## Response with Time Delay – step – ramp – sin

$$Y(s) = \underbrace{\left[ \frac{Ke^{-t_0 s}}{\tau s + 1} \right]}_{FOPDT} X(s)$$

The term in brackets is an important transfer function used to approximate the response of higher-order processes. We call it a **first-order-plus-dead-time** (FOPDT) **transfer Function**.

The effect of the time delay on the three responses presented in this section is as follows:

### Step response

$$y(t) = K(\Delta x)u(t - t_0)\left(1 - e^{-(t-t_0)/\tau}\right)$$

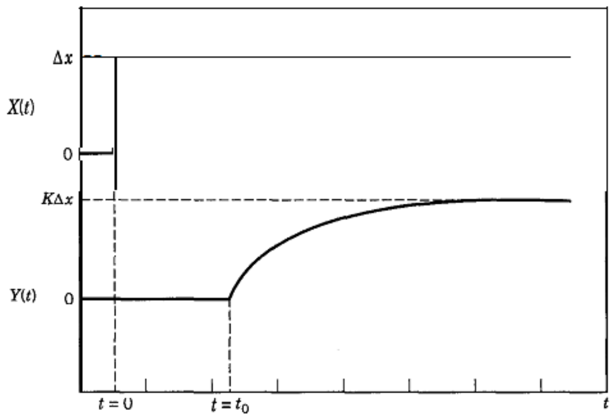


Figure 2-4.4 First-order step response with time delay $t_0$.

### Ramp response

$$y(t) = u(t - t_0)\left[ Kr\tau e^{-(t-t_0)/\tau} + Kr(t - t_0 - \tau) \right]$$

Note that the effect of the time delay in the long-term response is that the output ramp lags the input ramp by the sum of the time delay and the time constant.

**Sinusoidal response**

$$y(t) = u\left(t - t_0\right)\left\{\frac{KA\omega\tau}{1+\omega^2\tau^2} e^{-\left(t-t_0\right)/\tau} + \frac{KA}{\sqrt{1+\omega^2\tau^2}} \sin\left[\omega\left(t-t_0\right)+\theta\right]\right\}$$

The only effect of the time delay on the long-term response is to increase the phase lag by omega and t0,. This increase in phase lag is proportional to the frequency of the input sine wave. The phase lag 0 is the same as in Eq. 2-4.9.