## 4-2 Interactive systems

Interacting systems are more frequently encountered in industry than noninteracting systems; this section presents three examples. The differences in dynamic response between the noninteracting and interacting systems are also presented.

Let us rearrange the tanks of Fig. 4-1.1 to result in the new process shown in Fig. 4-2.1. In this case the pressure drop, AP(t), across the valve between the two tanks is given by:

$$P_u = P_{atm} + \rho g h_1(t)$$

$$P_d = P_{atm} + \rho g h_2(t)$$

$$P_u - P_d = P_{atm} + \rho g h_1(t) - \left[P_{atm} + \rho g h_2(t)\right]$$

$$\Delta P = \rho g h_1(t) - \rho g h_2(t) = \rho g\left[h_1(t) - h_2(t)\right]$$

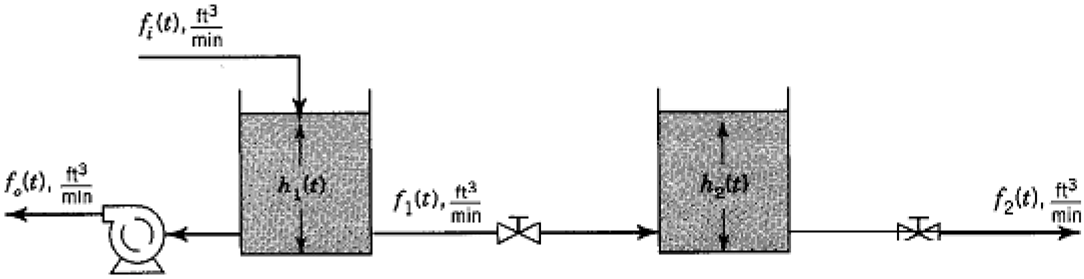



**Figure 4-2.1** Tanks in series-interacting system.

f(t) = flow through valve, $m^3/s$
$C_v$ = valve coefficient, $m^3/s\text{-}Pa^{1/2}$
$\Delta P(t)$ = pressure drop across valve, Pa
$G_f$ = specific gravity of liquid, dimensionless

$$f_1(t) = C_v\sqrt{\frac{\Delta P(t)}{G_f}} = C_v\sqrt{\frac{\rho g h_1(t) - \rho g h_2(t)}{G_f}} = C_v\sqrt{\frac{\rho g\left[h_1(t) - h_2(t)\right]}{G_f}} = C_{v1}\sqrt{h_1(t) - h_2(t)}$$

$$\rho f_i(t) - \rho f_1(t) - \rho f_o(t) = \rho A_1 \frac{dh_1(t)}{dt}$$

$$f_1(t) = C_{v1}\sqrt{h_1(t) - h_2(t)}$$

$$\rho f_1(t) - \rho f_2(t) = \rho A_2 \frac{dh_2(t)}{dt}$$

$$f_2(t) = C_{v1}\sqrt{h_2(t)}$$ despreciа la presión atmosférica

Linealizando f1 y f2:

$$f_1(t) = \bar{f}_1 + C_1\left[h_1(t) - \bar{h}_1\right] + C_2\left[h_2(t) - \bar{h}_2\right]$$

$$f_2(t) = \bar{f}_2 + C_3\left[h_2(t) - \bar{h}_2\right]$$

$$\rho \bar{f}_i - \rho \bar{f}_1 - \rho \bar{f}_o = \rho A_1 \frac{d\bar{h}_1}{dt}$$

$$\rho \bar{f}_1 - \rho \bar{f}_2 = \rho A_2 \frac{d\bar{h}_2}{dt}$$

$$C_1 = \left.\frac{\partial f_1(t)}{\partial h_1}\right|_{\bar{h}_1,\bar{h}_2} = \left[C_{v1}\frac{1}{2}\left(h_1(t) - h_2(t)\right)^{-\frac{1}{2}}\right]_{\bar{h}_1,\bar{h}_2}$$

$$\boxed{C_1 = C_{v1}\frac{1}{2}\left(\bar{h}_1 - \bar{h}_2\right)^{-\frac{1}{2}}}$$

$$C_2 = \left.\frac{\partial f_1(t)}{\partial h_2}\right|_{\bar{h}_1,\bar{h}_2} = -\left[C_{v1}\frac{1}{2}\left(h_1(t) - h_2(t)\right)^{-\frac{1}{2}}\right]_{\bar{h}_1,\bar{h}_2}$$

