3-4 TRANSFER FUNCTIONS AND BLOCK DIAGRAMS

$$G(s)=\frac{Y(s)}{X(s)}=\frac{K\left(a_m s^m+a_{m-1}s^{m-1}+\bullet\bullet\bullet+a_1 s+1\right)}{\left(b_n s^n+b_{n-1}s^{n-1}+\bullet\bullet\bullet+b_1 s+1\right)}e^{-t_0 s}$$

The transferfunction completely defines the steady-state and dynamic characteristics, for the total response, of a system described by a linear differential equation.

Block diagrams

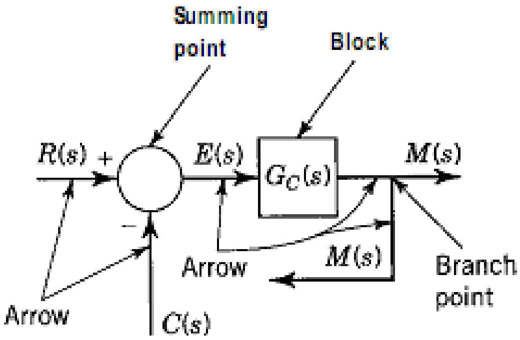


Figure 3-4.1 Elements of a block diagram.

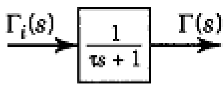


Figure 3-4.2 Block diagram of Eq. 3-2.12.

$$M(s) = G_c(s)E(s) = G_c(s)[R(s) - C(s)]$$

Example:

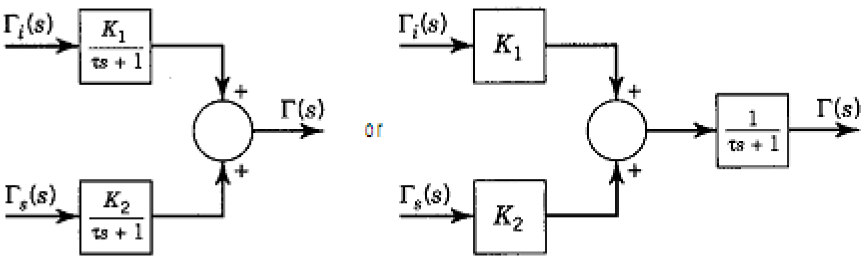
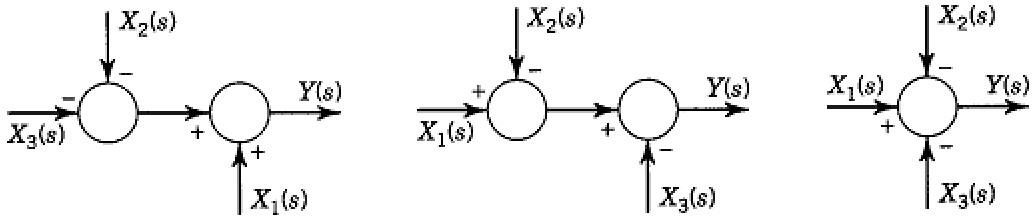


Figure 3-4.3 Block diagram of Eq. 3-2.24.

Principle of superposition:

$$\Gamma(s)=\Gamma_i(s)\frac{K_1}{\tau s+1}+\Gamma_s(s)\frac{K_2}{\tau s+1}=\frac{1}{\tau s+1}\left[K_1\Gamma_i(s)+K_2\Gamma_s(s)\right]$$

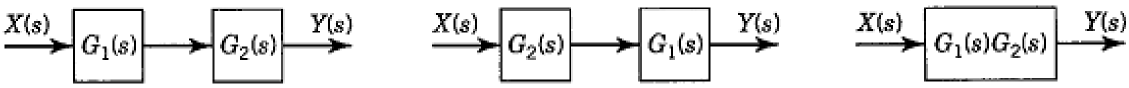
1. Y(S) = X,(S) $X_2(s) - X_3(s)$



$$Y(s) = X_1(s) + [-X_2(s) - X_3(s)]$$
$$Y(s) = -X_3(s) + X_1(s) - X_2(s)$$

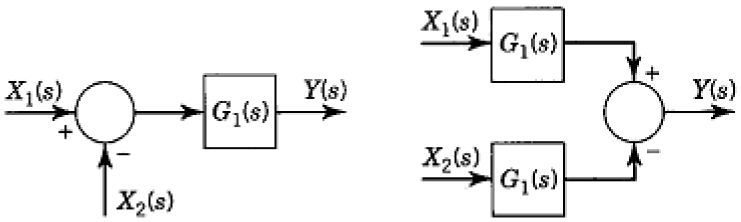
2. Associative and Commutative Properties:

$$Y(s) = G_1(s)\ G_2(s)\ X(s) = G_2(s)\ G_1(s)\ X(s)$$



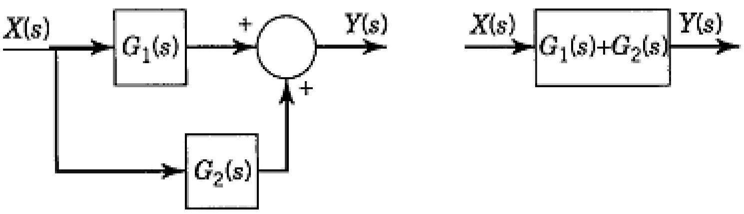
3. Distributive Property:

