Linearization

2-6.1 Linearization of Functions of One Variable

$$f\left(x(t)\right) = f\left(\bar{x}\right) + \left(\left.\frac{df}{dx}\right|_{\bar{x}}\right)\left(x(t) - \bar{x}\right)$$

This is the basic linearization formula.

$\bar{x} = cte$

The right hand side of the equation is linear in the variable x(t)

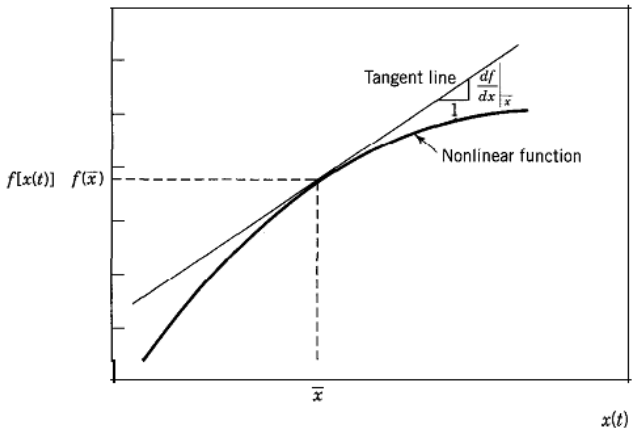

Figure 2-6.1 The linear approximation is the tangent to the nonlinear function at the base point $\bar{x}$.

Taylor serie:

$$f\left(x(t)\right) = f(a) + \left(\frac{\left.\frac{dx(t)}{dt}\right|_{t=a}}{1!}\right)(x-a) + \left(\frac{\left.\frac{d^2x(t)}{dt^2}\right|_{t=a}}{2!}\right)(x-a)^2 + \ldots$$

A es el punto alrededor donde se realizaraá la linealización

2-6.9. The linear approximation is a straight line passing through the point $[\bar{x}, f(\bar{x})]$ with slope $df/dx|_{\bar{x}}$. This line is by definition the tangent to $f(x)$ at $\bar{x}$. Note that the difference between the nonlinear function and its linear approximation is small near the base point $\bar{x}$ and becomes larger the farther x(t) is from $\bar{x}$. The width of the range in which the linear approximation is accurate depends on the function. Some functions are more curved than others and thus have a narrower range over which the linear approximation is accurate.

It is important to realize that what affects the parameters of the transfer function of a linearized system is the slope, $df/dx|_{\bar{x}}$, not the value of the function itself, $f(\bar{x})$. This will become obvious when we show how to apply the linearization technique to nonlinear differential equations. The following example illustrates the application of the linearization formula.

Example

Linearize the arrhenius equation for the temperature dependence of chemical reaction rate coefficients. For a reaction with a coefficient $k\left(\overline{T}\right) = 100\left[s^{-1}\right]$ and an energy of activation $E = 22,000\left[\frac{kcal}{kmole}\right]$, estime the error in the slope of the function in the range $\pm 10\left[°C\right]$ around $\overline{T} = 300°C_{(573K)}$

Apply the linearization formula, Eq. 2-6.9, to Eq. 2-6.6.

$$k\left(T\right) = k_0 e^{-\frac{E}{RT}}$$

Linearization:

$$k\left(T\right) \approx k\left(\overline{T}\right) + \left(\frac{\left.\frac{d\left[k\left(T\right)\right]}{dT}\right|_{T=\overline{T}}}{1!} \right)\left(T - \overline{T}\right)$$

$$\left.\frac{dk}{dT}\right|_{\overline{T}} = \left.\frac{d\left[k_0 e^{-\frac{E}{RT}}\right]}{dT}\right|_{T=\overline{T}} = k_0 e^{-\frac{E}{R\overline{T}}} \frac{E}{R\overline{T}^2} = k\left(\overline{T}\right)\frac{E}{R\overline{T}^2}$$

$$R = 1.987\left[\frac{kcal}{kmole - k}\right]$$

$$k\left(300\right) = 100$$

$$\left.\frac{dk}{dT}\right|_{300°C} = \left(100\right)\frac{22,000}{1.987\left(300+273\right)^2} = 3.37$$

And the line approximation of the fuction is:

$$k\left(T\right) \approx k\left(\overline{T}\right) + k\left(\overline{T}\right)\frac{E}{R\overline{T}^2}\left(T - \overline{T}\right)$$

$\boxed{k\left(T\right) \approx 100 + 3.37\left(T - 300\right)}$ recta tangente al punto T testada.

		Linear approximation	Original	Absolute error	Relative error
T=290°C	$k\left(290\right) \approx 100 + 3.37\left(290 - 300\right)$	66.3	70.95	6.55%	4.65
T=300°C	$k\left(300\right) \approx 100 + 3.37\left(300 - 300\right)$	100	100	0	0
T=310°C	$k\left(310\right) \approx 100 + 3.37\left(310 - 300\right)$	133.7	139.3	4.020%	5.6%

However, the error of 35% in the parameters is usually satisfactory for many control system calculations.