$$\boxed{C_2 = -C_{v1}\frac{1}{2}\left(\bar{h}_1 - \bar{h}_2\right)^{-\frac{1}{2}}}$$

$$\rho f_i(t) - \rho f_1(t) - \rho f_o(t) - \left[\rho \bar{f}_i - \rho \bar{f}_1 - \rho \bar{f}_o\right] = \rho A_1 \frac{dh_1(t)}{dt} - \rho A_1 \frac{d\bar{h}_1}{dt}$$

$$\rho f_i(t) - \rho f_1(t) - \rho f_o(t) - \rho \bar{f}_i + \rho \bar{f}_1 + \rho \bar{f}_o = \rho A_1 \frac{dh_1(t)}{dt} - \rho A_1 \frac{d\bar{h}_1}{dt}$$

$$\left[\rho f_i(t) - \rho \bar{f}_i\right] - \rho f_1(t) + \rho \bar{f}_1 - \rho f_o(t) + \rho \bar{f}_o = \rho A_1 \frac{d\left[h_1(t) - \bar{h}_1\right]}{dt}$$

$$\left[\rho f_i(t) - \rho \bar{f}_i\right] - \rho\left\{\bar{f}_1 + C_1\left[h_1(t) - \bar{h}_1\right] + C_2\left[h_2(t) - \bar{h}_2\right]\right\} + \rho \bar{f}_1 - \rho f_o(t) + \rho \bar{f}_o = \rho A_1 \frac{d\left[h_1(t) - \bar{h}_1\right]}{dt}$$

$$\left[\rho f_i(t) - \rho \bar{f}_i\right] - \rho \bar{f}_1 - \rho C_1\left[h_1(t) - \bar{h}_1\right] - \rho C_2\left[h_2(t) - \bar{h}_2\right] + \rho \bar{f}_1 - \rho f_o(t) + \rho \bar{f}_o = \rho A_1 \frac{d\left[h_1(t) - \bar{h}_1\right]}{dt}$$

$$\left[\rho f_i(t) - \rho \bar{f}_i\right] - \rho C_1\left[h_1(t) - \bar{h}_1\right] - \rho C_2\left[h_2(t) - \bar{h}_2\right] = \rho A_1 \frac{d\left[h_1(t) - \bar{h}_1\right]}{dt} + \rho f_o(t) - \rho \bar{f}_o$$

$$\left[F_i\right] - C_1\left[H_1\right] - C_2\left[H_2\right] = A_1 \frac{d\left[H_1\right]}{dt} + F_o$$

$$\left[F_i\right] - C_2\left[H_2\right] = A_1 \frac{d\left[H_1\right]}{dt} + C_1\left[H_1\right] + F_o$$

$$F_i(s) - C_2 H_2(s) = A_1 s H_1(s) + C_1 H_1(s) + F_o(s)$$

$$F_i(s) - C_2 H_2(s) = H_1(s)\left(A_1 s + C_1\right) + F_o(s)$$

$$F_i(s) - C_2 H_2(s) - F_o(s) = H_1(s)\left(A_1 s + C_1\right)$$

$$H_1(s) = \frac{F_i(s) - C_2 H_2(s) - F_o(s)}{\left(A_1 s + C_1\right)}$$

$$H_1(s) = \frac{F_i(s) - C_2 H_2(s) - F_o(s)}{C_1\left(A_1 s + 1\right)}$$

$$H_1(s) = \frac{\frac{1}{C_1}}{\left(\frac{A_1}{C_1} s + 1\right)} F_i(s) - \frac{\frac{C_2}{C_1}}{\left(\frac{A_1}{C_1} s + 1\right)} H_2(s) - \frac{\frac{1}{C_1}}{\left(\frac{A_1}{C_1} s + 1\right)} F_o(s)$$

$$H_1(s) = \frac{\frac{1}{C_1}}{\left(\tau_1 s + 1\right)} F_i(s) - \frac{\frac{C_2}{C_1}}{\left(\tau_1 s + 1\right)} H_2(s) - \frac{\frac{1}{C_1}}{\left(\tau_1 s + 1\right)} F_o(s)$$

$$H_1(s) = \frac{\frac{1}{C_1}}{\left(\tau s + 1\right)}\left[F_i(s) - F_o(s)\right] + \frac{1}{\left(\tau s + 1\right)} H_2(s)$$

$$\boxed{K_4 = \frac{1}{C_1}} \qquad \boxed{\tau_1 = \frac{A_1}{C_1}}$$

$$\boxed{H_1(s) = \frac{K_4}{\left(\tau_1 s + 1\right)}\left[F_i(s) - F_o(s)\right] + \frac{1}{\left(\tau_1 s + 1\right)} H_2(s)}$$