$$Y(s) = G_1(s)\ [X_1(s) - X_2(s)] = G_1(s)\ X_1(s) - G_1(s)\ X_2(s)$$



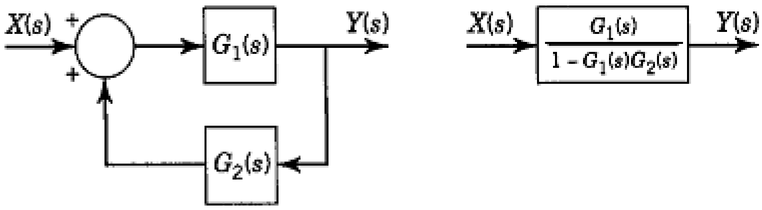
4. Blocks in Parallel:

$$Y(s) = [G_1(s) + G_2(s)]\ X(s) = G_1(s)\ X(s) + G_2(s)\ X(s)$$



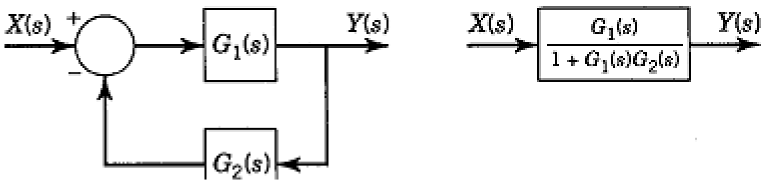
5. Positive Feedback Loop:

$$Y(s) = G_1(s)\,[X(s) + G_2(s)\,Y(s)] = \frac{G_1(s)}{1 - G_1(s)\,G_2(s)}X(s)$$



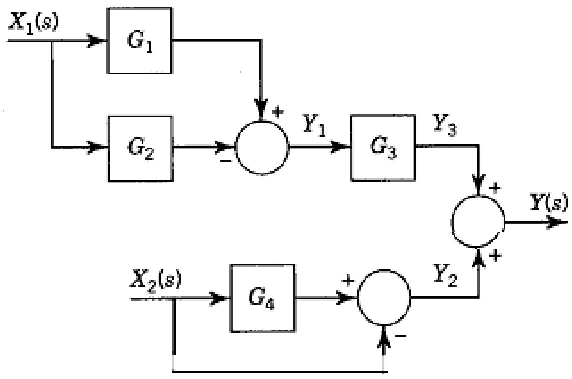
6. Negative Feedback Loop:

$$Y(s) = G_1(s)\,[X(s) + G_2(s)\,Y(s)] = \frac{G_1(s)}{1 + G_1(s)\,G_2(s)}X(s)$$



Example:

Determine the transfer functions relating Y(s) to X,(S) and X,(S) from the block diagram shown in Fig. 3-4.4a. That is, obtain $\frac{Y(s)}{X_1(s)}$ and $\frac{Y(s)}{X_2(s)}$



$$\left[\begin{aligned}
&Y_1 = X_1(s)\left[G_1 - G_2\right] \\
&Y_3 = Y_1 G_3 \\
&Y_2 = X_2(s)\,G_4 - X_2(s) = X_2(s)\left[G_4 - 1\right] \\
&Y(s) = Y_3 + Y_2 \\
&Y(s) = X_1(s)\left[G_1 - G_2\right]G_3 + X_2(s)\left[G_4 - 1\right]
\end{aligned}\right]$$

Superposition:

$$Y(s)\big|_{X_1(s)=0} = \cancel{X_1(s)}\left[G_1 - G_2\right]G_3 + X_2(s)\left[G_4 - 1\right]$$

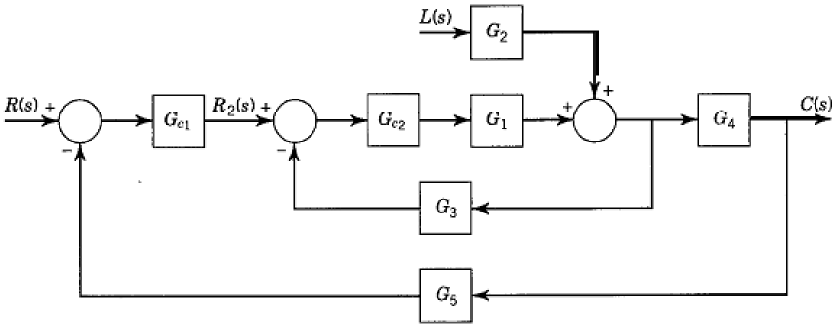
$$\boxed{\frac{Y(s)}{X_2(s)} = \left[G_4 - 1\right]}$$

$$Y(s)\Big|_{X_2(s)=0} = X_1(s)\left[G_1 - G_2\right]G_3 + \cancel{X_2(s)}\left[G_4 - 1\right]$$

$$\boxed{\frac{Y(s)}{X_1(s)} = \left[G_1 - G_2\right]G_3}$$

$$\boxed{Y(s) = X_1(s)\left[G_1 - G_2\right]G_3 + X_2(s)\left[G_4 - 1\right]}$$

Example



(a)

$$\frac{C(s)}{R(s)} = \frac{G_{c1}G_{c2}G_1G_4}{1 + G_{c2}G_1G_3 + G_{c1}G_{c2}G_1G_4G_5} \qquad \frac{C(s)}{L(s)} = \frac{G_2G_4}{1 + G_{c2}G_1G_3 + G_{c1}G_{c2}G_1G_4G_5}$$

