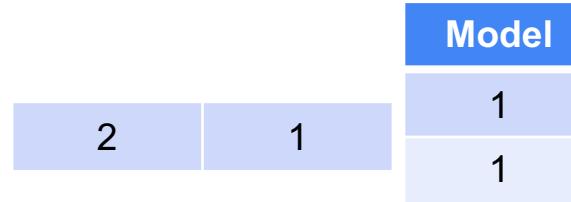
Spam	Lottery	Win
Yes	1	1
Yes	2	1
No	0	0
Yes	0	2
No	0	1
No	1	0
Yes	2	2
Yes	2	0
Yes	1	2

Model
1
1

Check: > 1.5?

Graphical natural language processing

Spam	Lottery	Win
Yes	1	1
Yes	2	1
No	0	0
Yes	0	2
No	0	1
No	1	0
Yes	2	2
Yes	2	0
Yes	1	2



Check: > 1.5?

Graphical natural language processing

Spam	Lottery	Win
Yes	1	1
Yes	2	1
No	0	0
Yes	0	2
No	0	1
No	1	0
Yes	2	2
Yes	2	0
Yes	1	2



Graphical natural language processing

Spam	Lottery	Win
Yes	1	1
Yes	2	1
No	0	0
Yes	0	2
No	0	1
No	1	0
Yes	2	2
Yes	2	0
Yes	1	2



Dot product between vectors

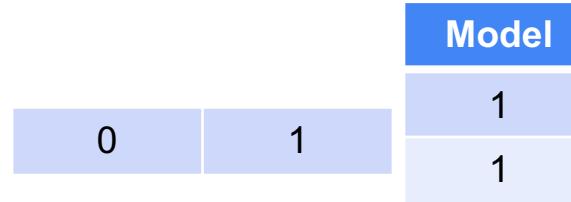
Spam	Lottery	Win
Yes	1	1
Yes	2	1
No	0	0
Yes	0	2
No	0	1
No	1	0
Yes	2	2
Yes	2	0
Yes	1	2

Model
1
1

Check: > 1.5?

Dot product between vectors

Spam	Lottery	Win
Yes	1	1
Yes	2	1
No	0	0
Yes	0	2
No	0	1
No	1	0
Yes	2	2
Yes	2	0
Yes	1	2



Check: $> 1.5?$

Dot product between vectors

Spam	Lottery	Win
Yes	1	1
Yes	2	1
No	0	0
Yes	0	2
No	0	1
No	1	0
Yes	2	2
Yes	2	0
Yes	1	2

Check: $> 1.5?$

$$\begin{matrix} & & \text{Model} \\ & 0 & | & 1 \\ & & + & \\ & & 1 & \\ & & | & \\ & & 1 & \end{matrix} = 1$$

Dot product between vectors

Spam	Lottery	Win
Yes	1	1
Yes	2	1
No	0	0
Yes	0	2
No	0	1
No	1	0
Yes	2	2
Yes	2	0
Yes	1	2

$$0 \quad | \quad 1 = \begin{matrix} \text{Model} \\ 1 \\ 1 \end{matrix}$$

Check: $> 1.5?$



Not spam

Matrix multiplication

Spam	Lottery	Win
Yes	1	1
Yes	2	1
No	0	0
Yes	0	2
No	0	1
No	1	0
Yes	2	2
Yes	2	0
Yes	1	2

Model
1
1

Matrix multiplication

Spam	Lottery	Win
Yes	1	1
Yes	2	1
No	0	0
Yes	0	2
No	0	1
No	1	0
Yes	2	2
Yes	2	0
Yes	1	2

=

Prod
2
3
0
2
1
1
4
2
3

Matrix multiplication

Spam	Lottery	Win
Yes	1	1
Yes	2	1
No	0	0
Yes	0	2
No	0	1
No	1	0
Yes	2	2
Yes	2	0
Yes	1	2

=

Model	Prod
1	2
1	3
0	0
2	2
1	1
1	1
4	4
2	2
3	3

Check: >1.5?