**Para el tanque 2**

$$\rho f_1(t) - \rho f_2(t) = \rho A_2 \frac{dh_2(t)}{dt}$$

$$\rho \bar{f}_1 - \rho \bar{f}_2 = \rho A_2 \frac{d\bar{h}_2}{dt}$$

Restando:

$$\rho f_1(t) - \rho f_2(t) - \left\{ \rho \bar{f}_1 - \rho \bar{f}_2 \right\} = \rho A_2 \frac{dh_2(t)}{dt} - \rho A_2 \frac{d\bar{h}_2}{dt}$$

$$\rho f_1(t) - \rho f_2(t) - \rho \bar{f}_1 + \rho \bar{f}_2 = \rho A_2 \frac{d\left[h_2(t) - \bar{h}_2\right]}{dt}$$

$$f_2(t) = C_{v2}\sqrt{h_2(t)}$$

$$f_2(t) = \bar{f}_2 + C_4\left(h_2(t) - \bar{h}_2\right)$$

$$\boxed{C_4 = \frac{1}{2} C_{v2} \bar{h}_2^{\,-\frac{1}{2}}}$$

$$\rho f_1(t) - \rho\left\{ \bar{f}_2 + C_4\left(h_2(t) - \bar{h}_2\right)\right\} - \rho \bar{f}_1 + \rho \bar{f}_2 = \rho A_2 \frac{d\left[h_2(t) - \bar{h}_2\right]}{dt}$$

$$\rho f_1(t) - \rho \bar{f}_2 - \rho C_4\left(h_2(t) - \bar{h}_2\right) - \rho \bar{f}_1 + \rho \bar{f}_2 = \rho A_2 \frac{d\left[h_2(t) - \bar{h}_2\right]}{dt}$$

$$\rho f_1(t) - \rho \bar{f}_1 - \rho C_4\left(h_2(t) - \bar{h}_2\right) = \rho A_2 \frac{d\left[h_2(t) - \bar{h}_2\right]}{dt}$$

$$\left[\left\{ \bar{f}_1 + C_1\left[h_1(t) - \bar{h}_1\right] + C_2\left[h_2(t) - \bar{h}_2\right]\right\} - \bar{f}_1\right] - C_4\left[h_2(t) - \bar{h}_2\right] = A_2 \frac{d\left[h_2(t) - \bar{h}_2\right]}{dt}$$

$$\bar{f}_1 + C_1\left[h_1(t) - \bar{h}_1\right] + C_2\left[h_2(t) - \bar{h}_2\right] - \bar{f}_1 - C_4\left[h_2(t) - \bar{h}_2\right] = A_2 \frac{d\left[h_2(t) - \bar{h}_2\right]}{dt}$$

$$\cancel{\bar{f}_1} + C_1\left[h_1(t) - \bar{h}_1\right] + C_2\left[h_2(t) - \bar{h}_2\right] \cancel{-\bar{f}_1} - C_4\left[h_2(t) - \bar{h}_2\right] = A_2 \frac{d\left[h_2(t) - \bar{h}_2\right]}{dt}$$

$$C_1\left[h_1(t) - \bar{h}_1\right] + C_2\left[h_2(t) - \bar{h}_2\right] - C_4\left[h_2(t) - \bar{h}_2\right] = A_2 \frac{d\left[h_2(t) - \bar{h}_2\right]}{dt}$$

$$C_1\left[H_1\right] + C_2\left[H_2\right] - C_4\left[H_2\right] = A_2 \frac{d\left[H_2\right]}{dt}$$

$$C_1\left[H_1\right] + C_2\left[H_2\right] - C_4\left[H_2\right] = A_2 \frac{d\left[H_2\right]}{dt}$$

$$C_1\left[H_1\right] = A_2 \frac{d\left[H_2\right]}{dt} + C_4\left[H_2\right] - C_2\left[H_2\right]$$

$$C_1\left[H_1\right] = A_2 \frac{d\left[H_2\right]}{dt} + C_4\left[H_2\right] - C_2\left[H_2\right]$$

