4-1.1 Noninteracting Level Process – tank – pump

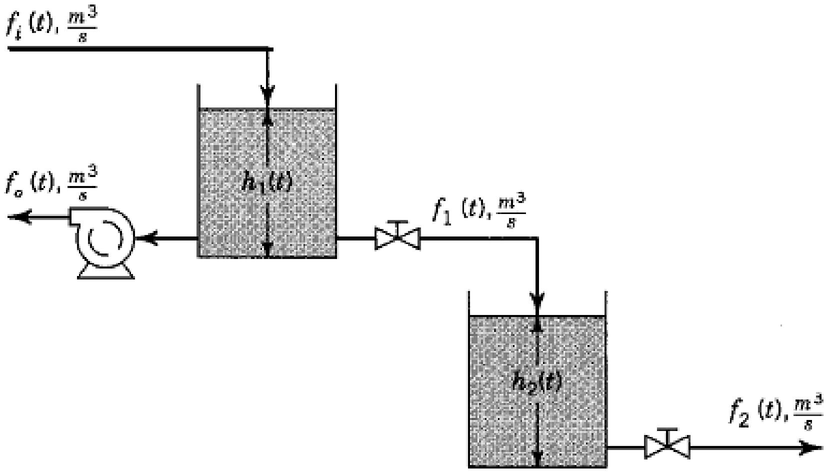

Figure 4-1.1 Tanks in series-noninteracting system.

f(t) = flow through valve, m^3/s

C_v = valve coefficient, $m^3/s\text{-}Pa^{1/2}$

$\Delta P(t)$ = pressure drop across valve, Pa

G_f = specific gravity of liquid, dimensionless

$$f(t) = C_v\sqrt{\frac{\Delta P(t)}{G_f}}$$

Because the tanks are open to the atmosphere and the valves discharge to atmospheric pressure, the pressure drop across each valve is given by:

$$\Delta P(t) = P_r(t) - P_d = P_a + \rho g h(t) - P_a$$

$$\Delta P(t) = \rho g h(t)$$

Asi la ecuacion de la valvula es:

$$\boxed{f(t) = C_v\sqrt{\frac{\rho g h(t)}{G_f}} = C_v'\sqrt{h(t)}}$$

$$\rho f_i(t) - \rho f_1(t) - \rho f_o(t) = \frac{dm_1(t)}{dt}$$

M1(t) es la masa acumulada en el tanque 1

$$m_1(t) = \rho A_1 h_1(t)$$

Sustituyendo:

$$\rho f_i(t) - \rho f_1(t) - \rho f_o(t) = \rho A_1\frac{dh_1(t)}{dt}$$

$$f_i(t) - f_1(t) - f_o(t) = A_1\frac{dh_1(t)}{dt}$$

$$f_1(t) = C_{v1}'\sqrt{h_1(t)}$$

Sustituyendo:

$$f_i(t) - f_o(t) - C_1'\sqrt{h_1(t)} = A_1\frac{dh_1(t)}{dt}$$

Para el segundo tanque:

$$f_1(t) - f_2(t) = A_1\frac{dh_1(t)}{dt}$$

$$f_2(t) = C_{v2}'\sqrt{h_2(t)}$$

Linealizando:

$$f_1(t) = \bar{f}_1(\bar{h}_1) + C_1(h_1(t) - \bar{h}_1)$$

$$f_2(t) = \bar{f}_2(\bar{h}_2) + C_2(h_2(t) - \bar{h}_2)$$

$$C_1 = \left.\frac{\partial f_1(t)}{\partial h_1(t)}\right|_{h=\bar{h}_1} = \frac{1}{2}C'_{v1}(\bar{h}_1)^{\frac{1}{2}}\left[\frac{m^3/s}{m}\right]$$

$$C_2 = \left.\frac{\partial f_2(t)}{\partial h_2(t)}\right|_{h=\bar{h}_2} = \frac{1}{2}C'_{v2}(\bar{h}_2)^{\frac{1}{2}}\left[\frac{m^3/s}{m}\right]$$

These equations provide a set of linea equations that describes the process around the linearization values h1 tasted and h2 tested

$$f_i(t) - \left[\bar{f}_1 + C_1(h_1(t) - \bar{h}_1)\right] - f_o(t) = A_1\frac{dh_1(t)}{dt}$$

$$f_i(t) - \left[\bar{f}_1 + C_1 H_1(t)\right] - f_o(t) = A_1\frac{dh_1(t)}{dt}$$

$$f_i(t) - \bar{f}_1 - C_1 H_1(t) - f_o(t) = A_1\frac{dh_1(t)}{dt}$$

$$\left(f_i(t)-\overline{f}_1\right)-C_1H_1(t)-f_o(t)=A_1\frac{dh_1(t)}{dt}$$

$$\left(f_i(t)-\overline{f}_1\right)-C_1H_1(t)-f_o(t)=A_1\frac{dh_1(t)}{dt}$$

$$\frac{\left(f_i(t)-\overline{f}_1\right)-C_1H_1(t)-f_o(t)}{A_1}=\frac{dh_1(t)}{dt}$$

$$\frac{1}{A_1}\left(f_i(t)-\overline{f}_1\right)-\frac{C_1}{A_1}H_1(t)-\frac{1}{A_1}F_o(t)=\frac{dh_1(t)}{dt}$$

Definiendo:

$$K_1=\frac{1}{A_1}\,;\ \tau_1=\frac{A_1}{C_1}$$

$$F_i(t)=f_i(t)-\overline{f}_i$$

$$F_o(t)=f_o(t)-\overline{f}_o$$

$$F_1(t)=f_1(t)-\overline{f}_1$$

$$F_2(t)=f_2(t)-\overline{f}_2$$

$$\tau_1\frac{dH_1(t)}{dt}+H_1(t)=K_1F_i(t)-K_1F_o(t)$$

$$\tau_1sH_1(s)+H_1(s)=K_1F_i(s)-K_1F_o(s)$$

$$H_1(s)(\tau_1s+1)=K_1F_i(s)-K_1F_o(s)$$

$$H_1(s)=\frac{K_1F_i(s)-K_1F_o(s)}{(\tau_1s+1)}$$

$$H_1(s)=\frac{K_1F_i(s)}{(\tau_1s+1)}-\frac{K_1F_o(s)}{(\tau_1s+1)}$$

$$H_1(s)=\frac{K_1}{(\tau_1s+1)}F_i(s)-\frac{K_1}{(\tau_1s+1)}F_o(s)$$