Matrix multiplication

Spam	Lottery	Win
Yes	1	1
Yes	2	1
No	0	0
Yes	0	2
No	0	1
No	1	0
Yes	2	2
Yes	2	0
Yes	1	2

Model = Prod

Model	Prod
1	2
1	3
1	0
1	2
1	1
1	1
4	4
2	2
3	3

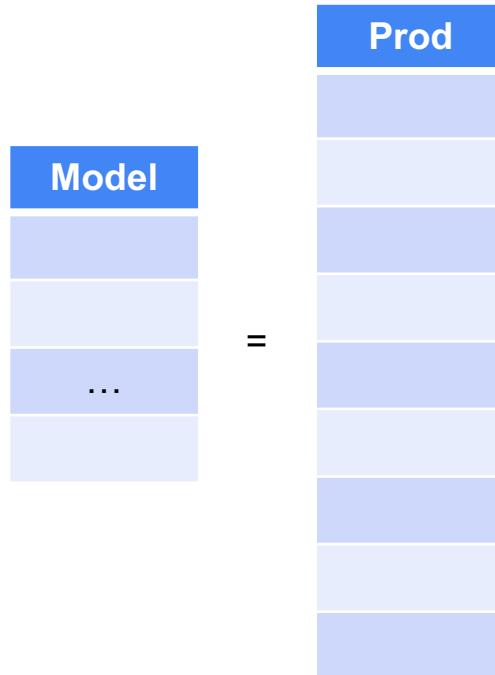
Check: >1.5?



Check
Yes
Yes
No
Yes
No
No
Yes
Yes
Yes

Perceptrons

Spam	Word1	Word2	...	WordN
Yes				
Yes				
No				
Yes				
No				
No				
Yes				
Yes				
Yes				



Check:
→

Check
Yes
Yes
No
Yes
No
No
Yes
Yes
Yes

Threshold and bias

Spam	Lottery	Win
Yes	1	1
Yes	2	1
No	0	0
Yes	0	2
No	0	1
No	1	0
Yes	2	2
Yes	2	0
Yes	1	2

Model
1
1

Check: > 1.5?

Threshold and bias

Spam	Lottery	Win
Yes	1	1
Yes	2	1
No	0	0
Yes	0	2
No	0	1
No	1	0
Yes	2	2
Yes	2	0
Yes	1	2

Check

$$1 \cdot \text{Win} + 1 \cdot \text{Lottery} - 1.5 > 0$$

Threshold

Model
1
1

Check: $> 1.5?$

Threshold and bias

Spam	Lottery	Win
Yes	1	1
Yes	2	1
No	0	0
Yes	0	2
No	0	1
No	1	0
Yes	2	2
Yes	2	0
Yes	1	2

Check

$$1 \cdot \text{Win} + 1 \cdot \text{Lottery} - 1.5 > 0$$

$$1 \cdot \text{Win} + 1 \cdot \text{Lottery} - 1.5 > 0$$

Threshold

Bias

Check: $> 1.5?$

Model
1
1

Threshold and bias

Spam	Lottery	Win	Bias
Yes	1	1	1
Yes	2	1	1
No	0	0	1
Yes	0	2	1
No	0	1	1
No	1	0	1
Yes	2	2	1
Yes	2	0	1
Yes	1	2	1

Check

$$1 \cdot \text{Win} + 1 \cdot \text{Lottery} - 1.5 > 0$$

$$1 \cdot \text{Win} + 1 \cdot \text{Lottery} - 1.5 > 0$$

Threshold

Bias

Check: $> 1.5?$

Model
1
1

Threshold and bias

Spam	Lottery	Win	Bias
Yes	1	1	1
Yes	2	1	1
No	0	0	1
Yes	0	2	1
No	0	1	1
No	1	0	1
Yes	2	2	1
Yes	2	0	1
Yes	1	2	1

Check

$$1 \cdot \text{Win} + 1 \cdot \text{Lottery} - 1.5 > 0$$

$$1 \cdot \text{Win} + 1 \cdot \text{Lottery} - 1.5 > 0$$

Threshold

Bias

Check: $> 1.5?$

Model
1
1
-1.5

Bias

Threshold and bias

Spam	Lottery	Win	Bias
Yes	1	1	1
Yes	2	1	1
No	0	0	1
Yes	0	2	1
No	0	1	1
No	1	0	1
Yes	2	2	1
Yes	2	0	1
Yes	1	2	1

Check

$$1 \cdot \text{Win} + 1 \cdot \text{Lottery} - 1.5 > 0$$

$$1 \cdot \text{Win} + 1 \cdot \text{Lottery} - 1.5 > 0$$

Threshold

Bias

Check: > 0?