$$C_1 H_1(s) = A_2 s H_2(s) + C_4 H_2(s) - C_2 H_2(s)$$

$$C_1 H_1(s) = H_2(s)\left[A_2 s + C_4 - C_2\right]$$

$$H_2(s) = \frac{C_1}{\left[A_2 s + (C_4 - C_2)\right]} H_1(s)$$

$$H_2(s) = \frac{C_1}{(C_4 - C_2)\left[\frac{A_2}{(C_4 - C_2)} s + 1\right]} H_1(s)$$

$$H_2(s) = \frac{\frac{C_1}{(C_4 - C_2)}}{\left[\frac{A_2}{(C_4 - C_2)} s + 1\right]} H_1(s)$$

$$H_2(s) = \frac{\frac{C_1}{(C_4 + C_1)}}{\left[\frac{A_2}{(C_4 + C_1)} s + 1\right]} H_1(s) \qquad \boxed{K_5 = \frac{C_1}{(C_4 + C_1)}} \qquad \boxed{\tau_2 \frac{A_2}{(C_4 + C_1)}}$$

$$\boxed{H_2(s) = \frac{K_5}{[\tau_2 s + 1]} H_1(s)}$$

$$H_2(s)\frac{[\tau_2 s + 1]}{K_5} = \frac{K_4}{(\tau_1 s + 1)}\left[F_i(s) - F_o(s)\right] + \frac{1}{(\tau_1 s + 1)} H_2(s)$$

$$H_2(s)\frac{[\tau_2 s + 1]}{K_5} - \frac{1}{(\tau_1 s + 1)} H_2(s) = \frac{K_4}{(\tau_1 s + 1)}\left[F_i(s) - F_o(s)\right]$$

$$H_2(s)\left(\frac{[\tau_2 s + 1]}{K_5} - \frac{1}{(\tau_1 s + 1)}\right) = \frac{K_4}{(\tau_1 s + 1)}\left[F_i(s) - F_o(s)\right]$$

$$H_2(s)\left[\frac{(\tau_2 s + 1)(\tau_1 s + 1) - K_5}{K_5(\tau_1 s + 1)}\right] = \frac{K_4}{(\tau_1 s + 1)}\left[F_i(s) - F_o(s)\right]$$

$$H_2(s) = \frac{\frac{K_4 K_5 \cancel{(\tau_1 s + 1)}}{\cancel{(\tau_1 s + 1)}}\left[F_i(s) - F_o(s)\right]}{(\tau_2 s + 1)(\tau_1 s + 1) - K_5}$$

$$H_2(s) = \frac{K_4 K_5}{(\tau_2 s + 1)(\tau_1 s + 1) - K_5}\left[F_i(s) - F_o(s)\right]$$

**desde aqui podemos identificar una realimentacion positiva**

$$H_2(s) = \frac{K_4 K_5 \left[F_i(s) - F_o(s)\right]}{\tau_1 \tau_2 s^2 + \tau_2 s + \tau_1 s + 1 - K_5}$$

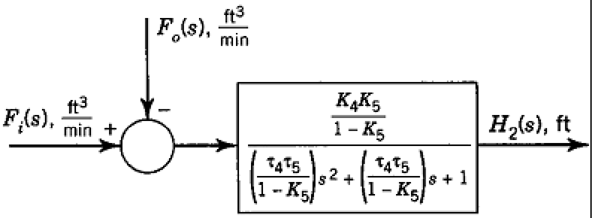
$$H_2(s) = \frac{K_4 K_5 \left[F_i(s) - F_o(s)\right]}{(1 - K_5)\left(\frac{\tau_1 \tau_2}{1 - K_5} s^2 + \frac{\tau_2}{1 - K_5} s + \frac{\tau_1}{1 - K_5} s + 1\right)}$$

$$H_2(s) = \frac{\dfrac{K_4K_5}{(1-K_5)}}{\left(\dfrac{\tau_1\tau_2}{1-K_5}s^2 + \dfrac{\tau_2}{1-K_5}s + \dfrac{\tau_1}{1-K_5}s + 1\right)}\left[F_i(s) - F_o(s)\right]$$

**Por superposición:**

$$\frac{H_2(s)}{F_i(s)} = \frac{\dfrac{K_4K_5}{(1-K_5)}}{\left(\dfrac{\tau_1\tau_2}{1-K_5}s^2 + \dfrac{\tau_2}{1-K_5}s + \dfrac{\tau_1}{1-K_5}s + 1\right)}$$

$$\frac{H_2(s)}{F_o(s)} = \frac{-\dfrac{K_4K_5}{(1-K_5)}}{\left(\dfrac{\tau_1\tau_2}{1-K_5}s^2 + \dfrac{\tau_2}{1-K_5}s + \dfrac{\tau_1}{1-K_5}s + 1\right)}$$




