

## Response of second-order systems

$$Y(s) = \left[ \frac{K}{\tau^2 s^2 + 2\zeta\tau s + 1} \right] X(s)$$

For The Response Is

$\zeta > 1$	<b>overdamped = monotonic and stable</b>
$0 < \zeta < 1$	<b>underdamped = oscillatory and stable</b>
$\zeta = 0$	<b>undamped = sustained oscillations</b>
$-1 < \zeta < 0$	<b>unstable = growing oscillations</b>
$\zeta \leq -1$	<b>run-away = monotonic unstable</b>

The case of  $\zeta = 1$  is sometimes called critically damped, but this is only the borderline case. Its response is monotonic and stable, just like the overdamped response.

For our purposes, we need consider only the two cases of real and complex roots, which we will call overdamped and underdamped, respectively. The following sections present the specific response equations for step, ramp, and sinusoidal inputs for both of these cases.

### Overdamped Responses

**Step Response**

**Ramp Response**

**Sinusoidal Response**