4-4.1 Integrating Processes: Level Process

Consider the process tank shown in Fig. 4-4.1. An input stream enters the tank freely, whereas the output stream depends on the speed of the pump. The pump speed is regulated by the signal m(t), %.

The relation between the output flow and the signal is given by:

$$\tau_p \frac{df_o(t)}{dt} + f_o(t) = K_p m(t)$$

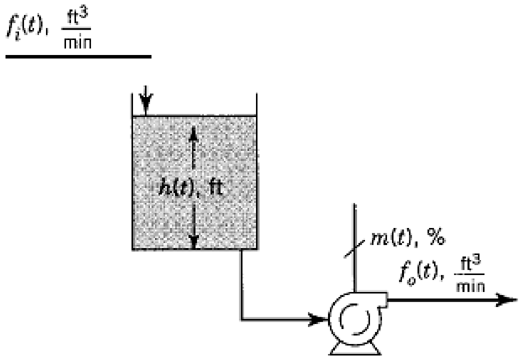

Figure 4-4.1 Process tank with pump manipulating outlet flow.

An unsteady-state mass balance around the tank provides the first equation needed:

$$\rho f_i(t) - \rho f_o(t) = \rho A \frac{dh(t)}{dt}$$

Only two equations are required to model this simple process.
Following the usual procedure, we obtain

$$\rho \bar{f}_i - \rho \bar{f}_o = \rho A \frac{d\bar{h}}{dt} = 0$$

Restando:

$$\rho f_i(t) - \rho f_o(t) - \left(\rho \bar{f}_i - \rho \bar{f}_o\right) =$$

$$\rho A \frac{dh(t)}{dt} - \rho A \frac{d\bar{h}}{dt}$$

$$\rho f_i(t) - \rho f_o(t) - \rho \bar{f}_i + \rho \bar{f}_o =$$

$$\rho A \frac{d\left[h(t) - \bar{h}\right]}{dt}$$

$$F_i(s) - F_o(s) = \rho A s H(s)$$

$$\frac{F_i(s) - F_o(s)}{As} = H(s)$$

$$\boxed{H(s) = \frac{1}{As}\left[F_i(s) - F_o(s)\right]}$$

where the deviation variables are

$$H(t) = h(t) - \bar{h}$$

$$F_i(t) = f_i(t) - \bar{f}_i$$

$$F_o(t) = f_o(t) - \bar{f}_o$$

$$\tau_p s F_o(s) + F_o(s) = K_p M(s)$$

$$\frac{F_o(s)\left(\tau_p s + 1\right)}{K_p} = M(s)$$

$$F_o(s) = \frac{K_p}{\left(\tau_p s + 1\right)} M(s)$$

where the new deviation variable is

$$M(t) = m(t) - \bar{m}$$

Sustituyendo:

$$\boxed{H(s) = \frac{1}{As}\left[F_i(s) - \frac{K_p}{\left(\tau_p s + 1\right)} M(s)\right]}$$

By superposition we can obtein transfer funtions desires:

$$\boxed{\frac{H(s)}{F_i(s)} = \frac{1}{As}}$$

$$\boxed{\frac{H(s)}{M(s)} = -\frac{K_p}{As\left(\tau_p s + 1\right)}}$$

S term in denominator indicates the "integrating" nature of the process

Let us develop the response of the system to a change of -B% in the signal m(t). That is,

$$m(t) = -Bu(t)$$

$$M(s) = -B\frac{1}{s}$$

$$H(s) = \frac{1}{As}\left[F_i(s) - \frac{K_p}{(\tau_p s + 1)}\left(-B\frac{1}{s}\right)\right]$$

$$H(s) = \frac{1}{As}\left[F_i(s) + \frac{BK_p}{s(\tau_p s + 1)}\right]$$

$$H(s) = \left[\frac{1}{As}F_i(s) + \frac{1}{As}\frac{BK_p}{s(\tau_p s + 1)}\right]$$

$$\boxed{H(s) = \frac{1}{As}F_i(s) + \frac{BK_p}{As^2(\tau_p s + 1)}}$$

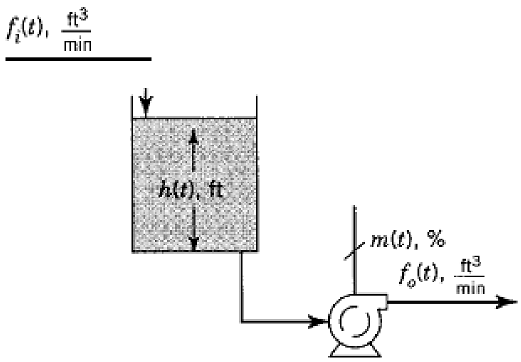

Figure 4-4.1 Process tank with pump manipulating outlet flow.