Liquid-level system with nonlinear resistance

$$q_o(h) = Ch^{\frac{1}{2}}$$

Balance

$$q_i(t) - q_o(h) = A\frac{d[h(t)]}{dt}$$

$$\boxed{q_i(t) - Ch^{\frac{1}{2}} = A\frac{d[h(t)]}{dt}}$$

$$f(x) = f(a) + \left(\frac{\left.\frac{dx(t)}{dt}\right|_{t=a}}{1!} \right)(x-a)$$

$$q_o(h) = q_o(\bar{h}) + \left(\left.\frac{dq_o(h)}{dt}\right|_{h=\bar{h}} \right)(h-\bar{h})$$

$$\left.\frac{dq_o(h)}{dt}\right|_{h=\bar{h}} = \left.\frac{1}{2}Ch^{-\frac{1}{2}}\right|_{h=\bar{h}} = \frac{1}{2}C\bar{h}^{-\frac{1}{2}}$$

$$q_o(h) = q_o(\bar{h}) + \left(\frac{1}{2}C\bar{h}^{-\frac{1}{2}} \right)(h-\bar{h})$$

Sustituyendo:

$$q_i(t) - \left[q_o(\bar{h}) + \left(\frac{1}{2}C\bar{h}^{-\frac{1}{2}} \right)(h-\bar{h}) \right] = A\frac{d[h(t)]}{dt}$$

este es una linea que aproxima al comportamiento real alrededor de del punto (hs,qos)

$$q_o(\bar{h}) = \left(C\bar{h}^{\frac{1}{2}} \right)$$

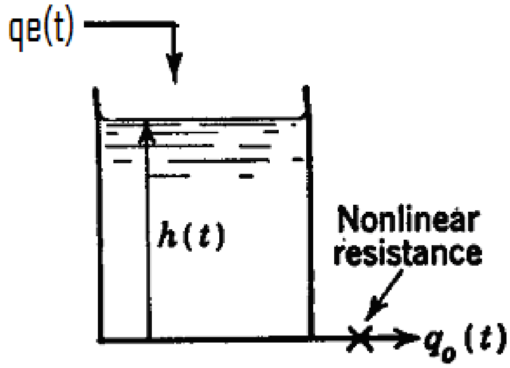
Definiendo:

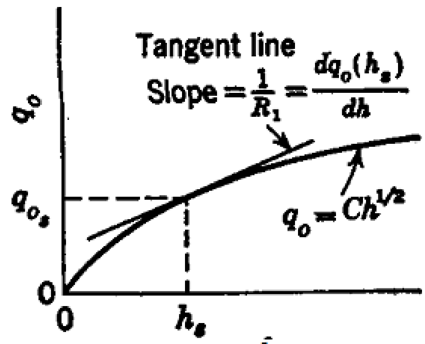
$$\frac{1}{R_1} = \frac{C}{2\sqrt{\bar{h}}}$$

En estado estable para el punto dado:

$$\bar{q}_i - \bar{q}_o = A\frac{d[\bar{h}]}{dt} = 0$$

Restando esta ecuacion a la original





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$$\boxed{\frac{H(s)}{Q(s)} = \left(\frac{R_1}{\tau s + 1} \right)}$$

The resistence R1 depends on the steady-state conditions around which the process operates.

$$q_i(t) - \left[q_o(\bar{h}) + \left(\frac{1}{2} C\bar{h}^{-\frac{1}{2}} \right)(h - \bar{h}) \right] =$$

$$A \frac{d[h(t)]}{dt}$$

$$\bar{q}_i - \bar{q}_o = A \frac{d[\bar{h}]}{dt} = 0$$

$$q_i(t) - \bar{q}_i - \left[q_o(\bar{h}) + \left(\frac{1}{2} C\bar{h}^{-\frac{1}{2}} \right)(h - \bar{h}) - \bar{q}_o \right] =$$

$$A \frac{d[h(t)]}{dt} - A \frac{d[\bar{h}]}{dt}$$

$$q_i(t) - \bar{q}_i - (C_1)(h - \bar{h}) = A \frac{d[h(t) - \bar{h}]}{dt}$$

$$[q_i(t) - \bar{q}_i] - (C_1)[h - \bar{h}] = A \frac{d[h(t) - \bar{h}]}{dt}$$

Definiendo las variables de desviación:

$$[Q_i] - (C_1)[H] = A \frac{d[H]}{dt}$$

Tomando la transformada de laplace:

$$L\{[Q_i] - (C_1)[H]\} = L\left\{ A \frac{d[H]}{dt} \right\}$$

$$[Q_i(s)] - (C_1)[H(s)] = AsH(s)$$

$$[Q_i(s)] = H(s)[C_1 + As]$$

$$\frac{1}{[C_1 + As]} = \frac{H(s)}{[Q_i(s)]}$$

$$\frac{H(s)}{[Q_i(s)]} = \frac{1}{[C_1 + As]}$$

$$\frac{H(s)}{[Q_i(s)]} = \frac{\frac{1}{C_1}}{\left[\frac{A}{C_1} s + 1 \right]}$$

$$\frac{H(s)}{[Q_i(s)]} = \frac{R_1}{[\tau s + 1]}$$

Process system analysis and control coughanowr

REHACIENDO EL CALCULO CON OTRO PUNTO DE VISTA:

$$\frac{dx}{dt} = f(x,u)$$

X: salida
U: entrada

Linearizar alrededor del punto (q_{os}, h_s)

$$q_o(h) = Ch^{\frac{1}{2}}$$

Qo: salida
H:entrada

$$f(q_o, h) = Ch^{\frac{1}{2}}$$

$$f(q_o, h) \approx f(q_o^s, h^s) + \left(\left.\frac{\partial f}{\partial q_o}\right|_{h^s, q_o^s}\right)(q_o - q_o^s) + \left(\left.\frac{\partial f}{\partial h}\right|_{h^s, q_o^s}\right)(h - h^s) + T.O.S$$

$\frac{dh_s}{dt} = f(h_s, q_o^s) = 0$ **es un estado estable**

$$\frac{dh}{dt} = \frac{d(h - h^s)}{dt} = \frac{d\overline{h}}{dt}$$

$\boxed{q(t) - Ch^{\frac{1}{2}} = A\frac{d[h(t)]}{dt}}$ **funcion no lineal**

$$\frac{d[h(t)]}{dt} = \frac{1}{A}q(t) - \frac{1}{A}Ch^{\frac{1}{2}}$$

$$\frac{dh}{dt} = \left(\left.\frac{\partial f}{\partial h}\right|_{h_s, q_o^s}\right)\overline{h} + \left(\left.\frac{\partial f}{\partial q_o}\right|_{h_s, q_o^s}\right)\overline{q}_o$$

$$\left.\frac{\partial f}{\partial h}\right|_{h_s, q_o^s} = -\frac{1}{2}\frac{C}{A}h^{-\frac{1}{2}}\Big|_{h_s, q_o^s} = -\frac{1}{2}\frac{C}{A}h_s^{-\frac{1}{2}}$$

$$\left.\frac{\partial f}{\partial q_o}\right|_{h_s, q_o^s} = \left.\frac{1}{A}\right|_{h_s, q_o^s} = \frac{1}{A}$$

$$\frac{d\overline{h}}{dt} = \left(\left.\frac{\partial f}{\partial h}\right|_{h_s, q_o^s}\right)\overline{h} + \left(\left.\frac{\partial f}{\partial q_o}\right|_{h_s, q_o^s}\right)\overline{q}_o = \left(-\frac{1}{2}\frac{C}{A}h_s^{-\frac{1}{2}}\right)\overline{h} + \left(\frac{1}{A}\right)\overline{q}_o$$

$\boxed{\frac{d\overline{h}}{dt} = \left(-\frac{1}{2}\frac{C}{A}h_s^{-\frac{1}{2}}\right)\overline{h} + \left(\frac{1}{A}\right)\overline{q}_o}$ **modelo linealizado**

$$s\overline{H}(s) = \left(-\frac{1}{2}\frac{C}{A}h_s^{-\frac{1}{2}}\right)\overline{H}(s) + \left(\frac{1}{A}\right)\overline{Q}_o(s) \qquad \frac{1}{R_1} = \frac{C}{2\sqrt{h_s}};\ \tau = R_1 A$$

$$\bar{H}(s)\left[s+\frac{1}{2}\frac{C}{A}h_s^{-\frac{1}{2}}\right]=\left(\frac{1}{A}\right)\bar{Q}_o(s)$$

$$\frac{\bar{H}(s)}{\bar{Q}_o(s)}=\frac{\left(\frac{1}{A}\right)}{s+\frac{1}{A}\frac{C}{2}h_s^{-\frac{1}{2}}}$$

$$\frac{\bar{H}(s)}{\bar{Q}_o(s)}=\frac{\left(\frac{1}{A}\right)}{s+\frac{1}{A}\frac{1}{R_1}}$$

$$\frac{\bar{H}(s)}{\bar{Q}_o(s)}=\frac{R_1A\left(\frac{1}{A}\right)}{R_1As+\frac{R_1A}{AR_1}}$$

$$\frac{\bar{H}(s)}{\bar{Q}_o(s)}=\frac{R_1}{R_1As+1}$$

$$\boxed{\frac{\bar{H}(s)}{\bar{Q}_o(s)}=\frac{R_1}{\tau s+1}}$$