Linearization - Jacobian Analisys

Equilibrium points

Consider a nonlinear differential equation

$$\dot{x}(t) = f\left(x(t), u(t)\right)$$

Where f is a function mapping Rn x Rm → Rn . A point $\overline{x} \in R^n$ is called an **equilibrium point** if there is an specific $\overline{u} \in R^m$ (**called the equilibrium input**) such that:

$$f\left(\overline{x}, \overline{u}\right) = 0_n$$

Suppose $\overline{x}$ is an equilibrium point (with equilibrium input $\overline{u}$). Consider starting the system $\dot{x}(t) = f\left(x(t), u(t)\right)$ from initial condition $x(t_0) = \overline{x}$, and applying the input $u(t) \equiv \overline{u}$ for all $t \quad t_0$. The resulting solution $x(t)$ satisfies:

$$x(t) = \overline{x}$$

For all $t \quad t_0$. That is why it is called an equilibrium point.