Model
1
1
-1.5

Bias

The AND operator

AND	x	y
No	0	0
No	1	0
No	0	1
Yes	1	1

The AND operator

AND	x	y
No	0	0
No	1	0
No	0	1
Yes	1	1

Model
1
1

The AND operator

AND	x	y
No	0	0
No	1	0
No	0	1
Yes	1	1

Model = Dot prod

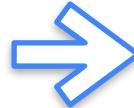
Model	Dot prod
1	0
1	1
1	1
2	2

The AND operator

AND	x	y
No	0	0
No	1	0
No	0	1
Yes	1	1

$$\begin{array}{c} \text{Model} \\ \hline 1 \\ 1 \end{array} = \begin{array}{c} \text{Dot prod} \\ \hline 0 \\ 1 \\ 1 \\ 2 \end{array}$$

Check: >1.5?



The AND operator

AND	x	y
No	0	0
No	1	0
No	0	1
Yes	1	1

$$\begin{matrix} \text{Model} \\ \begin{matrix} 1 \\ 1 \end{matrix} \end{matrix} = \begin{matrix} \text{Dot prod} \\ \begin{matrix} 0 \\ 1 \\ 1 \\ 2 \end{matrix} \end{matrix}$$

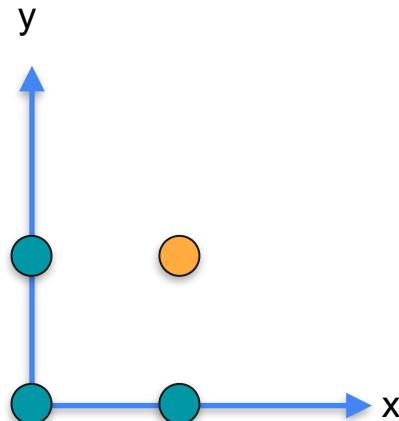
Check: >1.5?