(a)

**Otra manera mas corta de realizarlo es Mediante reconocer una realimentacion Positiva**

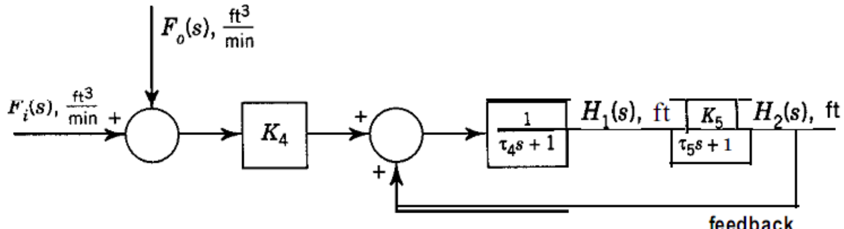
$$H_2(s) = \frac{K_4K_5}{(\tau_2 s+1)(\tau_1 s+1) - K_5}\left[F_i(s) - F_o(s)\right]$$

$$\frac{K_4K_5}{(\tau_2 s+1)(\tau_1 s+1) - K_5} = K_4\frac{K_5}{(\tau_2 s+1)(\tau_1 s+1)\left[1 - \dfrac{K_5}{(\tau_2 s+1)(\tau_1 s+1)}\right]}$$

$$\frac{K_4K_5}{(\tau_2 s+1)(\tau_1 s+1) - K_5} = K_4\frac{\dfrac{K_5}{(\tau_2 s+1)(\tau_1 s+1)}}{\left[1 - \dfrac{K_5}{(\tau_2 s+1)(\tau_1 s+1)}\right]}$$

Aqui vemos una realimentacion positiva:

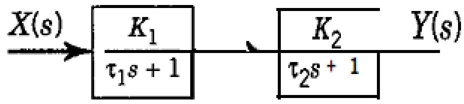
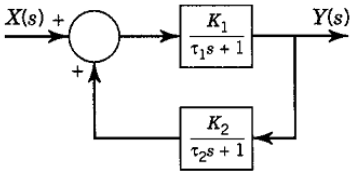
$$\frac{\dfrac{K_5}{(\tau_2 s+1)(\tau_1 s+1)}}{\left[1 - (1)\dfrac{K_5}{(\tau_2 s+1)(\tau_1 s+1)}\right]} = \frac{\dfrac{1}{(\tau_2 s+1)}\dfrac{K_5}{(\tau_1 s+1)}}{\left[1 - (1)\dfrac{1}{(\tau_2 s+1)}\dfrac{K_5}{(\tau_1 s+1)}\right]}$$




(b)

**Figure** 4-2.2 Block diagrams for an interacting two-tanks system.

H2(s) is another input to obtain H,(s), as indicated by the “feedback path.” Oftentimes, we referred to this type of system as “interacting lags.”

<table>
<tr>
<td>At this time, there are several things we can learn by comparing the transfer functions of the interacting and noninteracting systems. Consider Fig. 4-2.3, which shows a block diagram of a noninteracting system and one of an interacting system. For the noninteracting system, the transfer function is:<br><br>$$\frac{Y(s)}{X(s)} = \frac{K_1 K_2}{(\tau_1 s + 1)(\tau_2 s + 1)}$$<br><br>As presented in Section 2-5, the “effective” time constants are the negatives of the reciprocals of the roots of the denominator of the transfer function. For the foregoing transfer function, the effective time constants are equal to the individual $\tau$ values; that is, $\tau_{1eff} = \tau_1$, $\tau_{2eff} = \tau_2$</td>
<td><br><br>(a)</td>
</tr>
<tr>
<td>For the interacting system, the transfer function is<br><br>$$\frac{Y(s)}{X(s)} = \frac{K_1(\tau_2 s + 1)}{\tau_1\tau_2 + (\tau_1 + \tau_2)s + (1 - K_1 K_2)}$$</td>
<td><br><br>(b)<br>**Figure 4-2.3** (a) Block diagram of a noninteracting system. *(b)* Block diagram of an interacting system.</td>
</tr>
</table>