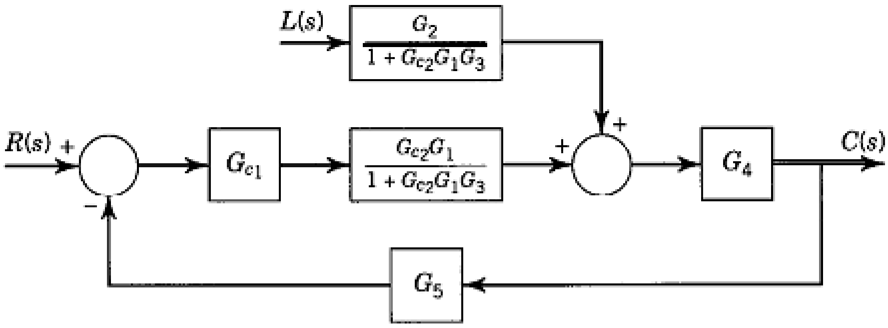
Observe que necesitamos hacer superposición, donde Z(s) es el punto despues de la sumatoria:

$$\left.\frac{Z(s)}{R_2(s)}\right|_{L(s)=0} = \frac{G_{c2}G_1}{1+G_{c2}G_1G_3} \qquad \left.\frac{Z(s)}{L(s)}\right|_{R_2(s)=0} = \frac{G_2}{1+G_{c2}G_1G_3}$$

Por lo que la suma de los efectos:

$$Z(s) = \frac{G_{c2}G_1}{1+G_{c2}G_1G_3}R_2(s) + \frac{G_2}{1+G_{c2}G_1G_3}L(s)$$

Y queda como acontinuación:



(b)

Nuevamente haciendo superposición:

$$\left.\frac{C(s)}{R(s)}\right|_{L(s)=0} = \frac{G_{c1}G_4\dfrac{G_{c2}G_1}{1+G_{c2}G_1G_3}}{1+G_5\left(G_{c1}G_4\dfrac{G_{c2}G_1}{1+G_{c2}G_1G_3}\right)}$$

Y también, con R(s)=0:

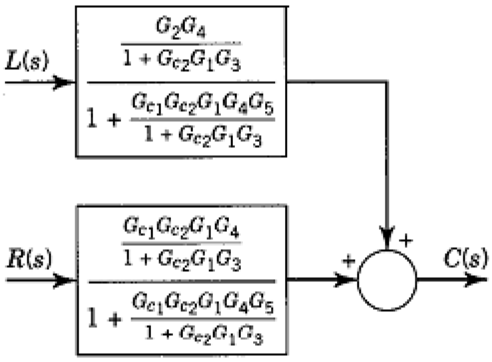
$$C(s) = G_4\left\{\frac{G_2}{1+G_{c2}G_1G_3}L(s) + \left(-G_5G_{c1}\frac{G_{c2}G_1}{1+G_{c2}G_1G_3}\right)C(s)\right\}$$

$$C(s)=\frac{G_4G_2}{1+G_{c2}G_1G_3}L(s)+\left(-G_4G_5G_{c1}\frac{G_{c2}G_1}{1+G_{c2}G_1G_3}\right)C(s)$$

$$\left.\frac{C(s)}{L(s)}\right|_{R(s)=0}=\frac{\dfrac{G_4G_2}{1+G_{c2}G_1G_3}}{1+G_4G_5G_{c1}\dfrac{G_{c2}G_1}{1+G_{c2}G_1G_3}}$$

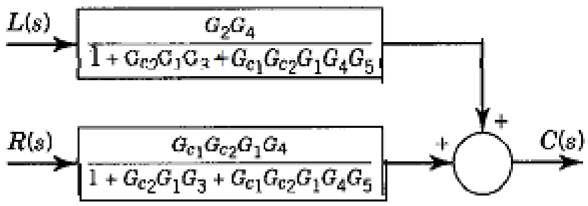
Sumando los efectos:

$$\boxed{C(s)=\frac{G_{c1}G_4\dfrac{G_{c2}G_1}{1+G_{c2}G_1G_3}}{1+G_5\left(G_{c1}G_4\dfrac{G_{c2}G_1}{1+G_{c2}G_1G_3}\right)}R(s)+\frac{\dfrac{G_4G_2}{1+G_{c2}G_1G_3}}{1+G_4G_5G_{c1}\dfrac{G_{c2}G_1}{1+G_{c2}G_1G_3}}L(s)}$$



(c)

Finaly



(d)

Figure 3-4.6 Block diagram of a cascade control system.