Check
No
No
No
Yes

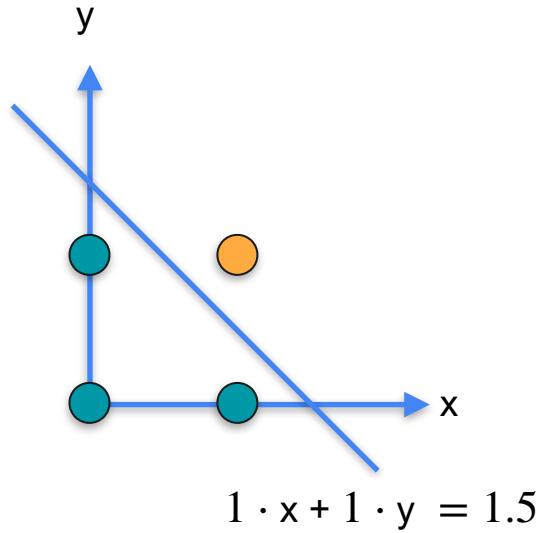
The AND operator

AND	x	y
No	0	0
No	1	0
No	0	1
Yes	1	1

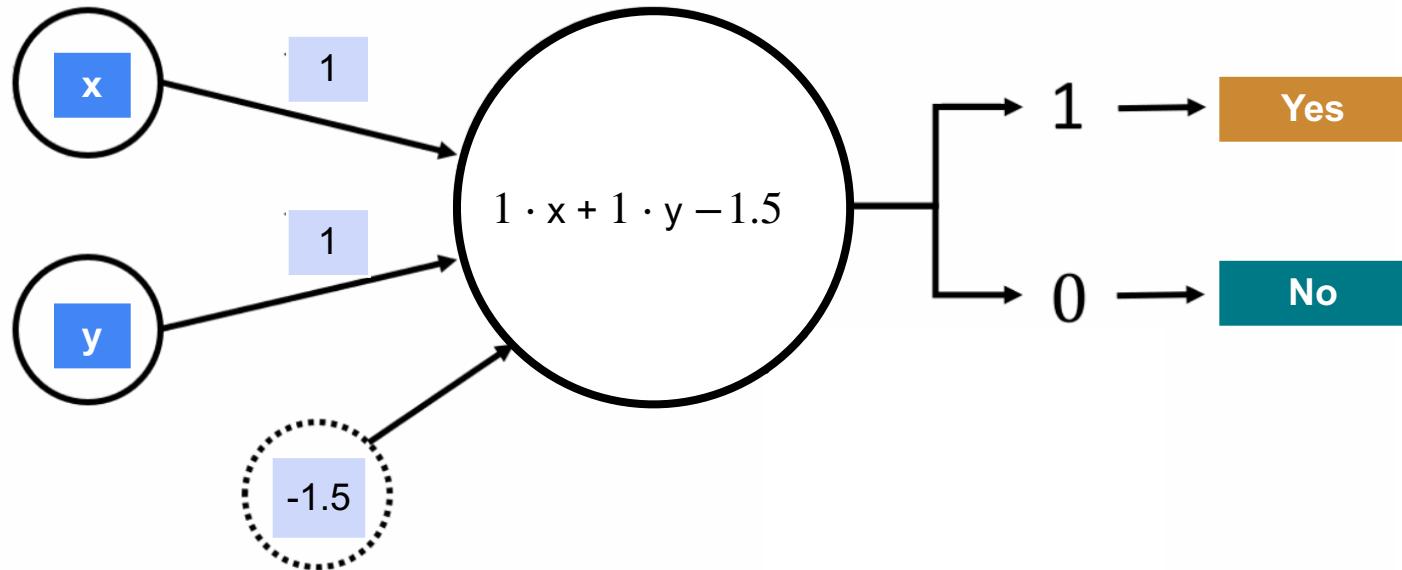


The AND operator

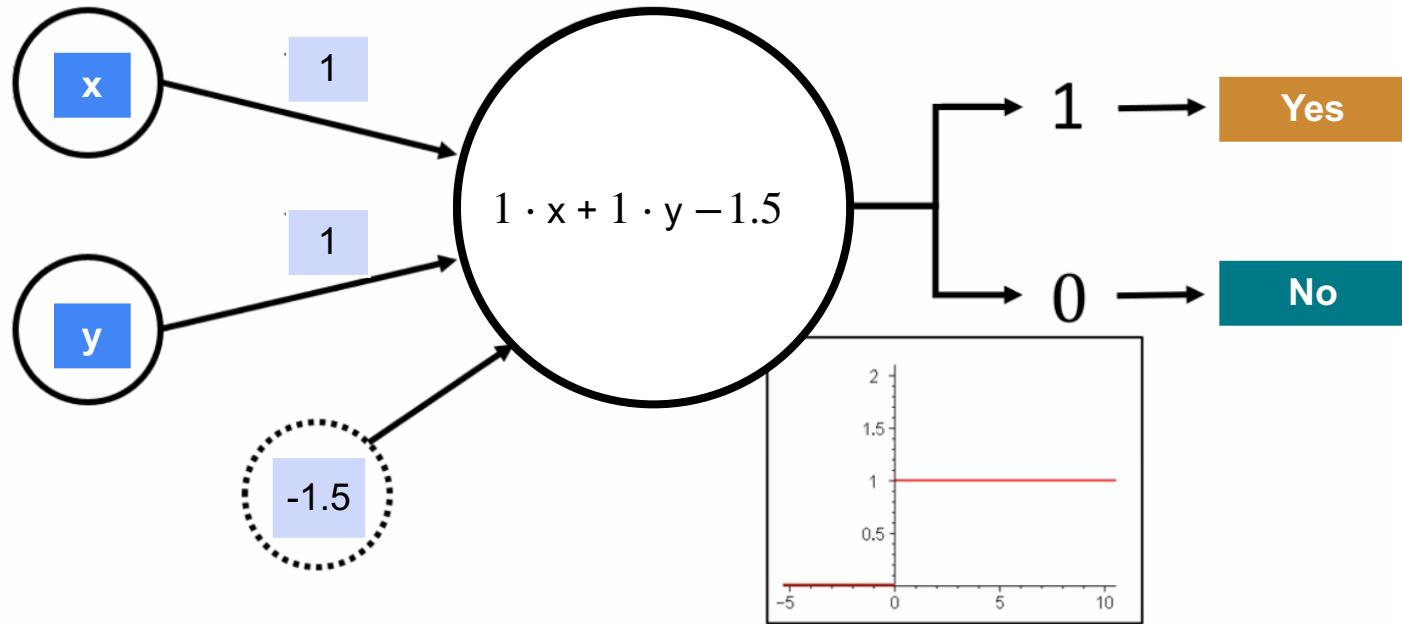
AND	x	y
No	0	0
No	1	0
No	0	1
Yes	1	1

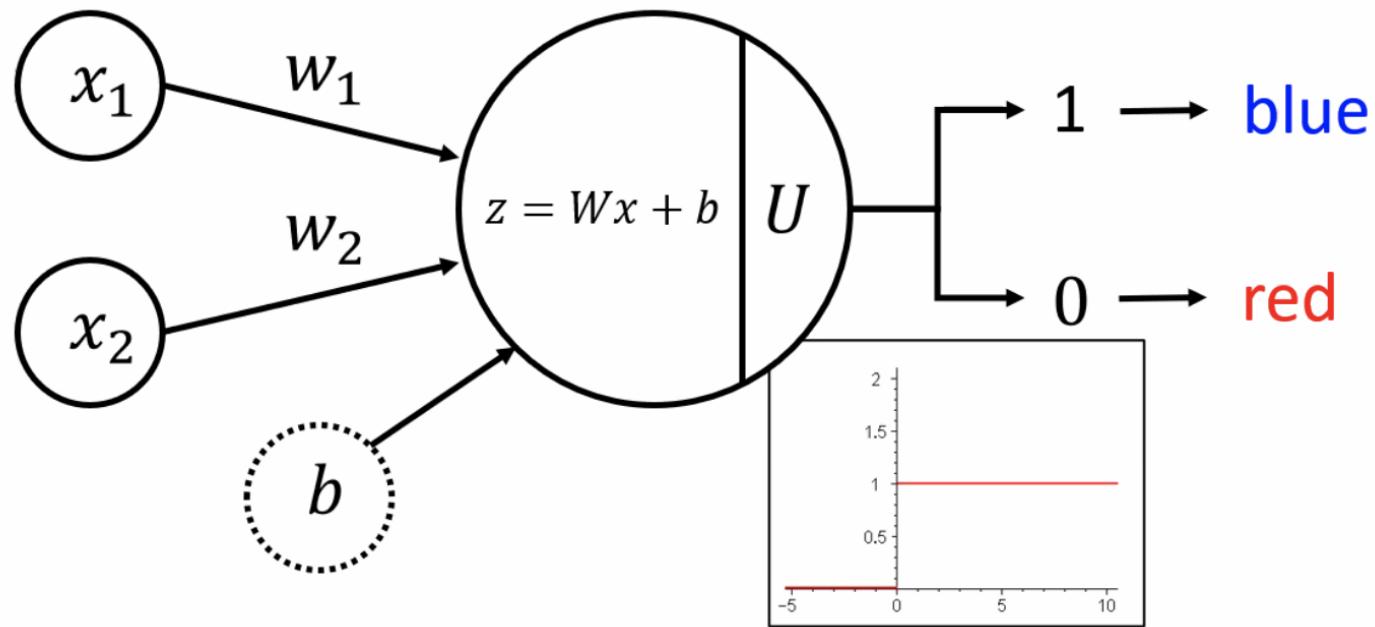


The perceptron



The perceptron







DeepLearning.AI

Vectors and Linear Transformations

Conclusion